How to add new degree of freedom in modified gravity : standard case

MASAHIDE YAMAGUCHI

(Tokyo Institute of Technology)

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$c = \hbar = M_G^2 = 1/(8\pi G) = 1$

Contents

- **Introduction**

  Why do we extend SM of particle physics and gravity ?

  General relativity : pros and cons

  Why do we consider a dynamical DE model ?

- **How to add new d.o.f to GR (standard way)**

  Top down, bottom up, etc

  Healthy higher order derivative system

- **Discussion and conclusions**

Introduction

Standard model of particle physics and gravity

$$
\begin{aligned}
S &= S_{\mathsf{g}} + S_{\mathsf{m}} \qquad (M_G^2 = 1/(8\pi G)) \\
&= \int d^4x\sqrt{-g}\,\frac{1}{2}M_G^2 R + \int d^4x\sqrt{-g}\,\mathcal{L}_{\mathsf{SM}}
\end{aligned}
$$

Einstein-Hilbert action (GR)

Action for SM of particle physics

Well tested by several experiments.

Well tested by accelerators like LHC.

General relativity is well tested !!

Classical tests :

- the perihelion precession of Mercury's orbit
- the deflection of light by the Sun
- the gravitational redshift of light

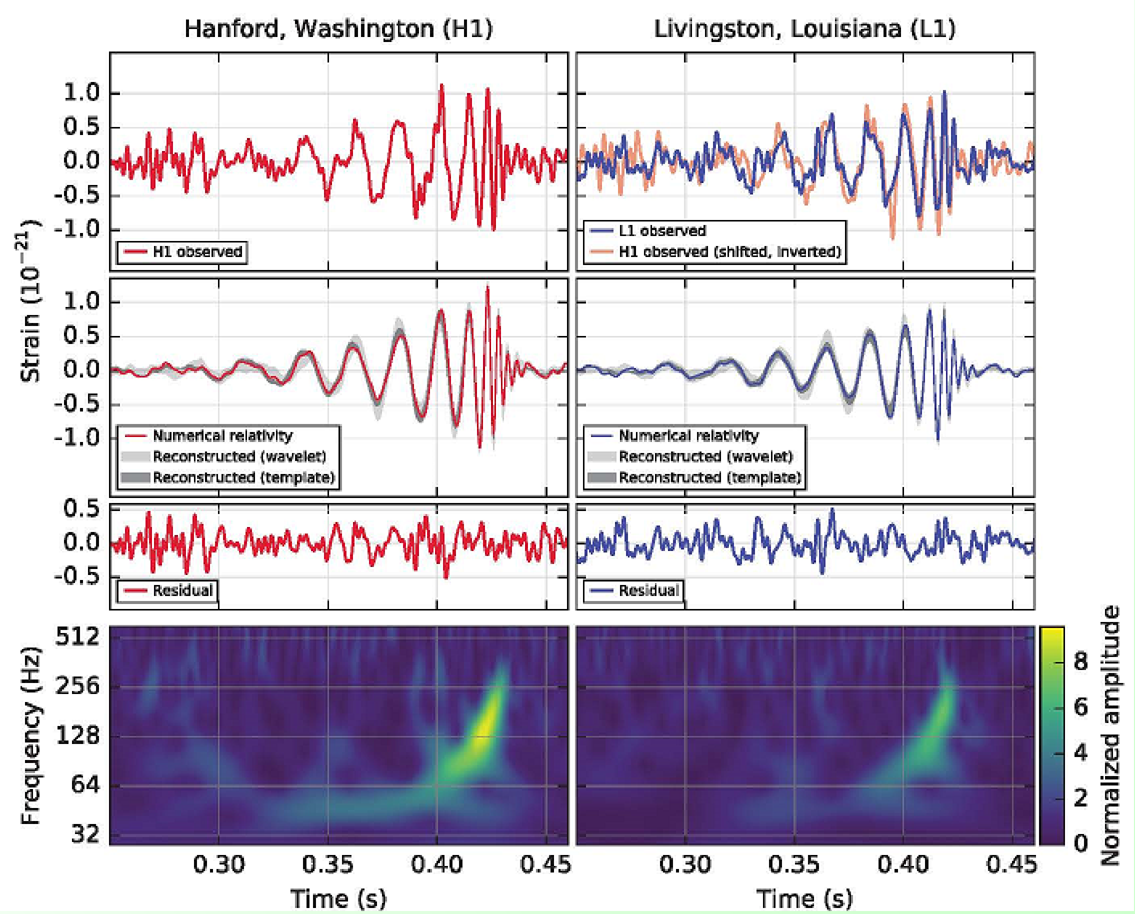
More modern tests :

e.g. see C. M. Will, Living Rev. Relativity 17 (2014), 4

- Post-Newtonian tests of gravity
- Gravitational lensing
- Shapiro time delay
- Indirect detection of GWs by Hulse & Taylor
- …

Direct detection of GWs

h ~ 10^{-21}



$$f \sim \frac{1}{t} \sim \frac{1}{(R/c)} \sim \frac{3 \times 10^{10}\mathrm{cm/s}}{30 \times 3 \times 10^{5}\mathrm{cm}} \sim 10^{3}\,\mathrm{Hz}$$

Necessity of the extension of standard model of particle physics and gravity

$$\begin{aligned} S &= S_g + S_m \\ &= \int d^4x\sqrt{-g}\,\frac{1}{2}M_G^2 R + \int d^4x\sqrt{-g}\,\mathcal{L}_m \end{aligned}$$

$(M_G^2 = 1/(8\pi G))$

Einstein-Hilbert action (GR)

Action for SM of particle physics

These actions might not be able to account for DE(inflation) and/or DM (in addition to the non-zero neutrino masses, baryon asymmetry, etc.)

⇨ Extension of GR and/or SM of particle physics

⇨ Add new d.o.f. responsible for DE(inflation) and/or DM.

General relativity needs to be modified !!

On (very) short distance scales (UV side) :

- GR is non-renormalizable.
- GR predicts singularities.
- …

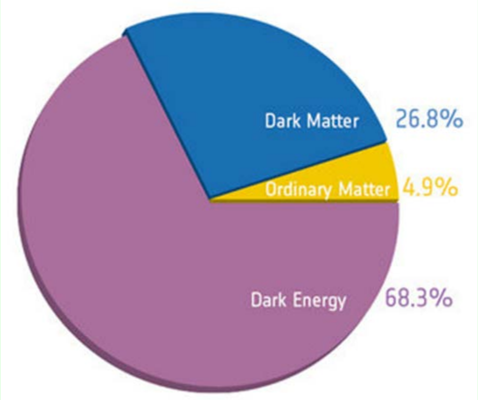
(Possibly) on very long distance scales (IR side) :

- The expansion of Universe is now accelerating.
- …

The presence of dark energy (current acceleration of the universe)

The expansion of the universe is now **accelerating** !!

(SN, CMB, BAO etc)



PLANCK

- **Dark Energy is introduced**

  or

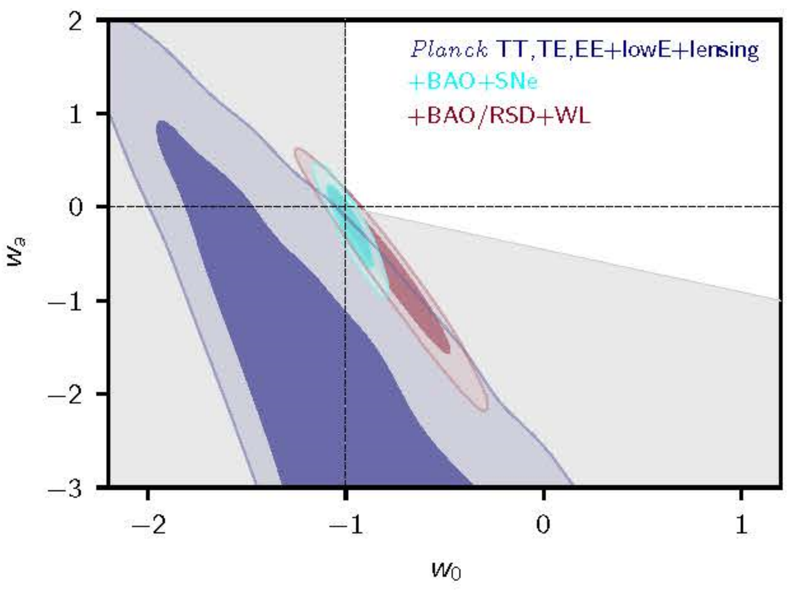
- **GR may be modified in the IR limit**

$$G_{\mu\nu} + \cdots = 8\pi G T_{\mu\nu} + \cdots$$

In either case, we need a (dynamical) degree of freedom responsible for current acceleration.

Dynamical or time-independent ?

Is dark energy dynamical (like inflation) or time-independent (Lambda, meta-stable state suggested by string landscape) ?



Parameter	*Planck*+SNe+BAO	*Planck*+BAO/RSD+WL
w_0	-0.961 ± 0.077	-0.76 ± 0.20
w_a	$-0.28^{+0.31}_{-0.27}$	$-0.72^{+0.62}_{-0.54}$
H_0 [km s^{-1} Mpc^{-1}]	68.34 ± 0.83	66.3 ± 1.8
σ_8	0.821 ± 0.011	$0.800^{+0.015}_{-0.017}$
S_8	0.829 ± 0.011	0.832 ± 0.013
$\Delta\chi^2$	-1.4	-1.4

$$w(a) = w_0 + w_a(1-a)$$

There is no strong constraint at present, though Lambda is consistent with observations.

If w_0 approaches minus unity within 1% by future observations, you may wonder if dark energy is almost Lambda-like and in a (meta)stable state.

But, this is not the case.

Dark energy view of inflation

(Ilic et al. arXiv:1002.4196)

Equation of state during inflation :

$$1+w=\frac{2}{3}\epsilon_H, \quad \epsilon_H \equiv -\frac{\dot{H}}{H^2} \simeq \epsilon \equiv \frac{1}{2}M_G^2\left(\frac{V'}{V}\right)^2$$

Tensor to scalar ratio : $r = 16\epsilon_H \lesssim 0.064$ (PLANCK with BICEP2/Keck Array BK14, 95%CL)

$$\Rightarrow \quad 1+w \lesssim 0.0027.$$

We have already had an example with w equal to -1 within 0.3% level.
However, it is not in a (meta)stable state but dynamical
because inflation must have ended to produce hot universe.

N.B. **Low scale inflation like new inflation** $\Rightarrow$ **$1+w_\varphi$: much smaller**

It's too early to conclude that
DE is cosmological constant.

➔

In my opinion, there is still large motivation to consider a dynamical model, which might require the modification of GR on large scales.

$$G_{\mu\nu} + \cdots = 8\pi G\, T_{\mu\nu} + \cdots$$

Cosmology offers a good opportunity to probe gravity both on short and long distance scales

On (very) short distance scales : (UV side) ⟵ **probed by inflation (and BH)**

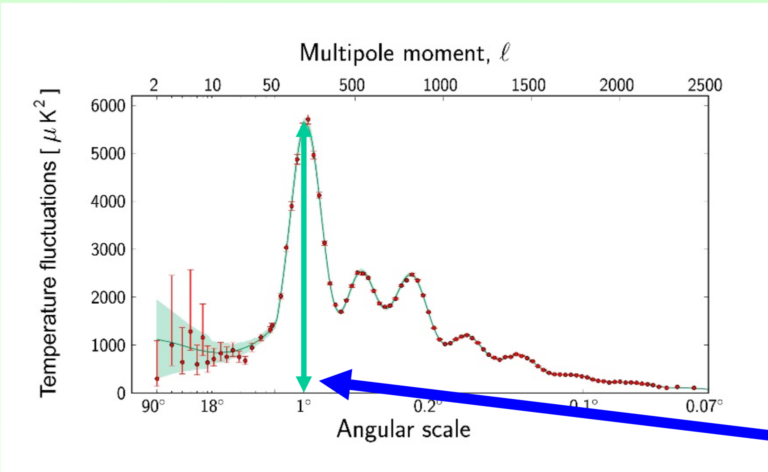
- GR is non-renormalizable.
- GR predicts singularities.
- …

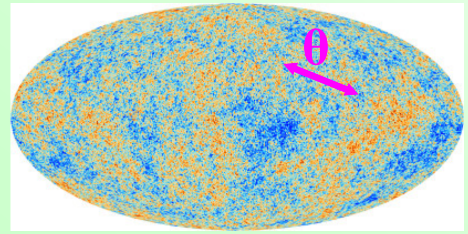
(Possibly) on very long distance scales : (IR side) ⟵ **probed by DE**

- The expansion of Universe is now accelerated again.
- …

Inflation is strongly supported by CMB observations

Planck TT correlation :





Green line : prediction by inflation

Red points : observation by PLANCK

Angle θ ~ 180° / l

Total energy density ←→ Geometry of our universe

- **Our universe is spatially flat, as predicted by inflation !!**
- **Primordial perturbations are generated during inflation.**

We need a (scalar-like) dynamical degree of freedom responsible for inflation.

We have almost confirmed

- **the presence of inflation & dark energy**

But, unfortunately, we do not know their identifications.

What we have to do next is to identify these new d.o.f.,

- **inflaton**
- **dark energy.**

Identifying them is equivalent to clarifying

- **what kind of a scalar-tensor theory (gravity theory) is realized in our Universe.**

(Possible) modification of GR :
How to extend GR (& SM)
by adding new d.o.f. ?

Identification methods

- **Top down approach :**

To construct the unique model from the ultimate theory like string theory.
(Recently, it may not be so actively studied.)

- **Bottom up approach**

To consider the most general model.
Then, we can constrain models (or to single out the true model finally) from the observational results.

Recently, this latter approach is significantly investigated.

Bottom up approach

- **Effective field theory approach :** **(Weinberg 2008, Cheung et al. 2008)**

  The low-energy effective theory (after integrating out heavy mode with its mass M).

  A ghost seems to appear around the cut-off scale M (>> E).

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{2}\frac{1}{M^2}(\Box\phi)^2$$

$$\partial_\mu\phi\partial^\mu\phi \sim E^2\phi^2 \qquad \frac{1}{M^2}(\Box\phi)^2 \sim \frac{E^4}{M^2}\phi^2$$

(E: the energy scale we pay attention to)

$$M \gg E \quad \Longleftrightarrow \quad \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \gg \frac{1}{2}\frac{1}{M^2}(\Box\phi)^2$$

- **Most general theory without ghost**

  (if we are interested in the case in which higher derivative terms play important roles in the dynamics.)

In this talk, we take a closer look at the latter approach.

Integrating out a heavy field

σ : a heavy field with mass M, φ: a light field

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}M^2\sigma^2 - \partial_\mu\sigma\partial^\mu\phi$$

$$\wr$$

$$E^2\sigma^2 \ll M^2\sigma^2$$

$$\uparrow$$

energy scale we are interested in (E << M)

$$\sim -\frac{1}{2}M^2\sigma^2 + \sigma\Box\phi = -\frac{1}{2}M^2\left(\sigma - \frac{\Box\phi}{M^2}\right)^2 + \frac{1}{2}\frac{1}{M^2}(\Box\phi)^2$$

Integrating out σ

$$\sim \frac{1}{2}\frac{1}{M^2}(\Box\phi)^2$$

The following question arises:

What is the most general scalar-tensor theory without ghost (to account for DE and/or inflation) ?

How to add new d.o.f. to GR & SM ?

Minimal (simple) scalar tensor theory :

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_G^2 R + X - V(\phi) + \mathcal{L}_{\rm SM}\right) \quad \left(X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right)$$

Non-minimal coupling :

(Brans-Dicke, Higgs inflation)

$$\mathcal{L} = f(\phi)R$$

k-inflation, k-essence :

(Armendariz-Picon et.al. 1999, Chiba, Okabe, MY 2000)

$$\mathcal{L} = K(\phi, X)$$

Higher derivative terms :

(Nicolis et.al. 2009)

$$\Delta\mathcal{L} = X\Box\phi$$

Even higher and more extensions ….

Theories with higher order derivatives are quite dangerous in general.

Lagrangian

Why does Lagrangian generally depend on only
a position q and its velocity dot{q} ?

Newton recognized that an acceleration, which is given by the second time derivative of a position, is related to the Force :

$$m\frac{\mathrm{d}^2 \boldsymbol{x}}{\mathrm{d}t^2} = \boldsymbol{F}\left(\boldsymbol{x}, \dot{\boldsymbol{x}}\right).$$

The Euler-Lagrange equation gives an equation of motion up to the second time derivative if a Lagrangian is given by L = L(q,dot{q},t).

$$\frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0, \quad \Longrightarrow \quad \ddot{q} = \ddot{q}\left(\dot{q}, q\right) \quad \Longrightarrow \quad q(t) = Q\left(\dot{q}_0, q_0, t\right).$$

(if $p := \frac{\partial L}{\partial \dot{q}}$ depends on dot{q} ⇔ non-degenerate condition.)

What happens if Lagrangian depends on higher derivative terms ?

Example with higher order (time) derivatives

- $L = \frac{1}{2}\ddot{q}^2(t)$ $\Longrightarrow$ (EL eq.) $q^{(4)} = 0$ **requires 4 initial conditions.**

  $\Updownarrow$

  2 (real) DOF

- $L^{(1)}_{\rm eq} = \ddot{q}u - \frac{1}{2}u^2$ $\Longrightarrow$ (EL eq.) $\begin{cases} \ddot{u} = 0, \\ \ddot{q} = u, \end{cases}$ $\Longrightarrow$ $q^{(4)} = 0$

  $x \equiv \frac{q-u}{\sqrt{2}}, \quad y \equiv \frac{q+u}{\sqrt{2}}$ $\Longrightarrow$ $L^{(1)}_{\rm eq} = -\dot{q}\dot{u} - \frac{1}{2}u^2 = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\dot{y}^2 - \frac{1}{4}(x-y)^2.$

  $\Longrightarrow$ $H = \frac{1}{2}p_x^2 - \frac{1}{2}p_y^2 + \frac{1}{4}(x-y)^2.$

  $(p_x \equiv \dot{x}, \ \ p_y \equiv \dot{y})$

  2 (real) DOF = 1 healthy & 1 ghost

- $L^{(2)}_{\rm eq} = \frac{1}{2}\dot{Q}^2 + \lambda(Q - \dot{q})$ $\Longrightarrow$ $p \equiv \frac{\partial L^{(2)}_{\rm eq}}{\partial \dot{q}} = -\lambda, \quad P \equiv \frac{\partial L^{(2)}_{\rm eq}}{\partial \dot{Q}} = \dot{Q}.$

  $\Longrightarrow$ $H = p\dot{q} + P\dot{Q} - L^{(2)}_{\rm eq} = \frac{1}{2}P^2 + pQ.$

Hamiltonian is unbounded through a linear momentum !!

Ostrogradski's theorem

(Ostrogradsky 1850)

Assume that $L = L(\ddot{q}, \dot{q}, q)$ **and** $\frac{\partial L}{\partial \ddot{q}}$ **depends on** $\ddot{q}$ **:**

(Non-degeneracy)

$$\frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) + \frac{\mathrm{d}^2}{\mathrm{d}^2 t}\left(\frac{\partial L}{\partial \ddot{q}}\right) = 0, \quad \Longrightarrow \quad q^{(4)} = q^{(4)}\left(q^{(3)}, \ddot{q}, \dot{q}, q\right).$$

Canonical variables :

$$\begin{cases} q, & p := \frac{\partial L}{\partial \dot{q}} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \ddot{q}} \quad \left(= \frac{\partial L_{\mathrm{eq}}}{\partial \dot{q}}\right), \\ Q := \dot{q}, & P := \frac{\partial L}{\partial \ddot{q}} \quad \left(= \frac{\partial L_{\mathrm{eq}}}{\partial \dot{Q}}\right). \end{cases}$$

$$L_{\mathrm{eq}} = L(\dot{Q}, \dot{q}, q) + \lambda(Q - \dot{q})$$

Non-degeneracy $\Leftrightarrow$ $\ddot{q} = \ddot{q}\left(q, \dot{q}, \frac{\partial L}{\partial \ddot{q}}\right)$ $\Leftrightarrow$ $\dot{Q} = \ddot{q} = \ddot{q}\,(q, Q, P)$

Hamiltonian:

$$\begin{aligned} H(q, Q, p, P) &:= p\dot{q} + P\dot{Q} - L \\ &= pQ + P\,\ddot{q}(q, Q, P) - L(q, Q, \ddot{q}(q, Q, P)). \end{aligned}$$

p depends linearly on H so that no system of this form can be stable !!

N.B. $\frac{\partial L}{\partial \phi} - \partial_\mu\left(\frac{\partial L}{\partial(\partial_\mu \phi)}\right) + \partial_\mu \partial_\nu\left(\frac{\partial L}{\partial(\partial_\mu \partial_\nu \phi)}\right) = 0.$ $\Longrightarrow$ $\frac{i}{(p^2 + m_1^2)(p^2 + m_2^2)} = \frac{1}{m_2^2 - m_1^2}\left(\frac{i}{p^2 + m_1^2} - \frac{i}{p^2 + m_2^2}\right).$

(propagators)

How to circumvent Ostrogradsky's arguments to obtain healthy higher order derivative theories ?

Loopholes of Ostrogradski's theorem

① **Abandon the non-degeneracy condition ➔ degenerate theory**

($\frac{\partial L}{\partial \ddot{q}}$ depends on ddot{q})

- **For L = L(q, dot{q}),**
  degeneracy => EOM is (less than) first order (constraint !!).

- **For L = L(q, dot{q}, ddot{q}),**
  degeneracy => EOM can be (more than) second order
  (still dynamics !!).

② **Abandon finite derivatives**

➔ infinite derivative (non-local) theory

① Generalized Galileon = Horndeski

Deffayet et al. 2009, 2011

prove

Kobayashi, MY, Yokoyama 2011

Horndeski 1974
Charmousis et al. 2011

$$
\begin{cases}
\mathcal{L}_2 = K(\phi, X), \\
\mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \\
\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\
\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\
\qquad -\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right].
\end{cases}
$$

$$\left(X = -\frac{1}{2}(\nabla\phi)^2, \quad G_{iX} \equiv \partial G_i/\partial X\right)$$

- A candidate of the most general (single-)scalar tensor theory whose Euler-Lagrange EOMs are up to second order.
- k-inflation, Galileon, Einstein-Hilbert action, non-minimal coupling, and so on are included as a part of this action.
- Horndeski already gave the most general action in 1974 !!
- What is the relation between Generalized Galileon and Horndeski models?

  ➔ Both models are completely equivalent !! (Kobayashi, MY, Yokoyama 2011)

How about cosmological perturbations ???

Cosmological perturbations in Horndeski theory

Kobayashi, MY, Yokoyama 2011

- **Tensor perturbations:**

$$S_T^{(2)} = \frac{1}{8}\int \mathrm{d}t\mathrm{d}^3x\, a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\nabla h_{ij})^2\right].$$

$$\left(G_{i\phi} \equiv \frac{\partial G_i}{\partial \phi}, \quad G_{iX} \equiv \frac{\partial G_i}{\partial X}\right)$$

$$\begin{cases} \mathcal{F}_T := 2\left[G_4 - X\left(\ddot{\phi}G_{5X} + G_{5\phi}\right)\right], \\ \mathcal{G}_T := 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\right] \end{cases}$$

$$c_T^2 := \frac{\mathcal{F}_T}{\mathcal{G}_T}$$

If this Horndeski field is responsible for dark energy, the sound velocity of tensor perturbations (GWs) must be very close to unity.

$$c_T^2 = c_{\mathrm{GW}}^2 \simeq 1.$$

(e.g. Creminelli & Vernizzi 2017)
(Kimura & Yamamoto 2012)

(GW170817 & GRB170817A)

(gravitational Cherenkov radiation)

$$\begin{aligned} \mathcal{L}_2 &= K(\phi, X), \\ \mathcal{L}_3 &= -G_3(\phi, X)\Box\phi, \\ \mathcal{L}_4 &= G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\ \mathcal{L}_5 &= G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ &\quad -\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]. \end{aligned}$$

$$G_{4X} \simeq 0, \quad G_5 \simeq 0$$

② Non-local (infinite derivative) theories

- **Scalar field Lagrangian:**

$$\mathcal{L} = -\frac{1}{2}\phi F(\Box)\phi - V(\phi),$$

↑

e.g. an entire function (good IR limit of F(☐) => - ☐ + m²)

- **Weierstrass's theorem:**

$$F(\Box) = e^{-\gamma(\Box)} \prod_{i=1}^{N} (-\Box + m_i^2)^{r_i},$$

γ(☐) : another entire function

- **N = 1, r_i =1, γ(☐) ≠ 0 :**
  (infinite derivative theory with one real zero)

$$F(\Box) = e^{-\gamma(\Box)}(-\Box + m^2) \Rightarrow \frac{ie^{\gamma(-p^2)}}{p^2 + m^2 - i\epsilon}$$

(propagators)

No ghost,
one healthy d.o.f.

One of them is ghost

N.B. $\frac{\partial L}{\partial \phi} - \partial_\mu \left(\frac{\partial L}{\partial(\partial_\mu \phi)} \right) + \partial_\mu \partial_\nu \left(\frac{\partial L}{\partial(\partial_\mu \partial_\nu \phi)} \right) = 0. \Rightarrow \frac{i}{(p^2 + m_1^2)(p^2 + m_2^2)} = \frac{1}{m_2^2 - m_1^2} \left(\frac{i}{p^2 + m_1^2} - \frac{i}{p^2 + m_2^2} \right).$

Non-local (infinite derivative) gravity theories

e.g. $$S_{\rm IDG} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left\{\mathcal{R} + G_{\mu\nu}\frac{e^{-\Box/M_s^2}-1}{\Box}\mathcal{R}^{\mu\nu}\right\}$$

The gauge independent part of the saturated propagator:

$$\Pi_{\rm IDG}(p) = e^{-p^2/M_s^2}\Pi_{GR}(p),$$

↑

Propagators of GR

Taking a static and spherically symmetric linearized metric : $(|\Phi|, |\Psi| \ll 1)$

$$ds^2 = -(1+2\Phi)dt^2 + (1-2\Psi)(dr^2 + r^2 d\Omega^2)$$

and considering a static point-like source with a mass m, $T_{\mu\nu} = m\delta^0_\mu\delta^0_\nu\delta^{(3)}(\vec{r})$

$$\Phi(r) = \Psi(r) = -\frac{Gm}{16\pi r}{\rm Erf}\left(\frac{rM_s}{2}\right) \Longrightarrow -\frac{GmM_s}{32\pi}$$

(r -> 0)

No divergence !!

Summary

- GR and SM of particle physics are very successful theories, but needs to be modified in order to explain DE (inflation), DM, and so on.
- Though the dark energy is consistent with cosmological constant, it is still too early to conclude it. A dynamical model is still worth studying.
- Cosmology provides a good opportunity to probe gravity both on short and long distance scales. (pros)
- It is quite useful to consider a general model (bottom-up approach) because it can accommodate many models in a unified way.
- Two ways to extend gravity theory with higher derivatives in a healthy (ghost-free) way is to consider (i) degenerate theory like Horndeski theory, (ii) infinite derivative (non-local) theory.
- The future observations including GWs will strongly constrain models.
- In this talk, we have added new d.o.f. to GR by hand in some sense.
  In my second one, I will introduce another method.