Inhomogeneous Couplings and Background Geometry in QFT

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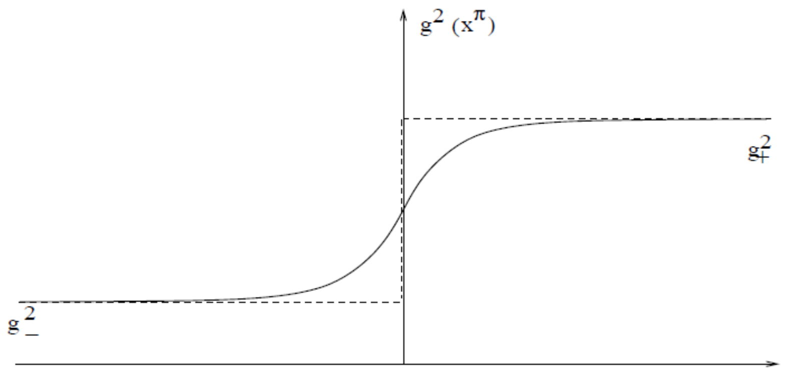
[Kyung Kiu Kim-**OK**, 1806.06963]
[Chanju Kim-Kyung Kiu Kim-Yoonbai Kim-**OK**, 1910.05044]
[Byoungjoon Ahn-Seungjoon Hyun-Kyung Kiu Kim-**OK**-Sang-A Park, 1911.05783, 1912.00784]
[Yoonbai Kim-**OK**-Dirba D. Tolla, 2008.00868]
[Chanju Kim-Yoonbai Kim-OK-Hanwool Song, 2110.13393, 2112.xxxx]
[Chanju Kim-Yoonbai Kim-OK-Hanwool Song-D.Tolla, 2201.xxxx]

Janus field theories Vs Inhomogeneous field theories

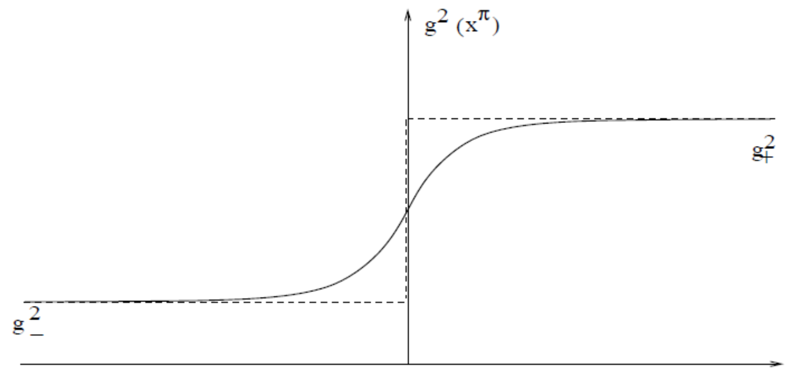
- spatially varying coupling parameters



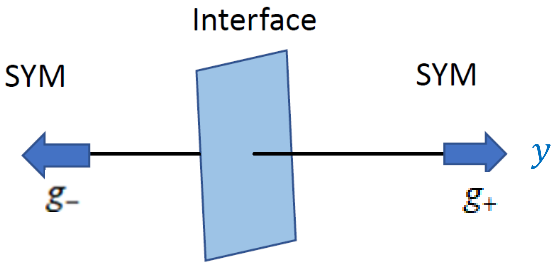
- space-dependent coupling parameter g=g(x)
- Inhomogeneous mass m=m(x)

Janus field theories Vs Inhomogeneous field theories

- spatially varying coupling parameters



Janus super Yang-Mills theory

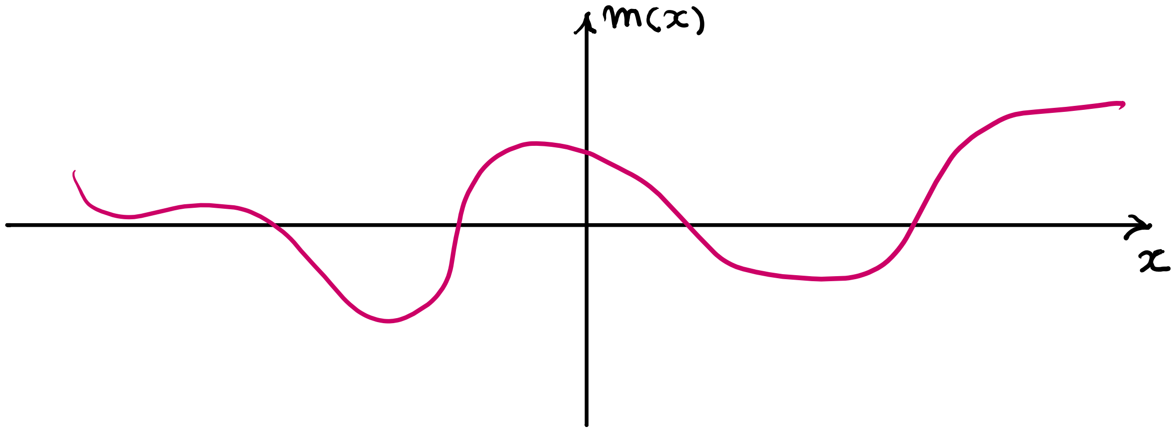


- usual coupling parameter g=g(x)
- Inhomogeneous mass m=m(x)

$$S_{\text{Janus}} = \int_{-\infty}^{0} \mathrm{dy} \int d^3x \mathcal{L}_{\text{SYM}}(g_-) + \int_{y=0} d^3x \mathcal{L}_{\text{interface}} + \int_{0}^{+\infty} \mathrm{dy} \int d^3x \mathcal{L}_{\text{SYM}}(g_+)$$

Janus field theories Vs Inhomogeneous field theories

- **spatially varying coupling parameters**



[Kyung Kiu Kim-**OK**, 1806.06963]

Inhomogenous mass-deformed ABJM theory

$$\mathcal{L}_{\text{ImABJM}} = \mathcal{L}_{\text{ABJM}} - \hat{V}_{\text{ferm}} - \hat{V}_{\text{flux}} - \hat{V}_{\text{mass}} - \hat{V}_{J}$$

$$\hat{V}_J = m'\text{tr}\left(Y_a^\dagger Y^a - Y_i^\dagger Y^i\right)$$

m=m(x)
arbitrary function

- usual coupling constant g=g(x)

- Inhomogeneous mass m=m(x)

Janus field theories Vs Inhomogeneous field theories

- **Dual gravity origin (AdS/CFT correspondence)**

[Bak- Gutperle-Hirano, 2003]

usual coupling constant g=g(x)

turning on spatially varying background **dilaton field** $\frac{g(x)^2}{4\pi} = e^{\phi(x)}$

Inhomogeneous mass m=m(x) [Kim-OK, 2018] [Kim-Kim-Kim-OK, 2019] [Arav et al, 2020] [Kim-OK-Tolla, 2020]

turning on spatially varying 4-form field strength in 11-dim. SUGRA (M-theory)

RR 7-form field strength (IIB SUGRA)

$$F_{AB\bar{C}\bar{D}} = T_{AB\bar{C}\bar{D}}(w_1)$$

$$T_{12\bar{1}\bar{2}} = -m, \quad T_{34\bar{3}\bar{4}} = m$$

Inhomogeneously mass-deformed ABJM (ImABJM)
Inhomogeneously mass-deformed SYM (ImSYM)

- **arbitrary mass functions m=m(x)** with some reduced supersymmetries
  ImABJM [Kim-OK,Kim-Kim-Kim-Kwon, 2018], **ImSYM** [Arav et al, Kim-OK-Tolla, 2020]
  and their dual gravities for special mass functions
  [Gauntlett-Rosen,2018,Arav-Gauntlett-Roberts-Rosen,2019
  Arav-Gheung-Gauntlett-Roberts-Rosen,2020]

- Even for arbitrariness of the mass function, higher supersymmetries exist. In the point of view of the gauge/gravity in the top-down approach, which has rare examples, the supersymmetric ImABJM and ImSYM can compensate the **disadvantage of the top-down approach**

➔ possible applications of gauge/gravity in condensed matter physics with various backgrounds

Inhomogeneously mass-deformed ABJM(ImABJM)

- **Reduction of supersymmetry** $\mathcal{N}=6 \longrightarrow \mathcal{N}=3$ [Kim-OK,2018]

$$\gamma^1\omega_{ab} = -\omega_{ab} \iff \omega^{ab}\gamma^1 = \omega^{ab},$$
$$\gamma^1\omega_{ai} = \omega_{ai} \iff \omega^{ai}\gamma^1 = -\omega^{ai},$$

$a = 1,2$ and $i = 3,4,$

$$\omega^{AB} = -\omega^{BA} = (\omega_{AB})^* = \frac{1}{2}\epsilon^{ABCD}\omega_{CD}.$$

- **Deformation of the Lagrangian:**

$$-\frac{4\pi m}{k}M_B{}^D\left(Y_C^\dagger Y^C Y_D^\dagger Y^B - Y^C Y_C^\dagger Y^B Y_D^\dagger\right)$$: **flux term**

$$(m^2\delta_A^B + \boxed{m' M_A{}^B)\, Y^A Y_B^\dagger}$$: **mass term**

Inhomogeneously mass-deformed ABJM(ImABJM)

- Reduction of supersymmetry $\mathcal{N} = 6 \longrightarrow \mathcal{N} = 3$

$$\gamma^1\omega_{ab} = -\omega_{ab} \quad \Longleftrightarrow \quad \omega^{ab}\gamma^1 = \omega^{ab},$$
$$\gamma^1\omega_{ai} = \omega_{ai} \quad \Longleftrightarrow \quad \omega^{ai}\gamma^1 = -\omega^{ai},$$

$a = 1, 2$ and $i = 3, 4,$

- Deformation of the Lagrangian:

$$-\frac{4\pi m}{k} M_B{}^D \left(Y_C^\dagger Y^C Y_D^\dagger Y^B - Y^C Y_C^\dagger Y^B Y_D^\dagger\right)$$: flux term

$$\left(m^2 \delta_A^B + m' M_A{}^B\right) Y^A Y_B^\dagger$$: mass term

- Shape of the mass function: *m = m(x)*

⇒ ***arbitrary*** **function but it depends on only**
one spatial coordinate

Inhomogeneously mass-deformed SYM

❑ $\mathcal{N} = 4$ super Yang-Mills theory

- SU(N) gauge field $A_{\mu=0,1,2,3}$; 6 Hermitian scalar fields $\phi_{a=1,\ldots,6}$; super partners
- dynamics of *N* D3-branes in IIB SUGRA theory
- dual to IIB SUGRA on $\mathrm{AdS}_5 \times S^5$

❑ Mass deformation (constant mass) $\mathcal{N} = 1^*$

- SUSY preserving mass deformation is not possible
- deformation of supersymmetric rule for fermion $\delta'_\epsilon \psi_p = \mu_{pq}\phi_a\left(\Gamma^{q4}_a P_+ + \bar{\Gamma}^{q4}_a P_-\right)\epsilon$
- deformation of Lagrangian

fermionic mass matrix: $\mu_{pq} = \mathrm{diag}(\mu_1, \mu_2, \mu_3, 0)$

$$\mathcal{L}_\mu = \mathrm{tr}\left(-i\mu_{pq}\bar{\psi}_p\psi_q - M_{ab}\phi_a\phi_b + igT_{abc}\phi_a[\phi_b,\, \phi_c]\right)$$

$$M_{ab} = \mathrm{diag}(\mu_1^2, \mu_3^2, \mu_2^2, \mu_1^2, \mu_3^2, \mu_2^2)$$

$$T_{234} = \frac{1}{3}(\mu_1 - \mu_2 - \mu_3), \quad T_{126} = \frac{1}{3}(\mu_1 - \mu_2 + \mu_3)$$

$$T_{135} = \frac{1}{3}(\mu_1 + \mu_2 - \mu_3), \quad T_{456} = \frac{1}{3}(\mu_1 + \mu_2 + \mu_3)$$

Inhomogeneously mass-deformed SYM (ImSYM)

- **Space-dependent mass deformation** $\mu_m = \mu_m(x), \qquad m = 1, 2, 3$
  - projection for spinor $\gamma^1 \epsilon = \epsilon$

$$\mathcal{N} = 2^* \text{ mSYM} \longrightarrow \mathcal{N} = 1 \text{ ImSYM}$$

$$\mathcal{N} = 1^* \text{ mSYM} \longrightarrow \mathcal{N} = \frac{1}{2} \text{ ImSYM}$$

[Arav-Cheung-Gauntlett-Roberts-Rosen, Kim-OK-Tolla, 2020]

  - additional deformation of Lagrangian: $\mathcal{L}_J = -\mathrm{tr}\left(J'_{ab}\phi_a\phi_b\right)$

$$J_{ab} = \mathrm{diag}(\mu_1, \mu_3, \mu_2, -\mu_1, -\mu_3, -\mu_2) \quad \mathcal{N} = \frac{1}{2} \text{ ImSYM}$$

$$J_{ab} = \mathrm{diag}(\mu, 0, \mu, -\mu, 0, -\mu) \quad \mathcal{N} = 1 \text{ ImSYM}$$

Gravity dual of the ImABJM

- N=3 Inhomogeneously mass-deformed ABJM (Janus ABJM) model

  $m = m(x)$: arbitrary mass function [**OK**-K.Kim, JHEP (2018.06)]
  [K.Kim-Y.Kim-**OK**-C.Kim, JHEP (2019)]

↕

- SUSU Q-lattice geometry in 11-dimensional gravity

  For a special mass function: [Gauntlett-Rosen, JHEP (2018.08)]
  $m(x) = m_0 \sin(kx)$ [Arav-Gauntlett-Roberts-Rosen, JHEP (2018.12)]

  - Black brane solution dual to the N=3 ImABJM at finite temperature with the mass function [Ahn-Hyun-**OK**-Park, JHEP 2020]

Inhomogeneous coupling constant deformations in 1+1 dimensions

- **Supersymmetric field theories with Poincare symmetry**

  - Energy is nonnegative definite ➔ vacuum energy is zero [Witten-Olive, 1978]

  - existence of BPS objects : first order Bogomolny equation, preserve part of supersymmetries, such as lumps, kinks, vortices, monopoles, moduli space

- **Supersymmetric field theories with explicitly broken translation symmetry?**

Inhomogeneous coupling constant deformations in 1+1 dimensions

- 2-dimensional *N*=1 supersymmetric real scalar field theory

$$S = \int d^2x \Big[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + i\bar{\psi}\gamma^\mu\partial_\mu\psi + iW''(\phi)\,\bar{\psi}\psi - \frac{1}{2}W'(\phi)^2 \Big] \qquad W' \equiv \frac{dW}{d\phi}$$

$$\delta\phi = i\psi\epsilon, \qquad \gamma^\mu = (i\sigma^2, \sigma^1) \text{ with } \mu = 0, 1$$

$$\delta\psi = -\frac{1}{2}\gamma^\mu\partial_\mu\phi\,\epsilon + \frac{1}{2}W'\epsilon$$

$$Q_\epsilon = \int dx J^0_\epsilon = i\epsilon_+ Q_+ + i\epsilon_- Q_- \quad \text{with} \quad Q_\pm = \int dx \Big((\partial_0\phi \pm \partial_1\phi)\psi_\pm \mp W'\psi_\mp \Big)$$

$$\bar{\epsilon}^\alpha = (\epsilon_+, \epsilon_-) \text{ with } \bar{\epsilon} \equiv \epsilon^\dagger = \epsilon^T$$

$$\{Q_\pm, Q_\pm^\dagger\} = 2(P^0 \mp P^1), \qquad \{Q_\pm, Q_\mp^\dagger\} = 2T$$

$$T = \int dx (\partial_1\phi) W'(\phi) = \int dx \frac{dW(\phi(x))}{dx} = W(\phi(\infty)) - W(\phi(-\infty))$$

Inhomogeneous coupling constant deformations in 1+1 dimensions

- **2-dimensional *N*=1 supersymmetric real scalar field theory**

$$E = P^0 = \frac{1}{4}\{Q_+ \pm Q_-, Q_+^\dagger \pm Q_-^\dagger\} \mp T$$

[Witten-Olive, 1978]

$$E \geq |T|$$

Inhomogeneous coupling constant deformations in 1+1 dimensions

- 2-dimensional *N*=1 supersymmetric real scalar field theory

$$E = P^0 = \frac{1}{4}\{Q_+ \pm Q_-, Q_+^\dagger \pm Q_-^\dagger\} \mp T$$

[Witten-Olive, 1978]

$$E \geq |T|$$

- Homogeneous QFT ➔ Inhomogeneous QFT (ImQFT)

$$W(\phi) = \sum_i m_i \tilde{W}(\phi) \implies W(\phi, x) = \sum_i m_i(x) \tilde{W}(\phi)$$

Inhomogeneous coupling constant deformations in 1+1 dimensions

$$\delta\psi = -\frac{1}{2}\gamma^\mu\partial_\mu\phi\,\epsilon + \frac{1}{2}W'\epsilon \quad \longrightarrow \quad \delta\psi = -\frac{1}{2}\gamma^\mu\partial_\mu\phi\,\epsilon + \frac{1}{2}\frac{\partial W}{\partial\phi}\epsilon$$

$W' \equiv \frac{dW}{d\phi}$

Projection: $\gamma^1\epsilon = \pm\epsilon$ $\quad \epsilon_- = \pm\epsilon_+$

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + i\bar{\psi}\gamma^\mu\partial_\mu\psi + i\frac{\partial^2 W}{\partial\phi^2}\bar{\psi}\psi - \frac{1}{2}\left(\frac{\partial W}{\partial\phi}\right)^2 \boxed{\mp\frac{\partial W}{\partial x}}$$

$$\bar{Q}_\epsilon = i\epsilon_+\bar{Q} \qquad \bar{Q} = \int dx\Big[(\partial_0\phi + \partial_1\phi - \partial_\phi W)\psi_+ - (\partial_0\phi - \partial_1\phi + \partial_\phi W)\psi_-\Big]$$

new term

[Kim-Kim-OK, arXiv:2110.13393]

Position dependent potential

$$V(\phi, x) \equiv \frac{1}{2}\left(\frac{\partial W}{\partial\phi}\right)^2 + \frac{\partial W}{\partial x} \qquad E = \frac{1}{4}\{\bar{Q}, \bar{Q}^\dagger\} + T$$

need not be nonnegative definite.

Inhomogeneous coupling constant deformations in 1+1 dimensions

- Bogomolny equation:

$$E = \int dx \left[\frac{1}{2}(\partial_0\phi)^2 + \frac{1}{2}(\partial_1\phi)^2 + \frac{1}{2}\left(\frac{\partial W}{\partial \phi}\right)^2 + \frac{\partial W}{\partial x}\right]$$

$$= \int dx \left[\frac{1}{2}(\partial_0\phi)^2 + \frac{1}{2}\left(\partial_1\phi - \frac{\partial W}{\partial \phi}\right)^2\right] + T.$$

$$\partial_1\phi - \frac{\partial W}{\partial \phi} = 0$$

- Extension to non-canonical case:

$$\mathcal{L} = -\frac{1}{2}K^2\partial_\mu\phi\partial^\mu\phi + i\bar{\psi}\gamma^\mu\partial_\mu\psi + \frac{i}{K}\frac{\partial}{\partial\phi}\left(\frac{1}{K}\frac{\partial W}{\partial \phi}\right)\bar{\psi}\psi - \frac{1}{2}\left(\frac{1}{K}\frac{\partial W}{\partial \phi}\right)^2 \boxed{\mp\frac{\partial W}{\partial x}}$$

$$\delta\phi = i\psi\epsilon,$$

$$\delta\psi = -\frac{1}{2}K^2\gamma^\mu\partial_\mu\phi\,\epsilon + \frac{1}{2}\frac{\partial W}{\partial \phi}\epsilon$$

$$K\partial_1\phi \mp \frac{1}{K}\frac{\partial W}{\partial \phi} = 0$$

$K = K(\phi, x)$ ➔ $K = 1$

Supersymmetric BPS solutions

- Position-dependent Rescaling of Superpotential

[Adam-Queiruga-Wereszczynski, 2019]

[Kim-Kim-OK, 2021]

$$W(\phi, x) = g(x) W_0(\phi)$$

$$\partial_1 \phi - \frac{\partial W}{\partial \phi} = 0 \quad \rightarrow \quad \phi'(x) - g(x) W_0'(\phi) = 0$$

$$\begin{aligned} E_{\phi=\phi_i} &= T \\ &= g(\infty) W_0(\phi(\infty)) - g(-\infty) W_0(\phi(-\infty)) \\ &= (g_R - g_L) W_0(\phi_i). \end{aligned}$$

constant configurations $\phi = \phi_i$

$g(\infty) = g_R \quad g(-\infty) = g_L$

Supersymmetric BPS solutions

- ϕ^4 **theory** $$V(\phi, x) = \frac{1}{2}g^2(x)(\phi^2 - v^2)^2 + \frac{1}{3}g'(x)(\phi^3 - 3v^2\phi)$$

$$W(\phi, x) = g(x)W_0(\phi) \qquad W_0 = \int (\phi^2 - v^2)d\phi = \frac{1}{3}\phi^3 - v^2\phi$$

W_0 involves two extrema $\phi = \pm v$ which trivially satisfy the Bogomolny equation

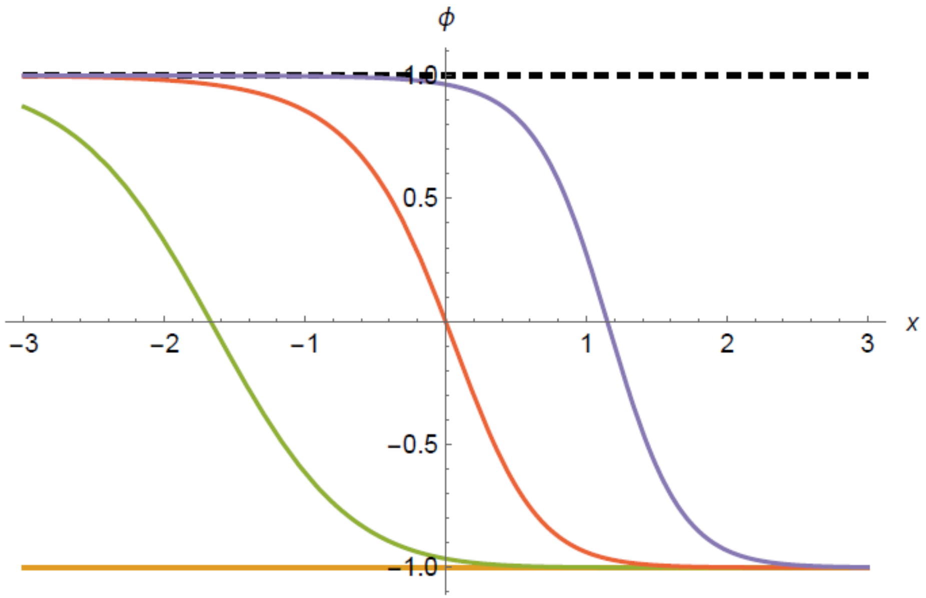
$$E_{\phi=\pm v} = \mp\frac{2}{3}v^3(g_R - g_L)$$

$$E_{\text{vac}} = -\frac{2}{3}v^3|g_R - g_L|$$

Constant vacuum solution with negative energy

Supersymmetric BPS solutions

- Space-dependent BPS solutions with non-negative energy:



$$g_L = 1 \text{ and } g_R = 2 \qquad v = a = 1$$

$$g(x) = \frac{1}{2}(g_R + g_L) + \frac{1}{2}(g_R - g_L)\tanh ax, \qquad (a > 0)$$

$$\phi(x) = -v\tanh(vG(x) - c)$$

$$G(x) = \int_0^x g(x')dx' \qquad |\phi(x)| < v$$

$$E_{\rm nc} = g(\infty)W_0(\phi(\infty)) - g(-\infty)W_0(\phi(-\infty))$$
$$= \frac{2}{3}v^3(|g_R| + |g_L|).$$

Supersymmetric BPS solutions

- Supersymmetric Sine-Gordon model

$$W(\phi, x) = m(x) W_0(\phi) \qquad V = 2\frac{m(x)^2}{\beta^2}\sin^2\frac{\beta\phi}{2} - 4\frac{m'(x)}{\beta^2}\cos\frac{\beta\phi}{2}$$

$$W_0(\phi) = -\frac{4}{\beta^2}\cos\frac{\beta\phi}{2} \qquad \phi = \frac{2n\pi}{\beta}, \qquad n \in \mathbb{Z} \qquad W_0\left(\frac{2n\pi}{\beta}\right) = (-1)^{n+1}\frac{4}{\beta^2}$$

Bogomolny equation: $$\frac{d\phi}{dx} = \frac{2m(x)}{\beta}\sin\frac{\beta\phi}{2}$$

Constant solution: $$E_{\phi_n} = (-1)^{n+1}\frac{4}{\beta^2}(m_R - m_L) \qquad m_R = m(\infty) \text{ and } m_L = m(-\infty)$$

Vacuum energy: $$E_{\text{vac}} = -\frac{4}{\beta^2}|m_R - m_L|$$

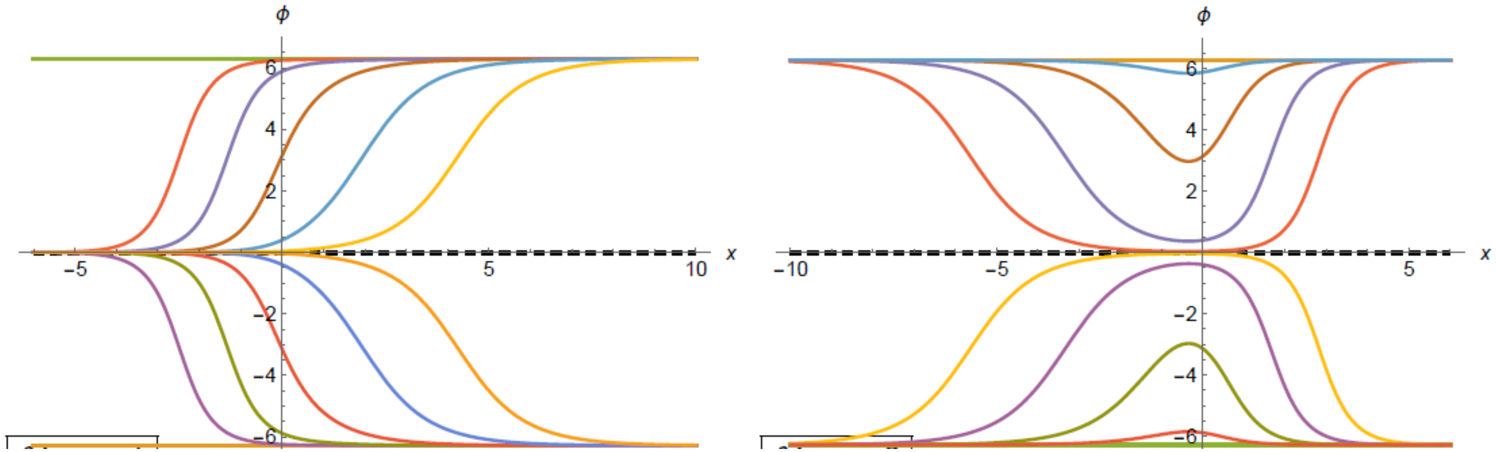
Supersymmetric BPS solutions

- Non-constant solutions

$$\phi(x) = \frac{4}{\beta}\tan^{-1}[c\, e^{\mu(x)}] \quad \left(\text{mod } \tfrac{4\pi}{\beta}\right) \qquad\qquad \mu(x) = \int_0^x m(x')dx'$$

$$m(x) = \frac{1}{2}(m_R + m_L) + \frac{1}{2}(m_R - m_L)\tanh ax$$

$$\begin{aligned} E_{\rm nc} &= m(\infty)W_0(\phi(\infty)) - m(-\infty)W_0(\phi(-\infty)) \\ &= \frac{4}{\beta^2}(|m_L| + |m_R|), \end{aligned}$$



$m_L = 2 > m_R = 1$

Supersymmetric BPS solutions

- Inhomogeneous Deformation of the Vacuum Expectation value in ϕ^6 theory

Inhomogeneous superpotential: $$W(\phi, x) = \frac{\lambda}{4}[\phi^4 - 2w(x)\phi^2]$$

Inhomogeneous potential: $$V(\phi, x) = \frac{\lambda^2}{2}\phi^2(\phi^2 - w(x))^2 - \frac{\lambda}{2}w'(x)\phi^2$$

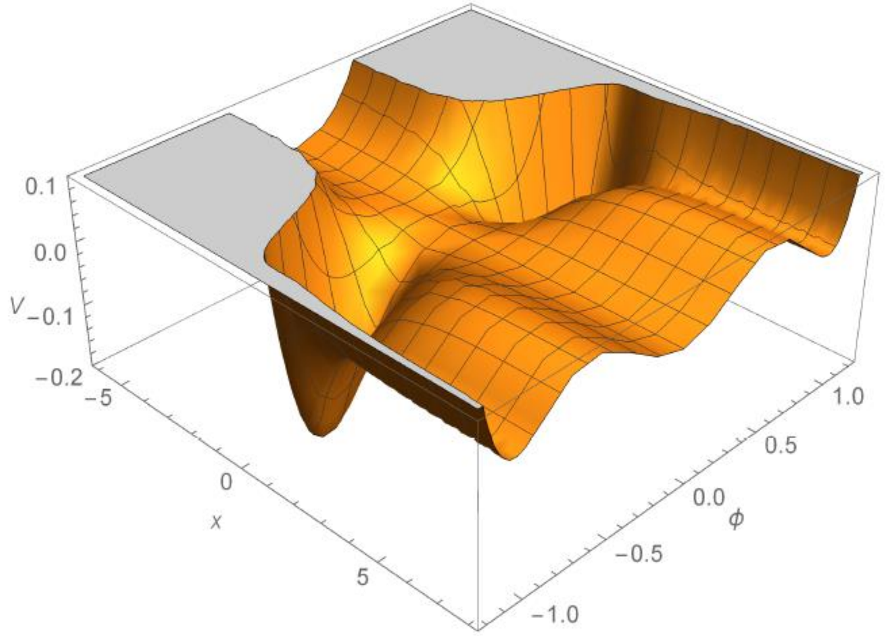
BPS equation: $$\phi'(x) - \lambda\phi(x)[\phi^2(x) - w(x)] = 0$$

$$W(0) = 0, \qquad W_L < 0 \text{ and } W_R < 0$$

Extremum values of $W(\phi, \pm\infty)$
$$W(\pm\sqrt{w_R}) = -\frac{\lambda}{4}w_R^2$$
$$W(\pm\sqrt{w_L}) = -\frac{\lambda}{4}w_L^2$$

Supersymmetric BPS solutions

- Shape of inhomogeneous potential:



$w(x) = \tanh x$ and $\lambda = v = 1$

An obvious solution is $\phi(x) = 0$ of which the energy vanishes.

Inhomogeneous solutions for general mass funciton m(x)

$$\phi^2(x) = \frac{e^{-2\lambda\Omega(x)}}{2\lambda(c-\xi(x))}$$

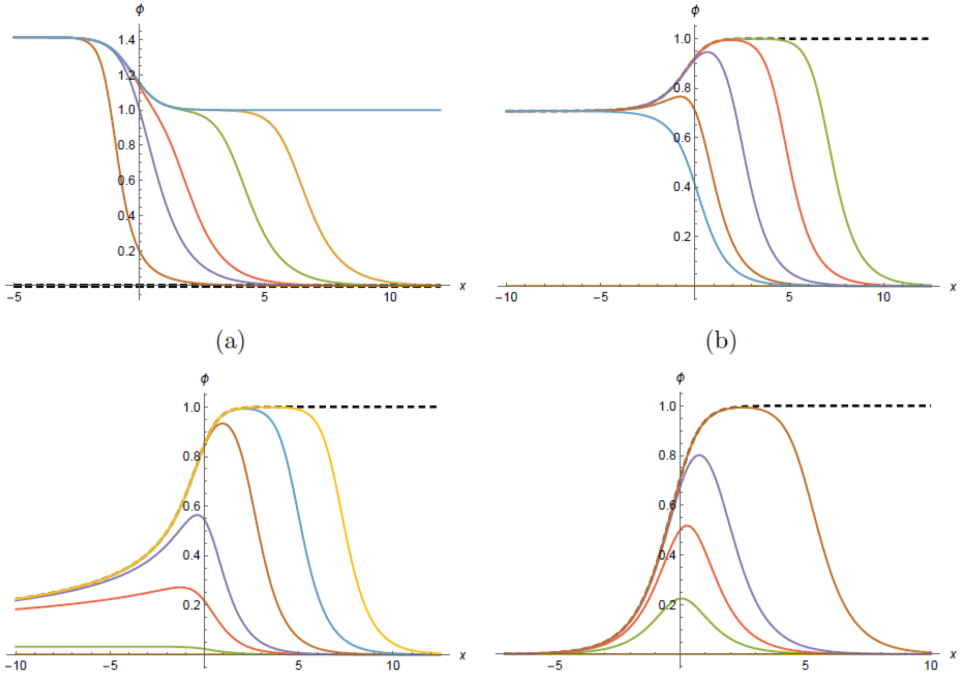
$$\Omega(x) = \int w(x)dx,$$

$$\xi(x) = \int e^{-2\lambda\Omega(x)}dx$$

Supersymmetric BPS solutions

- Classification of non-constant solutions: $W_R \geq 0$

w_L	c	$\phi^2(-\infty)$	$\phi^2(\infty)$	E
$w_L > 0$	$c > \xi(\infty)$	w_L	0	$\frac{\lambda}{4} w_L^2$
	$c = \xi(\infty)$	w_L	w_R	$\frac{\lambda}{4}(w_L^2 - w_R^2)$
$w_L \leq 0$	$c > \xi(\infty)$	0	0	0
	$c = \xi(\infty)$	0	w_R	$-\frac{\lambda}{4} w_R^2$



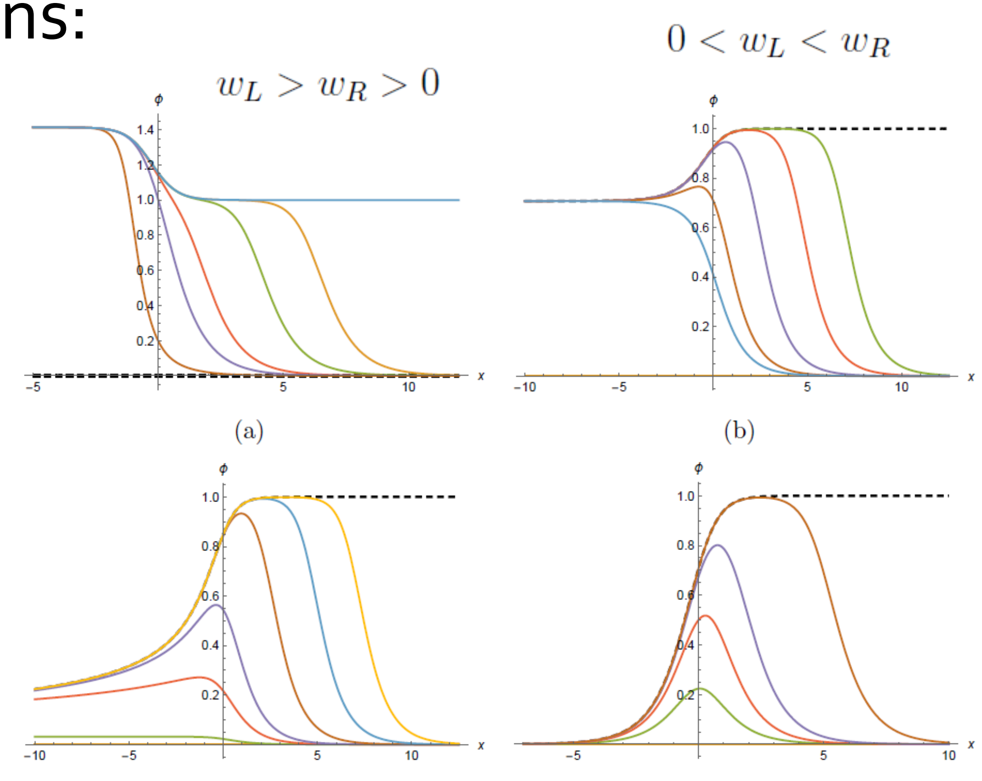
(a) (b) (c) (d)

Supersymmetric BPS solutions

- Classification of non-constant solutions:

w_L	c	$\phi^2(-\infty)$	$\phi^2(\infty)$	E
$w_L > 0$	$c > \xi(\infty)$	w_L	0	$\frac{\lambda}{4}w_L^2$
	$c = \xi(\infty)$	w_L	w_R	$\frac{\lambda}{4}(w_L^2 - w_R^2)$
$w_L \leq 0$	$c > \xi(\infty)$	0	0	0
	$c = \xi(\infty)$	0	w_R	$-\frac{\lambda}{4}w_R^2$

c : degenerate solutions ➔ zero modes



(a) (b)
(c) (d)

Higher dimensional well-known low supersymmetric theories

- Usual supersymmetric QFTs have translational symmetries, Poincare symmetry. ➔ vacuum is constant ➔ we can consider fluctuations on the homogeneous vacuum.
- Introducing space-dependent coupling parameters can be interpreted as introducing background fields with inhomogeneous profiles
- These background fields can interact with dynamical supersymmetric fields ➔ usually supersymmetry is broken without additional term
- Add a term ➔ half of supersymmetry is recovered ➔ BPS vacuum equation (first order different equation)
- Exact inhomogeneous vacuum solutions with negative energies
- Homogeneous vacuum + fluctuations and quantizations

➔ inhomogeneous vacuum + fluctuations (quantization??) : new approach is needed

Higher dimensional well-known lower supersymmetric QFTs

- Is this applicable to other well-known lower supersymmetric QFTs, such as N=2 Abelian Higgs model, N=2 Chern-Simons Higgs model in 2, 3, and 4-dimensions.
- The answer is yes.
- For instance, $\mathcal{N}=2$ Chern-Simons Higgs model in 3-dimensions

$$\bar{\mathcal{L}} = -D_\mu\bar{\phi}D^\mu\phi + i\bar{\psi}\gamma^\mu D_\mu\psi + \frac{k}{4\pi}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho + i\mu\bar{\psi}\psi - 3i\lambda|\phi|^2\bar{\psi}\psi - |\phi|^2\left(\lambda|\phi|^2-\mu\right)^2,$$

$$\lambda = \tfrac{2\pi}{k}e^2,\ \gamma^\mu = (i\sigma^2, \sigma^2, \sigma^3)$$

$$\delta\phi = i\bar{\epsilon}\psi$$
$$\delta\psi = -\gamma^\mu D_\mu\phi\epsilon - \lambda|\phi|^2\phi\epsilon + \mu\phi\epsilon,$$
$$\delta\alpha_\mu = -\frac{\lambda}{e}\left(\bar{\epsilon}\gamma_\mu\psi\bar{\phi} + \phi\bar{\psi}\gamma_\mu\epsilon\right),$$

[Kim-Kim-OK-Song, 2112.xxx]

introduce inhomogeneous vacuum value $\mu = \mu(x)$

$$\mathcal{L} = \bar{\mathcal{L}} - \mu'|\phi|^2$$

Higher dimensional well-known lower supersymmetric QFTs

- Is this applicable to other well-known lower supersymmetric QFTs, such as N=2 Abelian Higgs model, N=2 Chern-Simons Higgs model in 2, 3, and 4-dimensions.
- The answer is yes.
- For instance, N=2 Abelian Higgs model in 3-dimensions

$$\begin{aligned}\bar{\mathcal{L}} =& -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{k}{4\pi}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - \frac{1}{2}\partial_\mu N\partial^\mu N + i\bar{\chi}\gamma^\mu\partial_\mu\chi \\ & - D_\mu\bar{\phi}D^\mu\phi + i\bar{\psi}\gamma^\mu D_\mu\psi + i\sqrt{2}\,e\left(\bar{\psi}\chi\phi + \bar{\chi}\psi\bar{\phi}\right) + ieN\bar{\psi}\psi \\ & - e^2N^2|\phi|^2 - \frac{1}{2}e^2\left(|\phi|^2 + \frac{k}{2\pi e}N - \mu\right)^2,\end{aligned}$$

[Kim-Kim-OK-Song, 2112.xxx]

$$\begin{aligned}&\delta\phi = i\sqrt{2}\bar{\epsilon}\psi, \qquad \delta N = i(\bar{\epsilon}\chi + \bar{\chi}\epsilon), \\ &\delta\psi = -\sqrt{2}\gamma^\mu D_\mu\phi\,\epsilon + \sqrt{2}eN\phi\epsilon, \\ &\delta\chi = -\gamma^\mu\partial_\mu N\epsilon + \frac{i}{2}\epsilon^{\mu\nu\rho}F_{\mu\nu}\gamma_\rho\epsilon + e\left(|\phi|^2 + \frac{k}{2\pi e}N - \mu\right)\epsilon \\ &\delta A_\mu = \bar{\epsilon}\gamma_\mu\chi + \bar{\chi}\gamma_\mu\epsilon,\end{aligned}$$

$$\mathcal{L} = \bar{\mathcal{L}} + e\mu' N$$

Discussions

- Inhomogeneously mass-deformed ABJM (N=3,2,1), Super Yang-Mills (N=1, ½)
- Real scalar supersymmetric model in 1+1 dimensions, ϕ^6-theory, Sine-Gordon
- Vacuum equation in the inhomogeneous background is analytically solvable
- Vacuum solution has space-dependent profile with negative vacuum energy

- Inhomogeneous mass deformation for lower supersymmetric well-known models, N=2, Chern-Simons Higgs, Abelian Higgs model in 2,3,4 dimensions. There were many trials to apply these models to condensed matter physics, such as QHE and superconductor model, etc.
- Inhomogeneous background ➔ space-dependent vacuum with negative energy

  ➔ physics can be changed, need to try to find another possibilities using inhomogeneous model ➔ QHE, superconductor, topological insulator, topological matter, etc
- Deformation of supersymmetric Sine-Gordon ➔ integrability for some special mass function?