COSMIC MICROWAVE BACKGROUND & BARYON ACOUSTIC OSCILLATION

CQUEST GROUP II WORKSHOP 2022
SEP. 28 ~ OCT. 02 2022, ONLINE & JEJU NATIONAL UNIVERSITY
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OCT.01. 11:00AM

OUTLINE

Evolution of perturbations

CMB

- Odd and even peaks
- Large-scale anisotropies
- Acoustic oscillations
- Diffusion damping

BAO

OBSERVABLES

- Primordial perturbations : generated during inflation
- Quantum fluctuations = Gaussianity (@ linear level)
- Matter & radiation perturbation : descendant of inflaton
- Evolutions of matter and photons appear as PSs
- Evolutions determined by Einstein-Boltzmann system

"the devil is in the detail"

look simple

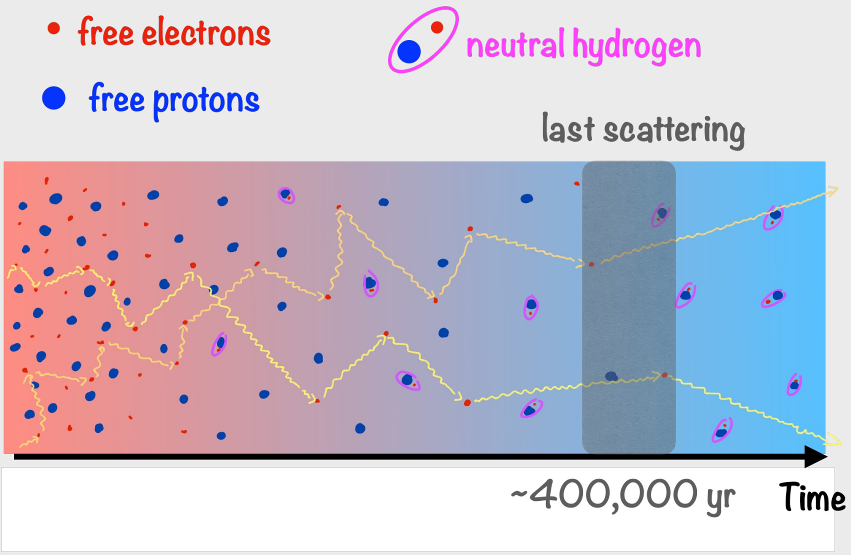
$$\underbrace{\mathcal{O}_k^X(\tau)}_{\text{link to Measurement}} = T_{\mathcal{O}}^X(k,\tau,\tau_*) \underbrace{\mathcal{R}_k(\tau_*)}_{\text{Curvature perturbation}} \xrightarrow{inflaton} T_{\mathcal{O}}^X(k,\tau,\tau_*) \underbrace{\frac{\mathcal{H}}{\bar{\phi}'}\delta\phi_k}_{\text{no evolution}}$$

$\langle \mathcal{O}_k^X \rangle = 0$: Quantum fluctuation $\rightarrow$ random distribution

$P_k^X \equiv \langle \mathcal{O}_k^X \mathcal{O}_k^X \rangle$: measurements, X : CMB (T, E, B), LSS 測量

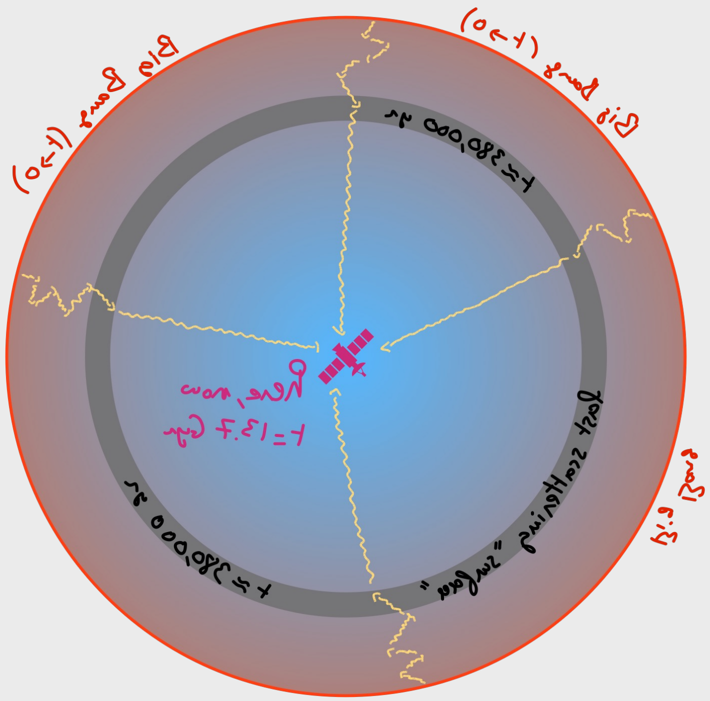
$B_k^X \equiv \langle \mathcal{O}_{k_1}^X \mathcal{O}_{k_2}^X \mathcal{O}_{k_3}^X \rangle = 0$: Gaussianity

become non-linear
at small scales
modes mixing



PHOTON PERTURBATIONS I

- Primordial perturbations appear in matter and radiation distributions
- Einstein-Boltzmann system : evolution of photon perturbations
- Two different evolutions of δ_γ before and after $z_{lss} \approx 1100$
- Before z_{lss} , γ, e^-, p were tightly coupled (the baryon-photon fluid = regarded as a single fluid)
- After z_{lss} , γ free-streamed from surface of last scattering
- Fourier modes = perturbations on different scales

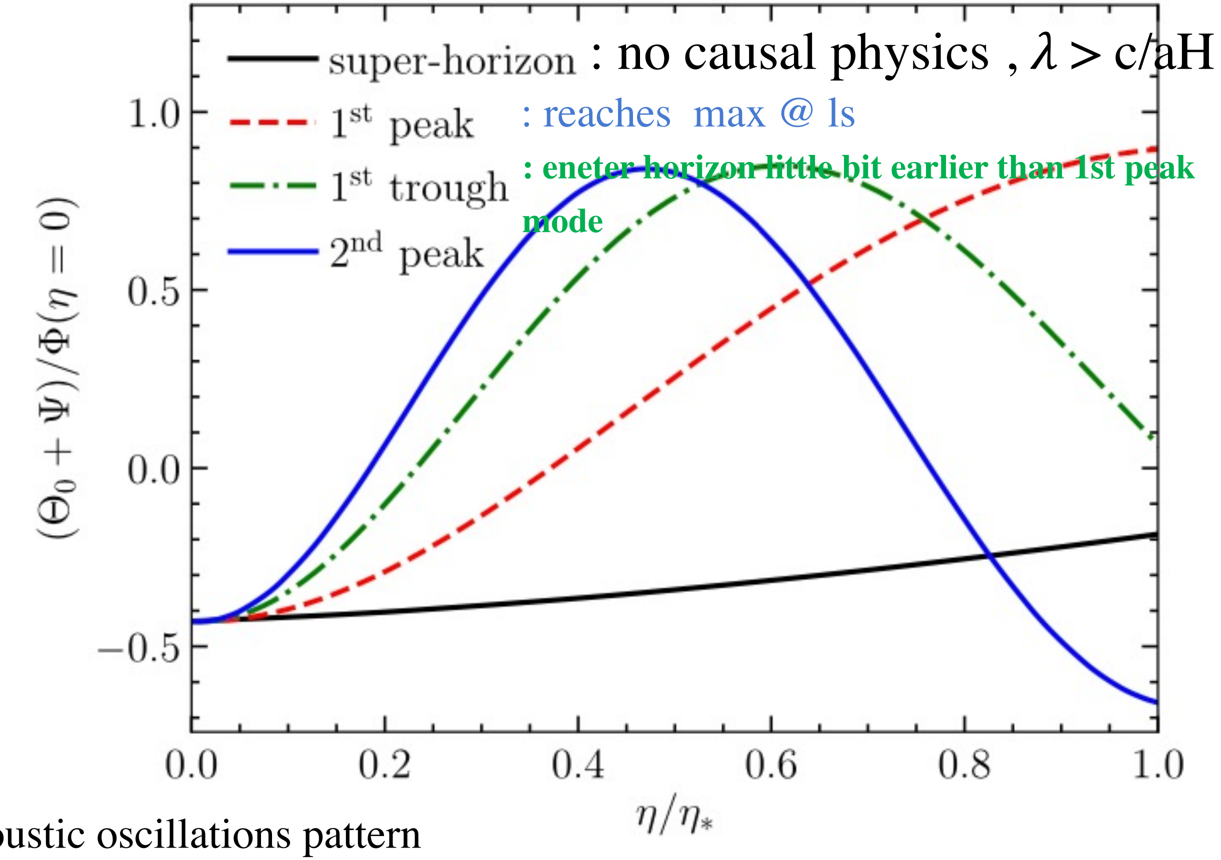


$$T\left(\mathbf{x},\hat{\mathbf{p}},\eta\right)=T^{(0)}(\eta)\left[1+\Theta\left(\mathbf{x},\hat{\mathbf{p}},\eta\right)\right]$$

$$f\left(\mathbf{x},p,\hat{\mathbf{p}},t\right)=\left(\exp\left[\frac{pc}{k_B T^{(0)}(t)\left[1+\Theta\left(\mathbf{x},\hat{\mathbf{p}},\eta\right)\right]}\right]-1\right)^{-1}$$

PHOTON PERTURBATIONS II (ACOUSTIC OSCILLATION)

$k^{-1}=c/(aH)$ k [h/Mpc] = 0.005, 0.02, 0.031, 0.039



Acoustic oscillations pattern

- Fourier modes = perturbations on different scales
- Evolution of perturbations of photons before $\eta=\eta_*$ (effective temperature perturbation : $\Theta_0+\Psi$)
- i) photon perturbations not grow after η_* ($\boldsymbol{\psi}$ too weak to trap γs) = free streaming to us : preserve perturbations at decoupling
- ii) perturbations of b & DM grows between η_* & η_0
- Sum of gravitational potential & photon monopole at η_* : present observed photons had to travel out of potentials they were in at η_*
- Normalized to $\boldsymbol{\psi}$ at the end of inflation (related to curvature perturbation $\mathfrak{R}$)

$$\Theta_0(\boldsymbol{x},t)\equiv\frac{1}{4\pi}\int d\Omega'\Theta(\hat{\boldsymbol{p}}',\boldsymbol{x},t).$$

$$\Theta_l(k,\eta)\equiv\frac{1}{(-i)^l}\int_{-1}^{1}\frac{d\mu}{2}\mathcal{P}_l(\mu)\Theta(\mu,k,\eta)$$

$$\Theta_1(k,\eta)\equiv i\int_{-1}^{1}\frac{d\mu}{2}\mu\Theta(\mu,k,\eta)$$

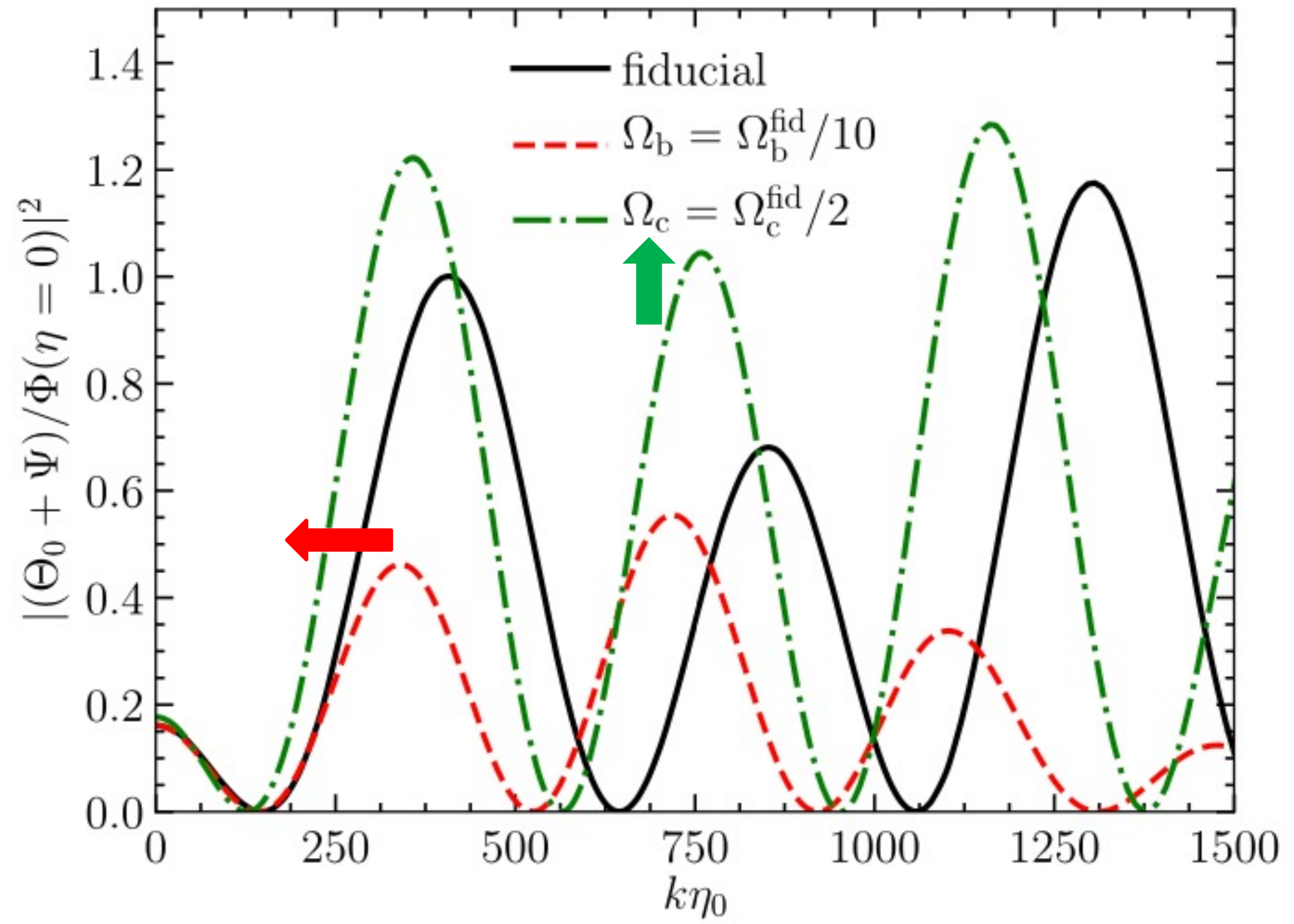
l-th multipole meoment

$$\Theta'+ik\mu\Theta=-\Phi'-ik\mu\Psi-\tau'\left[\Theta_0-\Theta+\mu u_b-\frac{1}{2}\mathcal{P}_2(\mu)\Pi\right]$$

$$\Pi=\Theta_2+\Theta_{P,2}+\Theta_{P,0}.$$

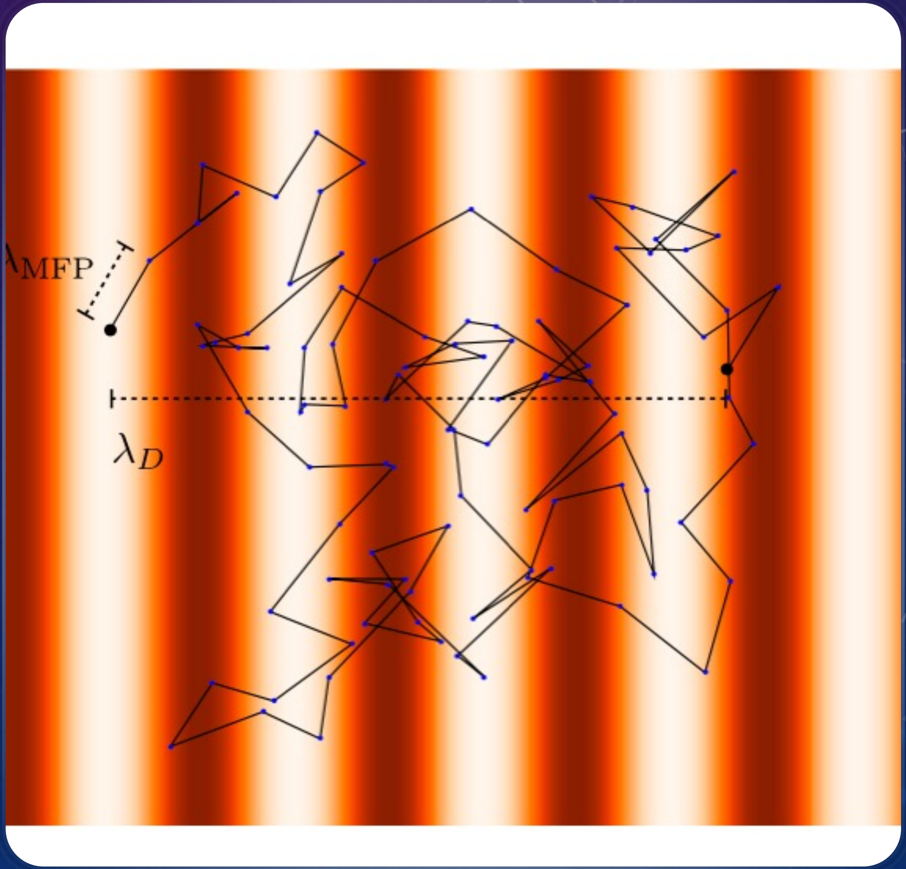
PHOTON PERTURBATIONS III (ODD, EVEN PEAKS)

- Spectrum of perturbations evaluated @ η_*
- odd peaks > even peaks (due to F)
- $\Theta_0'' + k^2 c_s^2 \Theta_0 = F$: forced HO
- i) $\omega = \sqrt{\frac{k}{m}}$: mass loading of fluid Ω_b
- ii) External force F (potential well generated by DM) sets asymmetry in odd & even peaks. (The larger F, and the lower ω), the larger the asymmetry
- iii) self-gravity & F act in consort leading to stronger contraction during contraction
- iv) self-gravity & F act in opposite when pressure wins leading to an underdensity
- Plots evaluated @ η_*, reduced baryons = increases m = increase ω , reduced DM = decreases F = suppressed asymmetry



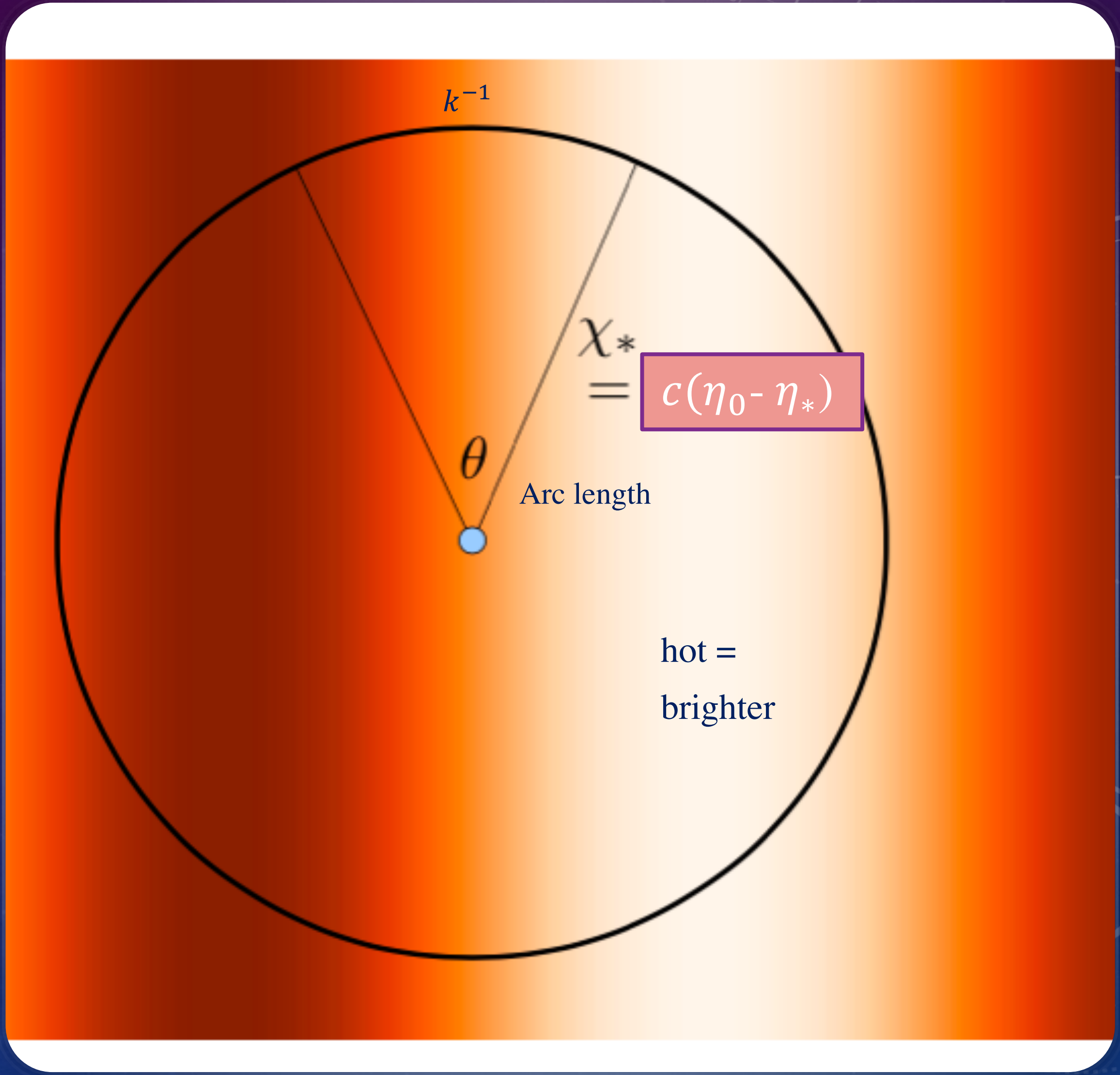
PHOTON PERTURBATIONS IV (DIFFUSION DAMPING)

- Beyond oscillations, damping on small scales $k\eta_0 \geq 500$ for low-Ω_b
- A single fluid approximation is valid only if $\Gamma \sim \infty$
- Reality : γs travel finite distances between scatters (τ : optical depth)
- $\lambda_{MFP} = -\frac{1}{\tau'} = \frac{1}{n_e \sigma_T a}$: γs' mean free path (a comoving distance btw each scatter)
- # of scatter during H^{-1} interval (total number of steps) : $n_e \sigma_T H^{-1}$
- Total comoving distance : $\lambda_D \sim \lambda_{MFP} \sqrt{n_e \sigma_T H^{-1}} = \frac{1}{a\sqrt{n_e \sigma_T H}}$
- Perturbations on scales smaller than λ_D are washed out because photon diffusing over λ_D restore the mean T_γ (= damping of high k modes)
- $\Omega_b \downarrow$: $n_b \downarrow$, λ_D $\uparrow$: stronger damping



PHOTON PERTURBATIONS V (FREE STREAMING)

- Relate perturbation $(\Theta_0+\Psi)(k,\eta_*)$ to the present observed anisotropies
- $\delta T(k_1,\eta_*)$: one Fourier mode (a plane-wave perturbation)
- Photons from hot & cold spots separated by a comoving distance $1/\,k_1$ travel to us coming from an angular separation $\theta = 1/(\chi_*\, k_1)$ where $\chi_* = c(\eta_0 \text{-}\, \eta_*)$
- If decompose δT into multipole moments Θ_l, then $\theta \sim 1/l$
- Project inhomogeneities on scales k_1 onto anisotropies on angular scales $l \sim \theta^{-1} = \chi_*\, k_1 \approx c\, \eta_0\, k_1$
- But interrupt during free journey
    - Gravitational potentials evolve at early (recombination) (due to radiation) and at late (due to DE) : Integrated Sachs-Wolfe (ISW) effect
    - Not neutral $z \leq 10$ due to reionization (damping anisotropies)



ANISOTROPIES FROM INFLATION

- Expansion of Θ in terms of spherical harmonics

$$\Theta(\mathbf{x},\hat{\mathbf{p}},\eta)=\sum_{l=1}^{\infty}\sum_{m=-1}^{l}a_{lm}(\eta,\mathbf{x})\,Y_{lm}(\hat{\mathbf{p}})$$

$$a_{lm}(\eta,\mathbf{x})=\int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{k}\cdot\mathbf{x}}\int d\Omega Y^*_{lm}(\hat{\mathbf{p}})\,\Theta(\mathbf{x},\hat{\mathbf{p}},\eta)$$

$$\langle a_{lm}\rangle=0\quad,\quad\langle a_{lm}a^*_{l'm'}\rangle\equiv\delta_{ll'}\delta_{mm'}C(l)$$

- Relation between the present temperature fluctuations and the primordial one

$$\Theta(\mathbf{k},\hat{\mathbf{p}},\eta_0)=\mathcal{T}(\mathbf{k},\hat{\mathbf{p}})\,\mathcal{R}(\mathbf{k})=\mathcal{T}\left(k,\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}\right)\mathcal{R}(\mathbf{k})$$

$$\langle\Theta(\mathbf{k},\hat{\mathbf{p}})\,\Theta(\mathbf{k},\hat{\mathbf{p}})\rangle\equiv(2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}')P_{\mathcal{R}}(k)\mathcal{T}\left(k,\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}\right)\mathcal{T}\left(k,\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}'\right)$$

$$C(l)=\frac{2}{\pi}\int_0^{\infty}dk\,k^2P_{\mathcal{R}}(k)|\mathcal{T}_l|^2\quad\text{where}\quad\mathcal{T}\left(k,\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}\right)=\sum_l(-i)^l(2l+1)\mathcal{P}_l(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})\mathcal{T}_l(k)$$

$$\Theta_l(k,\eta_0)\simeq\left[\Theta_0(k,\eta_*)+\Psi(k,\eta_*)\right]j_l\left[k(\eta_0-\eta_*)\right]\quad\text{monopole}$$

$$\text{dipole}\quad+3\Theta_1(k,\eta_*)\left(j_{l-1}\left[k(\eta_0-\eta_*)\right]-(l+1)\frac{j_l\left[k(\eta_0-\eta_*)\right]}{k(\eta_0-\eta_*)}\right)$$

$$\text{ISW}\quad+\int_0^{\eta_0}d\eta\,e^{-\tau}\left[\Psi'(k,\eta)-\Phi'(k,\eta)\right]j_l\left[k(\eta_0-\eta)\right].$$

- So far, investigate (monoploe & dipole) perturbations to photons Θ_0 & Θ_1 at η_* (i.e., last scattering epoch)
- Need to relate them ($\Theta_0(\eta_*)$ & $\Theta_1(\eta_*)$) to $\Theta_l(\eta_0)$ at η_0 (present epoch) in order to use for observables
- Angular power spectrum (PS) of photon perturbations

CMB POLARIZATIONS

- Polarization tensor I_{ij} :

$$I_{ij} = \begin{pmatrix} I+Q & U \\ U & I-Q \end{pmatrix} = I\delta_{ij} + I_{ij}^{T} \quad \text{where} \quad T(\mathbf{l}) = \int d^2\theta T(\theta) e^{i\mathbf{l}\cdot\theta}$$

$$I_{ij}^{T}(\mathbf{l}) = 2\left(\frac{l_i l_j}{l^2} - \frac{1}{2}\delta_{ij}\right) E(\mathbf{l}) + I_{ij}^{TT}(\mathbf{l})$$

$$E(\mathbf{l}) = \frac{l_i l_j}{l^2} I_{ij}^{T}(\mathbf{l}) = \cos 2\phi_l Q(\mathbf{l}) + \sin 2\phi_l U(\mathbf{l})$$

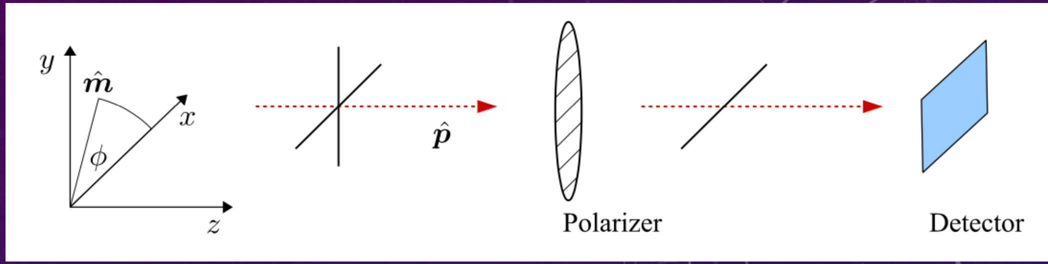
$$I_{12}^{TT}(\mathbf{l}) = I_{12}^{T}(\mathbf{l}) - 2\frac{l_1 l_2}{l^2} E(\mathbf{l}) = \cos 2\phi_l B(\mathbf{l})$$

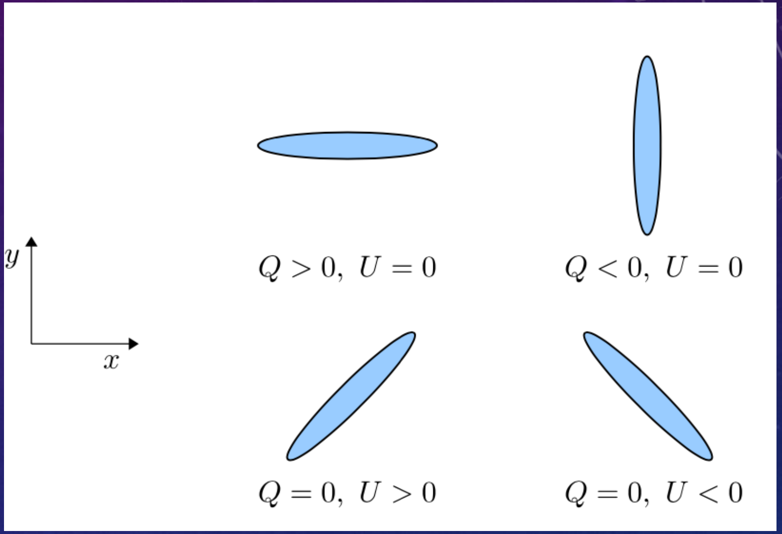
$$B(\mathbf{l}) = -\sin 2\phi_l Q(\mathbf{l}) + \cos 2\phi_l U(\mathbf{l})$$

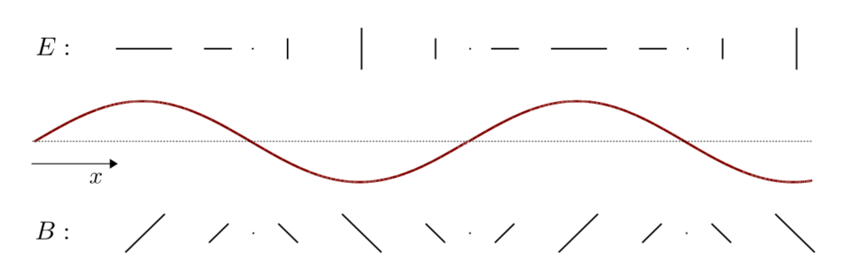
$$I_{ij}^{T}(\mathbf{l}) = \begin{pmatrix} \cos 2\phi_l & \sin 2\phi_l \\ \sin 2\phi_l & -\cos 2\phi_l \end{pmatrix} E(\mathbf{l}) + \begin{pmatrix} -\sin 2\phi_l & \cos 2\phi_l \\ \cos 2\phi_l & \sin 2\phi_l \end{pmatrix} B(\mathbf{l})$$

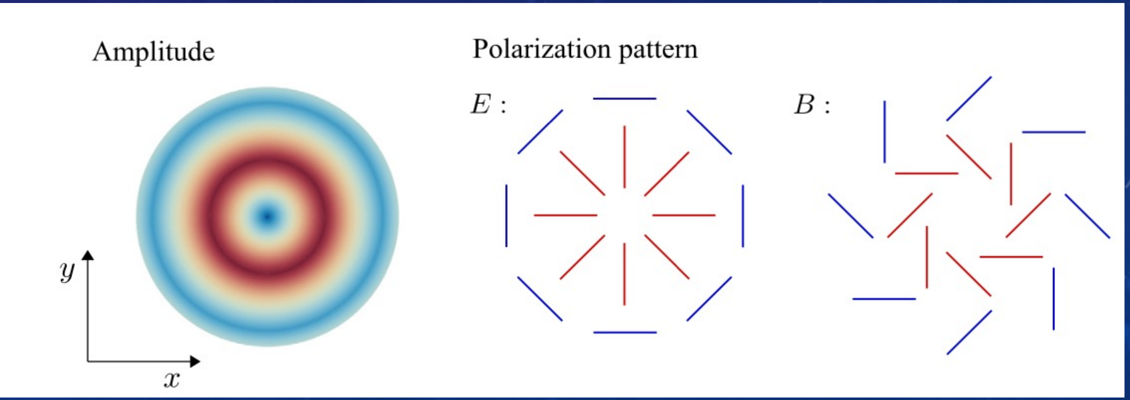
$$I_{ij}^{T}(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} e^{il_0\theta_x} E_0 + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} e^{il_0\theta_x} B_0$$

where Stokes parameters I : intensity (δT), Q and U : linear polarization, V : circular polarization

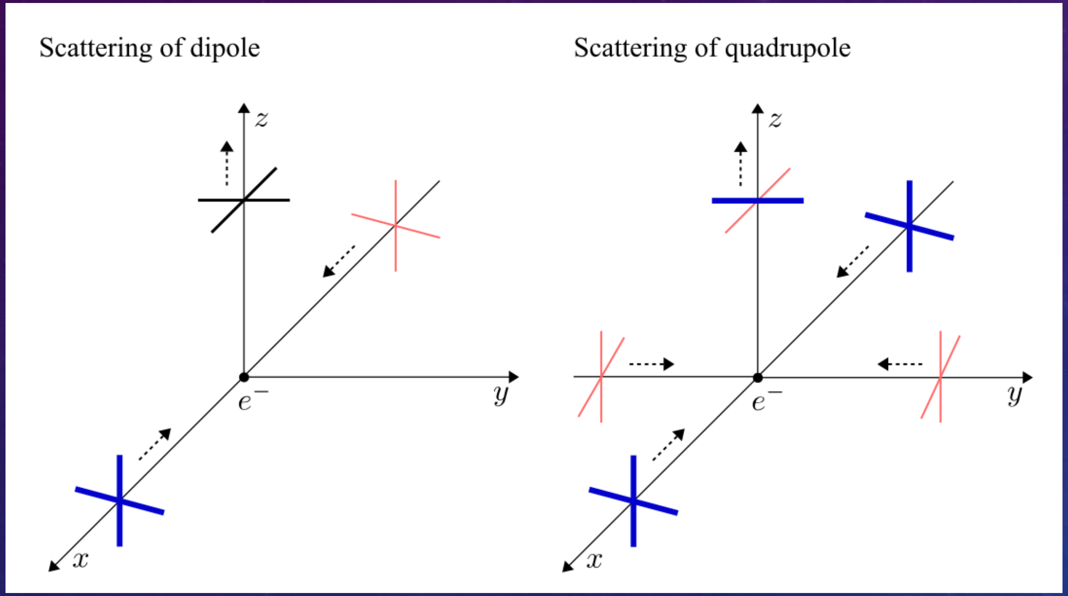


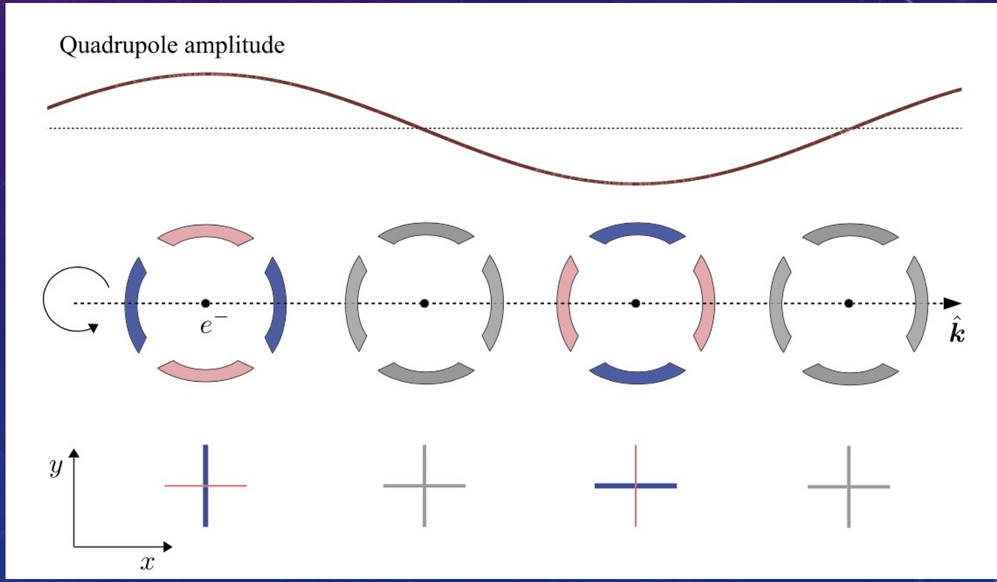


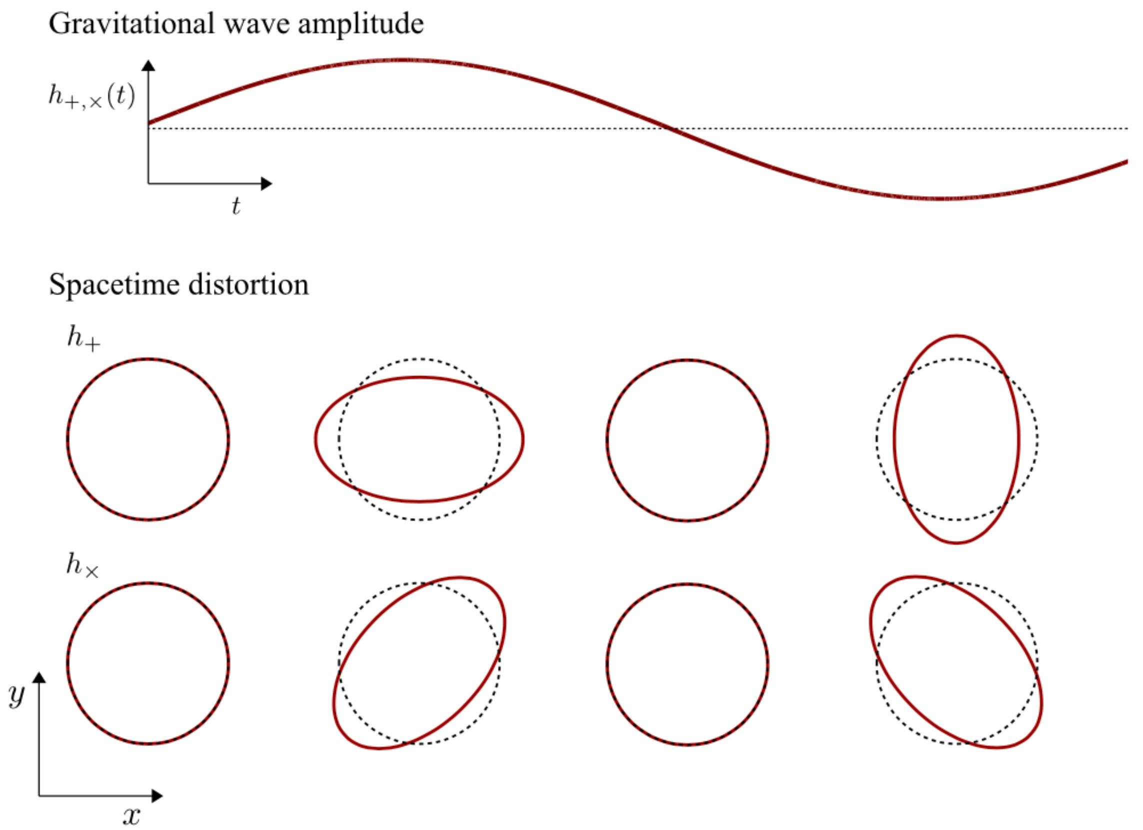


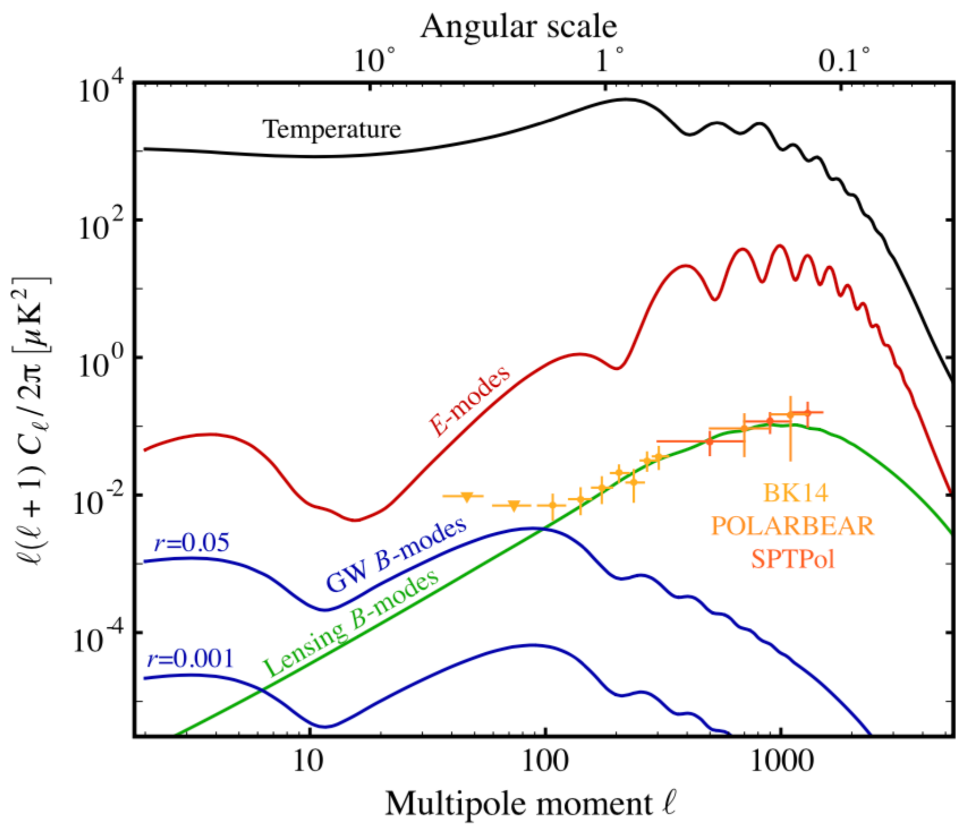


GENERATION OF CMB POLARIZATION I
E-MODE (FROM SCALAR)









GENERATION OF CMB POLARIZATION II

B-MODE (FROM TENSOR, GWS)

CMB POWER SPECTRA I (NUMERICAL TOOLS)

- List of cosmological Boltzmann codes
  - CMBFAST : out of date, https://ascl.net/9909.004
  - CAMB : https://camb.info/ (Fortran 90 , Python)
  - CLASS : https://lesgourg.github.io/class_public/class.html (C++, Python)
- Parameter estimation packages
  - CosmoMC
  - Slick Cosmological Parameter Estimator (SCoPE)

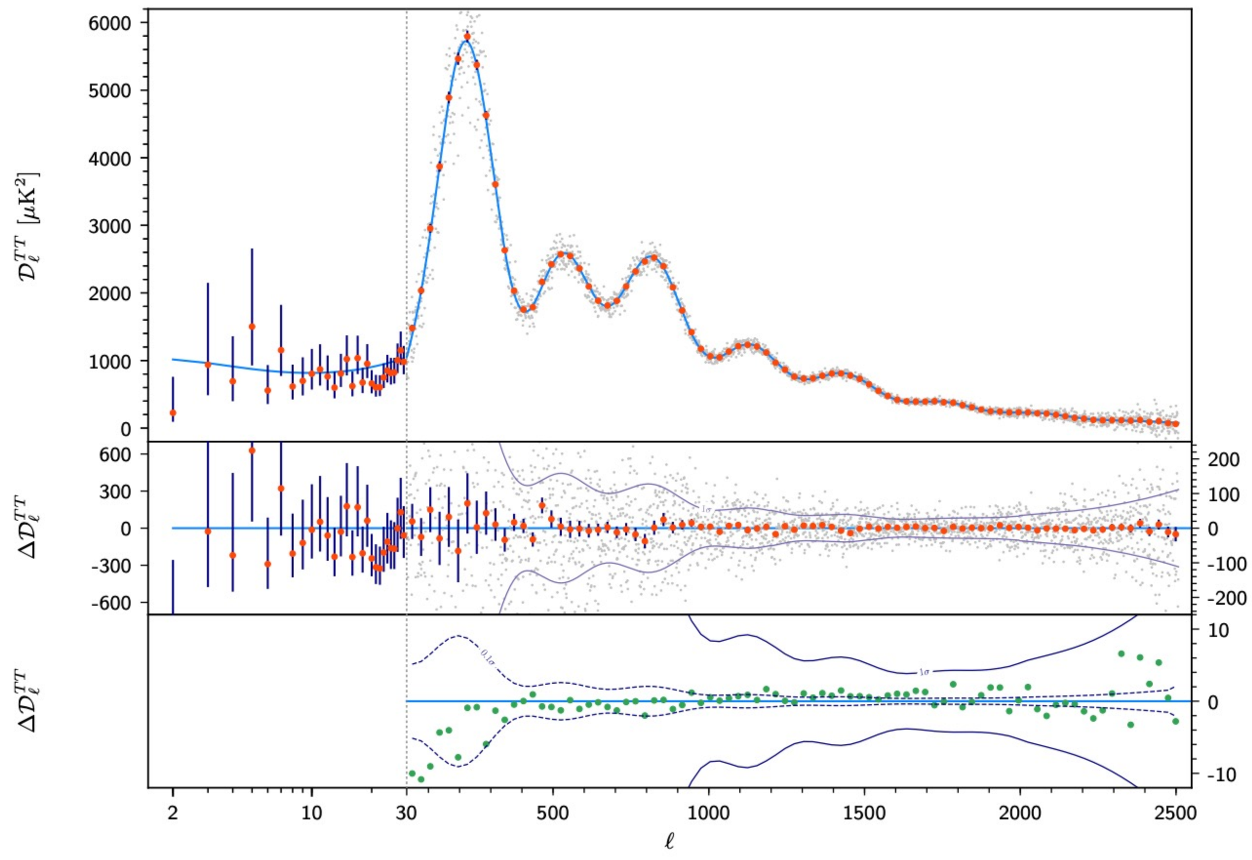


Fig. 57: *Planck* 2018 temperature power spectrum. At multipoles $\ell \geq 30$ we show the frequency-coadded temperature spectrum computed from the `Plik` cross-half-mission likelihood, with foreground and other nuisance parameters fixed to a best fit assuming the base-ΛCDM cosmology. In the multipole range $2 \leq \ell \leq 29$, we plot the power-spectrum estimates from the `Commander` component-separation algorithm, computed over 86 % of the sky (see Sect. 2.1.1). The base-ΛCDM theoretical spectrum best fit to the likelihoods is plotted in light blue in the upper panel. Residuals with respect to this model are shown in the middle panel. The vertical scale changes at $\ell = 30$, where the horizontal axis switches from logarithmic to linear. The error bars show $\pm 1\,\sigma$ diagonal uncertainties, including cosmic variance (approximated as Gaussian) and not including uncertainties in the foreground model at $\ell \geq 30$. The $1\,\sigma$ region in the middle panel corresponds to the errors of the unbinned data points (which are in grey). The bottom panel displays the difference between the 2015 and 2018 coadded high-multipole spectra (green points). The $1\,\sigma$ region corresponds to the binned data errors. The vertical scale differs from the one of the middle panel. The trend seen for $\ell < 300$ corresponds to the change in the dust correction model described in Sect. 3.3.2.

CMB POWER SPECTRA II (TE)

- $C_{TE}(l)$: cross correlation btw T and E-mode polarization

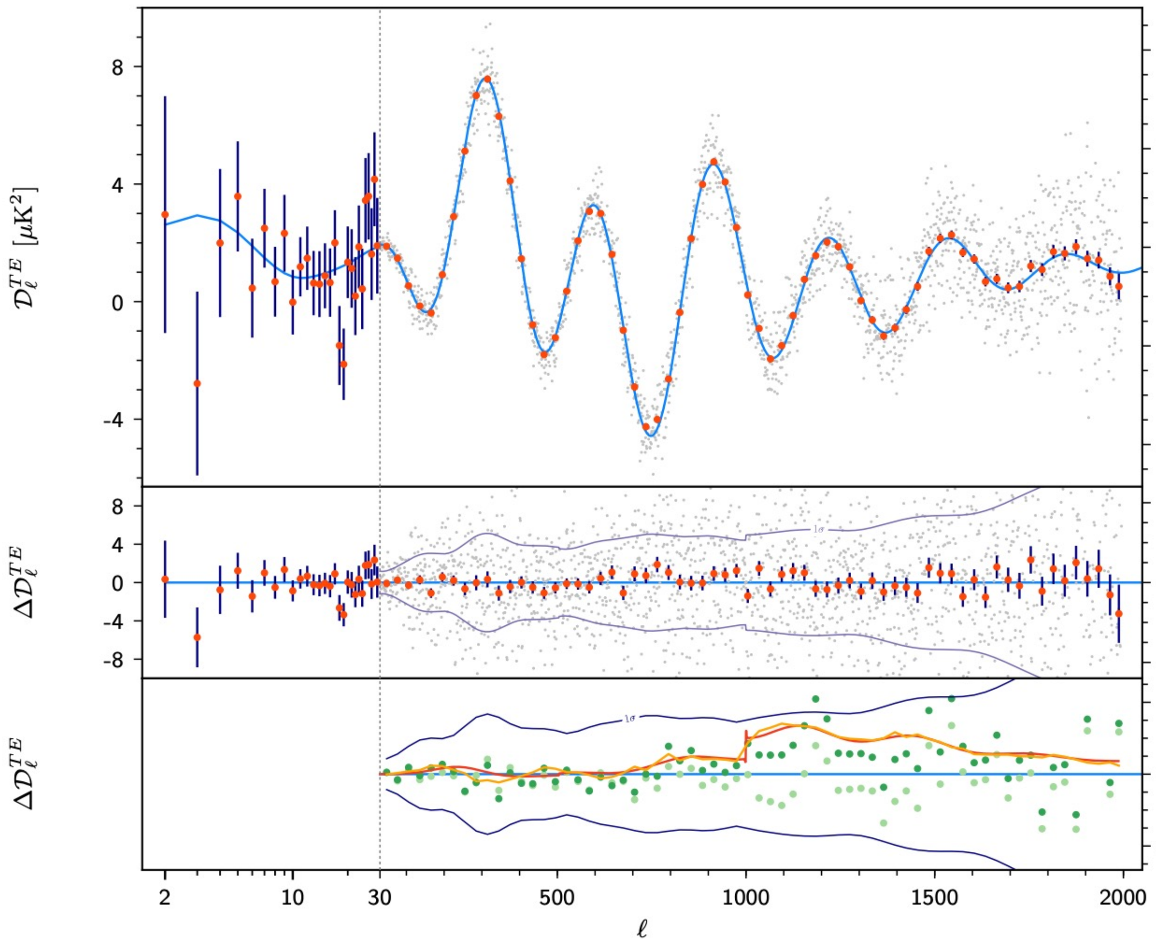


Fig. 58: *Planck* 2018 TE power spectrum. Figure conventions are similar to those of Fig. 57. In the multipole range $2 \leq \ell \leq 29$, we plot the power spectrum estimates from the SimAll likelihood (although this is not used in the baseline parameter analysis for $\ell \leq 29$; see the discussion of Sect. 2.2.6). The bottom panels display the difference between the 2015 and 2018 coadded high-multipole spectra (green points). The red and orange lines correspond to the effect of the beam-leakage correction and the addition of the beam-leakage and the polarization-efficiency corrections, respectively. Both corrections were absent in the 2015 data. The light green points show the difference between the 2015 and 2018 coadded spectrum, after correction of the 2015 data by these two effects. The difference here is dominated by the leakage correction.

CMB POWER SPECTRA III (EE)

- List of cosmological Boltzmann codes
- Parameter estimation packages

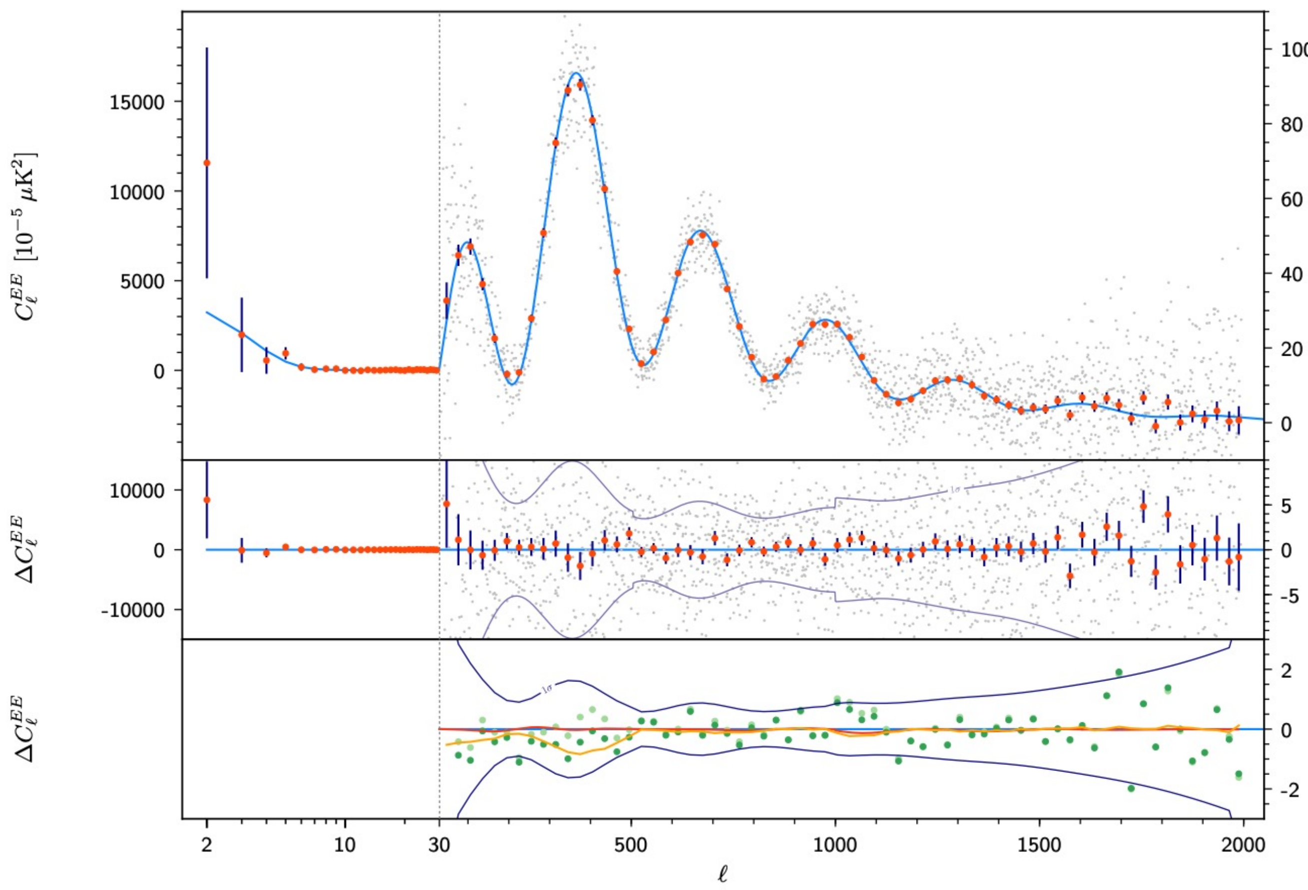


Fig. 59: *Planck* 2018 *EE* power spectrum. Figure conventions are similar to those of Fig. 57. In the multipole range $2 \leq \ell \leq 29$, we plot the power spectra estimates from the SimAll likelihood. The bottom panels display the difference between the 2015 and 2018 coadded high-multipole spectra (green points). The red and orange lines correspond to the effect of the beam-leakage correction and the addition of the beam-leakage and the polarization-efficiency corrections, respectively. Both corrections were absent in the 2015 data. The light green points show the difference between the 2015 and 2018 coadded spectra, after correction of the 2015 data by the two effects. The difference in *EE* is dominated by the polarization-efficiency correction.

CMB POWER SPECTRA IV

- Large scale mode : enter horizon only recently
  - Measure Ics (Inflation)
  - Neglect dipole & almost constant PS
- Acoustic peaks

$$\Theta_l(k,\eta_0) \simeq \left[\Theta_0(k,\eta_*) + \Psi(k,\eta_*)\right] j_l\left[k(\eta_0-\eta_*)\right] \quad \text{monopole}$$

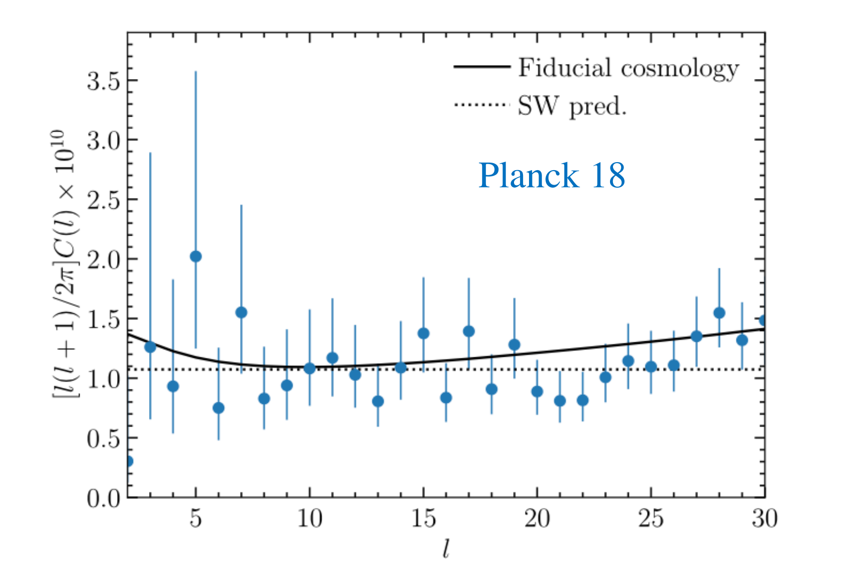
dipole $$+3\Theta_1(k,\eta_*)\left(j_{l-1}\left[k(\eta_0-\eta_*)\right] - (l+1)\frac{j_l\left[k(\eta_0-\eta_*)\right]}{k(\eta_0-\eta_*)}\right)$$

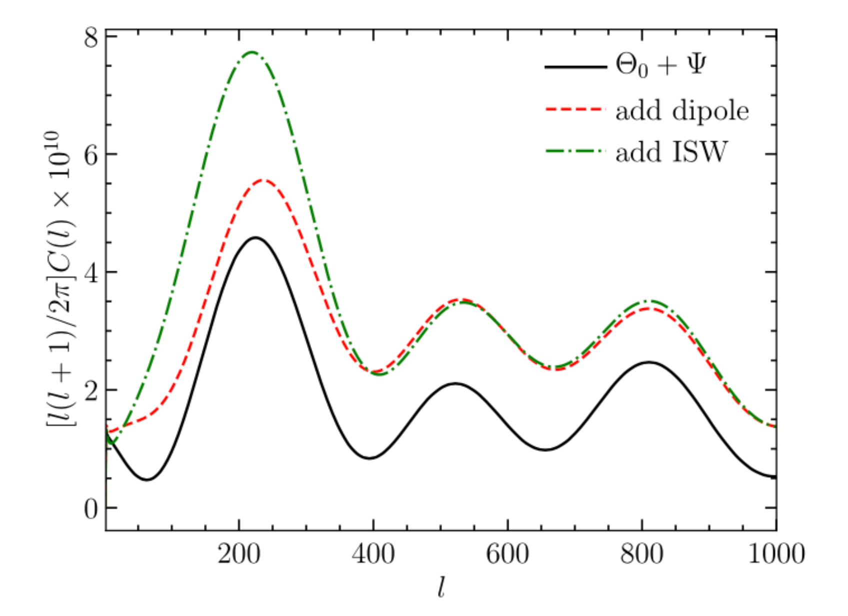
ISW $$+\int_0^{\eta_0} d\eta\, e^{-\tau}\left[\Psi'(k,\eta) - \Phi'(k,\eta)\right] j_l\left[k(\eta_0-\eta)\right].$$

$$C(l)^{\rm SW} \approx \frac{2}{25\pi}\int_0^\infty dk k^2 P_{\mathcal{R}}(k)\left|j_l\left[k(\eta_0-\eta_*)\right]\right|^2$$

$$\approx 2^{n_s-2}\frac{\pi^2}{25}A_s\,(\eta_0 k_p)^{1-n_s}\,\frac{\Gamma[l+\frac{n_s}{2}-\frac{1}{2}]}{\Gamma[l-\frac{n_s}{2}+\frac{5}{2}]}\,\frac{\Gamma[l3-n_s]}{\Gamma^2[2-\frac{n_s}{2}]}$$

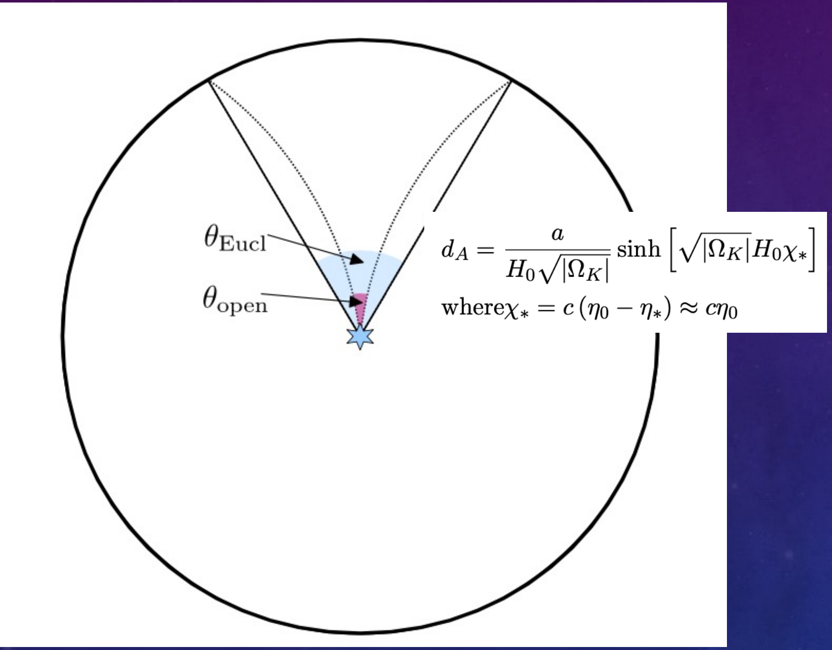
$$\overset{n_s=1}{=} \frac{8}{25}A_s\frac{1}{l(l+1)} \quad \text{thus} \quad D(l) \equiv l(l+1)C(l)$$

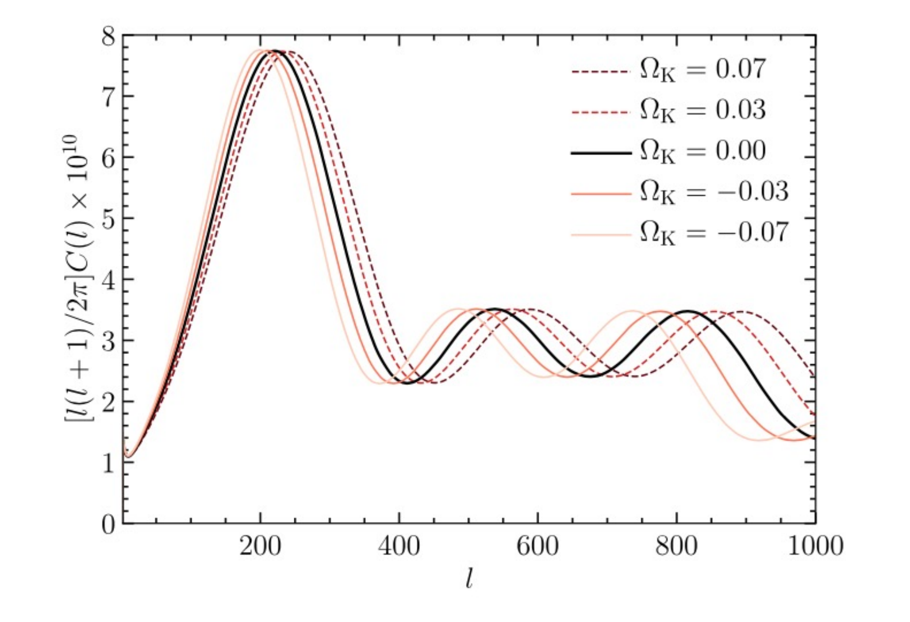




INTERPRETATION CMB TT SPECTRUM I

(EFFECT OF COSMOLOGICAL PARAMETERS)





K	$\Omega_K = -c^2K/(aH)^2$	d_A	$l \sim 1/\theta$
positive	negative	$< \chi_*$	decrease
0	0	χ_*	fiducial
negative	positive	$> \chi_*$	increase

$l \sim 1/\theta = kd_A$. Thus for the same k, $l \uparrow$ as $d_A \downarrow$.
k : wavenumber (scale) & K : (Gaussian) curvature

INTERPRETATION CMB TT SPECTRUM II
(EFFECT OF COSMOLOGICAL PARAMETERS)

- Effect of A_s and n_s

$$C(l)^{\mathrm{SW}} \approx \frac{2}{25\pi}\int_0^\infty dk k^2 P_{\mathcal{R}}(k)|j_l\left[k(\eta_0-\eta_*)\right]|^2$$

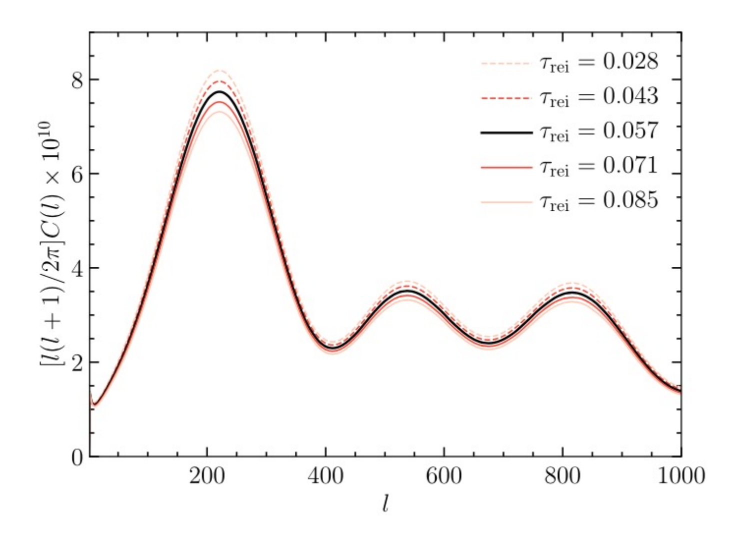
$$\approx 2^{n_s-2}\frac{\pi^2}{25}A_s\left(\eta_0 k_p\right)^{1-n_s}\frac{\Gamma[l+\frac{n_s}{2}-\frac{1}{2}]}{\Gamma[l-\frac{n_s}{2}+\frac{5}{2}]}\frac{\Gamma[l3-n_s]}{\Gamma^2[2-\frac{n_s}{2}]}$$

$$\overset{n_s=1}{=} \frac{8}{25}A_s\frac{1}{l(l+1)} \quad \text{thus} \quad D(l)\equiv l(l+1)C(l)$$

$C(l)\uparrow$ as $A_s\uparrow$
$\Delta C(l)$ by $(l/l_p)^\alpha$ if $n_s \to n_s+\alpha$

- Effect of τ (6 < z < 15)
- Reionization : CMB γ scatter off free e^- again & wash out primordial anisotropies (isotropy restored)

$$\underbrace{T\left(1+\Theta\right)e^{-\tau_{\mathrm{rei}}}}_{\text{escape reionized region}} + \underbrace{T\left(1-e^{-\tau_{\mathrm{rei}}}\right)}_{\text{scattered into the beam}} = \underbrace{T\left(\Theta e^{-\tau_{\mathrm{rei}}}\right)}_{\text{present T}}$$



INTERPRETATION CMB TT SPECTRUM III

(EFFECT OF COSMOLOGICAL PARAMETERS)

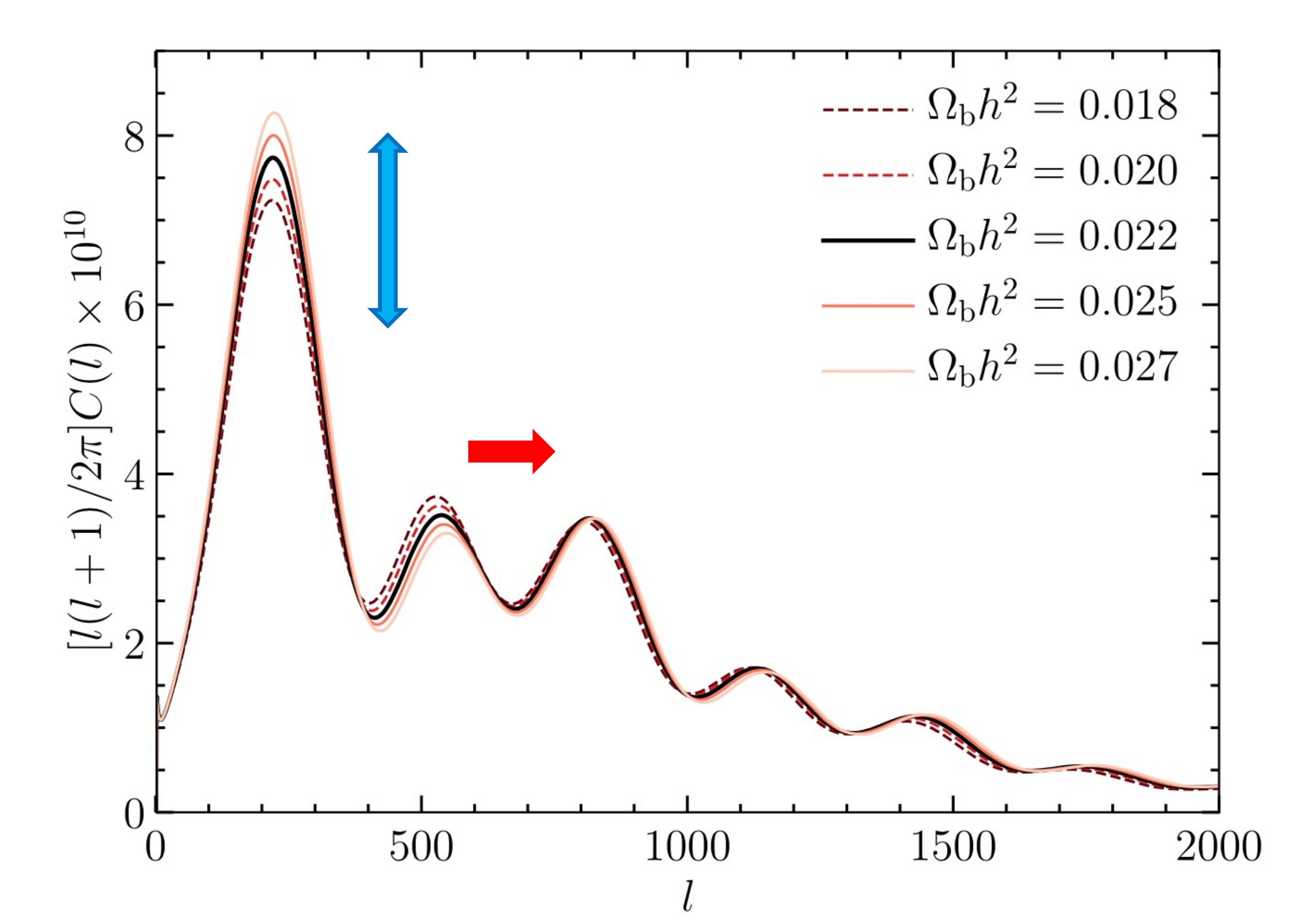
- Effect of $\Omega_b h^2$
- Assume flat Universe

$$l_{\rm pk} \simeq k_{\rm pk}\eta_0 \simeq n\pi\eta_0/r_s(\eta_*) \quad , \quad r_s(\eta_*) = \int_0^{\eta_*} d\eta c_s(\eta)$$

$$c_s(\eta) = \sqrt{\frac{1}{3(1+R(\eta))}} \quad , \quad R = \frac{3\rho_b}{4\rho_\gamma} \quad , \quad k_D \simeq \sqrt{\frac{n_e\sigma_T a}{\eta}}$$

As $\Omega_b \uparrow$:

- $R \uparrow, c_s \downarrow, r_s \downarrow, l_p \uparrow$
- $h_{\rm odd}/h_{\rm even} \uparrow$: increasing effective mass (fall deeper)
- $n_e \uparrow, k_{\rm D} \uparrow$: damping moves smaller angular scales

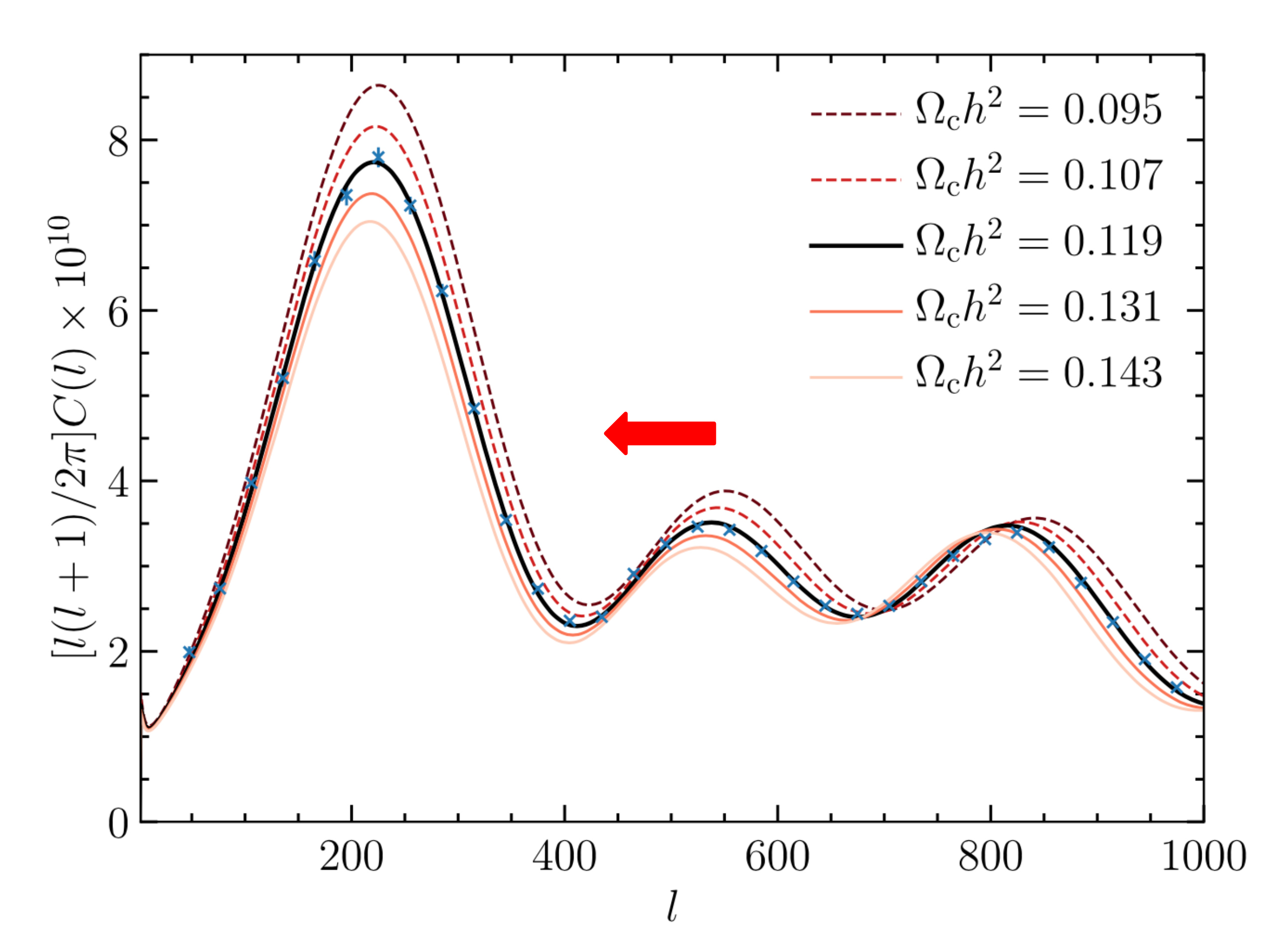


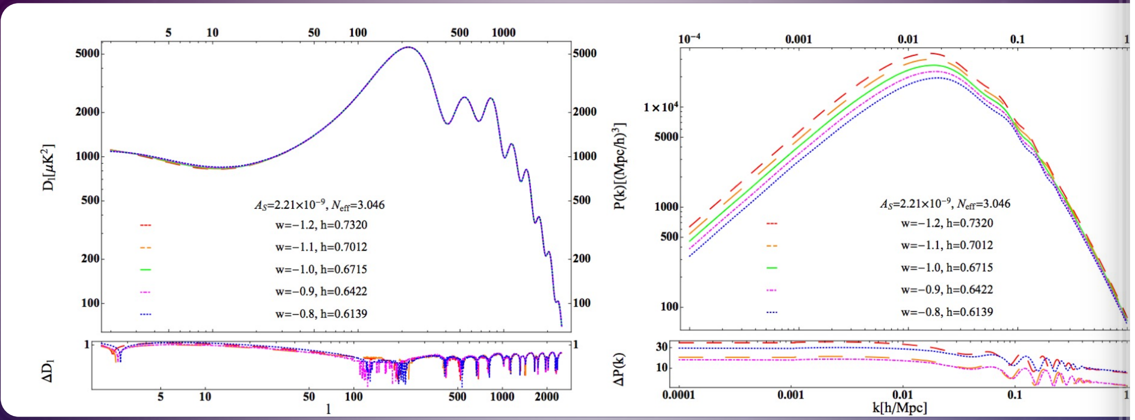
INTERPRETATION CMB TT SPECTRUM IV
(EFFECT OF COSMOLOGICAL PARAMETERS)

- Effect of $\Omega_{DM} h^2$
- Assume flat Universe

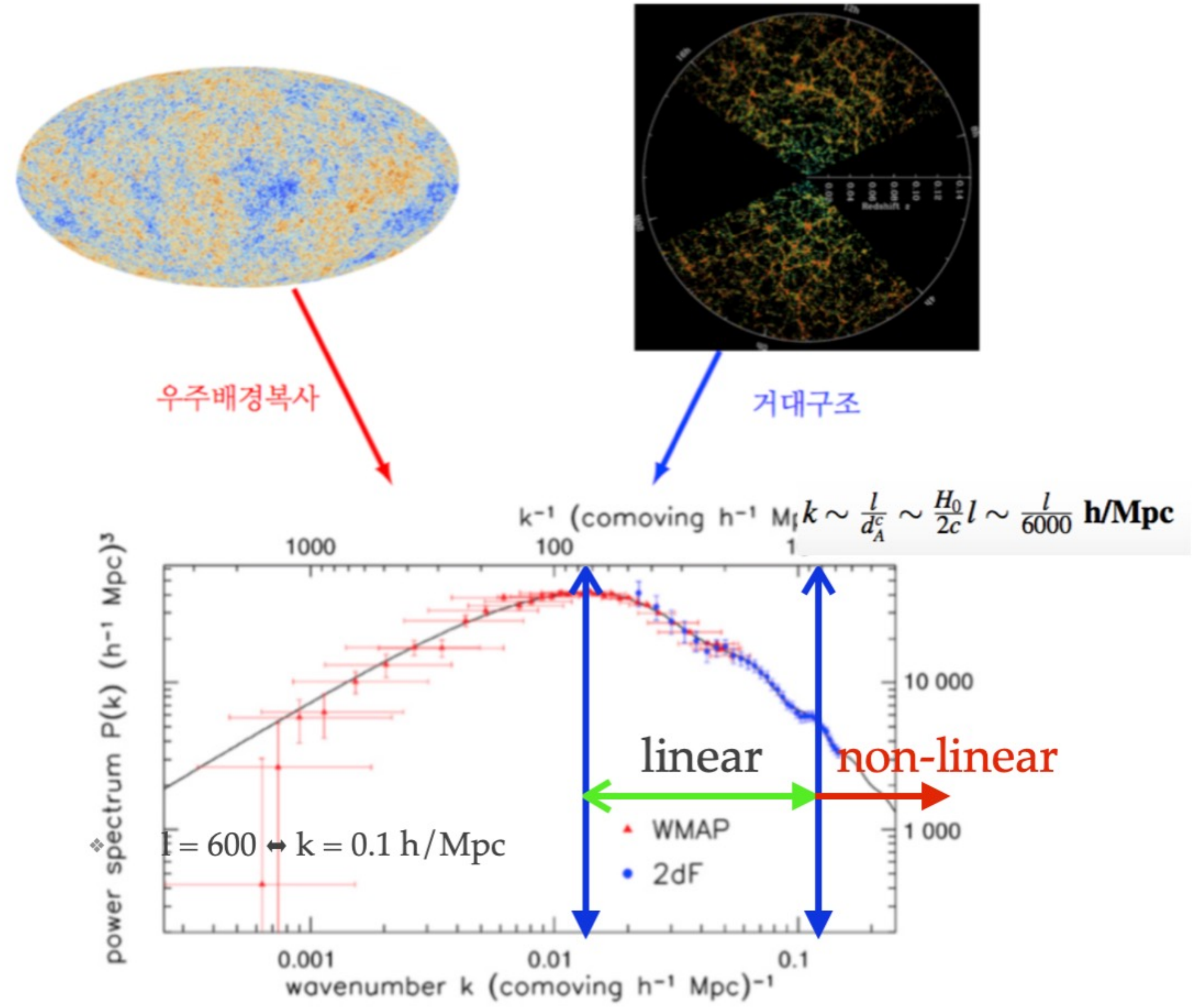
As $\Omega_c \uparrow$:

- $z_{\text{eq}} \uparrow$ more growth
- $\Phi' \downarrow$: less early ISW

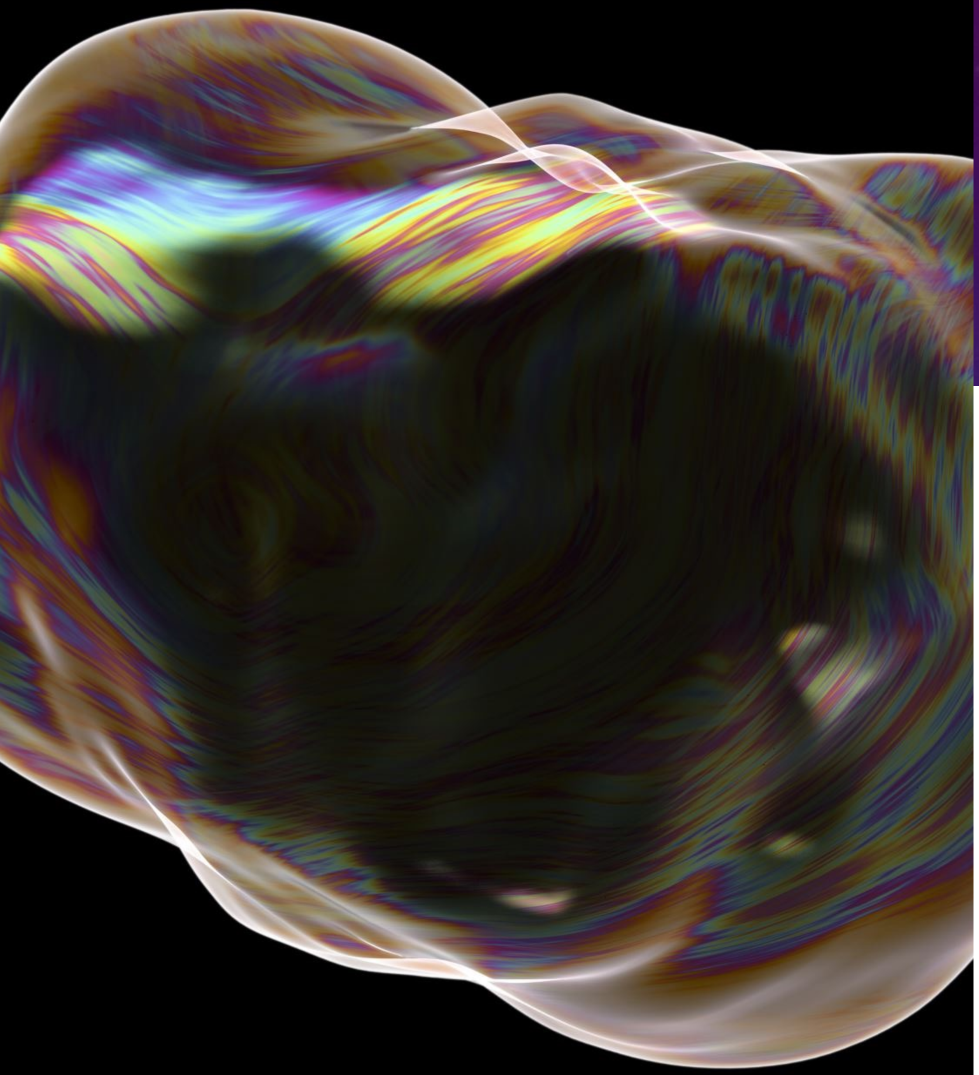




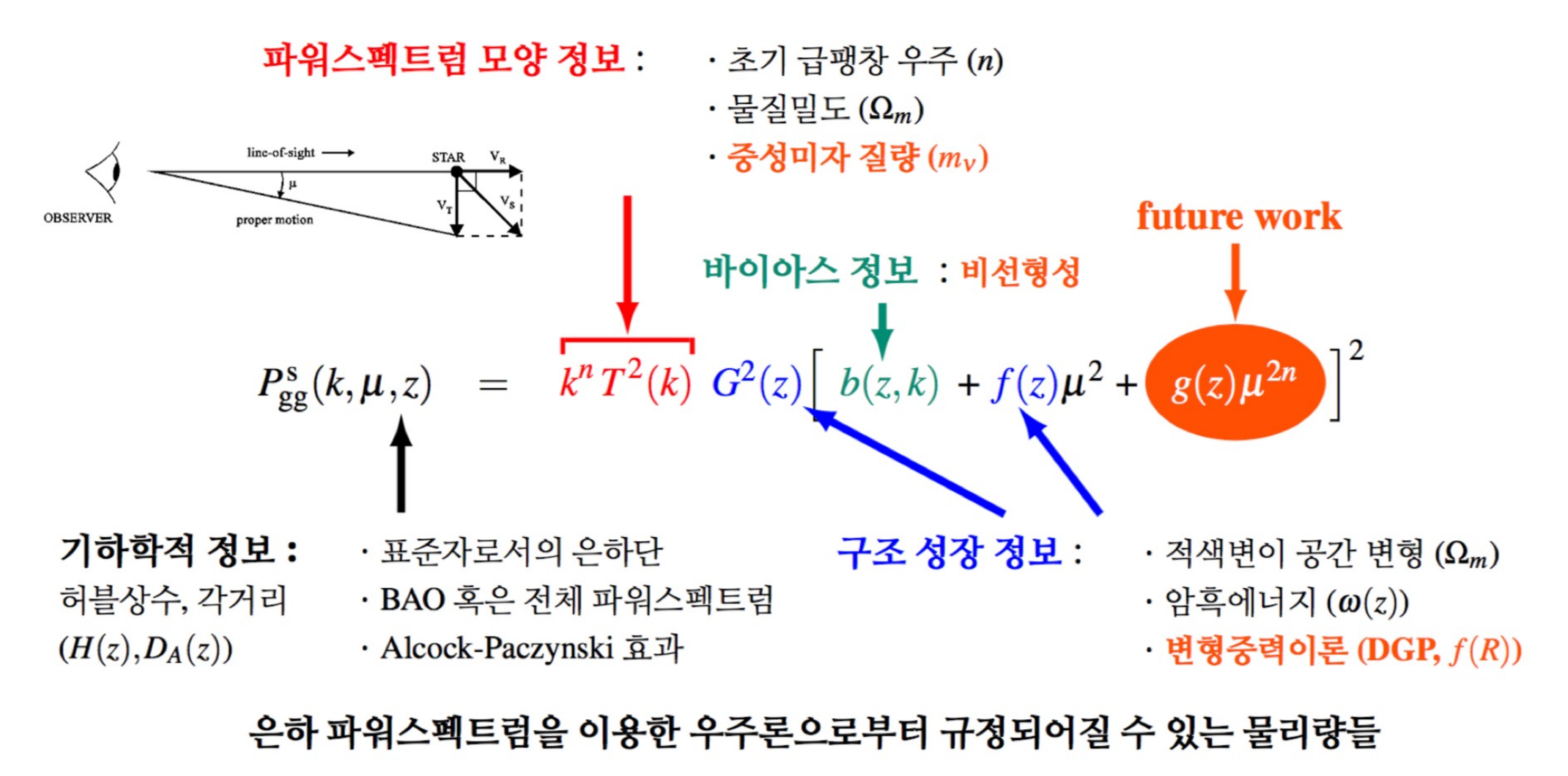
LARGE SCALE STRUCTURE



우주배경복사 (CMB)와 은하 거대구조 (LSS)을 이용한 관측량과 이론에서 얻어진 물질 파워스펙트럼

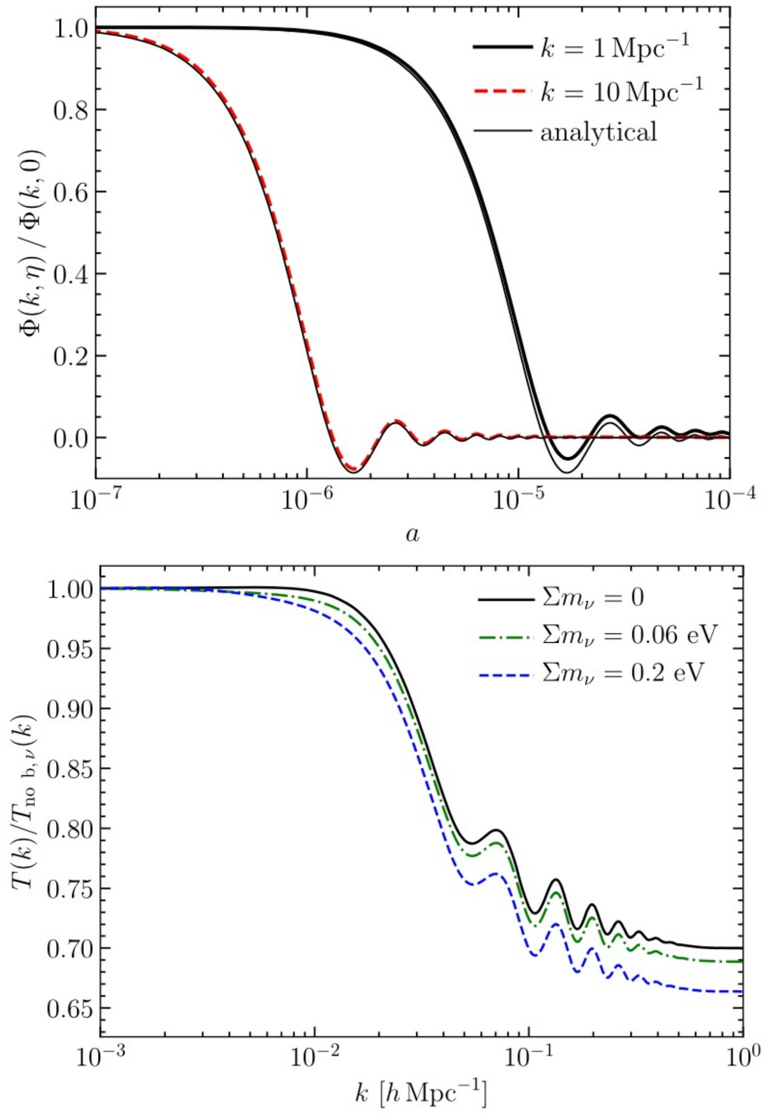


LARGE SCALE STRUCTURE (MATTER POWER SPECTRUM)



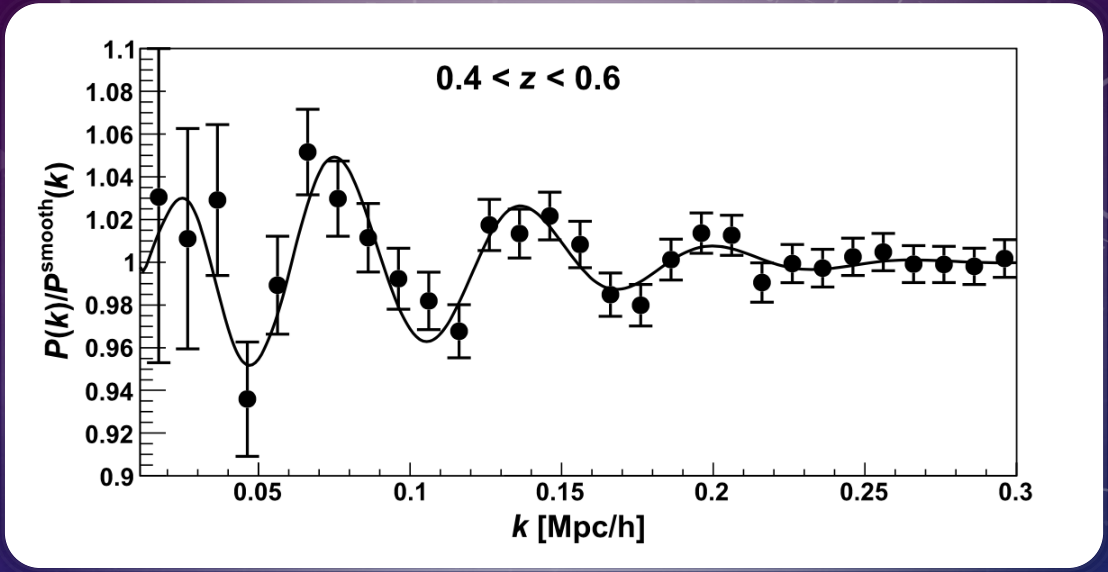
BARYON ACOUSTIC OSCILLATION I

- DM is the main matter component : T(k)
- Still about 16% baryon : affect to T(k)
- Baryon overdensity suppressed compared to DM because of coupling
- Lead to small oscillations in T(k) around k~0.1 h/Mpc
- Roughly form cos(kr_s) where $r_s \approx 105$ Mpc/h : sound horizon at recombination (a standard ruler)
- This feature was imprinted only baryon at early universe, but transferred to late time PS of matter due to coupled by gravity



BARYON ACOUSTIC OSCILLATION II

AS A STANDARD RULER, ONE CAN DERIVE BOTH H AND DA



$$\Delta\theta = \frac{\Delta\chi}{d_A} \quad , \quad d_A = \frac{1}{1+z}\frac{c}{H_0}\int_0^z \frac{dz'}{H(z')}$$