Regularized 4DEGB and non-minimal coupling gravity

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- Modified Gravity theories
- Novel 4D EGB gravity VS Lovelock’s theorem
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- Regularized scalar-tensor theory
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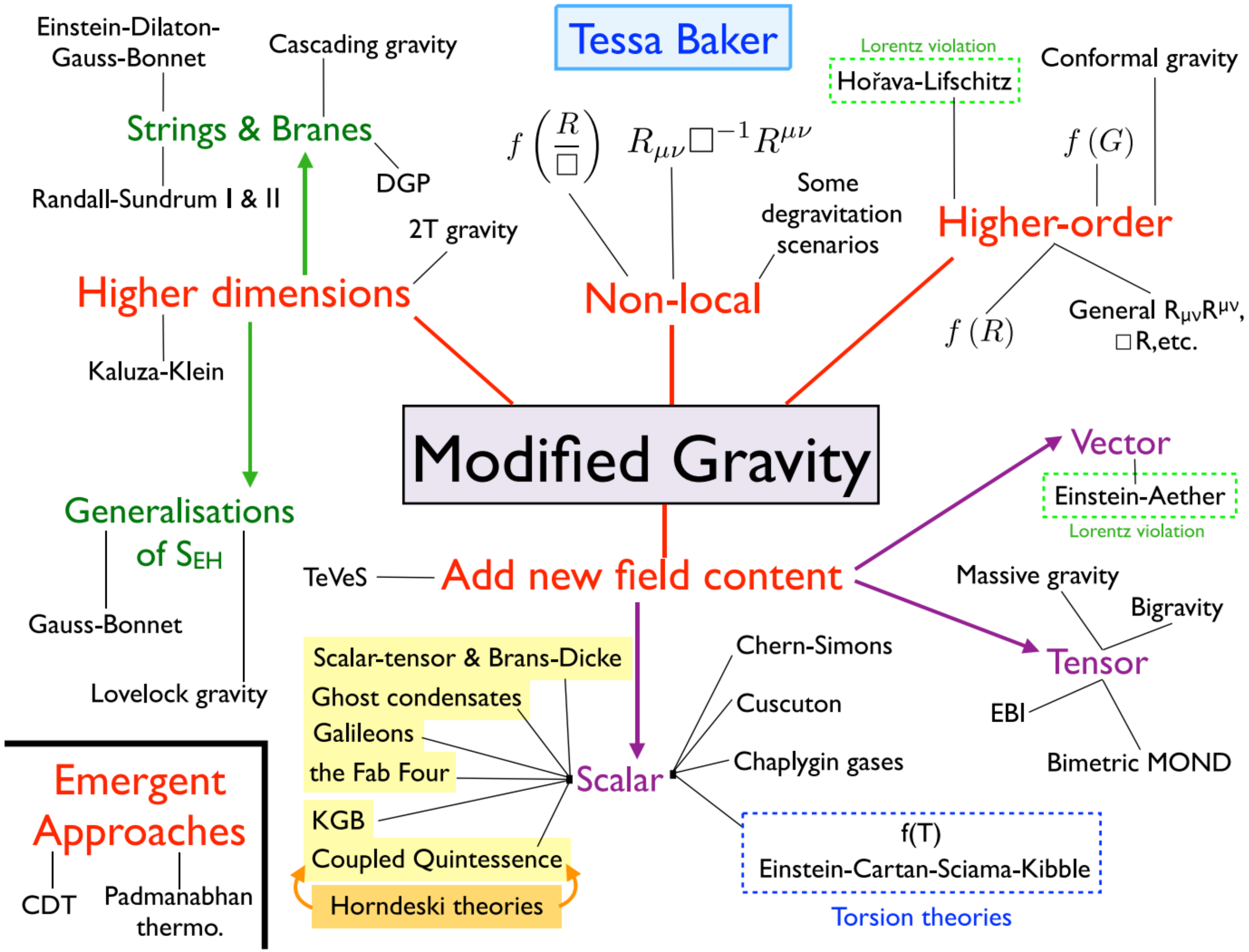
Modified Gravity theories

P. Bull et al. / Physics of the Dark Universe 12 (2016) 56–99



Tessa Baker

Modified Gravity

Higher dimensions

Strings & Branes

Einstein-Dilaton-Gauss-Bonnet

Cascading gravity

Randall-Sundrum I & II

DGP

2T gravity

Kaluza-Klein

Generalisations of S_{EH}

Gauss-Bonnet

Lovelock gravity

Non-local

$f\left(\frac{R}{\Box}\right)$

$R_{\mu\nu}\Box^{-1}R^{\mu\nu}$

Some degravitation scenarios

Higher-order

Lorentz violation

Hořava-Lifschitz

Conformal gravity

$f(G)$

$f(R)$

General $R_{\mu\nu}R^{\mu\nu}$, $\Box R$, etc.

Add new field content

TeVeS

Vector

Einstein-Aether

Lorentz violation

Tensor

Massive gravity

Bigravity

EBI

Bimetric MOND

Scalar

Scalar-tensor & Brans-Dicke

Ghost condensates

Galileons

the Fab Four

KGB

Coupled Quintessence

Horndeski theories

Chern-Simons

Cuscuton

Chaplygin gases

f(T)

Einstein-Cartan-Sciama-Kibble

Torsion theories

Emergent Approaches

CDT

Padmanabhan thermo.

Novel 4D EGB gravity VS Lovelock's theorem[1]

$$S_{\rm GB}[g_{\mu\nu}] = \int d^D x\sqrt{-g}\,\alpha\mathcal{G} \;\Rightarrow\; \frac{g_{\nu\rho}}{\sqrt{-g}}\frac{\delta S_{\rm GB}}{\delta g_{\mu\rho}} = 15\alpha\delta^{\mu}_{[\nu}R^{\rho\sigma}_{\rho\sigma}R^{\alpha\beta}_{\alpha\beta]} \;\Rightarrow\; \frac{g_{\mu\nu}}{\sqrt{-g}}\frac{\delta S_{\rm GB}}{\delta g_{\mu\nu}} = (D-4)\times\frac{\alpha}{2}\mathcal{G}$$

what if $\alpha \to \alpha/(D-4)$ and consider the limit $D \to 4$

$$S[g_{\mu\nu}] = \int d^D x\sqrt{-g}\left[\frac{M_P^2}{2}R - \Lambda_0 + \frac{\alpha}{D-4}\mathcal{G}\right]$$

<u>Maximally Symmetric</u>: $\frac{g_{\nu\rho}}{\sqrt{-g}}\frac{\delta S_{\rm GB}}{\delta g_{\mu\rho}} = \frac{\alpha}{D-4}\times\frac{(D-2)(D-3)(D-4)}{2(D-1)M_P^4}\times\Lambda^2\delta^{\mu}_{\nu}\,,\ \Lambda_{\pm} \equiv M_P^2 R/D = \frac{3M_P^4}{4\alpha}\left[-1\pm\sqrt{1+\frac{8\alpha\Lambda_0}{3M_P^4}}\right]$

<u>FLRW Cosmology:</u>

$S = S_{\rm EH} + S_{\rm GB} + S_{\phi}$

$$3M_P^2H^2 + \boxed{6\alpha H^4}^{[2]} = \frac{1}{2}\dot{\phi}^2 + V(\phi),\ -M_P^2\Gamma\dot{H} = \frac{1}{2}\dot{\phi}^2\,,$$

Scalar field equation

<u>Metric Fluctuation:</u> $\ddot{\gamma}_{ij} + 3H\left(1 - \frac{8\alpha\epsilon H^2}{3M_P^2\Gamma}\right)\dot{\gamma}_{ij} - c_s^2\frac{\partial^2\gamma_{ij}}{a^2} = 0$

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[1] Glavan D. and Lin C. "Einstein-Gauss-Bonnet Gravity in Four-Dimensional Spacetime", Phys. Rev. Lett. 124, no.8, 081301 (2020)

[2] 2012.15219 [gr-qc]

Novel 4D EGB gravity VS Lovelock’s theorem

$$ds^2 = -e^{2\omega}dt^2 + e^{2\lambda}dr^2 + r^2 d\Omega^2_{D-2}$$

$$-g_{00} = e^{2\omega} = e^{-2\lambda} = 1 + \frac{r^2}{32\pi\alpha G}\left[1 \pm \left(1 + \frac{128\pi\alpha G^2 M}{r^3}\right)^{1/2}\right]$$

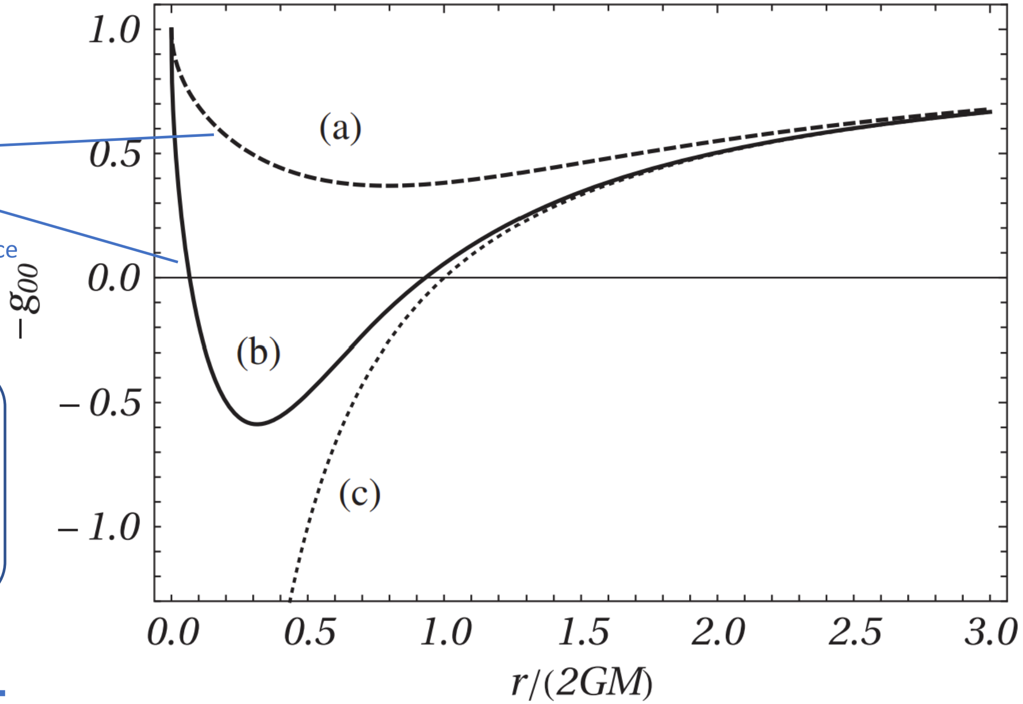
Conformal Anomaly

R. G. Cai, L. M. Cao, N. Ohta, 0911.4379[hep-th]

SSS metric

Y. Tomozawa,
Quantum corrections to gravity,
1107.1424 [gr-qc]

N. D. Birrell and P. C. W. Davis, Quantum Fields in Curved Space



$D > 4$

$$-g_{00} = 1 + \frac{r^2}{32\pi\alpha G(D-3)(D-4)}\left[1 \pm \left(1 + \frac{128\pi\alpha G^2 M(D-3)(D-4)}{r^{D-1}}\right)^{1/2}\right]$$

D. G. Boulware and S. Deser, “String-Generated Gravity Models”, Phys. Rev. Lett. 55 (1985) 2656

Glavan D. and Lin C. “Einstein-Gauss-Bonnet Gravity in Four-Dimensional Spacetime”, Phys. Rev. Lett. 124, no.8, 081301 (2020)

Comments on Novel 4D EGB gravity: Concerns

- Taking limit VS Discrete Index
- Restricted to EOM level: Boundary Terms? ⇒ Variational Principle?

  $$S_{Total} = S_{EGB}^{\text{on-shell}} + S_{GH}^{S.T} + S_{G.B}^{S.T}$$

  S. Mahapatra, Eur. Phys. J. C (2020) 80:992
  - BT does not affect EOM, but must be fixed and finite ⇒ Counterterms in D

    R. Olea, JHEP 0704, 073 (2007)
  - [1]Mixed Boundary Condition? $\delta S_{\text{Dir}} = EOM + \frac{1}{2}\int_{\partial M} \mathrm{d}^d x \sqrt{-g_{(0)}}\, T^{ij}\, \delta g_{(0)ij}, \quad \frac{2}{\sqrt{-g_{(0)}}} \frac{\delta S_{\text{Bndy grav}}}{\delta g_{(0)ij}} + T^{ij} = 0$
- Symmetry Assumptions: Highly Constrained
- Bypass Lovelock's theorem

Doesn't destroy the Theorem

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[1] G. Compere, D. Marolf, Class. Quant. Grav. 25, 195014 (2008), arXiv: 0805.1902 [hep-th]

Comments on Novel 4D EGB gravity: Regularized scalar-tensor theory

Glavan D. and Lin C. “Einstein-Gauss-Bonnet Gravity in Four-Dimensional Spacetime”, Phys. Rev. Lett. 124, no.8, 081301 (2020)

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = \alpha\mathcal{H}_{\mu\nu} + T_{\mu\nu}$$

$$\mathcal{H}_{\mu\nu} = 15\delta_{\mu[\nu}R^{\rho\sigma}{}_{\rho\sigma}R^{\alpha\beta}{}_{\alpha\beta]}$$

$$\mathcal{H}^{\mu}{}_{\mu} = \frac{1}{2}(D-4)\mathcal{G}$$

Only trace is considered

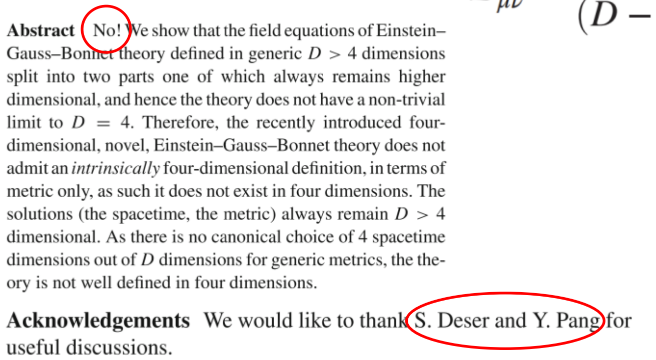
Gurses M., T. C. Sisman, B. Tekin, “Is there a novel Einstein-Gauss-Bonnet Theory in 4 Dimensions?” 2004.03390 [gr-qc]

$$\mathcal{H}_{\mu\nu} = -2(\mathcal{L}_{\mu\nu} + \mathcal{Z}_{\mu\nu})$$

$$\mathcal{L}_{\mu\nu} = C_{\mu\alpha\beta\sigma}C_{\nu}{}^{\alpha\beta\sigma} - \frac{1}{4}g_{\mu\nu}C_{\alpha\beta\rho\sigma}C^{\alpha\beta\rho\sigma}$$

Vanish in 4D ⟶ $\mathcal{L}_{\mu\nu} = (D-4)\cancel{\mathcal{S}}_{\mu\nu}$

$$\mathcal{Z}_{\mu\nu} = \frac{(D-4)(D-3)}{(D-1)(D-2)}\left[-2\frac{(D-1)}{(D-3)}C_{\mu\rho\nu\sigma}R^{\rho\sigma} - 2\frac{(D-1)}{(D-2)}R_{\mu\rho}R^{\rho}{}_{\nu} + \frac{D}{(D-2)}R_{\mu\nu}R + \frac{1}{(D-2)}g_{\mu\nu}\left((D-1)R_{\rho\sigma}R^{\rho\sigma} - \frac{(D+2)}{4}R^2\right)\right]$$



Abstract No! We show that the field equations of Einstein–Gauss–Bonnet theory defined in generic $D > 4$ dimensions split into two parts one of which always remains higher dimensional, and hence the theory does not have a non-trivial limit to $D = 4$. Therefore, the recently introduced four-dimensional, novel, Einstein–Gauss–Bonnet theory does not admit an *intrinsically* four-dimensional definition, in terms of metric only, as such it does not exist in four dimensions. The solutions (the spacetime, the metric) always remain $D > 4$ dimensional. As there is no canonical choice of 4 spacetime dimensions out of D dimensions for generic metrics, the theory is not well defined in four dimensions.

Acknowledgements We would like to thank S. Deser and Y. Pang for useful discussions.

Bianchi Identity can’t be satisfied

Comments on Novel 4D EGB gravity: Regularized scalar-tensor theory

P. G.S. Fernandes, P. Carrilho, T. Clifton, D. J. Mulryne, Phys. Rev. D **120**, no.2, 024025 (2020)

Regularization in 2D

$\tilde{g}_{\mu\nu} = e^{2\phi} g_{\mu\nu}$

$$S = \alpha \int_{\mathcal{M}} d^D x \sqrt{-g} R + S_m - \alpha \int_{\mathcal{M}} d^D x \sqrt{-\tilde{g}} \tilde{R}$$

Regularized action

$$S = \hat{\alpha} \int_{\mathcal{M}} d^2 x \sqrt{-g} (\phi R + (\nabla \phi)^2) + S_m$$

Field equations

$$R = \frac{2}{\hat{\alpha}} T \quad \text{and} \quad \nabla_\mu \phi \nabla_\nu \phi - \nabla_\mu \nabla_\nu \phi + g_{\mu\nu} \left(\Box \phi - \frac{1}{2} (\nabla \phi)^2 \right) = \frac{1}{\hat{\alpha}} T_{\mu\nu}$$

Regularization in 4D

$$S = \int_{\mathcal{M}} d^D x \sqrt{-g} (R + \alpha \mathcal{G}) + S_m - \alpha \int_{\mathcal{M}} d^D x \sqrt{-\tilde{g}} \tilde{\mathcal{G}}$$

Regularized action

$$S = \int_{\mathcal{M}} d^4 x \sqrt{-g} [R + \hat{\alpha} (4 G^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - \phi \mathcal{G} + 4 \Box \phi (\nabla \phi)^2 + 2 (\nabla \phi)^4)] + S_m$$

Horndeski Class of theories (G. W. Horndeski, Int. J. Theor. Phys. **10**, 363-384 (1974))

$$G_{\mu\nu} = \hat{\alpha} \hat{\mathcal{H}}_{\mu\nu} + T_{\mu\nu}$$

$$R + \frac{\hat{\alpha}}{2} \mathcal{G} = -T$$

Same as original 4DEGB

- This class is more general than the original one in terms of symmetry
- They are not necessarily equivalent

Hidden Scalar
Degree of freedom

Regularized scalar-tensor theory

P. G.S. Fernandes, P. Carrilho, T. Clifton, D. J. Mulryne, Phys. Rev. D **120**, no.2, 024025 (2020)

$$S = \int_{\mathcal{M}} d^4x\sqrt{-g}[R + \hat{\alpha}(4G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \phi\mathcal{G} + 4\Box\phi(\nabla\phi)^2 + 2(\nabla\phi)^4)] + S_m$$

H. Lu, Y. Pang, Phys. Lett. B **809** (2020) 135717

$$ds_D^2 = ds_p^2 + e^{2\phi}d\Sigma^2_{D-p,\lambda}\,, \qquad R_{abcd} = \lambda(g_{ac}g_{bd} - g_{ad}g_{bc})$$

- KK reduction, topological GB for $p \leq 4$ and scaling $\alpha \to \alpha/(D-p)$

$$-\frac{\alpha}{16\pi G_p}\int d^px\sqrt{-g}\,\mathrm{GB}$$

added by hand

$$S_p = \int d^px\sqrt{-g}\Big[R - 2\Lambda_0 + \alpha\Big(\phi\,\mathrm{GB} + 4G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2\lambda Re^{-2\phi} - 4(\partial\phi)^2\Box\phi$$
$$+2((\partial\phi)^2)^2 - 12\lambda(\partial\phi)^2e^{-2\phi} - 6\lambda^2e^{-4\phi}\Big)\Big]$$

Phys. Rev. D **102**, 024029

$$\xrightarrow[\alpha \to (D-4)\tilde{\alpha}]{} S = M_P^2\int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\partial\hat{\phi})^2 + \tilde{\alpha}(\partial\hat{\phi})^4\right]$$

Without strong coupling

$$S_p = \int d^px\sqrt{-g}e^{\phi}\Big[R - 2\Lambda_0$$
$$+\alpha\Big(-4G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 2\lambda Re^{-2\phi} + 2(\partial\phi)^2\Box\phi + 4\lambda(\partial\phi)^2e^{-2\phi} + 2\lambda^2e^{-4\phi}\Big)\Big]$$

$\alpha \to \alpha/(D-p-1)\,, p \leq 3$

$$S_p = \int d^px\sqrt{-g}e^{2\phi}\Big[R - 2\Lambda_0 + 2(\partial\phi)^2 + 2\lambda e^{-2\phi} + 2\alpha\Big(2\lambda\phi Re^{-2\phi} - 2(\partial\phi)^2\Box\phi$$
$$-((\partial\phi)^2)^2 - 2\lambda(\partial\phi)^2e^{-2\phi} - \lambda^2e^{-4\phi}\Big)\Big].$$

$\alpha \to \alpha/(D-p-2)\,, p \leq 2$

Motivation

- Combination of Regularized Scheme and Non-minimal coupling
- Dynamical Equation and its stability (DSA)?
- Cosmological evolution and observational constraints?

Combination of Regularized Scheme and Non-minimal coupling

$$S = \int \mathrm{d}^D x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - \frac{\alpha}{2} \xi(\phi) \mathcal{G} \right] + S_{\mathrm{m,r}}$$

$$\alpha \to \alpha/(D-4), \quad \xi(\phi) \to \xi^{(D-4)}(\phi)$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa^2 \left(T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{(\mathrm{m,r})} + T_{\mu\nu}^{(\mathrm{GB})} \right)$$

$$\Box\phi - V_\phi - \frac{\alpha}{2(D-4)} (D-4) \xi^{(D-5)} \xi_\phi \mathcal{G} = 0$$

$$T_{\mu\nu}^{(\mathrm{GB})} = \frac{\alpha}{(D-4)} \Big\{ \xi^{(D-4)} \left(-4 R_\mu{}^{\alpha} R_{\nu\alpha} + 2 R R_{\mu\nu} - 4 R^{\alpha\beta} R_{\mu\alpha\nu\beta} + 2 R_\mu{}^{\alpha\beta\gamma} R_{\nu\alpha\beta\gamma} - \frac{1}{2} g_{\mu\nu} \mathcal{G} \right)$$
$$+ (D-4)(D-5) \xi_\phi^2 \xi^{(D-6)} \Big[g_{\mu\nu} \left(2 R \nabla^\alpha \phi \nabla_\alpha \phi - 4 R_{\alpha\beta} \nabla^\alpha \phi \nabla^\beta \phi \right) - 4 R_{\mu\nu} \nabla^\alpha \phi \nabla_\alpha \phi \quad + \ldots \Big\}$$

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)(\mathrm{d}\chi_1^2 + \mathrm{d}\chi_2^2 + \mathrm{d}\chi_3^2 + \ldots + \mathrm{d}\chi_{D-1}^2)$$

- Taking limit

$$3H^2 = \kappa^2 (\rho_{\mathrm{m}} + \rho_{\mathrm{r}} + \rho_\phi)$$

$$2\dot{H} + 3H^2 = -\kappa^2 \left(\frac{1}{3} \rho_{\mathrm{r}} + p_\phi \right)$$

$$\dot{\rho_\phi} + 3H(1 + w_\phi)\rho_\phi = 0$$

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V + 12\alpha \frac{\dot{\xi}}{\xi} H^3 + 3\alpha H^4,$$

$$p_\phi = \frac{\dot{\phi}^2}{2} - V - 8\alpha \frac{\dot{\xi}}{\xi} H(\dot{H} + H^2) - 3\alpha H^4 - 4\alpha H^2 \dot{H} - 4\alpha \left(-\frac{\dot{\xi}^2}{\xi^2} + \frac{\ddot{\xi}}{\xi} \right) H^2$$

Dynamical Equation and its stability (DSA)

- Dimensionless Variable

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H}, \quad y = \frac{\kappa\sqrt{V}}{\sqrt{3}H}, \quad \alpha_{\mathrm{GB}} = \sqrt{\alpha}\kappa H, \quad \mu = -\frac{\sqrt{6}\xi_\phi}{\kappa\xi}, \quad \lambda = -\frac{V_\phi}{\sqrt{6}\kappa V}, \quad \epsilon = -\frac{\dot{H}}{H^2}$$

$$1 = \frac{\kappa^2\rho_{\mathrm{m}}}{3H^2} + \frac{\kappa^2\rho_{\mathrm{r}}}{3H^2} + \frac{\kappa^2\dot{\phi}^2}{6H^2} + \frac{\kappa^2 V}{3H^2} + \frac{4\alpha\kappa^2\dot{\xi}H}{\xi} + \alpha\kappa^2 H^2$$

$$\begin{aligned}
1 &= \Omega_{\mathrm{m}} + \Omega_{\mathrm{r}} + \Omega_\phi, \\
\Omega_{\mathrm{m}} &= \frac{\kappa^2\rho_{\mathrm{m}}}{3H^2}, \\
\Omega_{\mathrm{r}} &= \frac{\kappa^2\rho_{\mathrm{r}}}{3H^2}, \\
\Omega_\phi &= x^2 + y^2 + \alpha_{\mathrm{GB}}^2 - 4x\alpha_{\mathrm{GB}}^2\mu.
\end{aligned}$$

$$\xrightarrow[\xi(\phi) = \xi_0 e^{-\kappa\mu\phi/\sqrt{6}}]{V(\phi) = V_0 e^{-\sqrt{6}\kappa\lambda\phi}}$$

Fixed Points and Cosmological Parameters

- Linear stability theory
  - Analyzing the signs of eigenvalues of Jacobian matrix

Dynamical Equation and its stability (DSA Scheme)

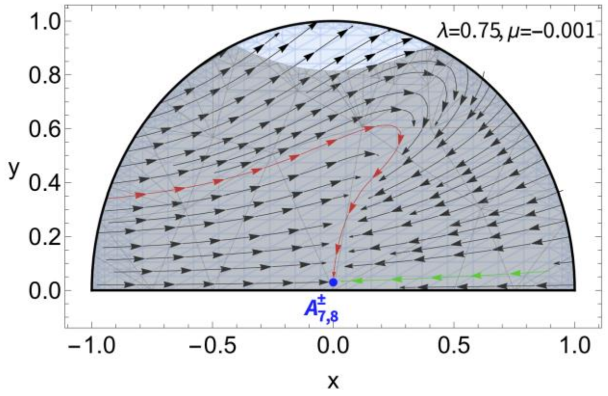
- Linear stability theory
  - Define dimensionless variables
  - Introducing $N = \ln a$ and take derivative WRT the variables
  - Obtain systems equations
  - Analyzing the signs of eigenvalues of the Jacobian matrix

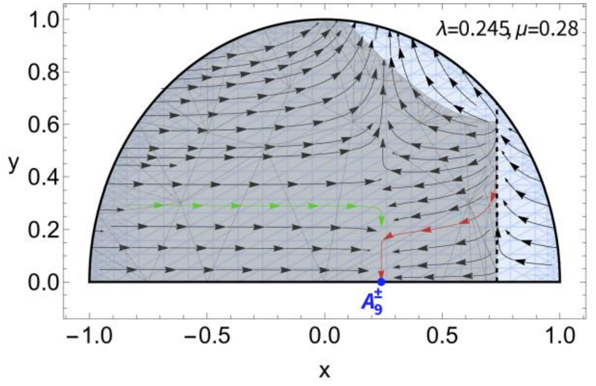
Fixed Points

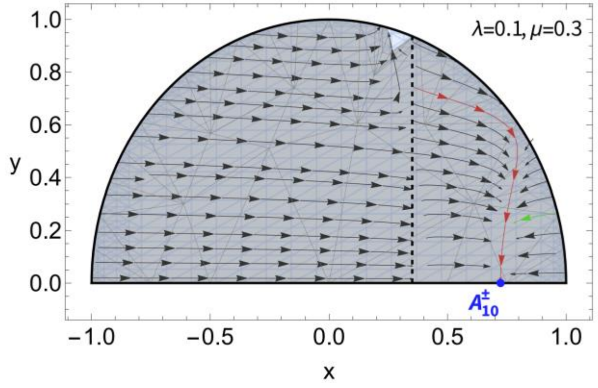
Points	x	y	$\alpha_{\rm GB}$	Ω_r	Ω_m	Ω_ϕ	Existence	$\omega_{\rm eff}=\omega_\phi$
$A_1^\pm$	$\frac{1}{2\lambda}$	$\pm\frac{1}{2\lambda}$	0	0	$1-\frac{1}{2\lambda^2}$	$\frac{1}{2\lambda^2}$	$\lambda^2>1/2$	0
$A_2^\pm$	$\frac{2}{3\lambda}$	$\pm\frac{\sqrt{2}}{3\lambda}$	0	$1-\frac{2}{3\lambda^2}$	0	$\frac{2}{3\lambda^2}$	$\lambda^2>2/3$	$1/3$
$A_3^\pm$	±1	0	0	0	0	1	$\forall\lambda$	1
A_4	0	0	0	0	1	0	$\forall\lambda$	0
A_5	0	0	0	1	0	0	$\forall\lambda$	$1/3$
$A_6^\pm$	λ	$\pm\sqrt{1-\lambda^2}$	0	0	0	1	$\lambda^2<1$	$-1+2\lambda^2$
$A_7^\pm$	0	$-\sqrt{\frac{2\mu}{2\mu-3\lambda}}$	$\pm\sqrt{\frac{3\lambda}{3\lambda-2\mu}}$	0	0	1	$(\mu\geq0,\lambda<0)$ or $(\mu\leq0,\lambda>0)$	-1
$A_8^\pm$	0	$\sqrt{\frac{2\mu}{2\mu-3\lambda}}$	$\pm\sqrt{\frac{3\lambda}{3\lambda-2\mu}}$	0	0	1	$(\mu\geq0,\lambda<0)$ or $(\mu\leq0,\lambda>0)$	-1
$A_9^\pm$	$\frac{3-\sqrt{9-80\mu^2}}{20\mu}$	0	$\pm\sqrt{\frac{6}{3+\sqrt{9-80\mu^2}}}$	0	0	1	$-\frac{3}{4\sqrt{5}}\leq\mu\leq\frac{3}{4\sqrt{5}},\mu\neq0$	-1
$A_{10}^\pm$	$\frac{3+\sqrt{9-80\mu^2}}{20\mu}$	0	$\pm\sqrt{\frac{9+3\sqrt{9-80\mu^2}}{40\mu^2}}$	0	0	1	$-\frac{3}{4\sqrt{5}}\leq\mu\leq\frac{3}{4\sqrt{5}},\mu\neq0$	-1

Points	Eigenvalues	Stability
$A_1^\pm$	$\left\{-\frac{3}{2},-1,-\frac{3}{4}\left(1+\sqrt{\frac{4}{\lambda^2}-7}\right),-\frac{3}{4}\left(1-\sqrt{\frac{4}{\lambda^2}-7}\right)\right\}$	stable for: $1/2<\lambda^2\leq4/7$, stable-focus for: $\lambda^2>4/7$, saddle for: $\lambda^2<1/2$
$A_2^\pm$	$\left\{-2,+1,-\frac{1}{2}\left(1+\sqrt{\frac{32}{3\lambda^2}-15}\right),-\frac{1}{2}\left(1-\sqrt{\frac{32}{3\lambda^2}-15}\right)\right\}$	saddle.
$A_3^\pm$	$\{-3,+3,+2,3\pm3\lambda\}$	saddle.
A_4	$\{-\frac{3}{2},-\frac{3}{2},+\frac{3}{2},-1\}$	saddle.
A_5	$\{-2,+2,-1,+1\}$	saddle.
$A_6^\pm$	$\{-3\lambda^2,3\left(\lambda^2-1\right),6\lambda^2-3,6\lambda^2-4\}$	stable for: $0<\lambda^2<1/2$; otherwise saddle.
$A_7^\pm$	$\left\{-4,-3,-\frac{3}{2}\left(1-\sqrt{1+\frac{32\lambda^2\mu^2-48\lambda^3\mu}{4\mu^2-\lambda^2(9-36\mu^2)}}\right),-\frac{3}{2}\left(1+\sqrt{1+\frac{32\lambda^2\mu^2-48\lambda^3\mu}{4\mu^2-\lambda^2(9-36\mu^2)}}\right)\right\}$	stable for: ${\rm con}_1$, stable focus for: $\frac{3\lambda^2}{-8\lambda^3+2\sqrt{16\lambda^6+17\lambda^4+\lambda^2}}<\mu<\frac{3\lambda}{2\sqrt{1+9\lambda^2}}$, saddle for: $\mu>\frac{3}{2}\sqrt{\frac{\lambda^2}{1+9\lambda^2}}$ if $\lambda<0$ or $\mu<-\frac{3}{2}\sqrt{\frac{\lambda^2}{1+9\lambda^2}}$ if $\lambda>0$.
$A_8^\pm$	$\left\{-4,-3,-\frac{3}{2}\left(1-\sqrt{1+\frac{32\lambda^2\mu^2-48\lambda^3\mu}{4\mu^2-\lambda^2(9-36\mu^2)}}\right),-\frac{3}{2}\left(1+\sqrt{1+\frac{32\lambda^2\mu^2-48\lambda^3\mu}{4\mu^2-\lambda^2(9-36\mu^2)}}\right)\right\}$	
$A_9^\pm$	$\left\{-\frac{3\lambda}{20\mu}\left(3-\sqrt{9-80\mu^2}\right),-4,-3,-3\right\}$	stable for: ${\rm con}_2$; otherwise saddle.
$A_{10}^\pm$	$\left\{-4,-3,-3,-\frac{3\lambda}{20\mu}\left(3+\sqrt{9-80\mu^2}\right)\right\}$	stable for: ${\rm con}_2$; otherwise saddle.

$$x=\frac{\kappa\dot{\phi}}{\sqrt{6}H},\quad y=\frac{\kappa\sqrt{V}}{\sqrt{3}H},\quad \alpha_{\rm GB}=\sqrt{\alpha}\kappa H,\quad \mu=-\frac{\sqrt{6}\xi_\phi}{\kappa\xi},\quad \lambda=-\frac{V_\phi}{\sqrt{6}\kappa V},\quad \epsilon=-\frac{\dot{H}}{H^2}$$

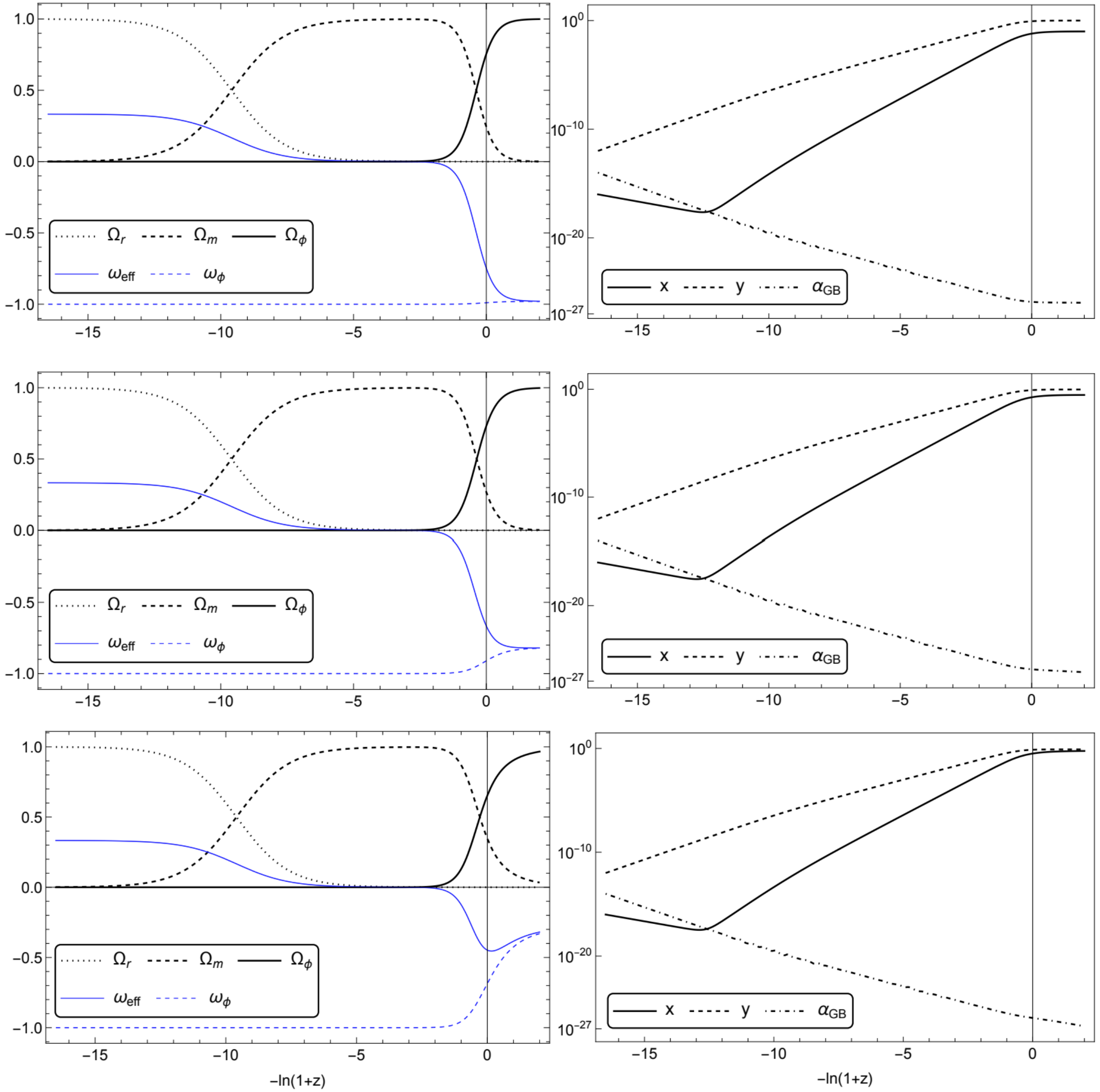






Fixed Points

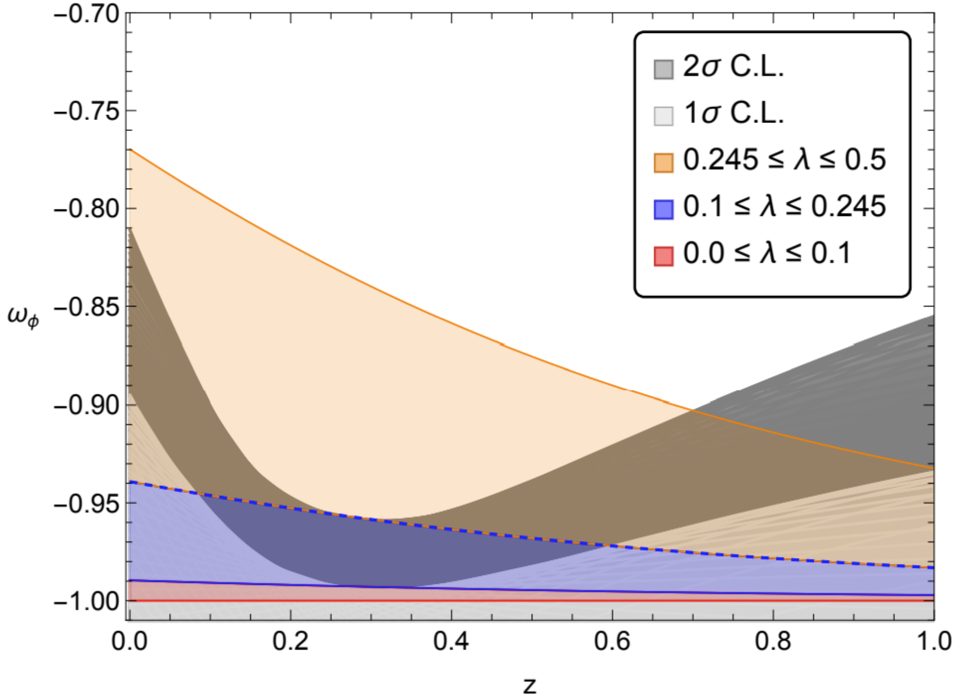
$\mu = 0.3$ and acceleration if $\lambda < 0.746$



$\lambda = 0.1$

$\lambda = 0.3$

$\lambda = 0.6$



$$\text{For } A^{\pm}_{7,8}: \quad -0.2268 \lesssim \mu < 0 \quad \text{when} \quad 0 < \lambda \leq 0.3166\,,$$

$$\text{For } A^{\pm}_{9,10}: \quad 0 < \mu \lesssim \frac{3}{4\sqrt{5}} \quad \text{when} \quad 0 < \lambda \leq 0.3166\,.$$

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H}, \quad y = \frac{\kappa\sqrt{V}}{\sqrt{3}H}, \quad \alpha_{\rm GB} = \sqrt{\alpha}\kappa H.$$

Conclusions

- We use a regularization scheme in EDGB gravity and obtain well-defined EoMs
- There exist stable fixed points for GB branch for a preferred potential and coupling function
- No missing cosmological history
- We obtain parameter space for non-minimal coupling function and potential by fitting against the data

Speculations

- It would be interesting to
  - regularize the action
  - consider the linear and second-order perturbation (2004.12998[gr-qc])
  - Early Universe?
  - consider the black hole
    - AdS would be interesting but counterterm in arbitrary D-dim is definitely non-trivial
    - or a regularized action should be considered first and introducing ad hoc counterterm after fixing the dimension
    - Holographic Transports?

Thank You!