Rectifying No-Hair Theorems in Gauss-Bonnet Theory

in collaboration with Alexandros Papageorgiou and Chan Park (IBS-CTPU) [1]

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Uniqueness Theorem

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- By Mazur in1982 and Bunting in 1983, it is studied for stationary and axisymmetric case. the Kerr-Newman solutions.

Israel-Carter Conjecture

Nonmeasurability of the Baryon Number of a Black-Hole (*).

C. TEITELBOIM (**)

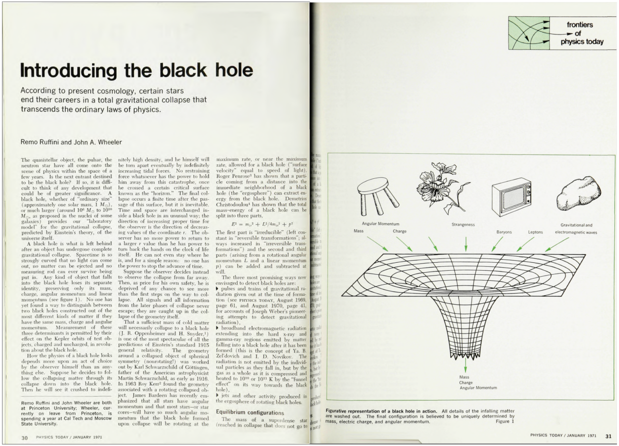
Joseph Henry Laboratories, Princeton University - Princeton, N. J.

(ricevuto il 3 Dicembre 1971)

One of the most astonishing features of gravitational collapse is the presumed « ideal perfection » of the final state. All theoretical evidence favors the Israel-Carter conjecture, which says that the most general final configuration of gravitational collapse is a Kerr-Newman black-hole. If this conjecture is true, then it follows that the only measurable quantum numbers of a black-hole are mass, charge and angular momentum—these three quantities being the only adjustable parameters appearing in the Kerr-Newman metric. Any other particularity that the collapsing matter had fades away.

Now, baryon number, one of the key quantities of particle physics is not in the list. The validity of the Israel-Carter conjecture implies then that the baryon number

No-Hair Conjecture by Ruffini and Wheeler in 1971



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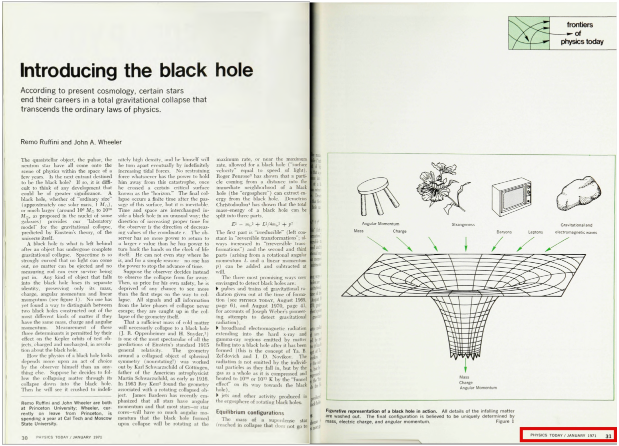

frontiers of physics today

Introducing the black hole

According to present cosmology, certain stars end their careers in a total gravitational collapse that transcends the ordinary laws of physics.

Remo Ruffini and John A. Wheeler

The quasistellar object, the pulsar, the neutron star have all come onto the scene of physics within the space of a few years. Is the next entrant destined to be the black hole? If so, it is difficult to think of any development that could be of greater significance. A black hole, whether of "ordinary size" (approximately one solar mass, 1 $M_\odot$), or much larger (around 10^6 $M_\odot$ to 10^{10} $M_\odot$, as proposed in the nuclei of some galaxies) provides our "laboratory model" for the gravitational collapse, predicted by Einstein's theory, of the universe itself.

A black hole is what is left behind after an object has undergone complete gravitational collapse. Spacetime is so strongly curved that no light can come out, no matter can be ejected and no measuring rod can ever survive being put in. Any kind of object that falls into the black hole loses its separate identity, preserving only its mass, charge, angular momentum and linear momentum (see figure 1). No one has yet found a way to distinguish between two black holes constructed out of the most different kinds of matter if they have the same mass, charge and angular momentum. Measurement of these three determinants is permitted by their effect on the Kepler orbits of test objects, charged and uncharged, in revolution about the black hole.

How the physics of a black hole looks depends more upon an act of choice by the observer himself than an anything else. Suppose he decides to follow the collapsing matter through its collapse down into the black hole. Then he will see it crushed to indefinitely high density, and he himself will be torn apart eventually by indefinitely increasing tidal forces. No restraining force whatsoever has the power to hold him away from this catastrophe, once he crossed a certain critical surface known as the "horizon." The final collapse occurs a finite time after the passage of this surface, but it is inevitable. Time and space are interchanged inside a black hole in an unusual way; the direction of increasing proper time for the observer is the direction of decreasing values of the coordinate r. The observer has no more power to return to a larger r value than he has power to turn back the hands on the clock of life itself. He can not even stay where he is, and for a simple reason: no one has the power to stop the advance of time.

Suppose the observer decides instead to observe the collapse from far away. Then, as price for his own safety, he is deprived of any chance to see more than the first steps on the way to collapse. All signals and all information from the later phases of collapse never escape; they are caught up in the collapse of the geometry itself.

That a sufficient mass of cold matter will necessarily collapse to a black hole (J. R. Oppenheimer and H. Snyder,[1]) is one of the most spectacular of all the predictions of Einstein's standard 1915 general relativity. The geometry around a collapsed object of spherical symmetry (nonrotating!) was worked out by Karl Schwarzschild of Göttingen, father of the American astrophysicist Martin Schwarzschild, as early as 1916. In 1963 Roy Kerr[2] found the geometry associated with a rotating collapsed object. James Bardeen has recently emphasized that all stars have angular momentum and that most stars—or star cores—will have so much angular momentum that the black hole formed upon collapse will be rotating at the maximum rate, or near the maximum rate, allowed for a black hole ("surface velocity" equal to speed of light). Roger Penrose[3] has shown that a particle coming from a distance into the immediate neighborhood of a black hole (the "ergosphere") can extract energy from the black hole. Demetrios Christodoulou[4] has shown that the total mass-energy of a black hole can be split into three parts,

$$E^2 = m_{ir}^2 + L^2/4m_{ir}^2 + p^2$$

The first part is "irreducible" (left constant in "reversible transformations"; always increased in "irreversible transformations") and the second and third parts (arising from a rotational angular momentum L and a linear momentum p) can be added and subtracted at will.

The three most promising ways now envisaged to detect black holes are:

- pulses and trains of gravitational radiation given out at the time of formation (see PHYSICS TODAY, August 1969, page 61, and August 1970, page 41, for accounts of Joseph Weber's pioneering attempts to detect gravitational radiation),
- broadband electromagnetic radiation extending into the hard x-ray and gamma-ray regions emitted by matter falling into a black hole after it has been formed (this is the concept of Ya. B. Zel'dovich and I. D. Novikov. The radiation is not emitted by the individual particles as they fall in, but by the gas as a whole as it is compressed and heated to 10^{10} or 10^{11} K by the "funnel effect" on its way towards the black hole),
- jets and other activity produced in the ergosphere of rotating black holes.

Equilibrium configurations

The mass of a superdense star (reached in collapse that does not go to

Remo Ruffini and John Wheeler are both at Princeton University; Wheeler, currently on leave from Princeton, is spending a year at Cal Tech and Moscow State University.

30 PHYSICS TODAY / JANUARY 1971

Figurative representation of a black hole in action. All details of the infalling matter are washed out. The final configuration is believed to be uniquely determined by mass, electric charge, and angular momentum. Figure 1

PHYSICS TODAY / JANUARY 1971 31

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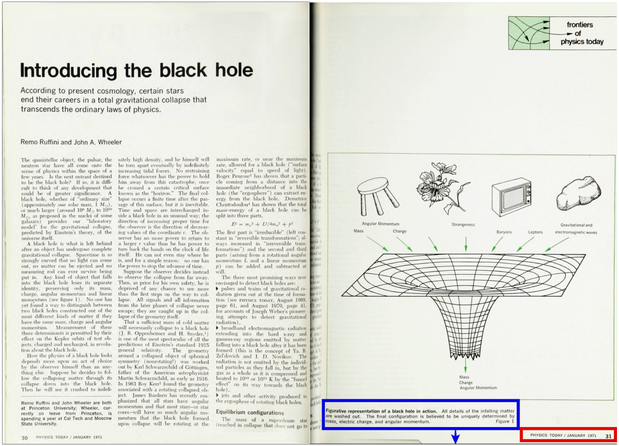

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Recent interests

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  - thus supported by No-hair theorem.

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Old No-Hair Theorems by J. Bekenstein in 1972

1. Klein-Gordon equation

$$\Box\psi(r) - m^2\psi(r) = 0$$

2. manipulate as follows

$$\begin{aligned}&\int \mathrm{d}^4x\sqrt{-g}\;\psi(r)\Big[\partial^\mu\partial_\mu\psi(r) - m^2\psi(r)\Big]\\ &= -\int_{\mathcal{V}} \mathrm{d}^4x\sqrt{-g}\Big[\partial^\mu\psi\partial_\mu\psi + m^2\psi^2\Big] + \int_{\partial\mathcal{V}} \mathrm{d}^3x\sqrt{-h}n^\mu\psi\partial_\mu\psi\end{aligned}$$

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3. Since $g^{rr}(\partial_r\psi)^2 > 0$ and $m^2\psi^2 > 0$, the integral can vanish only if ψ vanishes indentically in the black hole's exterior.
4. The proofs for vector and spin-2 fields are more complicated than the above, but the basic idea is the same.

Novel No-Hair theorem by J. Bekenstein in 1995

- The previous old no-hair theorem fails for a general potential, for example, a double well potential.

$$\int_{\mathcal{V}} \mathrm{d}^4x\sqrt{-g}\left(\partial^\mu\psi\partial_\mu\psi + V'(\psi^2)\psi^2\right) = 0$$

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$$S_{\psi} = -\frac{1}{2}\int \left[\psi_{,\alpha}\psi_{,}{}^{\alpha} + V(\psi^2)\right](-g)^{1/2}\mathrm{d}^4x$$

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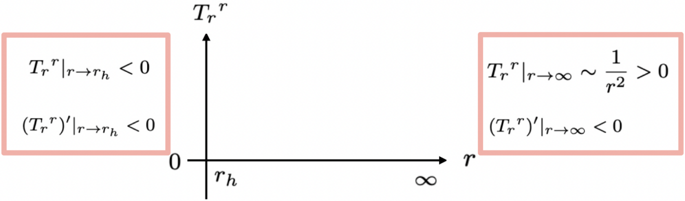

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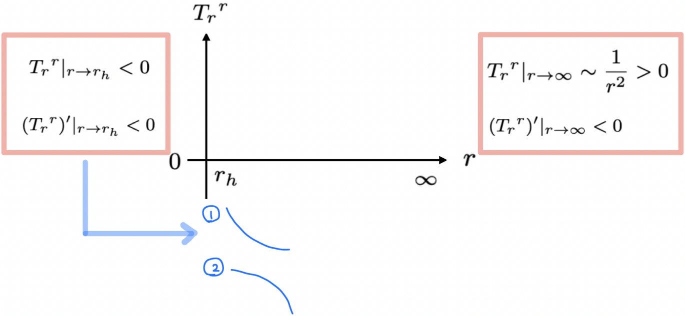

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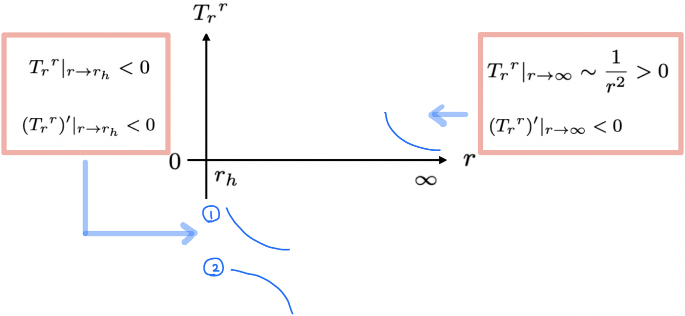

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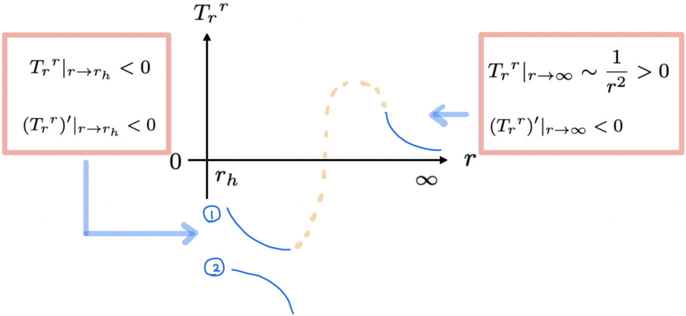

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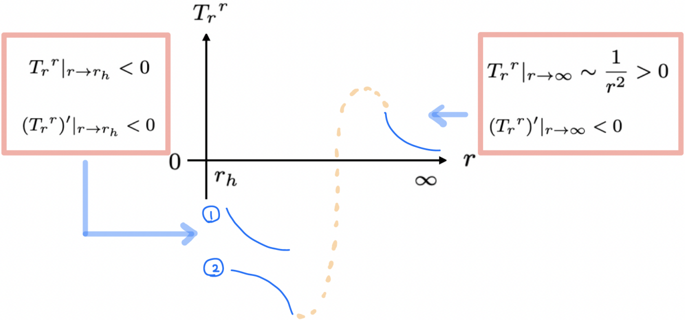

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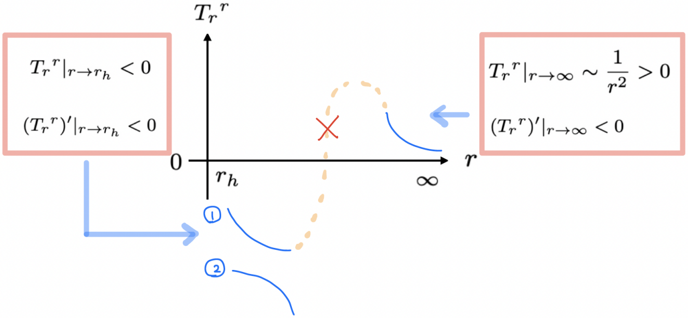

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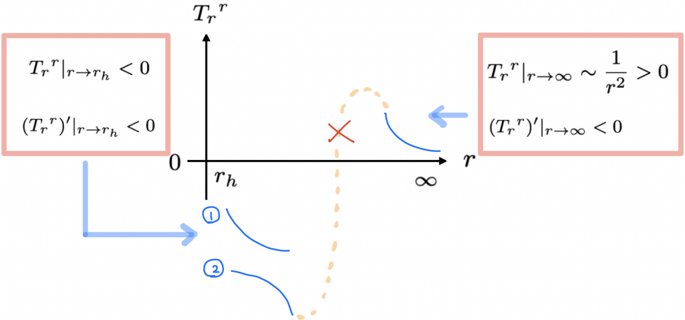

Evasion of No-Hair theorem

The desire to find new black hole solutions led to the discovery of several kinds of non-trivial field

- black holes with YM fields
- Skyrmion hair black holes

Evasion of the old no-hair theorem in ESGB[4]

Einstein-Scalar-Gauss-Bonnet Theory

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \frac{1}{2} \nabla_\alpha \varphi \nabla^\alpha \varphi + f(\varphi) \mathcal{G} \right], \qquad 2\kappa^2 = 1$$

$$\mathcal{G} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

[4] G. Antoniou, A. Bakopoulos, and P. Kanti, Phys. Rev. Lett. 120, 131102 (2018)

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Old no-hair theorem

$$\int_{\mathcal{V}} \mathrm{d}^4x\sqrt{-g}f(\varphi)\left(\nabla^2\varphi + \dot{f}(\varphi)\mathcal{G}\right) = 0$$

$$= -\int_{\mathcal{V}} \mathrm{d}^4x\sqrt{-g}\dot{f}(\varphi)\left(\partial^\mu\varphi\partial_\mu\varphi - f(\varphi)\mathcal{G}\right) + \int_{\partial\mathcal{V}} \mathrm{d}^3x\sqrt{-h}f(\varphi)n^\mu\partial_\mu\varphi$$

[4] G. Antoniou, A. Bakopoulos, and P. Kanti, Phys. Rev. Lett. 120, 131102 (2018)

Evasion of the old no-hair theorem in ESGB[4]

Einstein-Scalar-Gauss-Bonnet Theory

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}\nabla_\alpha\varphi\nabla^\alpha\varphi + f(\varphi)\mathcal{G}\right], \qquad 2\kappa^2 = 1$$

$$\mathcal{G} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

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$$f(\varphi) > 0$$

The Old No-Hair theorem is evaded, since $g^{rr}\partial_r\varphi\partial_r\varphi > 0$ **and** $\mathcal{G} > 0$

[4] G. Antoniou, A. Bakopoulos, and P. Kanti, Phys. Rev. Lett. 120, 131102 (2018)

Rectifying the old no-hair theorem in ESGB[5]

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[5] Alexandros Papageorgiou, Chan Park, and Miok Park, 2202.00907

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$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{1}{B(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2, \qquad \varphi \sim \varphi_\infty + \frac{\varphi_1}{r} + \cdots$$

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If $f(\varphi_\infty) = 0$ or $\varphi_1 = 0$, the no-hair theorem is evaded when

$$f(\varphi) > 0$$

If $f(\varphi_\infty) \neq 0$ and $\varphi_1 \neq 0$, the no-hair theorem fails. Black hole solution might exist

$$f(\varphi) > 0 \;\; \text{and} \;\; f(\varphi) < 0$$

[5] Alexandros Papageorgiou, Chan Park, and Miok Park, 2202.00907

Evasion of the novel no-hair theorem in ESGB[6]

Assumptions

- minimal coupling to gravity.
- the energy density ($= T_{\alpha\beta}U^{\alpha}U^{\beta}$) carried by the scalar field is non-negative.

Near horizon and infinity expansion of energy-momentum tensor

$$T_t{}^t = T_r{}^r|_{r\to r_h} > 0, \qquad (T_r{}^r)'|_{r\to r_h} : \text{undetermined}$$
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$$(T_r{}^r)'|_{r\to r_h} = \frac{BA'}{A}\left[\frac{4\,(rB'+1)\,\dot{f}\varphi'}{r^3} - \frac{r\varphi'^2}{4(r+2\dot{f}\varphi')} - \frac{2(\ddot{f}\varphi'^2+\dot{f}\varphi'')}{r(r+2\dot{f}\varphi')}\right] + \mathcal{O}(\epsilon)$$

Authours claim

there would be smooth matching of $T_r{}^r$ at both asymptotics if $V = 2(\ddot{f}\varphi'^2 + \dot{f}\varphi'') > 0$

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$$\begin{aligned} &{T_t}^t = {T_r}^r|_{r\to r_h} > 0, \qquad &({T_r}^r)'|_{r\to r_h} : \text{undetermined} \\ &{T_r}^r|_{r\to\infty} > 0, \qquad &({T_r}^r)'|_{r\to\infty} < 0 \end{aligned}$$

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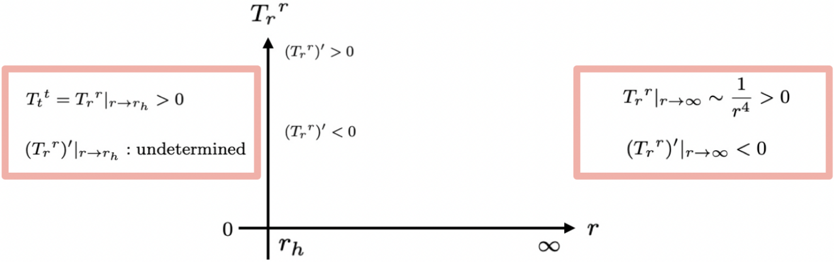

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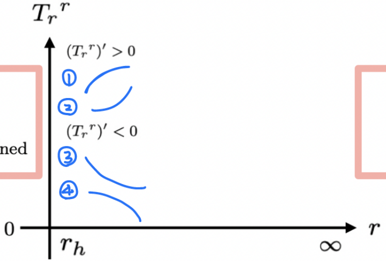

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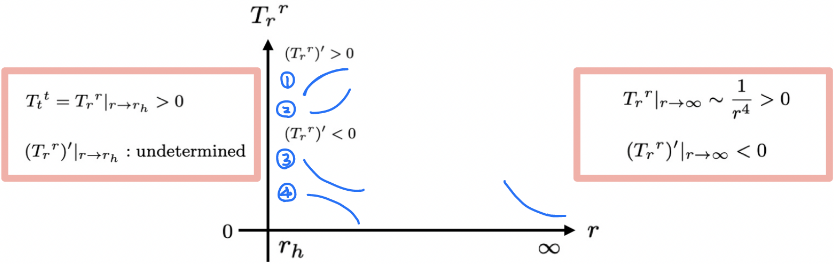

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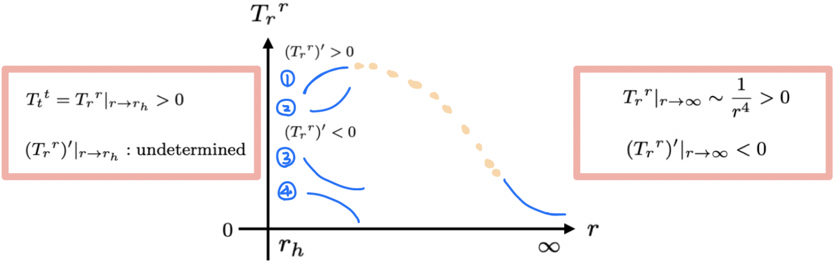

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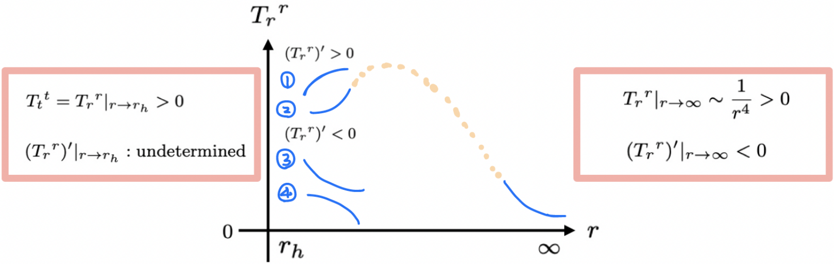

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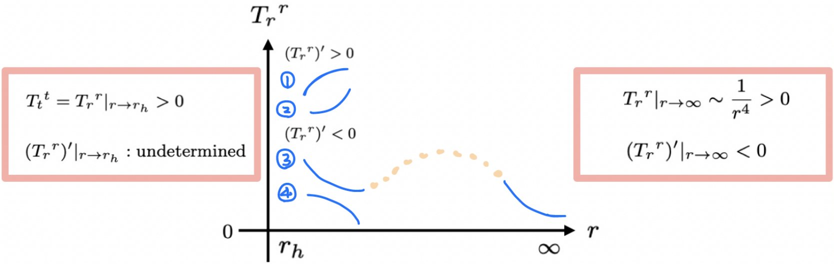

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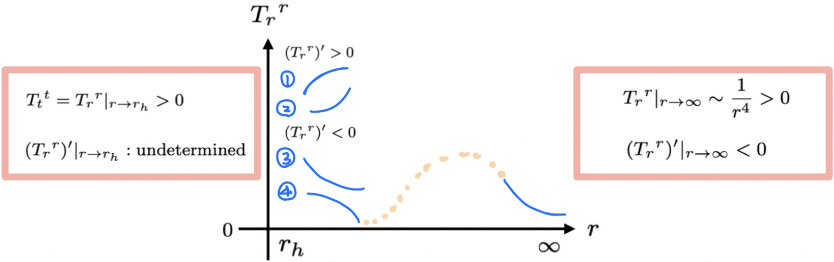

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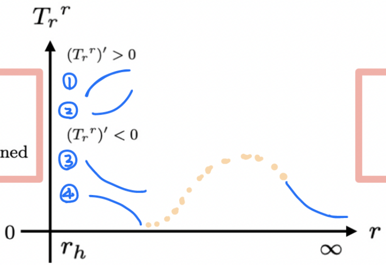

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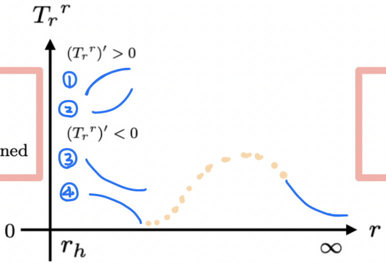

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No constraints are needed.

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- then the theorem loses its universal power.
- This conflict occurred because of omitting the surface term.

Hairy Black Holes for in ESGB

- Our metric ansatz

$$ds^2 = -A(r)dt^2 + \frac{1}{B(r)}dr^2 + r^2 d\Omega_2,$$

- Boundary conditions

$$\text{Near horizon}: \; A(r) \sim A_h\epsilon, \quad B(r) \sim B_h\epsilon, \quad \varphi(r) \sim \varphi_h + \varphi_{h,1}\epsilon$$

$$\text{Near infinity}: \; A(r) \sim 1 + \frac{A_1}{r}, \quad B(r) \sim 1 + \frac{A_1}{r}, \quad \varphi(r) \sim \varphi_\infty + \frac{\varphi_1}{r}$$

where

$$\varphi'(r_h) = \varphi_{h,1} = -\frac{r_h}{4\dot{f}_h}\left(1 \mp \sqrt{1 - \frac{96}{r_h^4}\dot{f}_h^2}\right), \qquad B_h = \frac{2}{r_h}\left(1 \pm \sqrt{1 - \frac{96}{r_h^4}\dot{f}_h^2}\right)^{-1}$$

- To avoid $\varphi''(r_h)$ being divergent the inside of the root should not be zero, namely

$$\dot{f}_h^2 < \frac{r_h^4}{96}.$$

Hairy Black Holes for $f = \alpha e^{\gamma\varphi}$ in ESGB

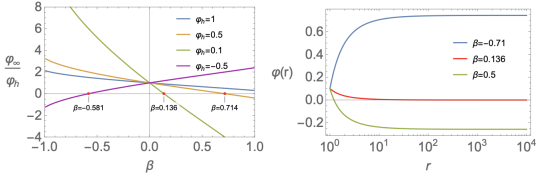

FIG. 2. For $f = \alpha e^{\gamma\varphi}$ (left) φ_∞/φ_h vs β and (right) $\varphi(r)$ for different values of β fixing $\varphi_h = 0.1$

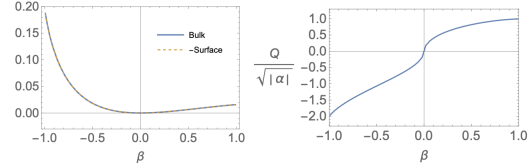

FIG. 3. Old no-hair theorem: For $f = \alpha e^{\gamma\varphi}$ with $\varphi_h = 0.1$ (left) plot of bulk and surface term (right) the scalar charge $Q/\sqrt{|\alpha|}$ vs β

Hairy Black Holes for $f = \alpha\varphi^2$ in ESGB

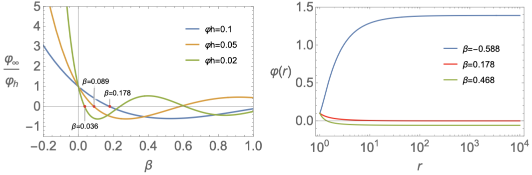

FIG. 4. For $f = \alpha\varphi^2$, (left) φ_∞/φ_h vs β and (right) $\varphi(r)$ for different values of β fixing $\varphi_h = 0.1$

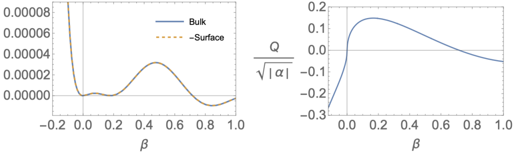

FIG. 5. Old no-hair theorem: For $f = \alpha\varphi^2$ with $\varphi_h = 0.1$ (left) plot of bulk and surface term (right) the scalar charge $Q/\sqrt{|\alpha|}$ vs β

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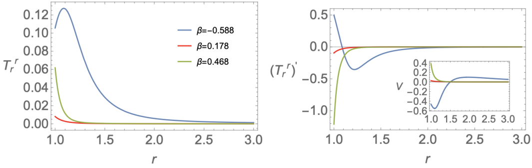

FIG. 6. Novel no-hair theorem: For $f = \alpha\varphi^2$ and $\varphi_h = 0.1$, (left) $T_r{}^r$ and (right) $(T_r{}^r)'$ and V.

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Thank You!