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About Symmetry in Horava Gravity: A New Interpretation

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Based on 2112.00576 [hep-th]

- In 2019, Horava discovered that gravity can be "power-counting renormalizable" without ghost problem once Lorentz symmetry is broken in UV [arXiv:0901.3775].

- Horava(2019)-Lifshitz(1941)-DeWitt(1967) gravity:

$$S_g = \int d\eta d^3x \sqrt{g} N \left[\frac{2}{\kappa^2} \left(K_{ij}K^{ij} - \lambda K^2 \right) + \mathcal{V} \right],$$

$$-\mathcal{V} = \sigma + \xi R + \alpha_1 R^2 + \alpha_2 R_{ij}R^{ij} + \alpha_3 \frac{\epsilon^{ijk}}{\sqrt{g}} R_{il} \nabla_j R^l{}_k$$

$$+ \alpha_4 \nabla_i R_{jk} \nabla^i R^{jk} + \alpha_5 \nabla_i R_{jk} \nabla^j R^{ik} + \alpha_6 \nabla_i R \nabla^i R,$$

- Some progress on Horava gravity as a modified gravity theory, beyond GR:

(1) Non-GR black hole/neutron-star solutions: GR test via gravitational waves.

(2) Non-standard cosmology with dynamical dark energy and (spatially) non-flatness (i.e., closed universe) [Nilsson-MIP]: Replacing LCDM.

Some progress on the proof of renormalization (for some toy models [Barvinsky et al]:Projectable).

But, there are some puzzles!

- 1. Does Black hole radiates, even without Lorentz-invariant horizon?

- 2. Horava gravity=GR+effective matter:

Bianchi identity of GR -> Conservation of effective matter! => Coupling with conserved physical matters, as in GR !?

- 3. Hamiltonian constraint becomes second-class but one "can" have the same DOF as in GR=>Hint of symmetry higher than FPDiff of Horava gravity?

- **Understanding of the full symmetry would be the key to resolve puzzles in Horava gravity.**

1. Symmetry in GR

- **Under the general coordinate transformations,** $x^\mu \to x'^\mu(x)$ **scalar quantity is transformed as**

$$\phi'(x') = \phi(x)$$

For the (first-order) infinitesimal transf, $\delta x^\mu = x'^\mu(x) - x^\mu = -\xi^\mu(x)$

$$\delta_\xi \phi(x) = \phi'(x) - \phi(x) = \xi^\mu \partial_\mu \phi(x)$$

Tensor transforms as

$$\delta_\xi g_{\mu\nu}(x) = g'_{\mu\nu}(x) - g_{\mu\nu}(x) = \xi^\mu \partial_\mu g_{\mu\nu}(x) + \partial_\mu \xi^\sigma g_{\nu\sigma} + \partial_\nu \xi^\sigma g_{\mu\sigma}$$

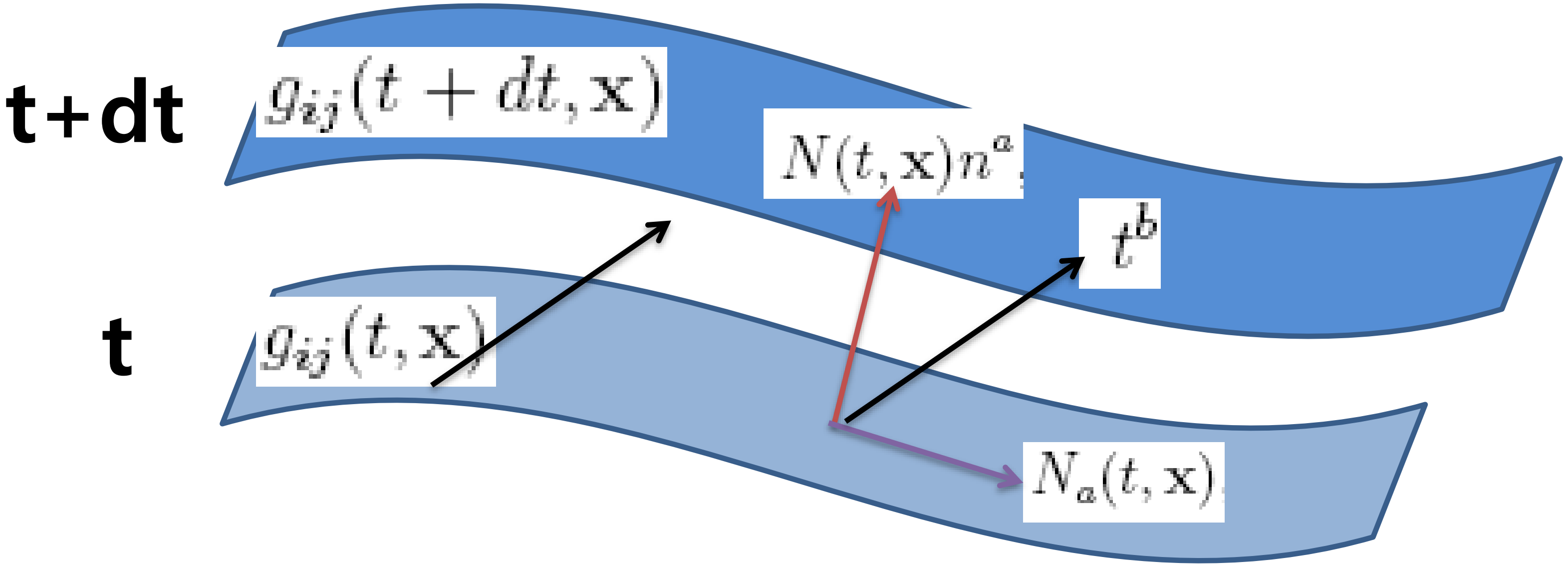
- **Under the space-time (ADM) decomposition,**

$$ds^2 = -N^2c^2dt^2 + g_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

- **The coord. Trasnf (Diff) becomes**

$$\begin{aligned}
\delta x^i &= -\zeta^i(t,\mathbf{x}),\ \ \delta t = -\zeta^t(t,\mathbf{x}),\\
\delta g_{ij} &= \partial_i\zeta^k g_{jk} + \partial_j\zeta^k g_{ik} + \zeta^k\partial_k g_{ij} + \zeta^t\partial_t g_{ij} + [N_j\partial_i\zeta^t + N_i\partial_j\zeta^t],\\
\delta N_i &= \partial_i\zeta^j N_j + \zeta^j\partial_j N_i + \partial_t\zeta^j g_{ij} + \zeta^t\partial_t N_i + \partial_t\zeta^t N_i + [(-N^2 + N_j N^j)\partial_i\zeta^t],\\
\delta N &= \zeta^j\partial_j N + \zeta^t\partial_t N + \partial_t\zeta^t N - [N N^i\partial_i\zeta^t].
\end{aligned}$$

Here, $N(t,\mathbf{x}), N_i(t,\mathbf{x})$ **determine the time foliations**



- Cf.

$$
\begin{aligned}
\delta x^i &= -\zeta^i(\mathbf{x}), \\
Scalar: \delta\phi &= \zeta^j \partial_j \phi, \\
Vector: \delta v_i &= \partial_i \zeta^j v_j + \zeta^j \partial_j v_i, \\
Tensor: \delta t_{ij} &= \partial_i \zeta^k t_{jk} + \partial_j \zeta^k t_{ik} + \zeta^k \partial_k t_{ij}
\end{aligned}
$$

- The space-time decomposition does not mean "breaking the general covariance".
- It is reflected in the "arbitrary time-foliation" symmetry t -> t'(t,x^i): No preferred reference frames!

The GR Action Construction:

- **Einstein-Hilbert action:**

$\mathcal{L}_{EH}$ — Scalar density

$$S_{EH} = \frac{1}{16\pi G_N} \int dx^4 \sqrt{-g^{(4)}} \left(R^{(4)} - 2\Lambda \right)$$

Lorentz invariant !

Lorentz scalars

$$= \frac{1}{16\pi G_N} \int d^4x \, \sqrt{g} \, N \left\{ \left(K_{ij} K^{ij} - K^2 \right) + R - 2\Lambda \right\}$$

$R^{(4)}$

in ADM decomposition

$$ds^2 = -N^2 c^2 dt^2 + g_{ij} \left(dx^i + N^i dt \right) \left(dx^j + N^j dt \right)$$

- Then, EH action should transform as (D: space dim.)

$$\delta_\xi S_{EH} = \int dt d^D x \, \partial_\mu \left[\xi^\mu(\mathbf{x},t) \mathcal{L}_{EH} \right]$$

Scalar density

- Actual calculation:

$$\delta_\xi S = \int dt d^D x \left[-\mathcal{H}\delta_\xi N - \mathcal{H}^i \delta_\xi N_i + \mathbf{E}^{ij}\delta_\xi g_{ij} + \partial_\mu \Theta^\mu(\delta_\xi N_i, \delta_\xi g_{ij}) \right]$$

$$= \int dt d^D x \left[\xi^0 \mathcal{I}_0 + \xi^i \mathcal{I}_i + \partial_\mu \Psi^\mu(\delta_\xi N_i, \delta_\xi g_{ij}) \right],$$

$\hat{\nabla}_\mu G^{\mu 0} = 0$ $\quad$ $\hat{\nabla}_\mu G^{\mu i} = 0$ $\quad$ $\xi^\mu(\mathbf{x},t)\mathcal{L}_{EH}$

(contracted) Bianchi Identities

- Diff-invariant EH action, due to Bianchi identities: General covariance is reflected in Bianchi identities.

- **Here,**

$$\mathcal{H} \equiv -\frac{\delta S}{\delta N} = \sqrt{g}\left[\left(\frac{2}{\kappa^2}\right)\left(K_{ij}K^{ij} - \lambda K^2\right) + \mathcal{V}\right]$$

$$\mathcal{H}^i \equiv -\frac{\delta S}{\delta N_i} = -2\sqrt{g}\left(\frac{2}{\kappa^2}\right)\nabla_j\left(K^{ij} - \lambda g^{ij}K\right),$$

$$\mathbf{E}^{ij} \equiv \frac{\delta S}{\delta g_{ij}}$$

2. Symmetries in Horava gravity [2112.00576]

- In order to realize the "absence of ghost" in the propagator $\frac{1}{\omega^2 - \mathbf{k}^2 - G(\mathbf{k}^2)^z}$, Horava considered foliation preserving Diff.

$$
\begin{aligned}
\delta x^i &= -\zeta^i(t,\mathbf{x}), \;\; \delta t = -f(t), \\
\delta g_{ij} &= \partial_i \zeta^k g_{jk} + \partial_j \zeta^k g_{ik} + \zeta^k \partial_k g_{ij} + f \dot{g}_{ij}, \\
\delta N_i &= \partial_i \zeta^j N_j + \zeta^j \partial_j N_i + \dot{\zeta}^j g_{ij} + f \dot{N}_i + \dot{f} N_i \\
\delta N &= \zeta^j \partial_j N + f \dot{N} + \dot{f} N.
\end{aligned}
$$

- Cf. Under the full Diff.

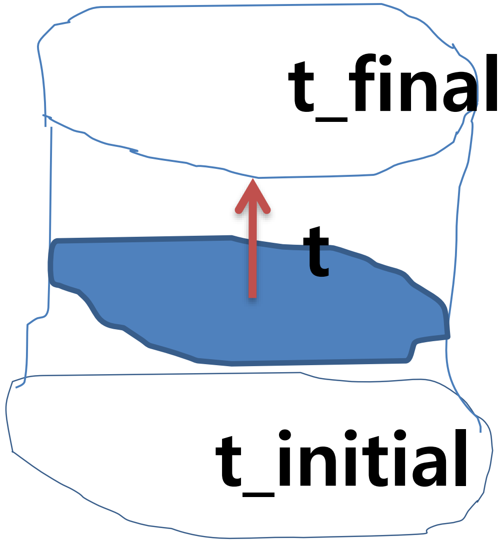
$$
\delta x^i = -\zeta^i(t,\mathbf{x}), \;\; \delta t = -\zeta^t(t,\mathbf{x}),
$$

the foliation is not preserved generally (no absolute time).

- **The general action that has this FPDiff symmetry is (with the detailed balance condition) [Horava, 2009],**

$$S=\int dt\, d^3x\,\sqrt{g}\,N\left\{\frac{2}{\kappa^2}\left(K_{ij}K^{ij}-\lambda K^2\right)-\frac{\kappa^2}{2w^4}C_{ij}C^{ij}+\frac{\kappa^2\mu}{2w^2}\varepsilon^{ijk}R_{i\ell}\nabla_j R^{\ell}_{k}\right.$$
$$\left.-\frac{\kappa^2\mu^2}{8}R_{ij}R^{ij}+\frac{\kappa^2\mu^2}{8(1-3\lambda)}\left(\frac{1-4\lambda}{4}R^2+\Lambda_W R-3\Lambda_W^2\right)\right\}.$$

$$C^{ij}=\varepsilon^{ik\ell}\nabla_k\left(R^j_\ell-\frac{1}{4}R\delta^j_\ell\right)$$



Extrinsic Curvature

$$K_{ij}=\frac{1}{2N}\left(\dot g_{ij}-\nabla_i N_j-\nabla_j N_i\right)$$

- For $\lambda \neq 1$ or higher-energy (UV) regime, the Lorentz symmetry is broken explicitly ($\lambda \neq 1$) [DeWitt(1967)] or dynamically [Lifshitz(1941), Horava(2009)].

- For $\lambda = 1/3$, the theory becomes singular but needs a separate consideration: Conformal symmetry.

- **For actual calculations, we consider (in (D+1)-dim)**

$$S = \int_{\mathcal{M}} dtd^D x \sqrt{g} N \left\{ \frac{2}{\kappa^2} \left(K^{ij} K_{ij} - \lambda K^2 \right) - \mathcal{V}[g^{ij}, R^i{}_{jkl}, \nabla_i] \right\}$$

$$-\mathcal{V} = \Lambda + \xi R + \alpha R^n + \beta \left(R_{ij} R^{ij} \right)^s + \gamma (R^i{}_{jkl} R_i{}^{jkl})^r + \cdots ,$$

- **without (spatially-covariant) derivatives ∇_i, for simplicity.**
- **Then, under arbitrary variations of ADM variables,**

$$\delta S = \int dtd^D x \left[-\mathcal{H} \delta N - \mathcal{H}^i \delta N_i + \mathrm{E}^{ij} \delta g_{ij} + \partial_\mu \Theta^\mu (\delta N_i, \delta g_{ij}) \right] ,$$

$$\Theta^0 \equiv \sqrt{g} \left(\frac{2}{\kappa^2} \right) \left(K^{ij} - \lambda g^{ij} K \right) \delta g_{ij},$$

$$\Theta^i \equiv \sqrt{g} \left(\frac{2}{\kappa^2} \right) \left(2N^l G^{ijkm} K_{km} \delta g_{jl} - N^i G^{ljmn} K_{mn} \delta g_{jl} - 2G^{kjil} K_{kj} \delta N_l \right) + 2\sqrt{g} P^{jkil} N \nabla_k \delta g_{lj} - 2\sqrt{g} \delta g_{lj} \nabla_k (P^{jikl} N),$$

$$G^{ijkm} \equiv \delta^{ijkm} - \lambda g^{ij} g^{km}$$

$$P_i^{jkl} \equiv \left(\frac{\partial \mathcal{L}}{\partial R^i{}_{jkl}} \right)_{g^{mn}} = - \left(\frac{\partial \mathcal{V}}{\partial R^i{}_{jkl}} \right)_{g^{mn}}$$

- **and the bulk terms,**

$$\mathcal{H} \equiv -\frac{\delta S}{\delta N} = \sqrt{g}\left[\left(\frac{2}{\kappa^2}\right)(K_{ij}K^{ij} - \lambda K^2) + \mathcal{V}\right],$$

$$\mathcal{H}^i \equiv -\frac{\delta S}{\delta N_i} = -2\sqrt{g}\left(\frac{2}{\kappa^2}\right)\nabla_j\left(K^{ij} - \lambda g^{ij}K\right),$$

$$\mathbf{E}^{ij} \equiv \frac{\delta S}{\delta g_{ij}} = E^{ij}_{(0)} - \sqrt{g}\left[NP^{iklm}R^j{}_{klm} + \frac{1}{2}Ng^{ij}\mathcal{V}[g^{ij}, R^i{}_{jkl}] - 2\nabla_k\nabla_l(NP^{iklj})\right]$$

- **Let us consider transformation Horava-type action under the full Diff., as in GR**

$$\delta_\xi S = \int dtd^Dx\left[-\mathcal{H}\delta_\xi N - \mathcal{H}^i\delta_\xi N_i + \mathbf{E}^{ij}\delta_\xi g_{ij} + \partial_\mu\Theta^\mu(\delta_\xi N_i, \delta_\xi g_{ij})\right]$$

$$= \int dtd^Dx\left[\xi^0\mathcal{I}_0 + \xi^i\mathcal{I}_i + \partial_\mu\Psi^\mu(\delta_\xi N_i, \delta_\xi g_{ij})\right],$$

- , where

$$\mathcal{I}_0 \equiv N\dot{\mathcal{H}} - \nabla_m(NN^m\mathcal{H}) + N_i\dot{\mathcal{H}}^i + \nabla_m\left[\mathcal{H}^m(g^{jl}N_jN_l - N^2)\right] + \dot{g}_{ij}\mathbf{E}^{ij} - 2\nabla_m(N_i\mathbf{E}^{mi})$$
$$\mathcal{I}_i \equiv (g_{ij}\mathcal{H}^j)_{,0} + \nabla_m(\mathcal{H}^m N_i) - \mathcal{H}\nabla_i N - \mathcal{H}^j\nabla_i N_j - 2g_{ij}\nabla_m\mathbf{E}^{jm},$$
$$\Psi^0 \equiv -\xi^0\left(N\mathcal{H} + N_i\mathcal{H}^i\right) - \xi^j g_{ij}\mathcal{H}^i + \Theta^0,$$
$$\Psi^i \equiv \xi^0\left[NN^i\mathcal{H} - \mathcal{H}^i(g^{lj}N_lN_j - N^2) + 2N_j\mathbf{E}^{ij}\right] + \xi^j\left(-N_j\mathcal{H}^i + 2g_{jl}\mathbf{E}^{il}\right) + \Theta^i$$

- **Expressing time-derivatives by K_{ij} and then canonical momenta $\pi_{ij} = (2/\kappa^2)\sqrt{g}(K_{ij} - \lambda K g_{ij})$, we obtain**

$$\mathcal{I}_0 = \nabla_i\left\{2N^2\left[\nabla_j\pi^{ij} + \left(\frac{\kappa^2}{2}\right)\left(\frac{2\lambda}{\lambda D - 1}\left(\pi\nabla_l P^{kl}{}_k{}^i - P^{kl}{}_k{}^i\nabla_l\pi\right) + 2P_{jkl}{}^i\nabla^k\pi^{jl} - 2\pi^{jl}\nabla^k P_{jkl}{}^i\right)\right]\right\}$$
$$\equiv \nabla_i\Omega^i, \quad (28)$$
$$\mathcal{I}_i = 0, \quad (29)$$
$$\Psi^0 = \xi^0\mathcal{L} + \partial_i\mathcal{U}^{0i}, \quad (30)$$
$$\Psi^i = \xi^i\mathcal{L} + \Sigma^i + \partial_0\mathcal{U}^{i0} + \partial_j\mathcal{U}^{ij}, \quad (31)$$

- ,where

$$\Sigma^i = 2N^2\Bigg[\left(\frac{\kappa^2}{2}\right)\left(\frac{2\lambda}{\lambda D-1}\left(P^{li}{}_{lk}\nabla^k(\xi^0\pi)-\xi^0\pi\nabla^k P^{li}{}_{lk}\right)+2\xi^0\pi^{jl}\nabla^k P_{jkl}{}^i-2P_{jkl}{}^i\nabla^k(\xi^0\pi^{jl})\right)$$
$$+\ \pi^{ij}\nabla_j\xi^0-\xi^0\nabla_j\pi^{ij}\Bigg], \qquad (32)$$
$$\mathcal{U}^{0i} = -\mathcal{U}^{i0} = 2\sqrt{g}(\xi^0 N_j+\xi_j)\left(\frac{2}{\kappa^2}\right)\left(K^{ij}-\lambda g^{ij}K\right), \qquad (33)$$
$$\mathcal{U}^{ij} = -\mathcal{U}^{ji}. \qquad (34)$$

and $\mathcal{U}^{\mu\nu}=-\mathcal{U}^{\mu\nu}$ is known as "super-potential" in GR.

- **Now, we have an identity $\mathcal{I}_i=0$, as an analogue of $\hat{\nabla}_\mu G^{\mu i}=0$**

: Invariance under spatial-Diff (FPDiff)!

- However, there is no known identity for $\mathcal{I}_0$ and it may reflect the non-invariance of Horava action.

- But, what if we demand $\mathcal{I}_0 \equiv \nabla_i \Omega^i = 0$, as an analogue of $\hat{\nabla}_\mu G^{\mu 0} = 0$ in GR ??

- We propose $\mathcal{I}_0 \equiv \nabla_i \Omega^i = 0$, as a "super-condition" which super-selects the fully-Diff invariant sector in Horava gravity !
- Cf. Hamiltonian formalism (DD-MIP): $\mathcal{I}_0 = \Omega - \nabla_i (N^2 \mathcal{H}^i) = 0$

$$\mathcal{H}^i \equiv -2\nabla_j \pi^{ij} \approx 0 \qquad \Omega \equiv \nabla_i (N^2 C^i) \approx 0$$ (Tertiary constraint)

On the super-condition

- 1. This should be off-shell condition.
- 2. On can not derive (i.e., not a mathematical identity) as far as we know.
- 3. But we should check its consistency, if it is correct.
- 4. This might be a fundamentally new hyphothesis.
- 5. This might provide a new reinterpretation of the very meaning of Horava gravity.
- Etc…

- **Super-potential:** $\mathcal{U}^{ij} = -\mathcal{U}^{ji} \equiv \mathcal{A}^{ij}(\xi^0) + \mathcal{B}^{ij}(\xi^m)$

$$\mathcal{A}^{ij}(\xi^0) \equiv \mathcal{A}^{ij}_{(0)} + \xi \mathcal{A}^{ij}_{(1)} + \alpha\, \mathcal{A}^{ij}_{(2)} + \beta\, \mathcal{A}^{ij}_{(3)} + \gamma\, \mathcal{A}^{ij}_{(4)}, \tag{A20}$$

$$\mathcal{A}^{ij}_{(0)} = 2\sqrt{g}\left(\frac{2}{\kappa^2}\right)\left[2\xi^0 N_m N^{[j} G^{i]mkl} K_{kl}\right],$$

$$\mathcal{A}^{ij}_{(1)} = 2\sqrt{g}\left[g^{i[k} g^{l]j}\left(2\xi^0 N_l \nabla_k N + N\nabla_l(\xi^0 N_k)\right)\right],$$

$$\mathcal{A}^{ij}_{(2)} = 2n\sqrt{g}\left[R^{n-1} g^{i[k} g^{l]j}\left(2\xi^0 N_l \nabla_k N + N\nabla_l(\xi^0 N_k)\right) + 4\xi^0 N N_l g^{i[l} g^{k]j} \nabla_k R^{n-1}\right],$$

$$\mathcal{A}^{ij}_{(3)} = 2s\sqrt{g}\left[2s\mu^{s-1} g^{[j}_{[l} R^{i]}_{k]}\left(2\xi^0 N^l \nabla^k N + N\nabla^l(\xi^0 N^k)\right) + 8\xi^0 N N^l g^{[i}_{[l} \nabla^{|k|}(\mu^{s-1} R^{j]}_{k]})\right],$$

$$\mathcal{A}^{ij}_{(4)} = 2r\sqrt{g}\left[8\xi^0 N N^l \nabla^k\left(\rho^{r-1} R^{[j}{}_{kl}{}^{i]}\right) + 2\rho^{r-1} R^{ijkl}\left(2\xi^0 N_l \nabla_k N + N\nabla_l(\xi^0 N_k)\right)\right],$$

$$\mathcal{B}^{ij}(\xi^m) \equiv \mathcal{B}^{ij}_{(0)} + \xi \mathcal{B}^{ij}_{(1)} + \alpha\, \mathcal{B}^{ij}_{(2)} + \beta\, \mathcal{B}^{ij}_{(3)} + \gamma\, \mathcal{B}^{ij}_{(4)}, \tag{A21}$$

$$\mathcal{B}^{ij}_{(0)} = 2\sqrt{g}\left(\frac{2}{\kappa^2}\right)\left[2\xi_m N^{[j} G^{i]mkl} K_{kl}\right],$$

$$\mathcal{B}^{ij}_{(1)} = 2\sqrt{g}\left[2g^{l[i} g^{j]}{}_k (N\nabla^k \xi_l - 2\xi_l \nabla^k N)\right],$$

$$\mathcal{B}^{ij}_{(2)} = 2n\sqrt{g}\left[2g^{l[i} g^{j]}{}_k R^{n-1} N\nabla^k \xi_l + 4\xi_l \nabla^k (g^{l[j} g^{i]}{}_k R^{n-1} N)\right],$$

$$\mathcal{B}^{ij}_{(3)} = 2s\sqrt{g}\left[4\mu^{s-1} g^{[j[k} R^{i]l]} N\nabla_k \xi_l + 8\xi_l \nabla_k(\mu^{s-1} g^{[j[k} R^{i]l]} N)\right],$$

$$\mathcal{B}^{ij}_{(4)} = 2r\sqrt{g}\left[4\rho^{r-1} R^{k[ij]l} N\nabla_k \xi_l + 8\xi_l \nabla_k(\rho^{r-1} R^{k[ji]l} N)\right].$$

- **Eqn.:**

$$\mathbf{E}^{ij} \equiv E^{ij}_{(0)} + \xi E^{ij}_{(1)} + \alpha E^{ij}_{(2)} + \beta E^{ij}_{(3)} + \gamma E^{ij}_{(4)}, \tag{A17}$$

$$\begin{aligned}
E^{ij}_{(0)} =& \sqrt{g}\left(\frac{2}{\kappa^2}\right)\Big[-N^i\nabla_k K^{jk} - N^j\nabla_k K^{ik} + K^{ik}\nabla^j N_k + K^{jk}\nabla^i N_k + N^k\nabla_k K^{ij} \\
&+2NK^{ik}K^j{}_k - NKK^{ij} + \frac{1}{2}g^{ij}NK^{kl}K_{kl} - g^{ik}g^{jl}\dot{K}_{kl}\Big] \\
&+\lambda\sqrt{g}\left[\frac{1}{2}Ng^{ij}K^2 + N^j\nabla^i K + N^i\nabla^j K - g^{ij}N^k\nabla_k K - g^{ij}K^{kl}\dot{g}_{kl} + g^{ij}g^{kl}\dot{K}_{kl}\right], \\
E^{ij}_{(1)} =& \sqrt{g}\left[N\left(-R^{ij} + \frac{1}{2}Rg^{ij} + \frac{\Lambda}{\xi}g^{ij}\right) + \left(g^{il}g^{jk} - g^{ij}g^{kl}\right)\nabla_l\nabla_k N\right], \\
E^{ij}_{(2)} =& \sqrt{g}\left[N\left(-nR^{n-1}R^{ij} + \frac{1}{2}R^n g^{ij}\right) + n\left(g^{il}g^{jk} - g^{ij}g^{kl}\right)\nabla_l\nabla_k\left(NR^{n-1}\right)\right], \\
E^{ij}_{(3)} =& \sqrt{g}\Big[N\left(-2s\mu^{s-1}R^{ik}R^j{}_k + \frac{1}{2}\mu^s g^{ij}\right) \\
&+s\left(g^{ik}g^{jm}g^{ln} + g^{jk}g^{mi}g^{nl} - g^{kl}g^{mi}g^{nj} - g^{ij}g^{km}g^{ln}\right)\nabla_l\nabla_k\left(N\mu^{s-1}R_{mn}\right)\Big], \\
E^{ij}_{(4)} =& \sqrt{g}\left[N\left(-2r\rho^{r-1}R^{iklm}R^j{}_{klm} + \frac{1}{2}\rho^r g^{ij}\right) + 4r\nabla_k\nabla_l\left(\rho^{r-1}NR^{iklj}\right)\right],
\end{aligned}$$

$$\mu \equiv R_{ij}R^{ij}, \;\; \rho \equiv R_{ijkl}R^{ijkl},$$

3. Some consequences

- **1. Noether currents:**

$$\partial_\mu \mathcal{J}^\mu(\delta_\xi g) = \mathcal{H}\delta_\xi N + \mathcal{H}^i \delta_\xi N_i - \mathrm{E}^{ij}\delta_\xi g_{ij}.$$

$$\begin{aligned} \mathcal{J}^\mu(\delta_\xi g) &\equiv \Theta^\mu(\delta_\xi N_i, \delta_\xi g_{ij}) - \Psi^\mu(\delta_\xi N_i, \delta_\xi g_{ij}) \\ &= \Theta^\mu - \xi^\mu \mathcal{L} - \Sigma^\mu - \partial_\nu \mathcal{U}^{\mu\nu}, \end{aligned}$$

$$Q(\xi) = \int_\Sigma d^D x \sqrt{g}\; n_\mu \mathcal{J}^\mu(\delta_\xi g)$$

- **For (3+1)-dim, static black hole [KS, LMP, Park],**

$$N^2 = f = k + (\omega - \Lambda_W) r^2 + \epsilon \sqrt{r[\omega(\omega - 2\Lambda_W) r^3 + \beta]},$$

- $Q(\xi^0) = \hat{\alpha}\beta$, **which agrees with known mass,** $\breve{\mathcal{M}} \equiv Q(\xi^0)$

- 2. Only the covariantly-conserved matter can couple to Horava gravity, i.e., $\hat{\nabla}^{\mu} T_{\mu\nu} = 0$;

- 3. New light on the very meaning of black hole entropy and its thermodynamics:

Covariant black! hole horizon?=> Hawking radiations!?