CQUEST GROUP II WORKSHOP 2022 (CGII 2022)

SEP. 29~OCT. 2, 2022

REHEATING AFTER INFLATION:

WITH APPLICATION TO MODIFIED GRAVITY

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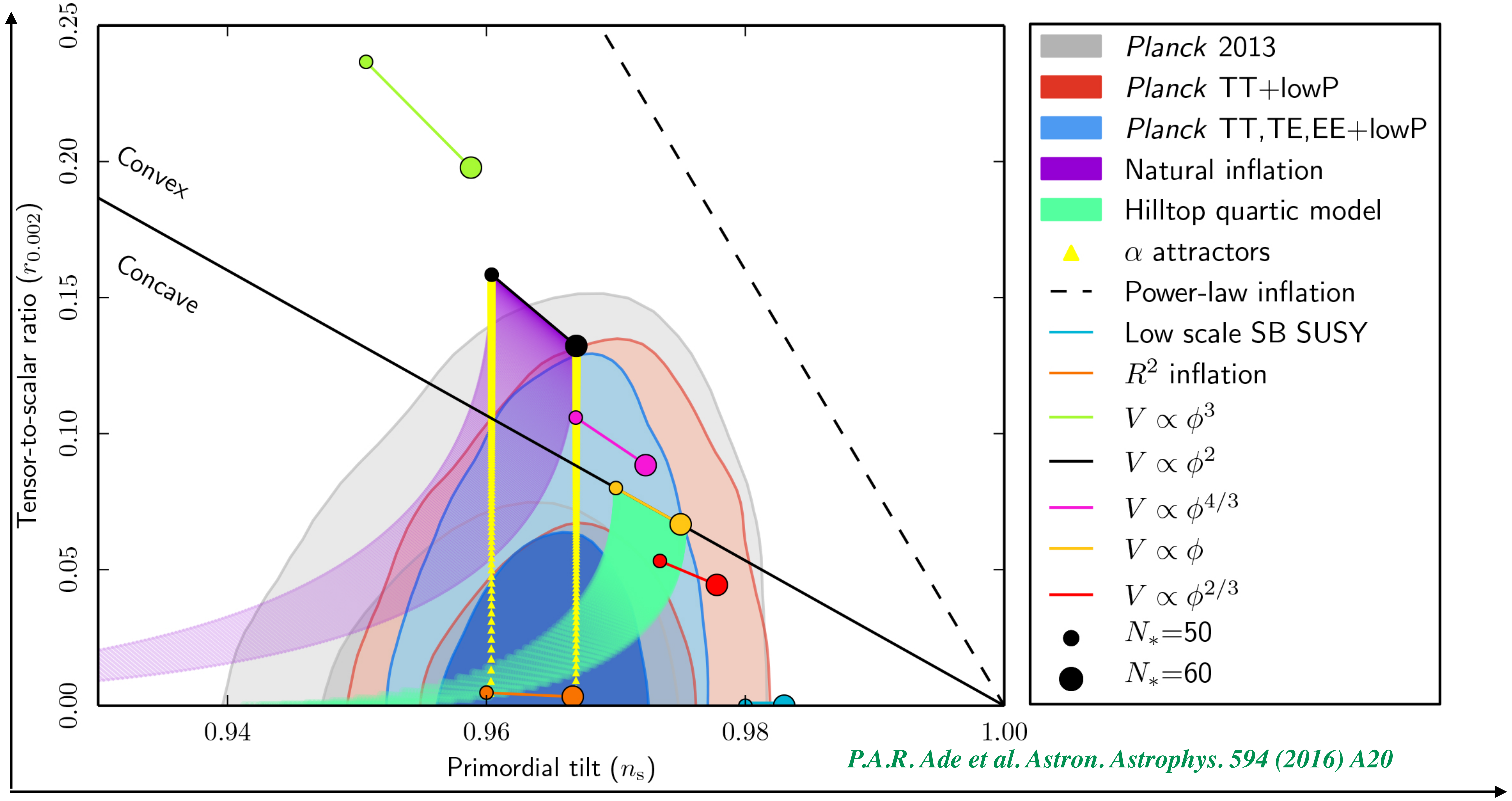
- ***There are hundreds of inflation models consistent with the observational data***

$r_{0.002} < 0.11$ (95 % CL)



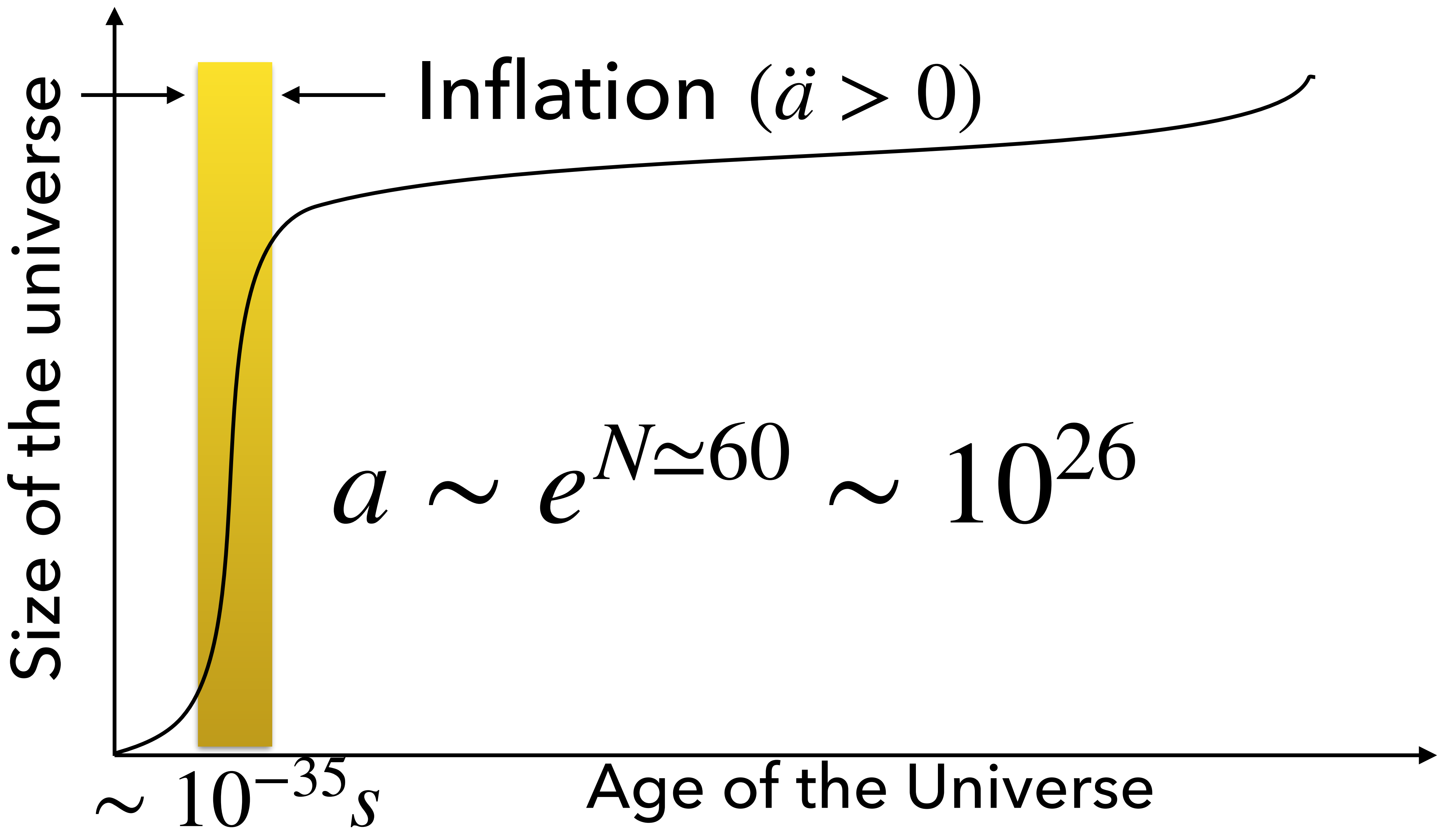
$n_s = 0.9655 \pm 0.0062$ (68 % CL, *Planck* TT+lowP)

- ***The simplest scenario is based upon a single scalar field minimally coupled to gravity***

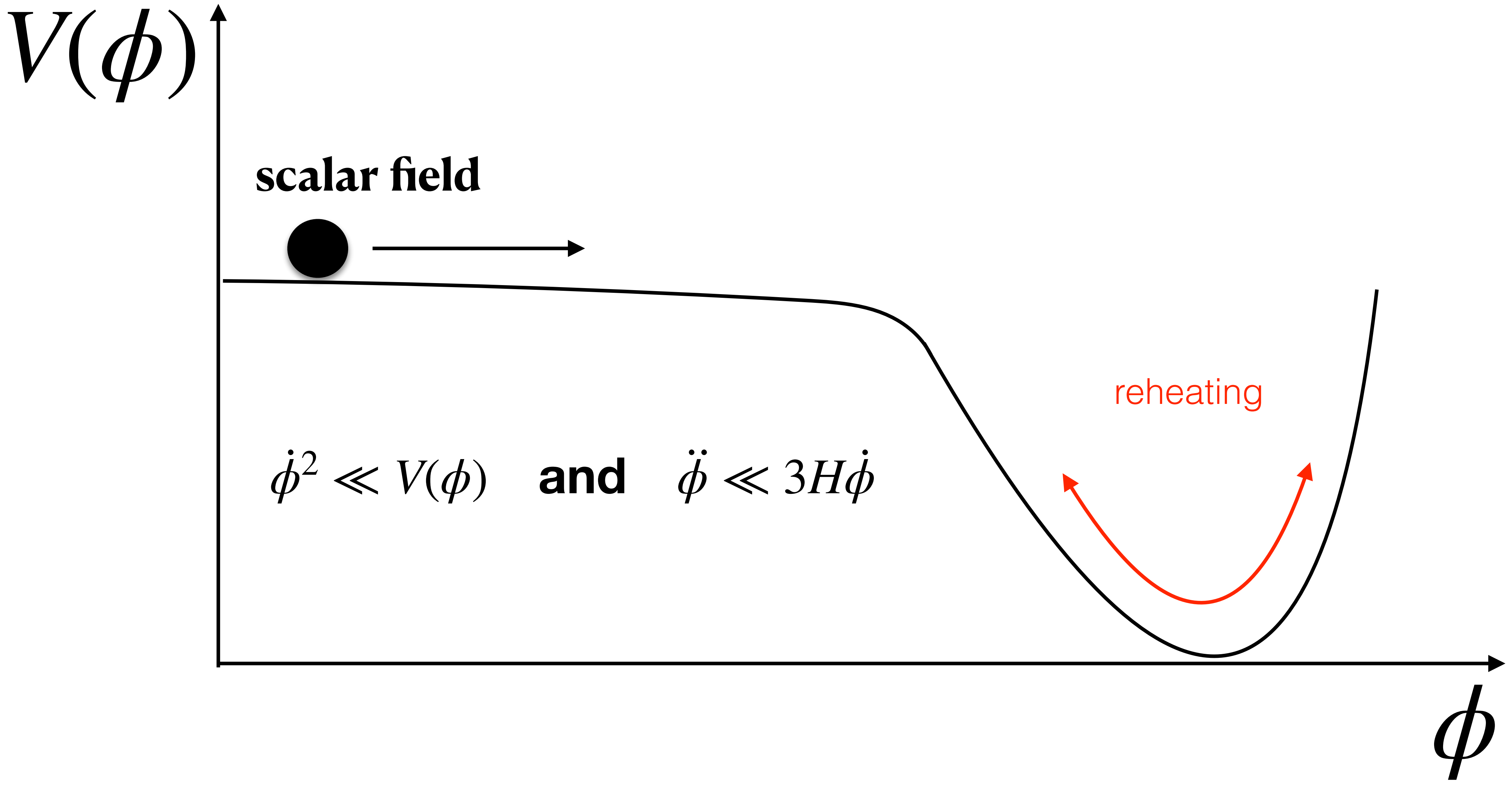
$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right],$$

Details of cosmic inflation:

Inflation is assumed to be driven by a scalar field ϕ slowly rolling down its potential.



$\Leftrightarrow$ Equivalent



Equations of motion: $H^2 = \frac{1}{3M_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$ **and** $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0\,.$

During inflation: $\boxed{H \simeq const\,. \longrightarrow a \sim e^{Ht}}$

cheating sheet for a slow-roll inflation

- Potential $V(\phi)$
- Slow-roll parameters: $\epsilon = \frac{M_P^2}{2}(\frac{V'}{V})^2, \eta = M_P^2 \left(\frac{V''}{V}\right)$
- End of inflation: $\mathrm{Max}[\epsilon, \eta] = 1$ @ $\phi = \phi_{end}$
- Number of efoldings, $N(\phi) = \int H dt = \frac{1}{M_P^2} \int_{\phi_{end}}^{\phi} d\phi \frac{V}{V'}$
- Pivot scale at CMB requesting $N(\phi_*) = 60$ => ϵ_*, η_*, V_*
- $\delta_s \sim 10^{-5} \Rightarrow \frac{V_*}{\epsilon_*} \approx \delta_s^2 M_P^4$
- $r \approx 16\epsilon_*, \quad n_s \approx 1 - 6\epsilon_* + 2\eta_*$
- Compare with DATA : $(\delta_s \sim 10^{-5}, n_s \approx 0.965, r < 0.1)$

credit: Seong Chan Park

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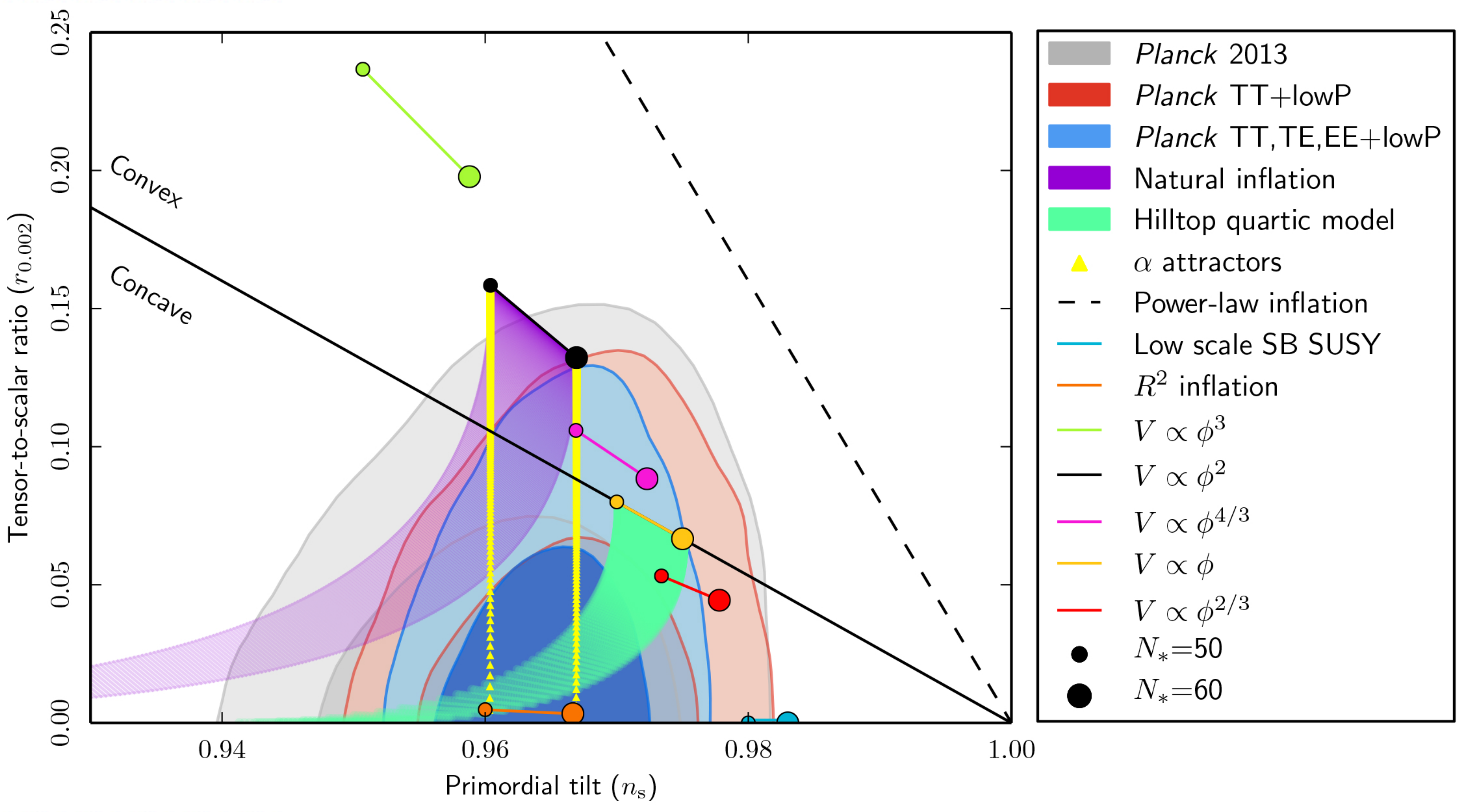

THERE ARE DEGENERACIES BETWEEN MODELS…

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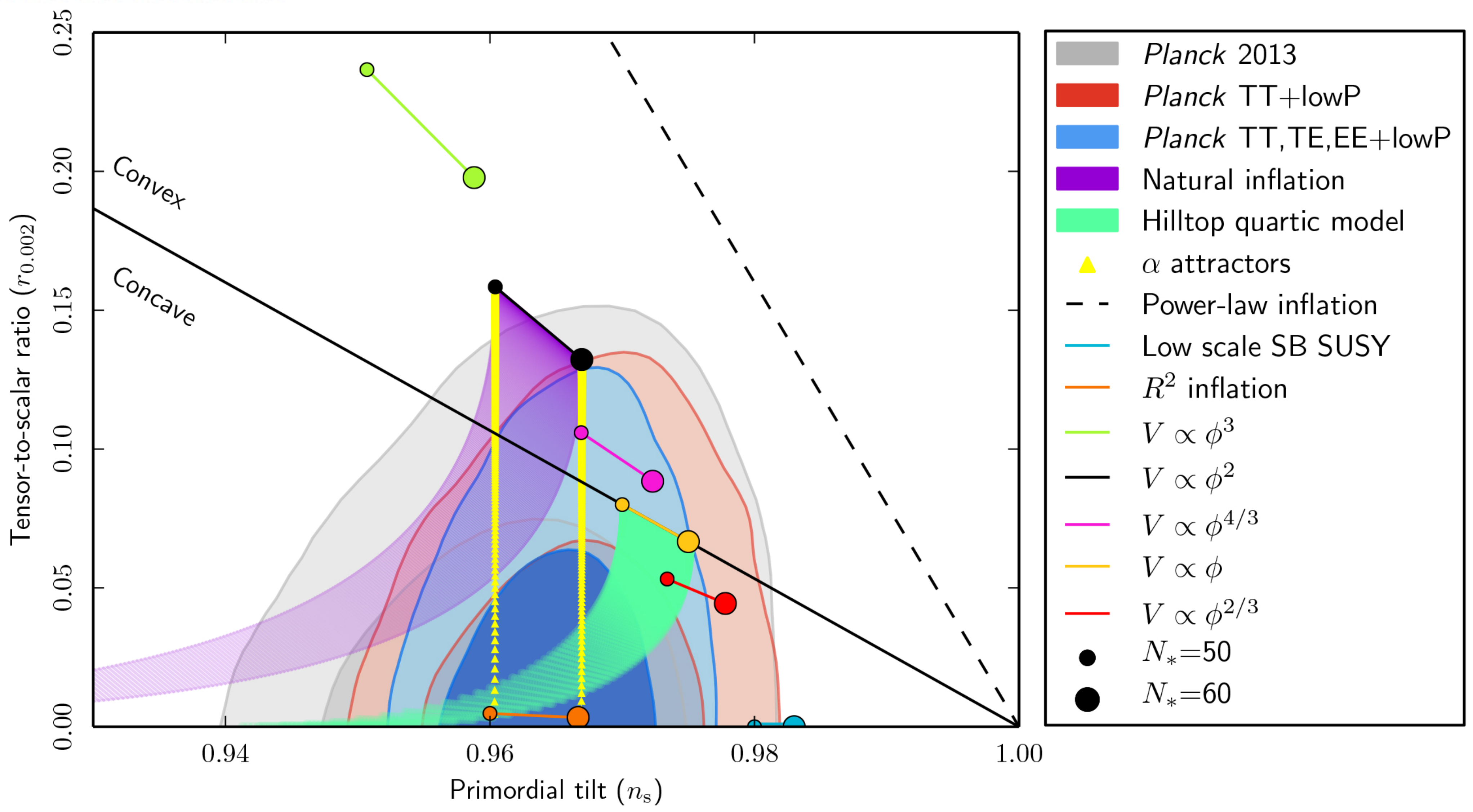

THERE ARE DEGENERACIES BETWEEN MODELS…

HOW TO BREAK THE DEGENERACIES BETWEEN THESE MODELS?

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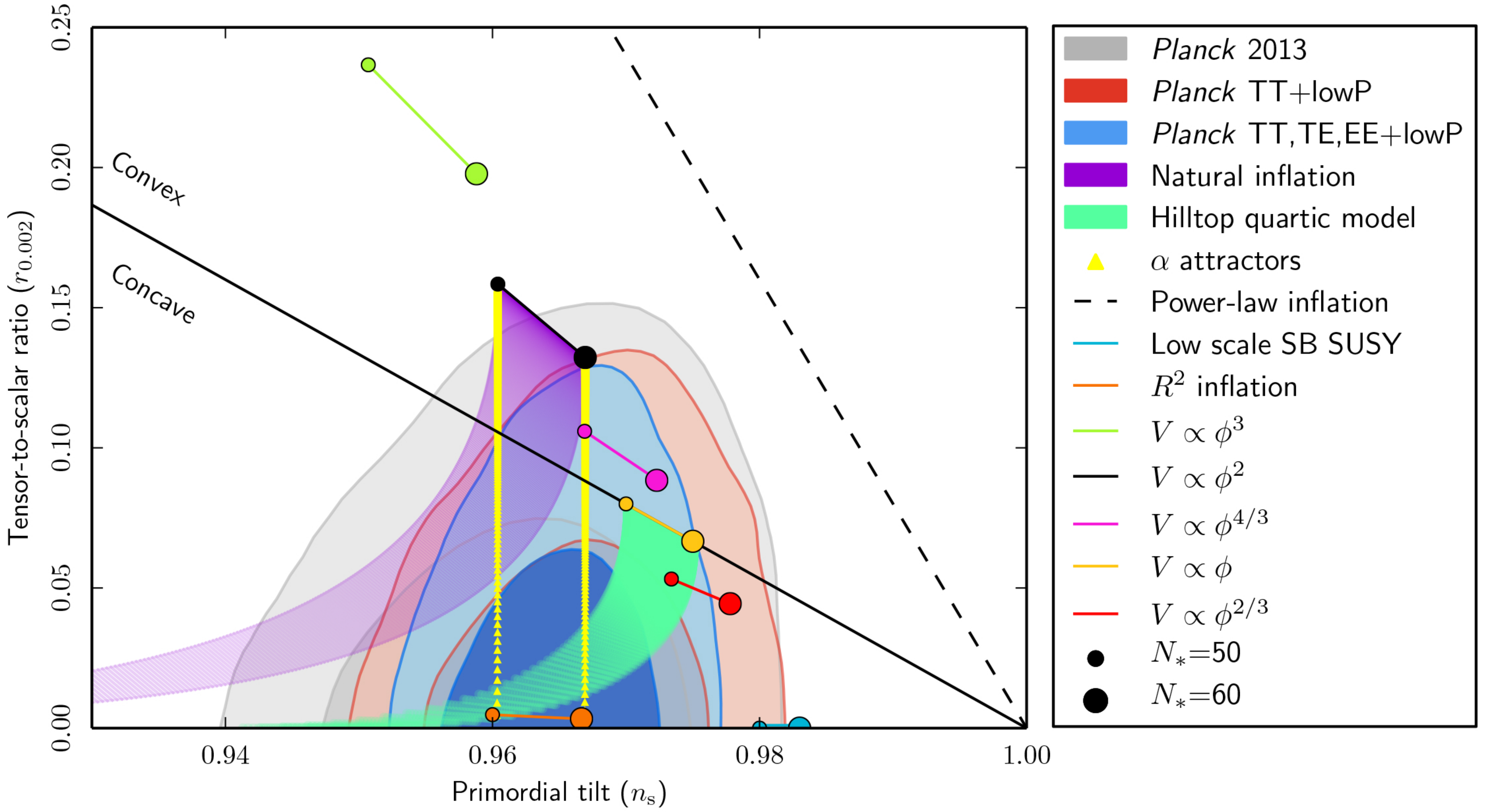

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REHEATING?!

REHEATING AFTER INFLATION:

~~WITH APPLICATION TO MODIFIED GRAVITY~~

REHEATING CONSTRAINTS TO INFLATIONARY MODELS:

- Reheating is a transition era between the end of inflation and the beginning of the radiation era during which the energy stored in the inflaton is converted to a plasma of relativistic particles.

- Although there are NO direct cosmological observables that are traceable this period of reheating, there are several reheating models have been proposed including perturbative decay of oscillating inflaton field at the end of inflation, and non-perturbative processes such as parametric resonance decay, and instant preheating.

- Depending upon the model, duration and final temperature of the reheating, as well as its equation of state, directly linked to inflationary observables if we approximate reheating by a constant EoS.

- Reheating can help to break degeneracies between inflation models that otherwise overlap in their predictions of n_s and r.

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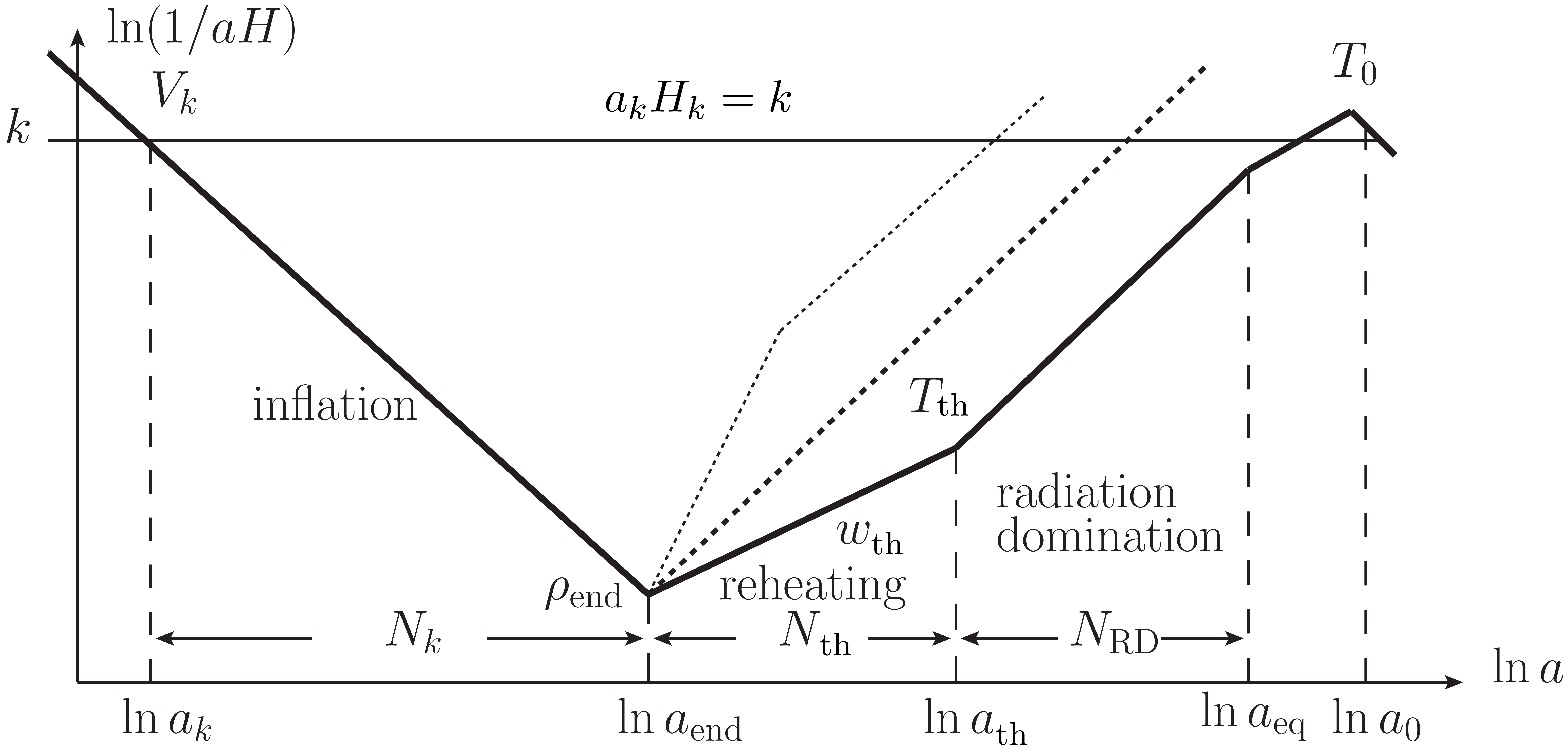
REHEATING CONSTRAINTS TO INFLATIONARY MODELS:

- Jessica L. Cook et al. JCAP 04 (2015), Liang Dai et al. PRL 113 041302 (2014), Julian B. Munoz et al. PRD91 (2015) 043521



$$\boxed{\frac{k}{a_0 H_0} = \frac{a_k}{a_{\rm end}} \frac{a_{\rm end}}{a_{\rm th}} \frac{a_{\rm th}}{a_{\rm eq}} \frac{a_{\rm eq} H_{\rm eq}}{a_0 H_0} \frac{H_k}{H_{\rm eq}},}$$

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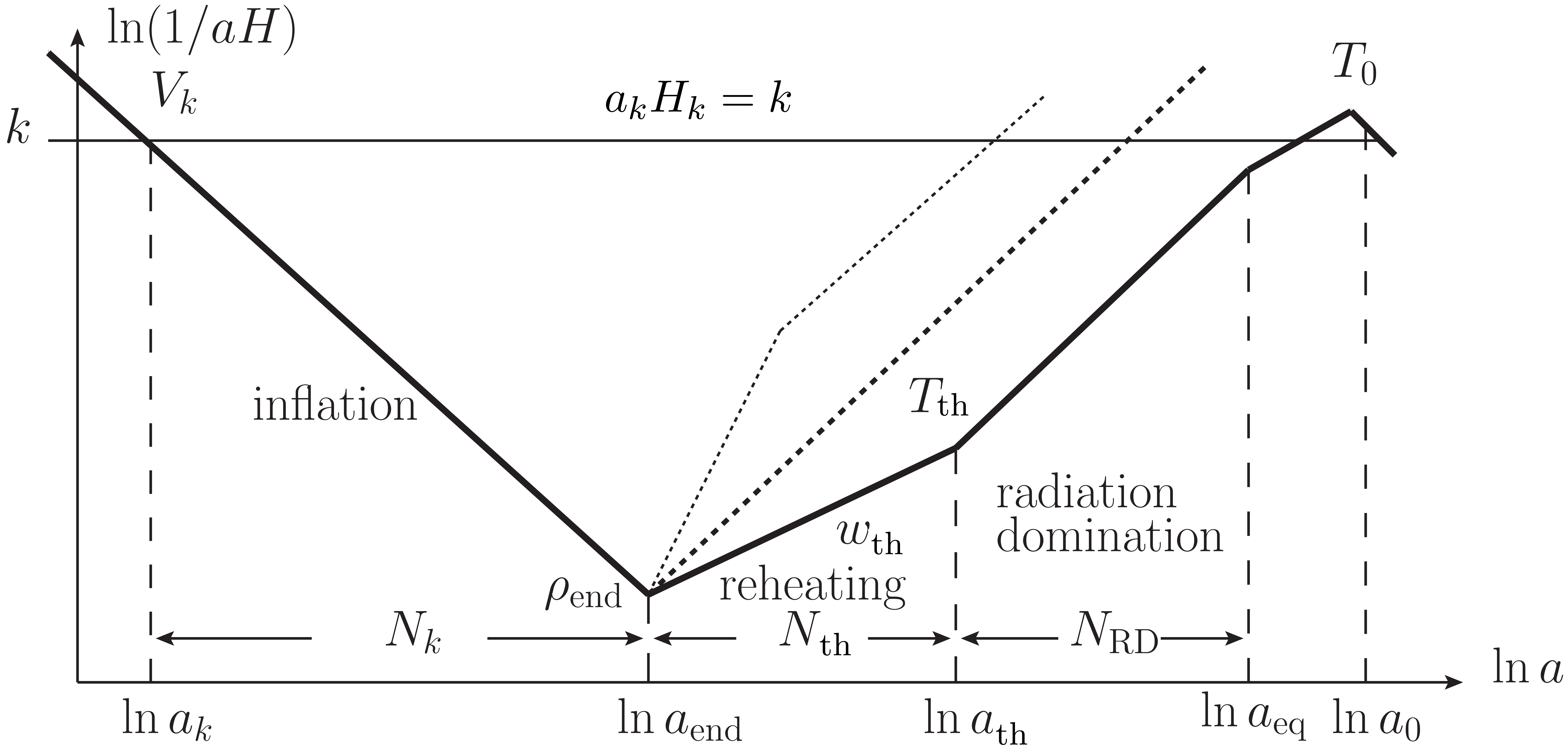

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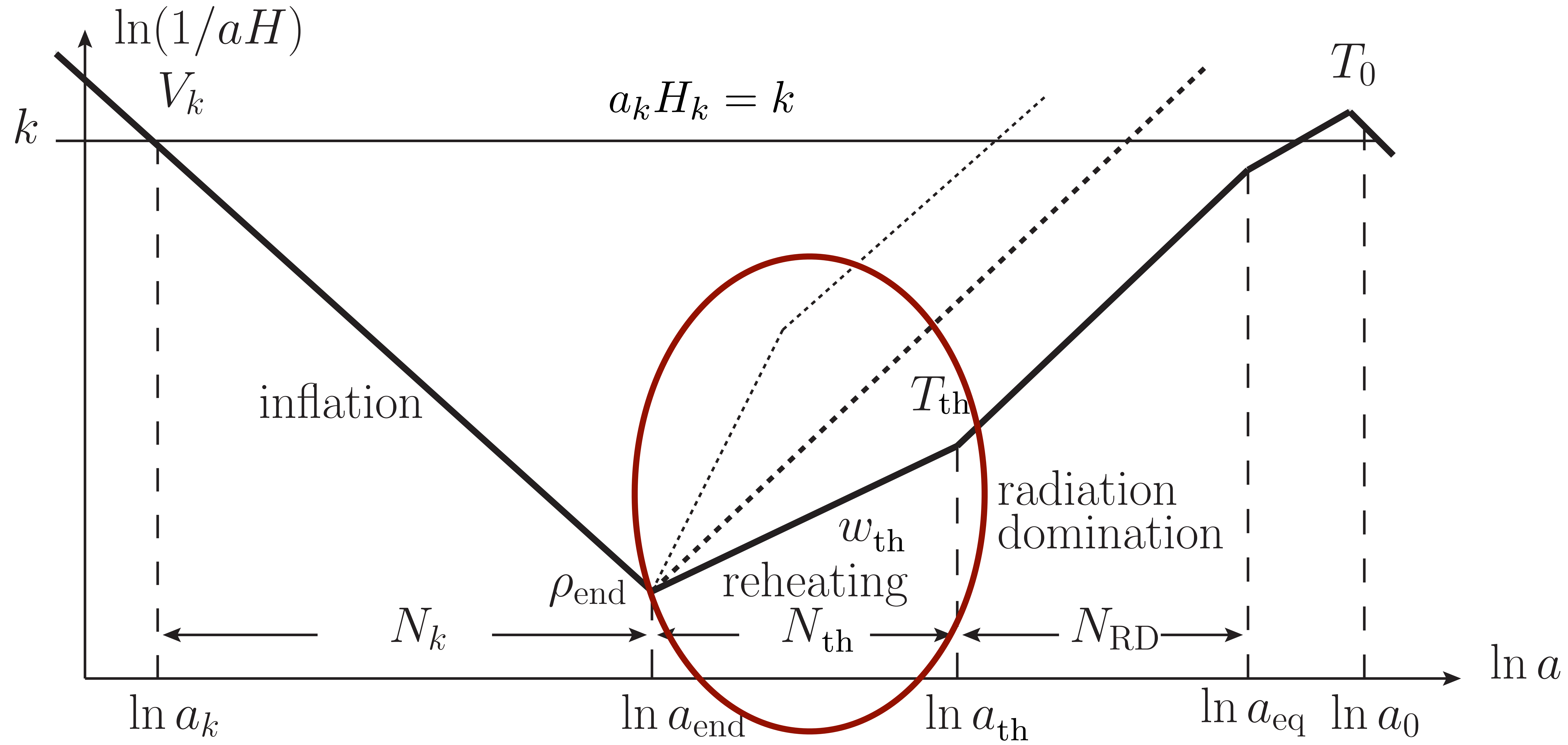
PRL **113,** 041302 (2014) Liang Dai et al.

REHEATING CONSTRAINTS TO INFLATIONARY MODELS:

- The energy density at the end of inflation can relate to that of at the end of reheating if the effective EoS is assumed to be constant.



$$\boxed{\frac{\rho_{\rm end}}{\rho_{\rm th}} = \left(\frac{a_{\rm end}}{a_{\rm th}}\right)^{-3(1+\omega_{\rm th})}}$$

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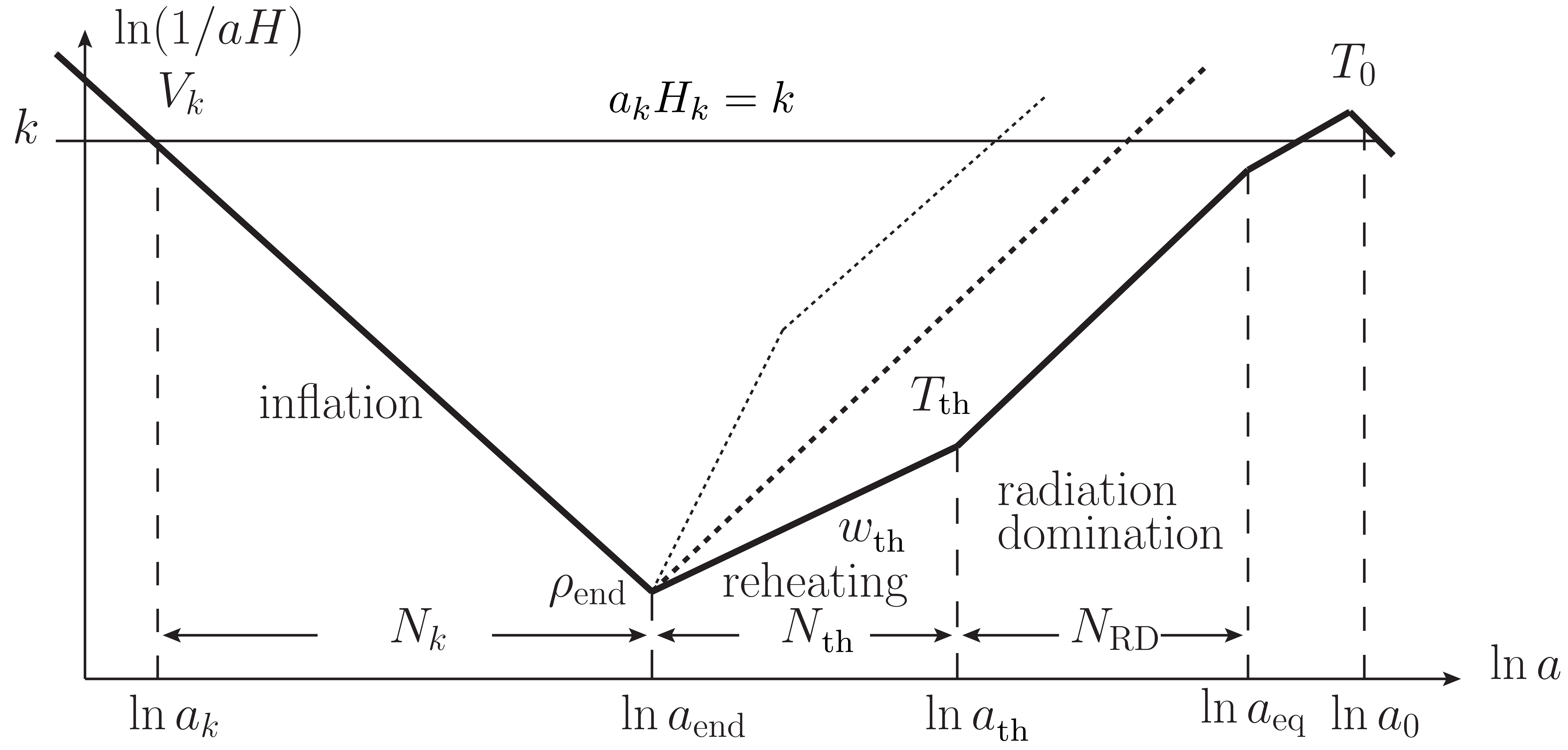


$$\rho_{\rm end} = (1+\lambda_{\rm eff})V_{\rm end} \qquad \boxed{\frac{\rho_{\rm end}}{\rho_{\rm th}} = \left(\frac{a_{\rm end}}{a_{\rm th}}\right)^{-3(1+\omega_{\rm th})}} \qquad \rho_{\rm th} = \frac{\pi^2}{30} g_{\rm th} T_{\rm th}^4$$

REHEATING CONSTRAINTS TO INFLATIONARY MODELS:

$$N_{\rm th} = \frac{1}{3(1+\omega_{\rm th})} \ln\left(\frac{\rho_{\rm end}}{\rho_{\rm th}}\right) \quad \Rightarrow \quad N_{\rm th} = \frac{1}{3(1+\omega_{\rm th})} \ln\left[(1+\lambda_{\rm eff})\frac{V_{\rm end}}{\rho_{\rm th}}\right]$$

- Assuming the entropy is preserved, no immense entropy production take place after the end of reheating.

$$g_{s\ \rm th} T_{\rm th}^3 = \left(\frac{a_0}{a_{\rm th}}\right)^3 \left(2\,T_0^3 + 6\cdot\frac{7}{8}T_{\nu 0}^3\right) \qquad T_{\nu 0} = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_0$$

$$T_{\rm th} = T_0 \left(\frac{43}{11 g_{\rm s,th}}\right)^{\frac{1}{3}} \frac{a_0}{a_{\rm eq}}\frac{a_{\rm eq}}{a_{\rm th}} \quad \Rightarrow \quad T_{\rm th} = \left(\frac{43}{11 g_{\rm s,th}}\right)^{\frac{1}{3}} \left(\frac{a_0 T_0}{k}\right) H_k e^{-N_k} e^{-N_{\rm th}}$$

$$\boxed{\begin{aligned} N_{\rm th} &= \frac{4}{3\omega_{\rm th}-1}\left[\ln\left(\frac{k}{a_0 T_0}\right) + \frac{1}{3}\ln\left(\frac{11 g_{\rm s,th}}{43}\right) + \frac{1}{4}\ln\left(\frac{30(1+\lambda_{\rm eff})}{\pi^2 g_{\rm th}}\right) + \ln\left(\frac{V_{\rm end}^{\frac{1}{4}}}{H_k}\right) + N_k\right], \\ T_{\rm th} &= \left[\frac{30(1+\lambda_{\rm eff})}{\pi^2 g_{\rm th}} V_{\rm end}\right]^{\frac{1}{4}} \exp\left[-\frac{3}{4}(1+\omega_{\rm th})N_{\rm th}\right]. \end{aligned}}$$

We set $g_{\rm s,th} = g_{\rm th}$, $1{\rm Mpc} = 3.0857\times 10^{24}{\rm cm}$, $T_0 = 2.725{\rm K}$ where $1{\rm K} = (0.23{\rm cm})^{-1}$ and $a_0 = 1$.

$$N_{\rm th} = \frac{4}{1-3\omega_{\rm th}}\left[60.77 - \frac{1}{4}\ln\left(\frac{3(1+\lambda_{\rm eff})}{10\pi^2}\right) - \ln\left(\frac{V_{\rm end}^{\frac{1}{4}}}{H_k}\right) - N_k\right]$$

REHEATING AFTER INFLATION:

IN MINIMAL (OR STANDARD) MODEL OF INFLATION

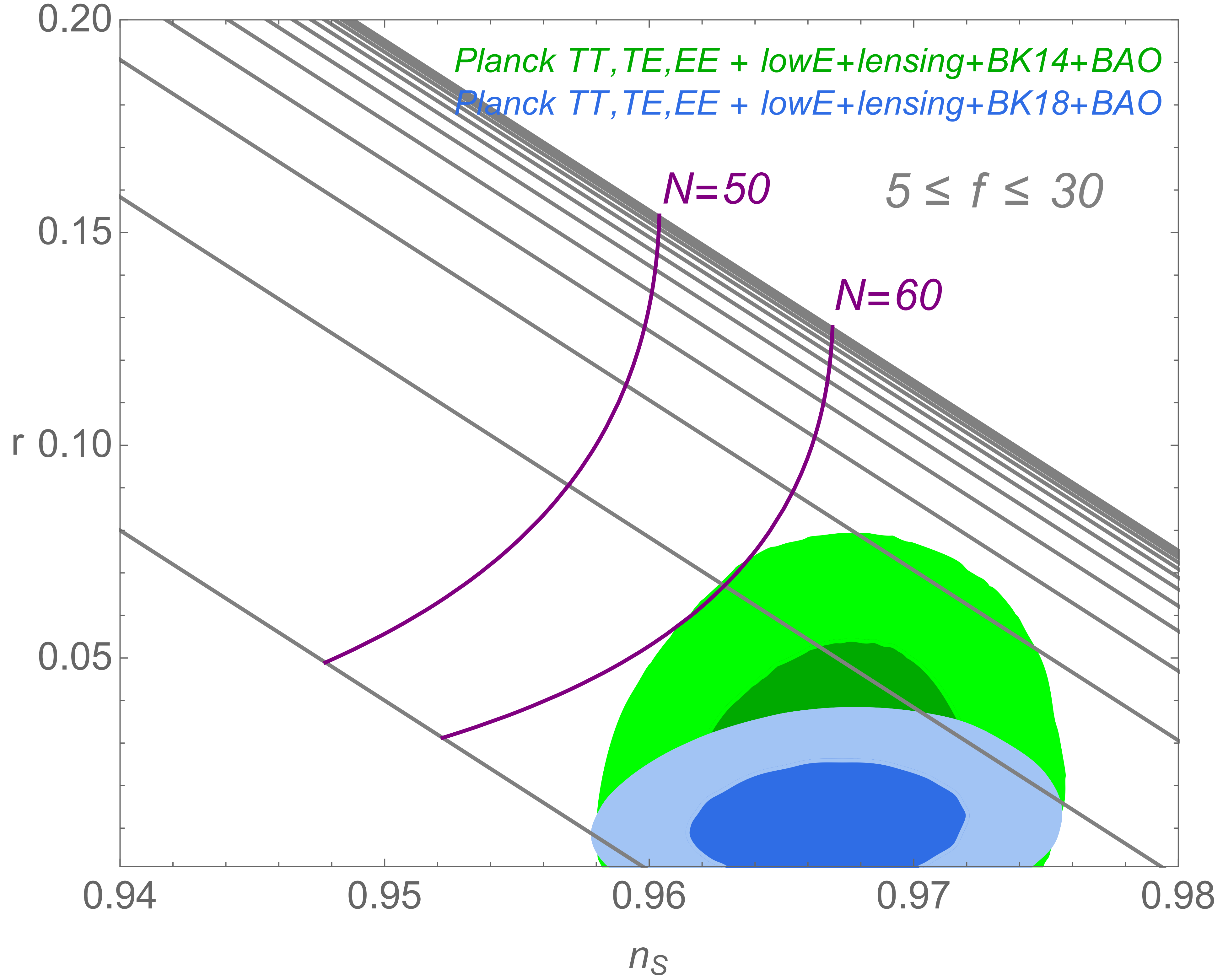
$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right],$$

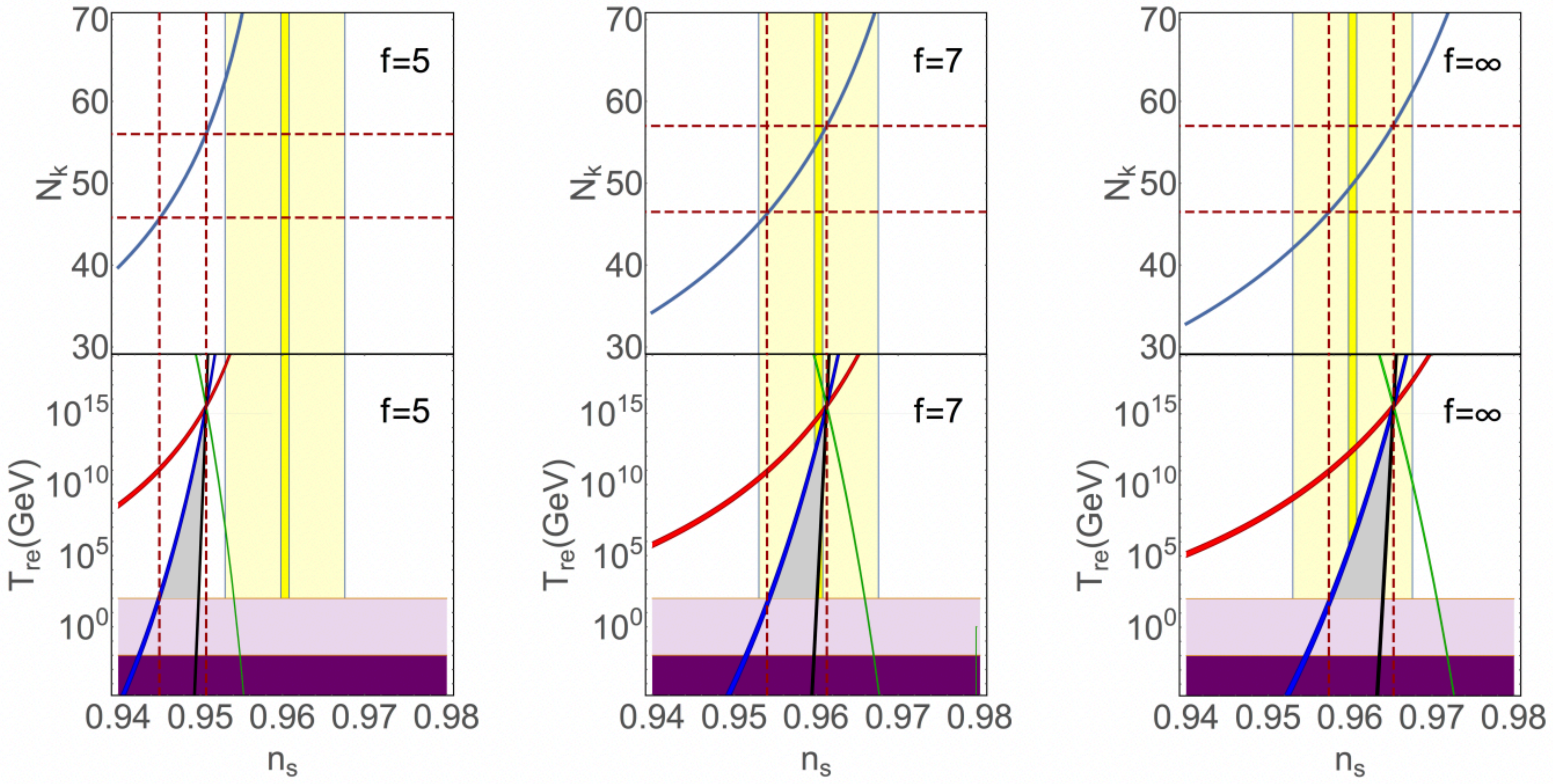
$$\textbf{Natural inflation: } V(\phi) = \frac{2\Lambda^4}{2^m}\left[1-\cos\left(\frac{\phi}{f}\right)\right]^m .$$

$$n_s - 1 = -\frac{m}{f^2} - \frac{2m(m+1)\,e^{-x}}{2f^2(1-e^{-x})+m} \qquad r = 8m^2\frac{e^{-x}}{2f^2(1-e^{-x})+m},$$

where $x = \frac{mN_k}{f^2}$,

Julian B. Munoz et al. PRD91 (2015) 043521





$$N_{\rm th} = \frac{4}{3\omega_{\rm th}-1}\left[\ln\left(\frac{k}{a_0T_0}\right)+\frac{1}{3}\ln\left(\frac{11g_{\rm s,th}}{43}\right)+\frac{1}{4}\ln\left(\frac{30(1+\lambda_{\rm eff})}{\pi^2 g_{\rm th}}\right)+\ln\left(\frac{V_{\rm end}^{\frac{1}{4}}}{H_k}\right)+N_k\right],$$

$$T_{\rm th} = \left[\frac{30(1+\lambda_{\rm eff})}{\pi^2 g_{\rm th}}V_{\rm end}\right]^{\frac{1}{4}}\exp\left[-\frac{3}{4}(1+\omega_{\rm th})N_{\rm th}\right].$$

REHEATING AFTER INFLATION:

WITH APPLICATION TO MODIFIED GRAVITY

The Horndeski theory, which is equivalent to the generalized Galileon theory!

Generalized Galileon Theory: $S = \int d^4x\sqrt{-g}\left(L_2 + L_3 + L_4 + L_5\right)$

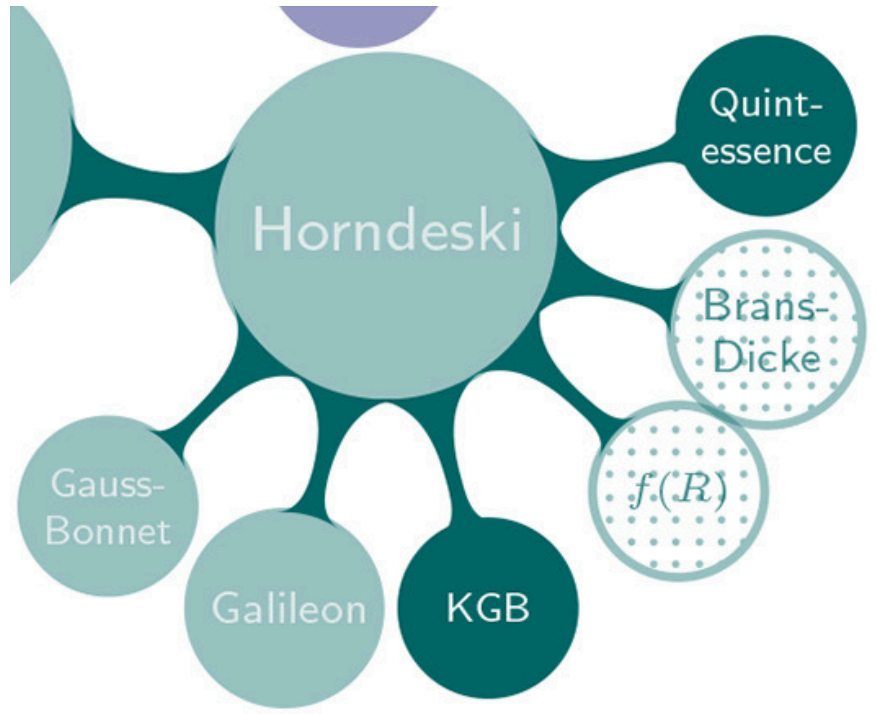
[G. Horndeski, "Second order scalar-tensor field equations in a 4D spacetime"];
C. Deffayet, X. Gao, D. A. Steer and G. Zahariade, PRD 84, 064039 (2011);
T. Kobayashi, M. Yamaguchi and J. Yokoyama, PTP 126, 511 (2011);
X. Gao, T. Kobayashi, M. Shiraishi, M. Yamaguchi, J. Yokoyama and S. Yokoyama, PTEP 2013, 053E03 (2013);

$L_2 = G_2(\phi, X)$ where $X = -\nabla_\mu\phi\nabla^\mu\phi/2$

$L_3 = G_3(\phi, X)\Box\phi$

$$L_4 = G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - \left(\nabla_\mu\nabla_\nu\phi\right)\left(\nabla^\mu\nabla^\nu\phi\right)\right]$$

$$L_5 = G_5(\phi, X)G_{\mu\nu}\left(\nabla^\mu\nabla^\nu\phi\right) - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3\Box\phi\left(\nabla_\mu\nabla_\nu\phi\right)\left(\nabla^\mu\nabla^\nu\phi\right) + 2\left(\nabla^\mu\nabla_\alpha\phi\right)\left(\nabla^\alpha\nabla_\beta\phi\right)\left(\nabla^\beta\nabla_\mu\phi\right)\right]$$



by choosing a certain combinations of $G_i(\phi, X)$ functions, one can construct a broad spectrum of cosmological models describing cosmic inflation (and dark energy).

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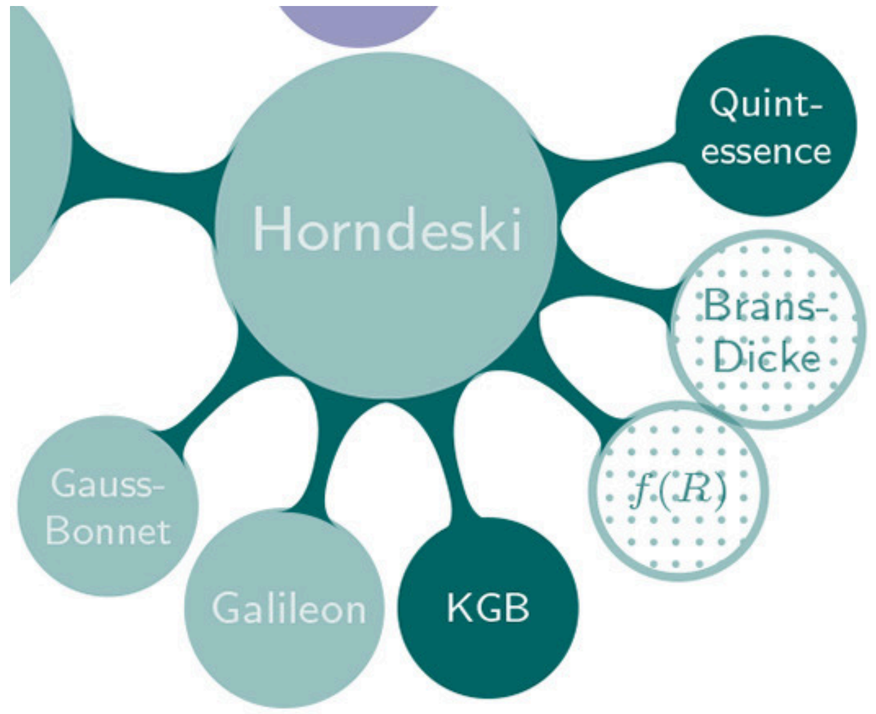


by choosing a certain combinations of $G_i(\phi, X)$ functions, one can construct a broad spectrum of cosmological models describing cosmic inflation (and dark energy).

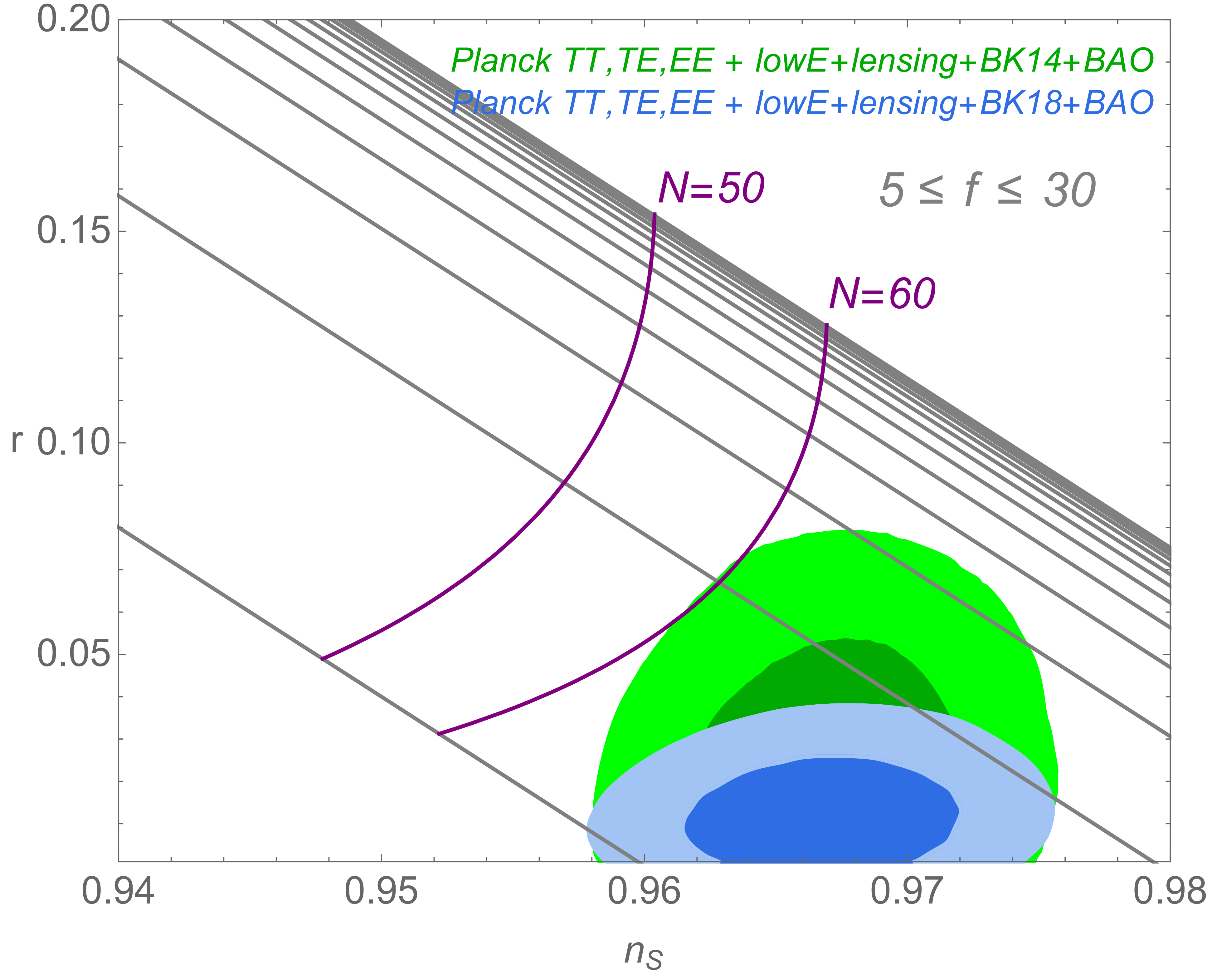
$$G_2(\phi, X) = X - V(\phi)\,,\; G_3(\phi, X) = \frac{\alpha}{M^3}\xi(\phi)X\,,\; G_4 = \frac{M_{pl}^2}{2}\,,\; G_5(\phi) = \frac{\beta}{2M^2}\phi$$

G. T, *Eur.Phys.J.C* 79 (2019) 11, 920, Chen-Hsu Chien, Seoktae Koh, G. T, *Eur.Phys.J.C* 82 (2022) 3, 268

Natural inflation: $V(\phi) = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]$

$$n_S = 1 - \frac{2[2-\cos(\phi/f)]}{\Delta[1+\cos(\phi/f)]^2}, \qquad r = \frac{8\left[1-\cos(\phi/f)\right]}{\Delta[1+\cos[\phi/f]]^2}.$$

without $\alpha-$ and $\beta-$ terms



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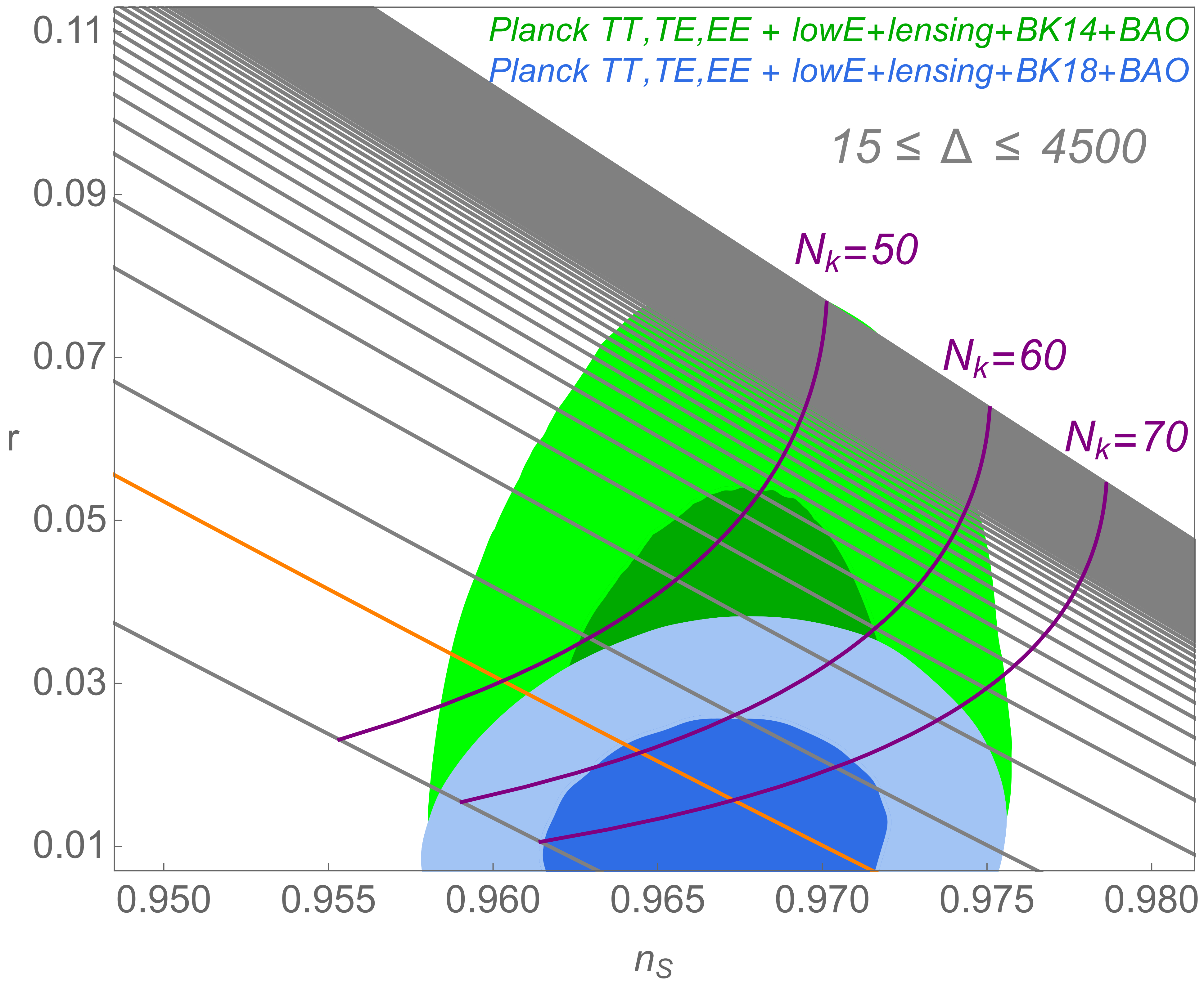

The number of e-folds:

$$\boxed{N_k = \int_{\phi}^{\phi_e} \frac{H}{\dot\phi} d\phi' \simeq \frac{1}{M_p^2} \int_{\phi_e}^{\phi} \frac{V}{V_{,\phi}} (1 + \mathscr{A}) d\phi',}$$

$$\cos\left(\frac{\phi}{f}\right) = 1 + 2\mathscr{W}\left(-e^{\frac{1}{2}\left(\mathscr{F}(\phi_e) - \frac{N_k+\Delta}{\Delta}\right)}\right) \quad \text{and} \quad \boxed{\Delta \equiv \beta(\gamma-1)\frac{f^2\Lambda^4}{M^2 M_p^4}.}$$

$$\mathscr{F}\left(\phi_e\right) = 2\ln\left[\frac{1+8\Delta-\sqrt{16\Delta+1}}{8\Delta}\right] - \frac{1+4\Delta-\sqrt{16\Delta+1}}{4\Delta}$$

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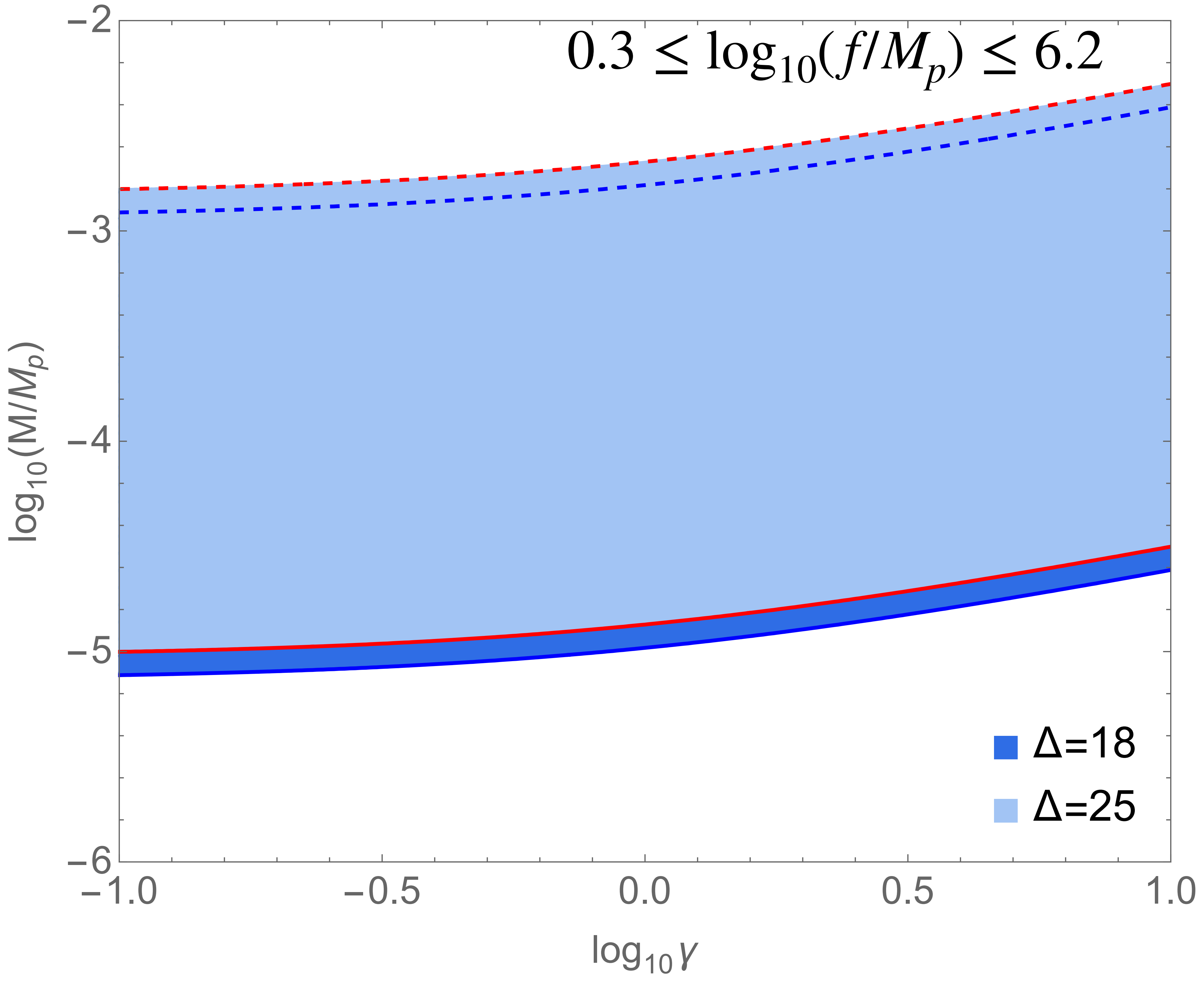
$n_S = 0.9649 \pm 0.0042\,,\ \mathscr{P}_S = 2.0989 \times 10^{-9}\,(TT, TE, EE + lowE + lensing)$



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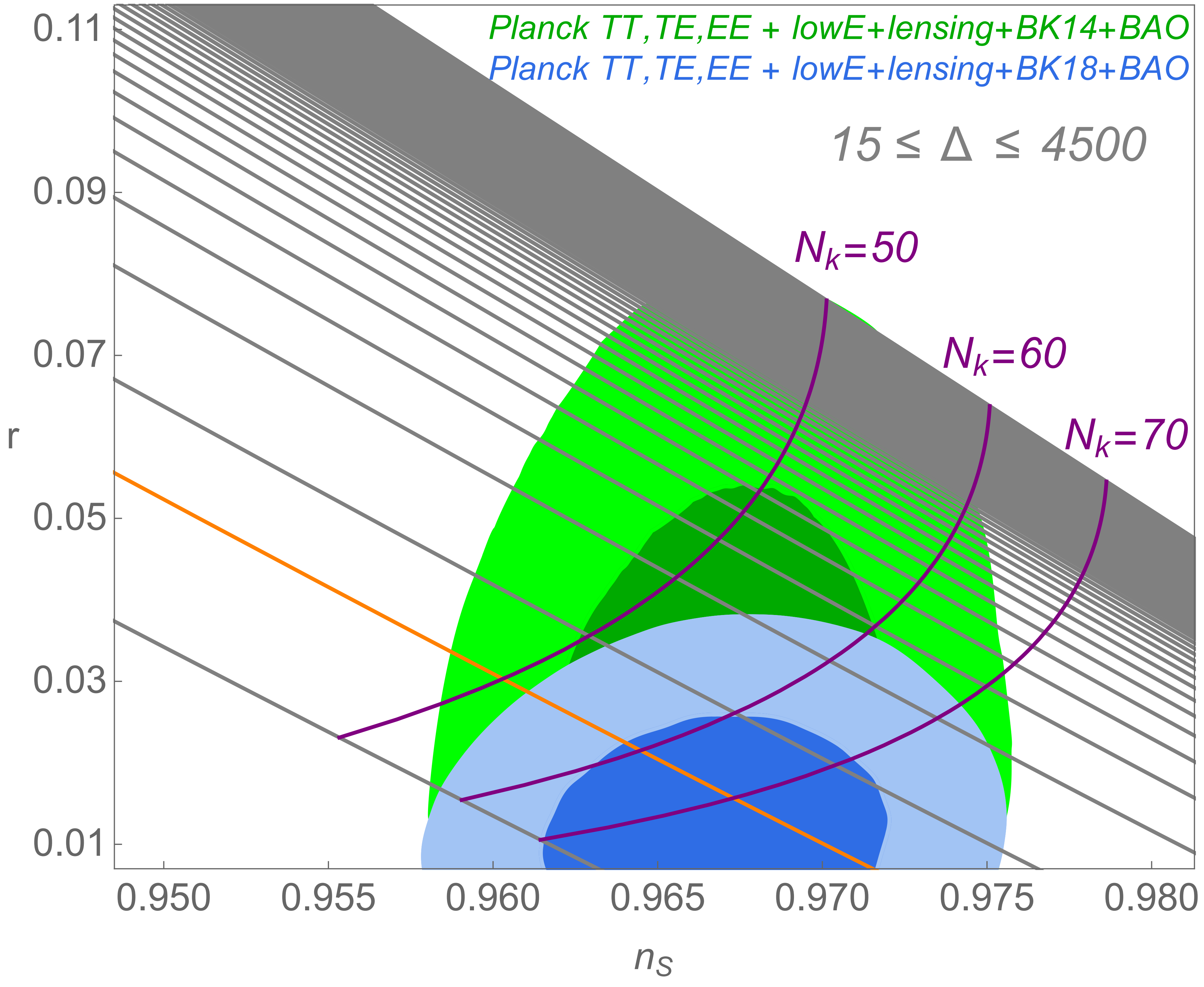

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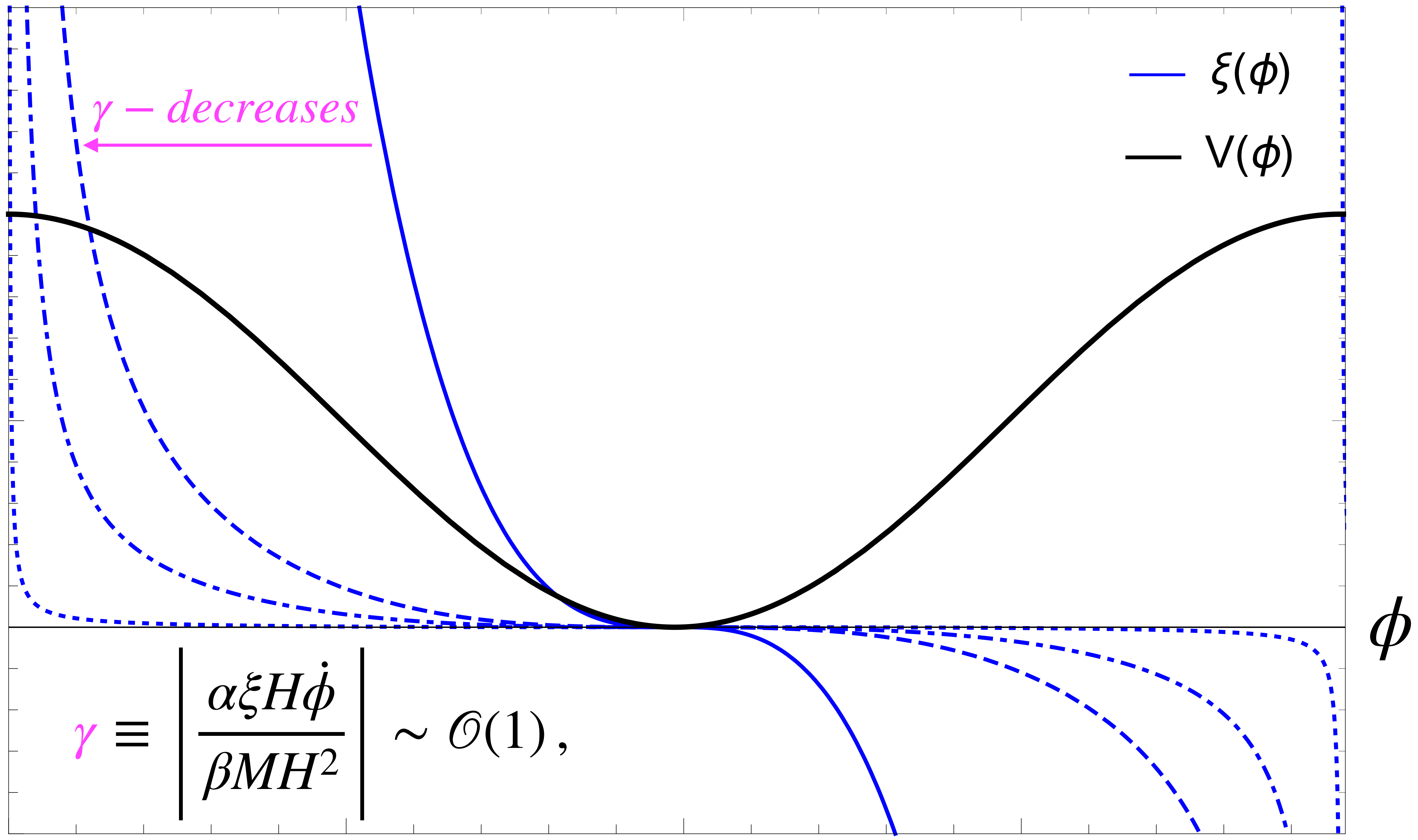
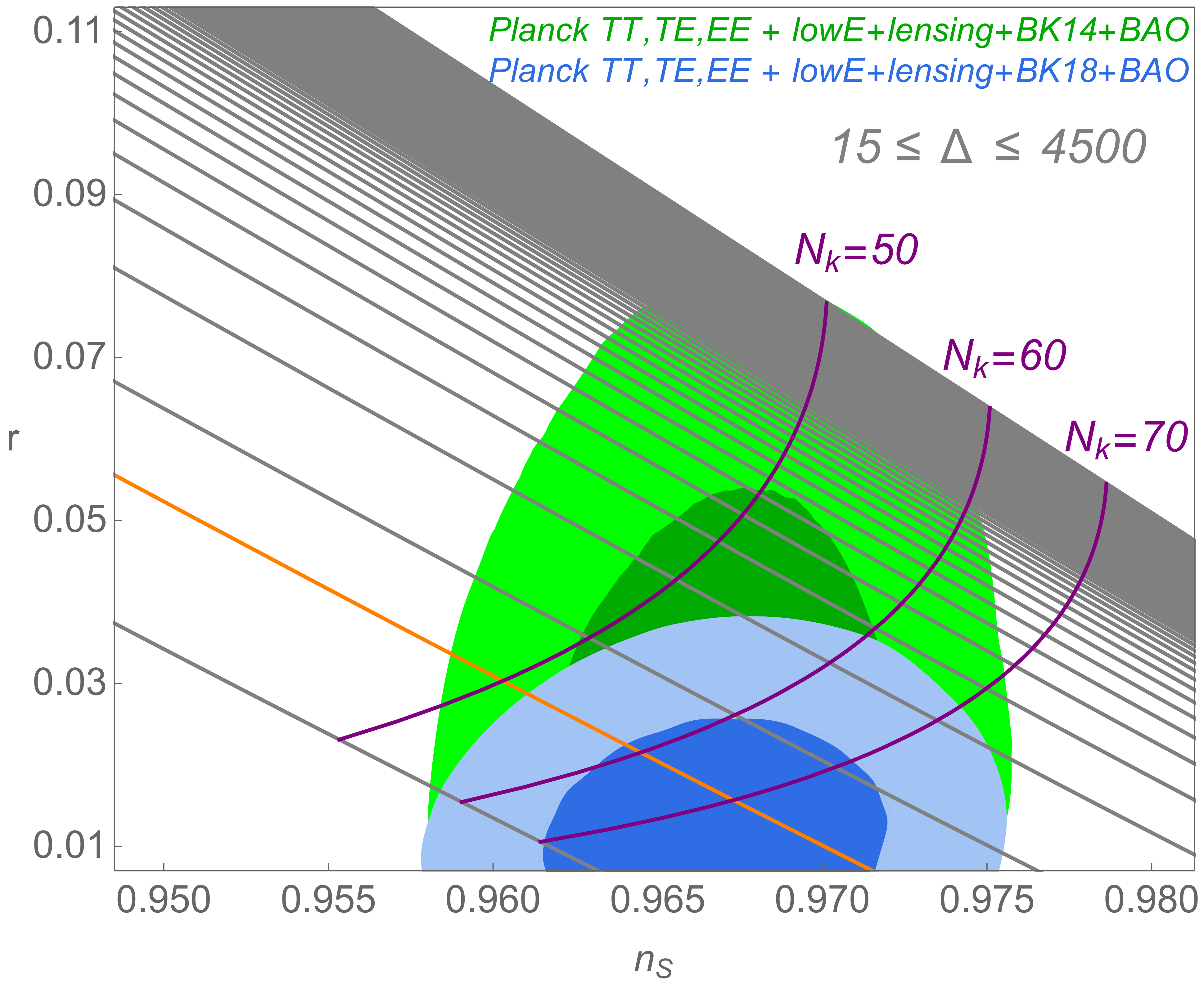
$$n_S = 1 - \frac{2[2-\cos(\phi/f)]}{\Delta[1+\cos(\phi/f)]^2}, \qquad r = \frac{8\,[1-\cos(\phi/f)]}{\Delta[1+\cos[\phi/f]]^2}.$$



with $\alpha-$ and $\beta-$ terms



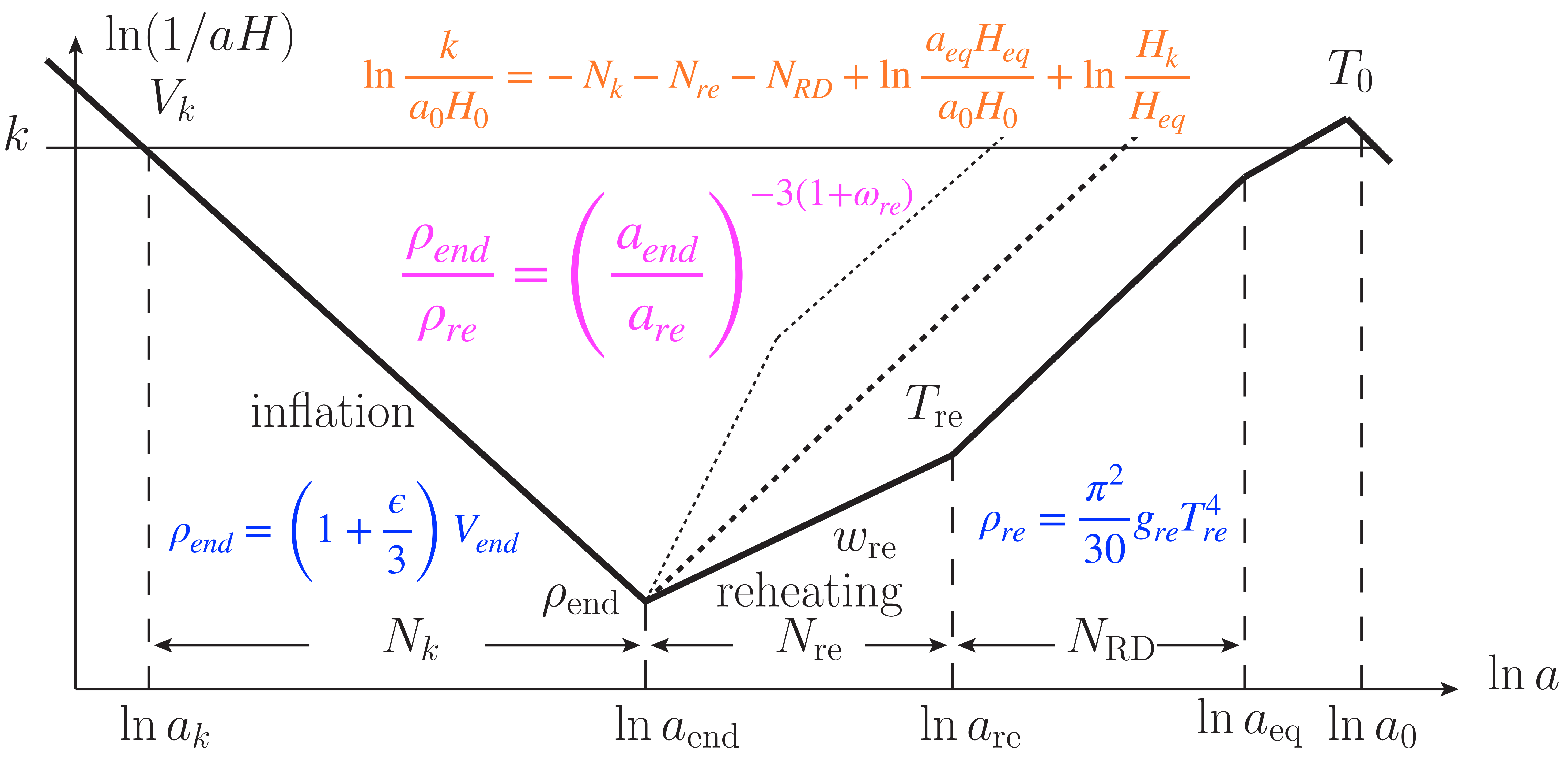
The number of e-folds:

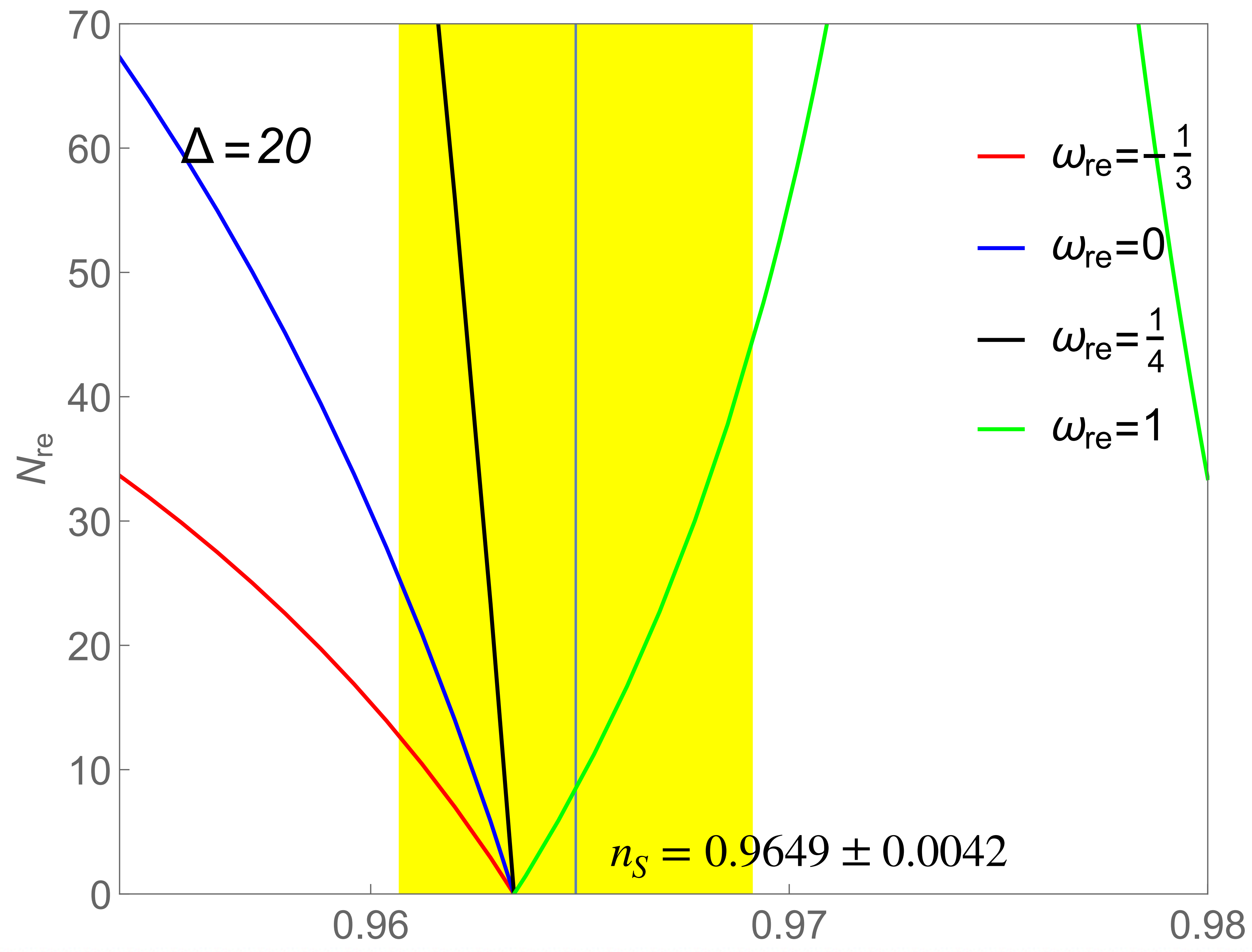
$$\boxed{N_k = \int_{\phi}^{\phi_e} \frac{H}{\dot{\phi}} d\phi' \simeq \frac{1}{M_p^2} \int_{\phi_e}^{\phi} \frac{V}{V_{,\phi}} (1 + \mathscr{A}) d\phi',}$$

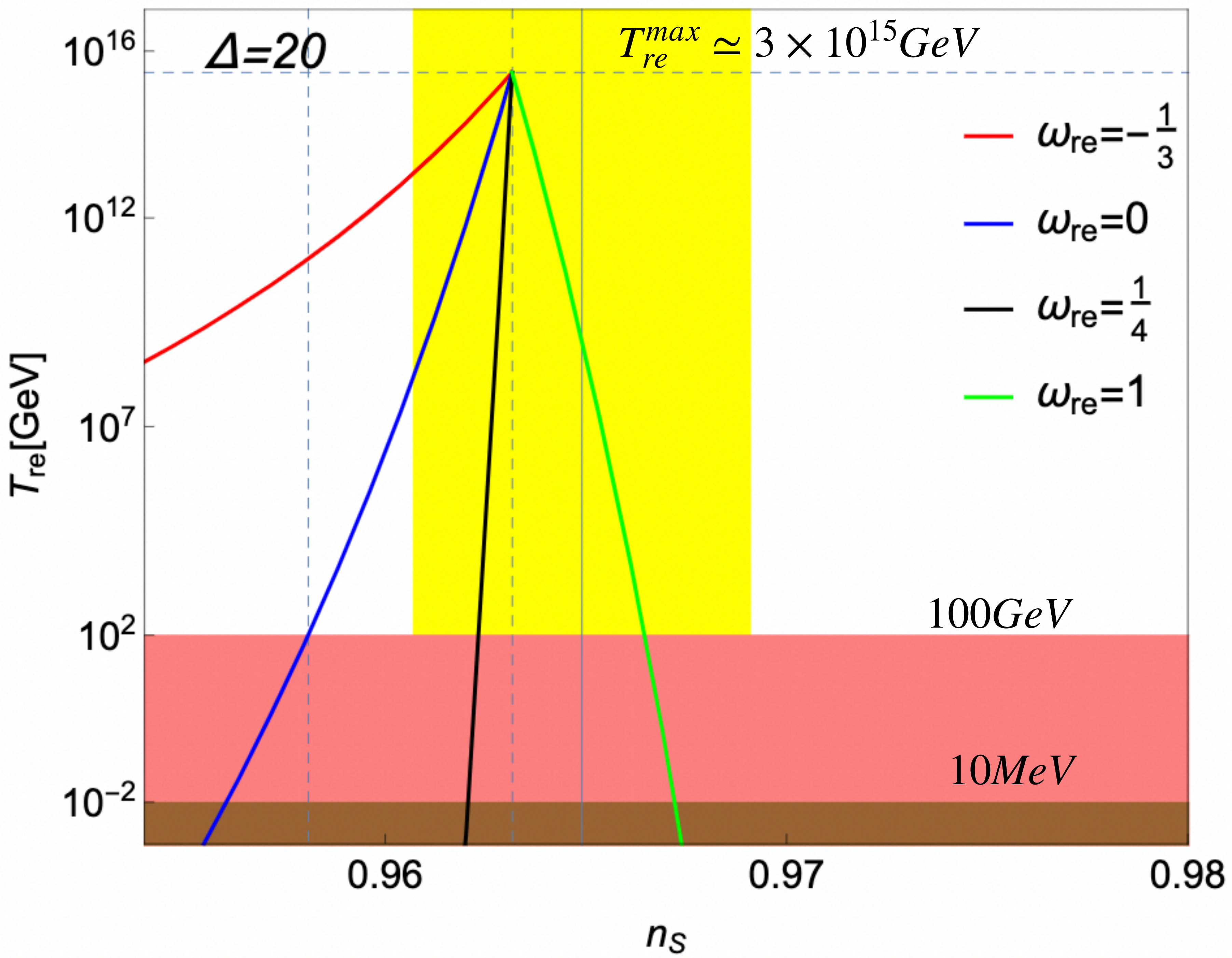
$$\cos\left(\frac{\phi}{f}\right) = 1 + 2\mathscr{W}\left(-e^{\frac{1}{2}\left(\mathscr{F}(\phi_e) - \frac{N_k + \Delta}{\Delta}\right)}\right) \quad \text{and} \quad \boxed{\Delta \equiv \beta(\gamma - 1)\frac{f^2 \Lambda^4}{M^2 M_p^4}}.$$

Calculating N_{re} and T_{re} after inflation:

If $\omega_{re} \approx$ const., the ρ_{end} at the end of inflation is related to that of reheating ρ_{re} :







Natural inflation: $V(\phi) = \Lambda^4 \left[1 - \cos\left(\phi/f\right)\right]$ **with $\alpha-$ and $\beta-$ terms:**

$$N_{re} = \frac{4}{1-3\omega_{re}} \left[-N_k - \ln \frac{k}{a_0 T_0} - \frac{1}{4} \ln \frac{30}{\pi^2 g_{re}} - \frac{1}{3} \ln \frac{11 g_{s,re}}{43} - \frac{1}{4} \ln V(\phi_e) + \frac{1}{2} \ln \left(2\pi^2 M_p^2 r \mathscr{P}_S\right) \right]$$

$$T_{re}^4 = \left(\frac{30}{\pi^2 g_{re}}\right) V(\phi_e) e^{-3(1+\omega_{re})N_{re}} .$$

$$T_{re}^4 = \left(\frac{30}{\pi^2 g_{re}}\right) V(\phi_e) e^{-3(1+\omega_{re})N_{re}}, N_{re} = \frac{4}{1-3\omega_{re}}\left[-N_k - \ln\frac{k}{a_0T_0} - \frac{1}{4}\ln\frac{30}{\pi^2 g_{re}} - \frac{1}{3}\ln\frac{11 g_{s,re}}{43} - \frac{1}{4}\ln V(\phi_e) + \frac{1}{2}\ln\left(2\pi^2 M_p^2 r \mathscr{P}_S\right)\right], n_S = 1 - \frac{2[2-\cos(\phi/f)]}{\Delta[1+\cos(\phi/f)]^2} \Longrightarrow r(n_S, T_{re})$$

The Horndeski theory and the generalized Galileon theory are equivalent!

[G. Horndeski, "Second order scalar-tensor field equations in a 4D spacetime"];
C. Deffayet, X. Gao, D. A. Steer and G. Zahariade, PRD 84, 064039 (2011);
T. Kobayashi, M. Yamaguchi and J. Yokoyama, PTP 126, 511 (2011);
X. Gao, T. Kobayashi, M. Shiraishi, M. Yamaguchi, J. Yokoyama and S. Yokoyama, PTEP 2013, 053E03 (2013);

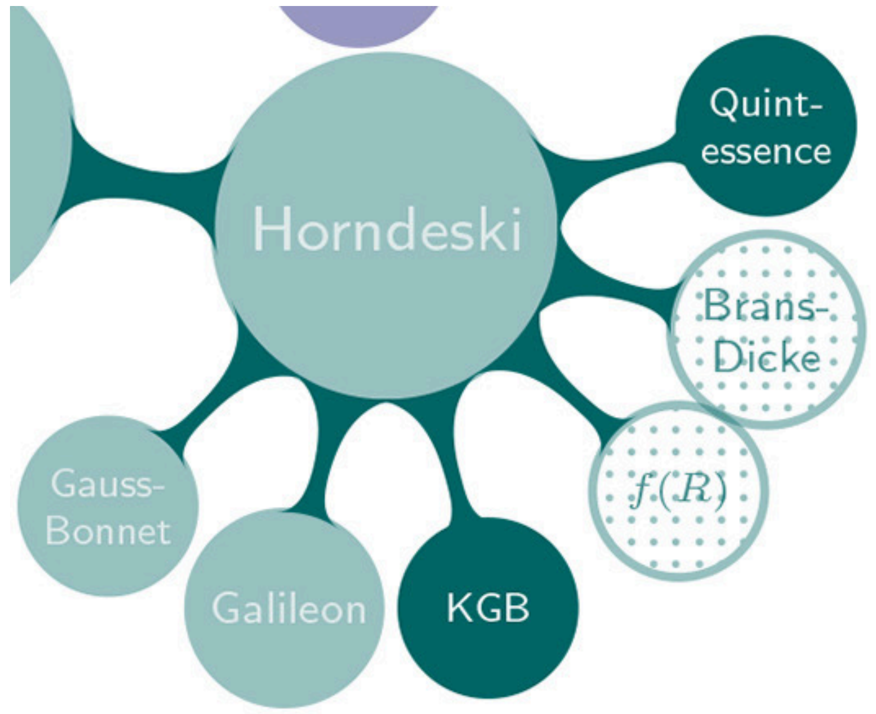
Generalized Galileon Theory: $S=\int d^4x\sqrt{-g}\,\left(L_2+L_3+L_4+L_5\right)$

$$L_2 = G_2(\phi, X)$$
$$L_3 = -G_3(\phi, X)\Box\phi$$

where $X\equiv-\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi\,,\quad \Box\phi\equiv\nabla_\mu\nabla^\mu\phi\,,\quad G_{i,X}\equiv\frac{dG_i}{dX}\,,\quad G_{\mu\nu}\equiv R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R$

$$L_4 = G_4(\phi,X)R+G_{4,X}\left[\left(\Box\phi\right)^2-\left(\nabla_\mu\nabla_\nu\phi\right)\left(\nabla^\mu\nabla^\nu\phi\right)\right]$$

$$L_5 = G_5(\phi,X)G_{\mu\nu}\left(\nabla^\mu\nabla^\nu\phi\right)-\frac{1}{6}G_{5,X}\left[(\Box\phi)^3-3\Box\phi\left(\nabla_\mu\nabla_\nu\phi\right)\left(\nabla^\mu\nabla^\nu\phi\right)+2\left(\nabla^\mu\nabla_\alpha\phi\right)\left(\nabla^\alpha\nabla_\beta\phi\right)\left(\nabla^\beta\nabla_\mu\phi\right)\right]$$



by choosing a certain combinations of $G_i(\phi, X)$ functions, one can construct a broad spectrum of cosmological models describing cosmic inflation (and dark energy).

The Horndeski theory and the generalized Galileon theory are equivalent!

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C. Deffayet, X. Gao, D. A. Steer and G. Zahariade, PRD 84, 064039 (2011);
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Generalized Galileon Theory: $S = \int d^4x\sqrt{-g}\,\left(L_2 + L_3 + L_4 + L_5\right)$

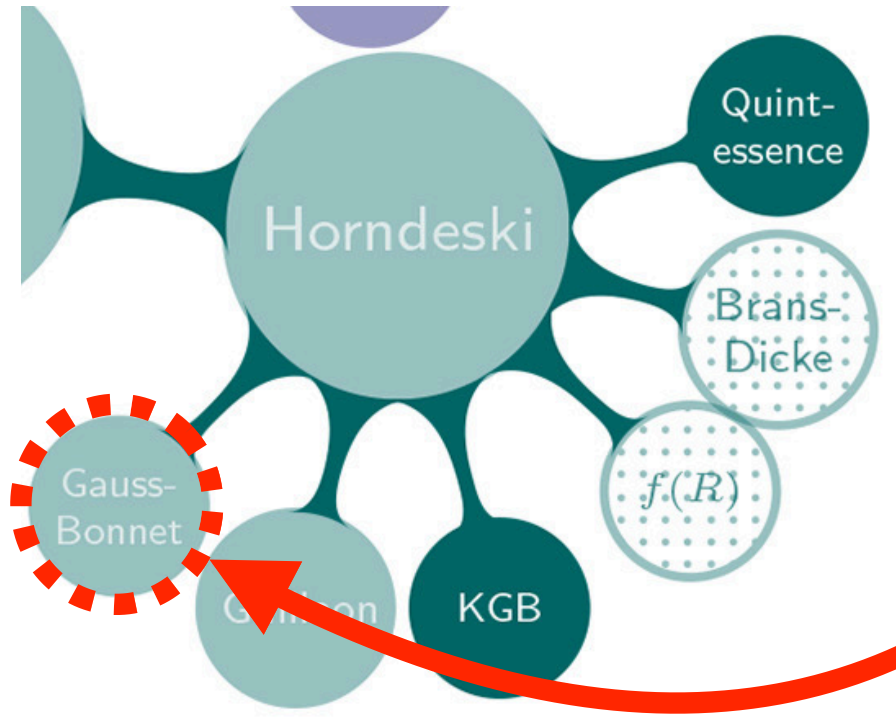
$$L_2 = G_2(\phi, X)$$

$$L_3 = -G_3(\phi, X)\Box\phi$$

where $X \equiv -\frac{1}{2}\nabla_\mu\phi\,\nabla^\mu\phi\,,\quad \Box\phi \equiv \nabla_\mu\nabla^\mu\phi\,,\quad G_{i,X} \equiv \frac{dG_i}{dX}\,,\quad G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$

$$L_4 = G_4(\phi, X)R + G_{4,X}\left[\left(\Box\phi\right)^2 - \left(\nabla_\mu\nabla_\nu\phi\right)\left(\nabla^\mu\nabla^\nu\phi\right)\right]$$

$$L_5 = G_5(\phi, X)G_{\mu\nu}\left(\nabla^\mu\nabla^\nu\phi\right) - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3\Box\phi\left(\nabla_\mu\nabla_\nu\phi\right)\left(\nabla^\mu\nabla^\nu\phi\right) + 2\left(\nabla^\mu\nabla_\alpha\phi\right)\left(\nabla^\alpha\nabla_\beta\phi\right)\left(\nabla^\beta\nabla_\mu\phi\right)\right]$$



The Gauss-Bonnet term:

$$\xi(\phi)\left(R^2 + 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)$$

$\Longleftrightarrow$

The choice:

$$G_2(\phi, X) = 8\xi^{(4)}X^2\left(3 - \ln X\right),$$

$$G_3(\phi, X) = 4\xi^{(3)}X\left(7 - \ln X\right),$$

$$G_4(\phi, X) = 4\xi^{(2)}X\left(2 - \ln X\right),$$

$$G_5(\phi, X) = -4\xi^{(1)}\ln X.$$

- ***In light of both the current and future observations, extended models of inflation seem to be more promising!***

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right],$$

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \boxed{- \frac{1}{2} \xi(\phi) R^2_{\rm GB}} \right],$$

$$R^2_{\rm GB} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

- ***In the early universe, approaching the Planck era, it is quite natural to consider corrections like this.***

We are interested in understanding the effects of this additional term
- ***during inflation and reheating***
- ***its contribution to the Primordial GW spectra***

S. Koh, B. H. Lee, W. Lee and GT, PRD 90, 063527 (2014)

- **Here, it is important to have:** $\xi(\phi) \neq const\,.$

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)-\frac{1}{2}\xi(\phi)R^2_{\rm GB}\right],$$

- **because of the background EoM in flat FRW universe:** $ds^2=-dt^2+a^2\left(dr^2+r^2d\Omega^2\right),$

$$H^2=\frac{\kappa^2}{3}\left(\frac{1}{2}\dot\phi^2+V+12\dot\xi H^3\right),$$
$$\dot H=-\frac{\kappa^2}{2}\left[\dot\phi^2-4\ddot\xi H^2-4\dot\xi H\left(2\dot H-H^2\right)\right],$$
$$\ddot\phi+3H\dot\phi+V_\phi+12\xi_\phi H^2\left(\dot H+H^2\right)=0\,,$$

- **Observable quantities are obtained as,**

$$\mathscr{P}_S(k)=\mathscr{P}_S(k_*)\left(\frac{k}{k_*}\right)^{n_S-1},\qquad \mathscr{P}_T(k)=\mathscr{P}_T(k_*)\left(\frac{k}{k_*}\right)^{n_T}$$

$$n_S-1\simeq-2\epsilon-\frac{2\epsilon(2\epsilon+\eta)-\delta_1(\delta_2-\epsilon)}{2\epsilon-\delta_1},\qquad n_T\simeq-2\epsilon\,,\qquad r\simeq 8(2\epsilon-\delta_1)\,,$$

where $\epsilon\equiv-\frac{\dot H}{H^2}\,,\quad \eta\equiv\frac{\ddot H}{H\dot H}\,,\quad \zeta\equiv\frac{\dddot H}{H^2\dot H}\,,\quad \delta_1\equiv 4\kappa^2\dot\xi H\,,\quad \delta_2\equiv\frac{\ddot\xi}{\dot\xi H}\,,\quad \delta_3=\frac{\dddot\xi}{\dot\xi H^2}\,.$

S. Koh, B. H. Lee, W. Lee and GT, PRD 90, 063527 (2014)

- **Here, it is important to have:** $\xi(\phi) \neq const\,.$

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{2}\xi(\phi)R^2_{\rm GB}\right],$$

- **because of the background EoM in flat FRW universe:** $ds^2 = -dt^2 + a^2\left(dr^2 + r^2 d\Omega^2\right),$

$$H^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}\dot\phi^2 + V + 12\dot\xi H^3\right),$$

$$\dot H = -\frac{\kappa^2}{2}\left[\dot\phi^2 - 4\ddot\xi H^2 - 4\dot\xi H\left(2\dot H - H^2\right)\right],$$

$$\ddot\phi + 3H\dot\phi + V_\phi + 12\xi_\phi H^2\left(\dot H + H^2\right) = 0,$$

- **Observable quantities are obtained as,**

$$\mathscr{P}_S(k) = \mathscr{P}_S(k_*)\left(\frac{k}{k_*}\right)^{n_S-1}, \qquad \mathscr{P}_T(k) = \mathscr{P}_T(k_*)\left(\frac{k}{k_*}\right)^{n_T}$$

$$n_S - 1 \simeq -2\epsilon - \frac{2\epsilon(2\epsilon+\eta) - \delta_1(\delta_2 - \epsilon\ \text{[illegible]}}{2\epsilon - \delta_1} \qquad n_T = -2\epsilon \qquad r \simeq 8(2\epsilon - \delta_1),$$

where $\epsilon \equiv -\frac{\dot H}{H^2}$, [illegible] $\frac{\text{[illegible]}}{\text{[illegible]}^2 H}$, $\delta_1 \equiv 4\kappa^2\dot\xi H$, $\delta_2 \equiv \frac{\ddot\xi}{\dot\xi H}$, $\delta_3 = \frac{\dddot\xi}{\dot\xi H^2}$.

S. K[illegible] H. Lee, W. Lee and GT, PRD 90, 063527 (2014)

n_T can be either negative or positive. If it is positive (negative), then the tensor power spectrum is called "blue-tilted" ("red-tilted").

Class-I: $n_T < 0, \longrightarrow \epsilon > 0,$

$$n_T = -2\epsilon$$

Class-II: $n_T > 0, \longrightarrow \epsilon < 0,$

$$\begin{cases} \xi_\phi > -\frac{3}{4\kappa^4}\frac{V_\phi}{V^2}, & \text{for} \quad V_\phi > 0, \\ \xi_\phi < -\frac{3}{4\kappa^4}\frac{V_\phi}{V^2}, & \text{for} \quad V_\phi < 0. \end{cases}$$

$$\epsilon = \frac{1}{2\kappa^2}\frac{V_\phi}{V}\left(\frac{V_\phi}{V} + \frac{4}{3}\kappa^4\xi_\phi V\right).$$

$$\begin{cases} \xi_\phi < -\frac{3}{4\kappa^4}\frac{V_\phi}{V^2}, & \text{for} \quad V_\phi > 0, \\ \xi_\phi > -\frac{3}{4\kappa^4}\frac{V_\phi}{V^2}, & \text{for} \quad V_\phi < 0. \end{cases}$$

Model-I: $V(\phi) = \frac{V_0}{\kappa^4}(\kappa\phi)^n, \quad \xi(\phi) = \xi_0(\kappa\phi)^{-n},$

Model-II: $V(\phi) = \frac{1}{\kappa^4}\left[\tanh(\kappa\phi) + \sqrt{\mu}\,\mathrm{sech}(\kappa\phi)\right]^2, \quad \xi(\phi) = \frac{3\left[\sinh^2(\kappa\phi) - \frac{1}{\sqrt{\mu}}\sinh(\kappa\phi)\right]}{4\left[\sqrt{\mu} + \sinh(\kappa\phi)\right]^2},$

S. Koh, B. H. Lee, W. Lee and GT, PRD 90, 063527 (2014)

Model-I: $$V(\phi) = \frac{V_0}{\kappa^4}(\kappa\phi)^n\,, \quad \xi(\phi) = \xi_0(\kappa\phi)^{-n}\,,$$

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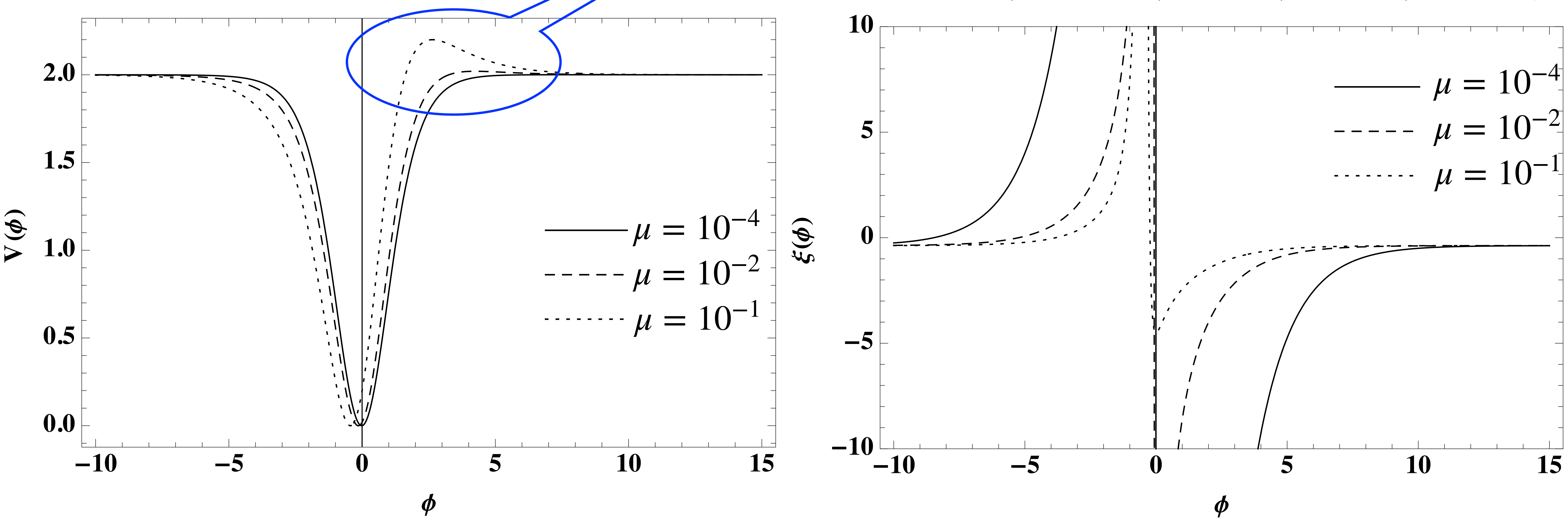
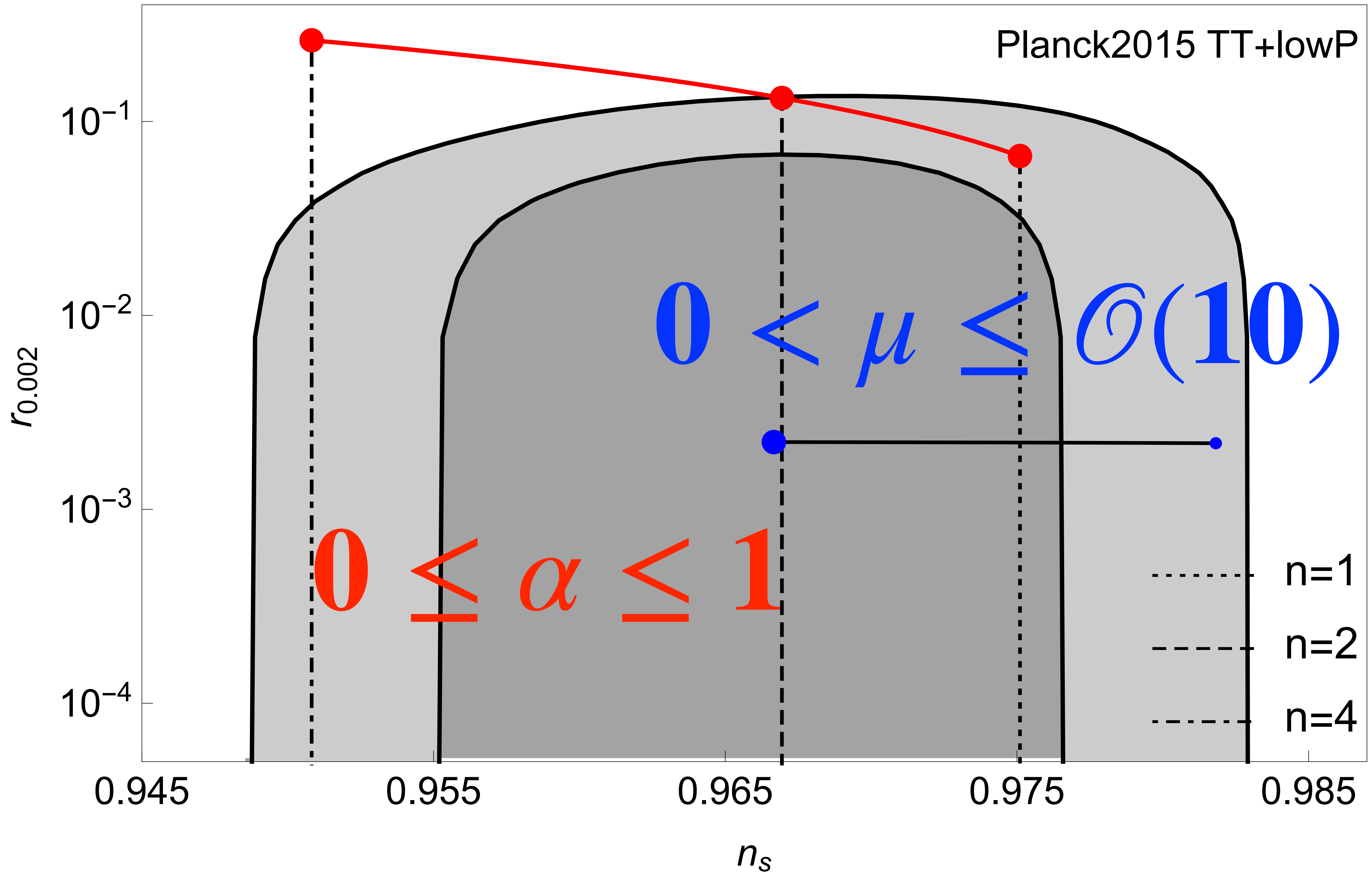
S. Koh, B. H. Lee, and GT, PRD 95, 123509 (2017)



Model-I: $V(\phi) = \frac{V_0}{\kappa^4}(\kappa\phi)^n\,, \quad \xi(\phi) = \xi_0(\kappa\phi)^{-n}\,, \quad \Longrightarrow \quad \alpha \equiv \frac{4}{3}V_0\xi_0$

Model-II: $V(\phi) = \frac{1}{\kappa^4}\left[\tanh(\kappa\phi) + \sqrt{\mu}\,\mathrm{sech}(\kappa\phi)\right]^2\,, \quad \xi(\phi) = \frac{3\left[\sinh^2(\kappa\phi) - \frac{1}{\sqrt{\mu}}\sinh(\kappa\phi)\right]}{4\left[\sqrt{\mu} + \sinh(\kappa\phi)\right]^2}\,,$



- ***In addition,*** $n_T < 0,$ ***and*** $n_T > 0.$

- ***By assuming,***
  - ***constant equation-of-state during reheating,*** $\omega_{\text{th}} = const,$
  - ***no entropy production after the end of reheating***

we calculate the duration of reheating and the thermalization temperature at the end of reheating,

$$N_{\text{th}} = \frac{4}{3\omega_{\text{th}} - 1}\left[\ln\left(\frac{k}{a_0 T_0}\right) + \frac{1}{3}\ln\left(\frac{11 g_{*s}}{43}\right) + \frac{1}{4}\ln\left(\frac{30\lambda_{\text{end}}}{\pi^2 g_*}\right) + \frac{1}{4}\ln\left(\frac{V_{\text{end}}}{H_*^4}\right) + N_*\right].$$

$$T_{\text{th}} = \left(\frac{30\lambda_{\text{end}} V_{\text{end}}}{\pi^2 g_*}\right)^{\frac{1}{4}} e^{-\frac{3}{4}(1+\omega_{\text{th}})N_{\text{th}}}.$$ ***where*** $$\lambda_{\text{end}} = \left.\frac{6}{6 - 2\epsilon - \delta_1(5 - 2\epsilon + \delta_2)}\right|_{\phi=\phi_{\text{end}}}.$$

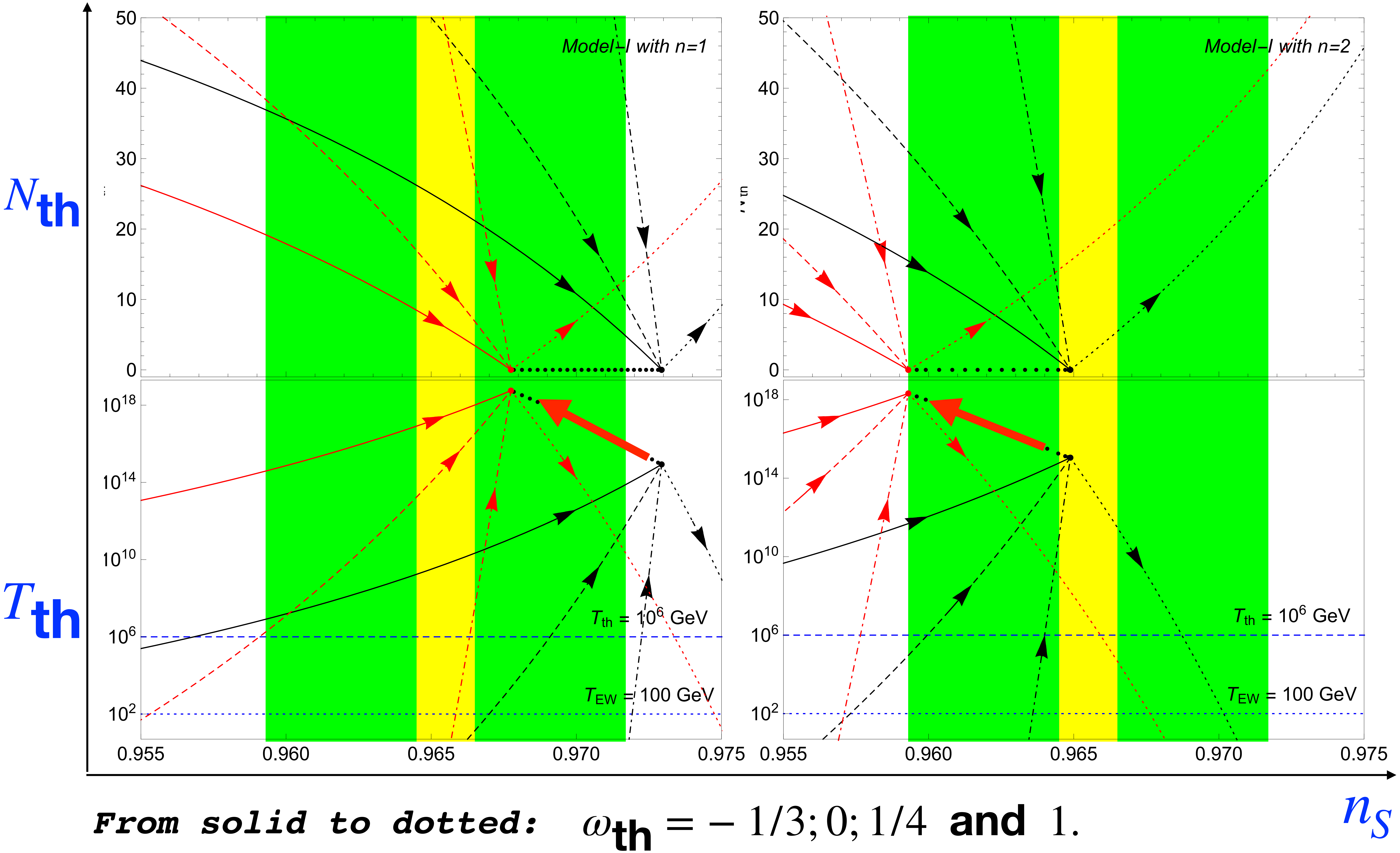
- ***In our numerical study, we consider following models:***

Model-I: $$V(\phi) = \frac{V_0}{\kappa^4}(\kappa\phi)^n, \quad \xi(\phi) = \xi_0(\kappa\phi)^{-n}, \quad \Longrightarrow \quad \alpha \equiv \frac{4}{3}V_0\xi_0$$

Model-II: $$V(\phi) = \frac{1}{\kappa^4}\left[\tanh(\kappa\phi) + \sqrt{\mu}\,\text{sech}(\kappa\phi)\right]^2, \quad \xi(\phi) = \frac{3\left[\sinh^2(\kappa\phi) - \frac{1}{\sqrt{\mu}}\sinh(\kappa\phi)\right]}{4\left[\sqrt{\mu} + \sinh(\kappa\phi)\right]^2},$$

- ***Numerical results:***

Model-I: $V(\phi) = \frac{V_0}{\kappa^4}(\kappa\phi)^n\,, \quad \xi(\phi) = \xi_0(\kappa\phi)^{-n}\,,$ ***with*** $\alpha \equiv \frac{4}{3}V_0\xi_0$



From solid to dotted: $\omega_{\rm th} = -1/3; 0; 1/4$ **and** $1.$

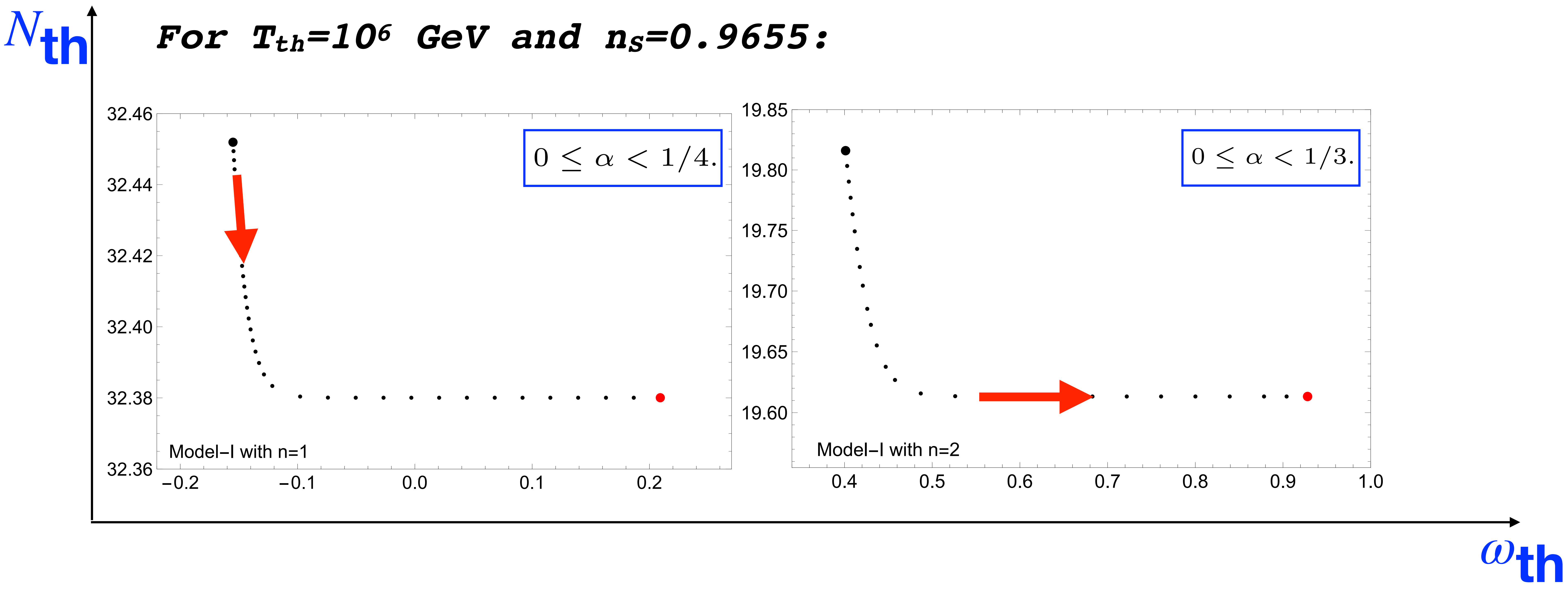
- ***Numerical results:***

Model-I: $V(\phi) = \frac{V_0}{\kappa^4}(\kappa\phi)^n\,, \quad \xi(\phi) = \xi_0(\kappa\phi)^{-n}\,,$ ***with*** $\alpha \equiv \frac{4}{3}V_0\xi_0$

$$N_{\rm th} = \frac{4}{3\omega_{\rm th}-1}\left[-60.0085 + \frac{1}{4}\ln\left(\frac{3\lambda_{\rm end}}{100\pi^2}\right) + \frac{1}{4}\ln\left(\frac{V_{\rm end}}{H_*^4}\right) + N_*\right]$$

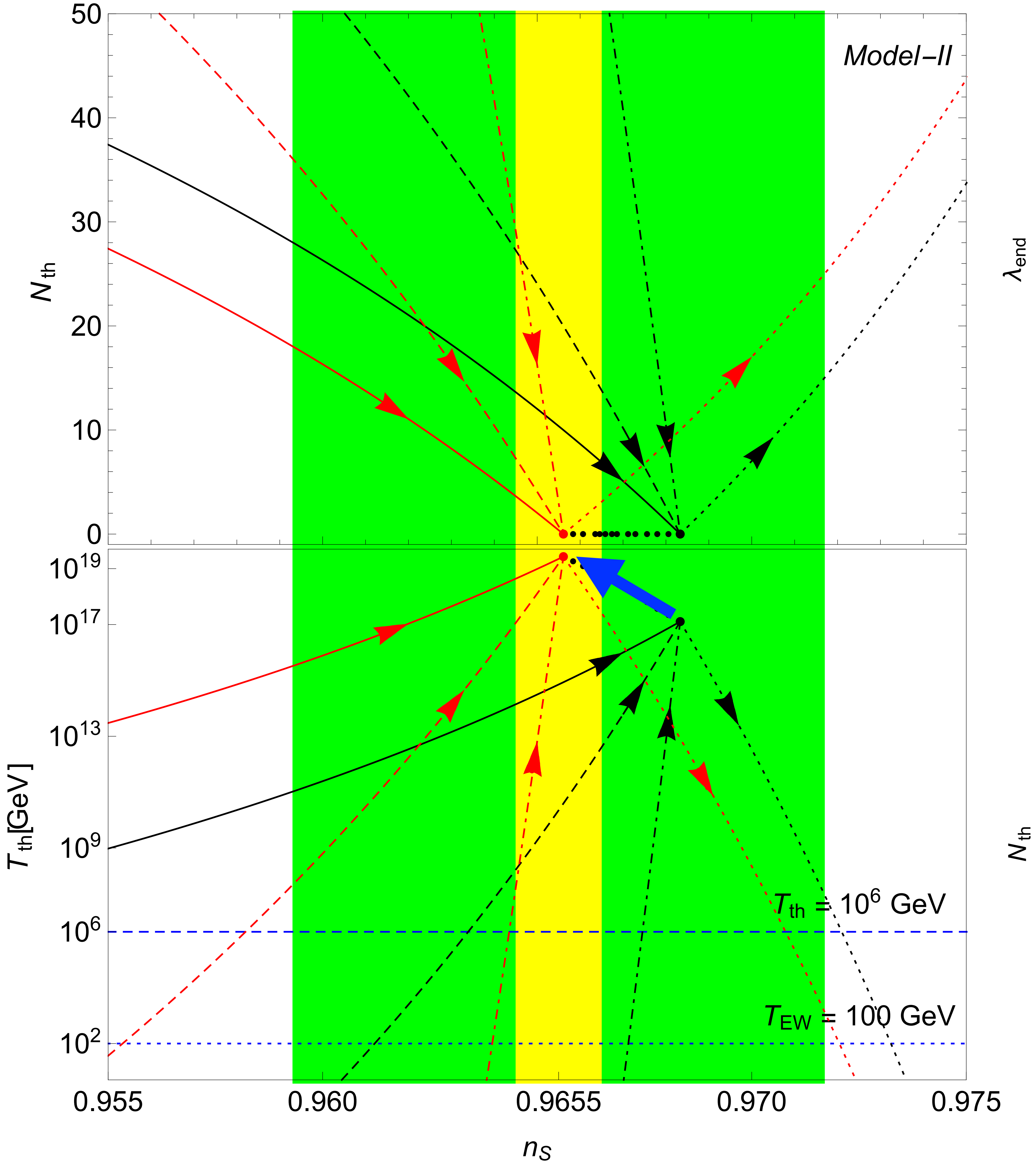
$$\lambda_{\rm end} = \left.\frac{6}{6-2\epsilon-\delta_1(5-2\epsilon+\delta_2)}\right|_{\phi=\phi_{\rm end}} > 0; \quad 0 \le \alpha \le 1 \;\Rightarrow\; 0 \le \alpha < \frac{n}{2n+2}\,.$$

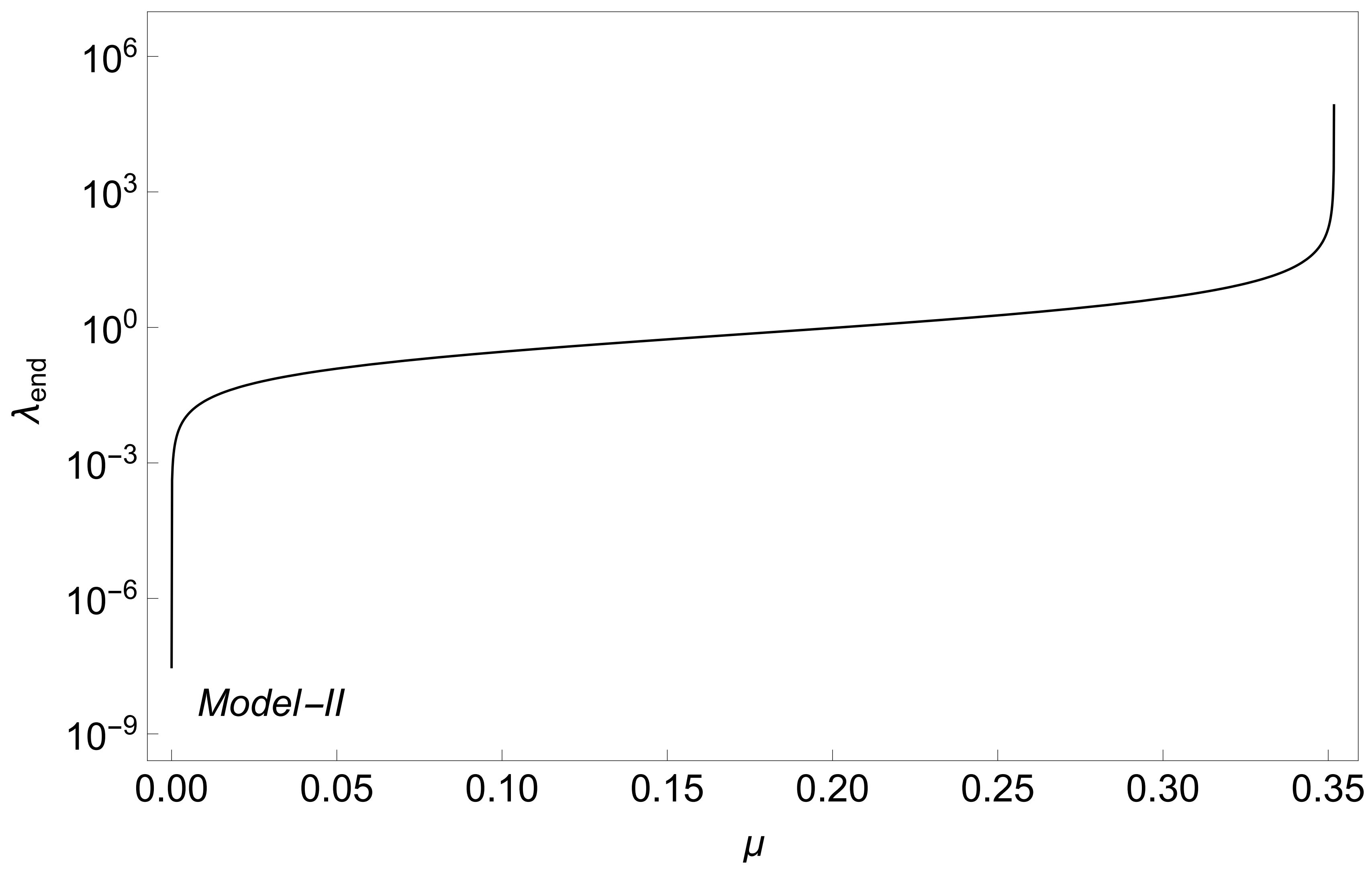
For $T_{th}=10^6$ GeV and $n_s=0.9655$:

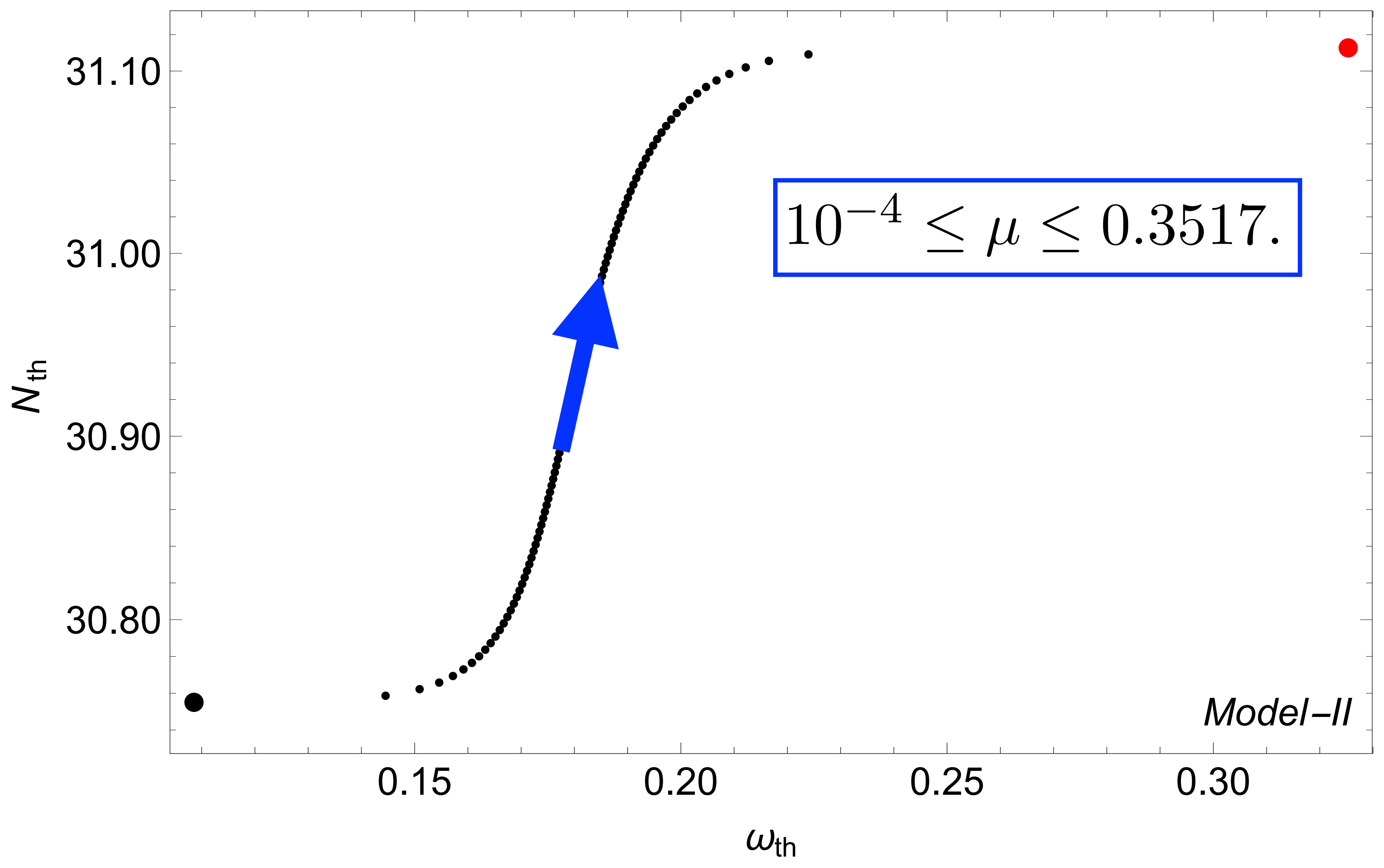


Numerical results:

Model-II: $V(\phi) = \frac{1}{\kappa^4}\left[\tanh(\kappa\phi) + \sqrt{\mu}\,\mathrm{sech}(\kappa\phi)\right]^2 , \quad \xi(\phi) = \frac{3\left[\sinh^2(\kappa\phi) - \frac{1}{\sqrt{\mu}}\sinh(\kappa\phi)\right]}{4\left[\sqrt{\mu} + \sinh(\kappa\phi)\right]^2} ,$







THANK YOU

FOR YOUR KIND ATTENTION!