Impurity Driven Metal-Insulator Transition In Holography

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Based on the on-going work with Kyung Kiu Kim, Sang-Jin Sin, Keun-Young Kim and Yongjun Ahn

Introduction

Introduction

■ Metal

- Freely moving electrons
- Described by Drude model: conventional metal $\rho \sim T^2$
- Bad metal : $\rho \sim T$
- Resistivity increases as temperature increased

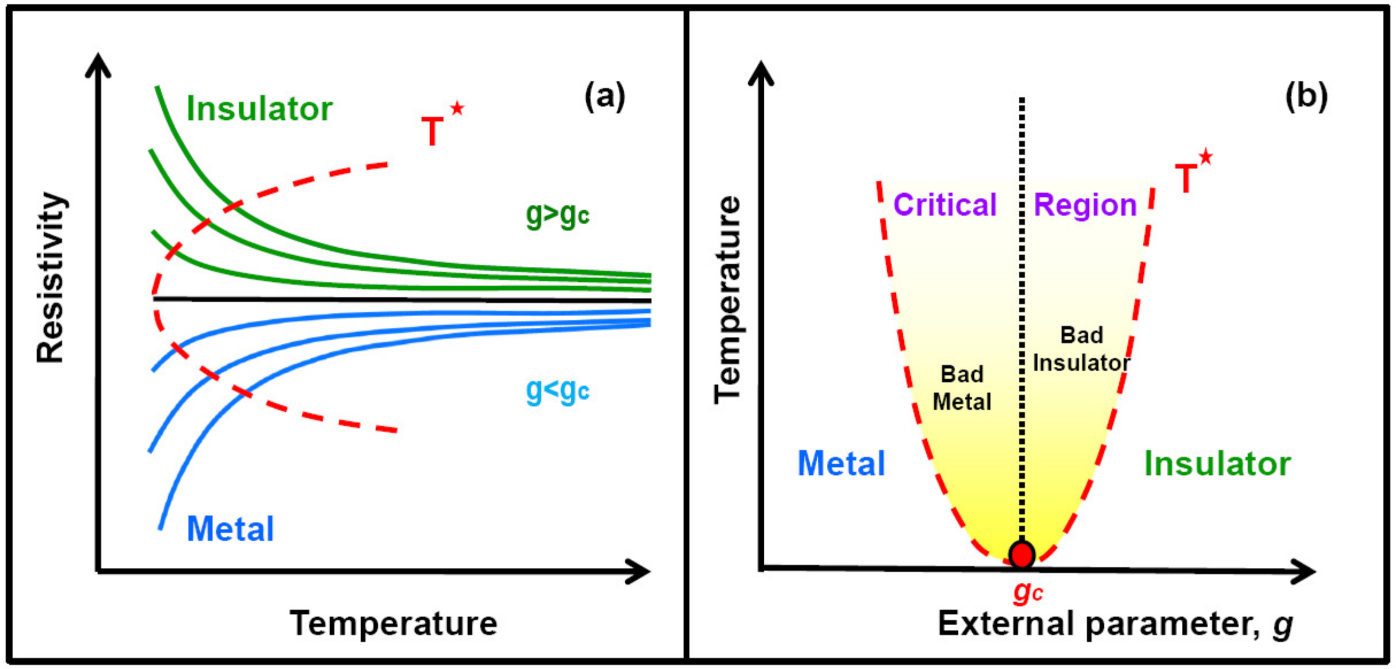
Introduction

- Insulator
  - No freely moving electrons
  - Strong electron-electron interaction: Mott
  - Strong electron-disorder interaction: Anderson
  - Resistivity decreases as temperature increased

Introduction

■ Metal-Insulator Transition(MIT)

- The MIT is one of the oldest, not yet understood well in condensed matter physics
- MIT can capture properties of quantum critical point
- It is very hart to describe different excitations in one model

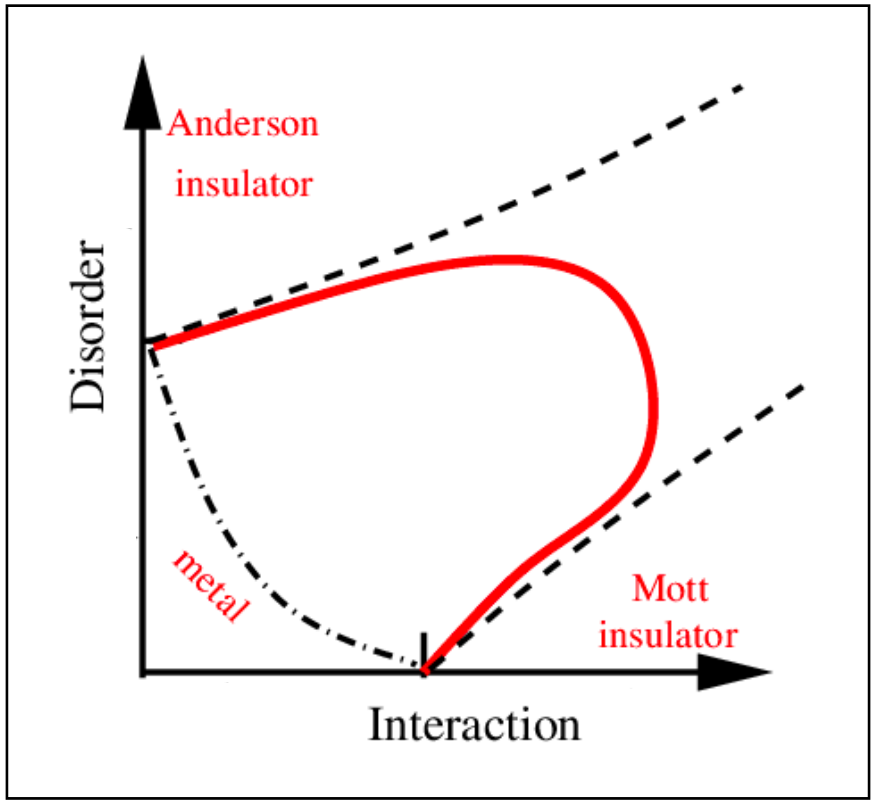


1112.6166: Dovrosavljevic

Introduction

■ Insulating mechanism

- Interaction induced insulator: Mott insulator
- Impurity(or disorder) induced insulator: Anderson insulator



Byczuk et al, IJMPB(2010)

Introduction

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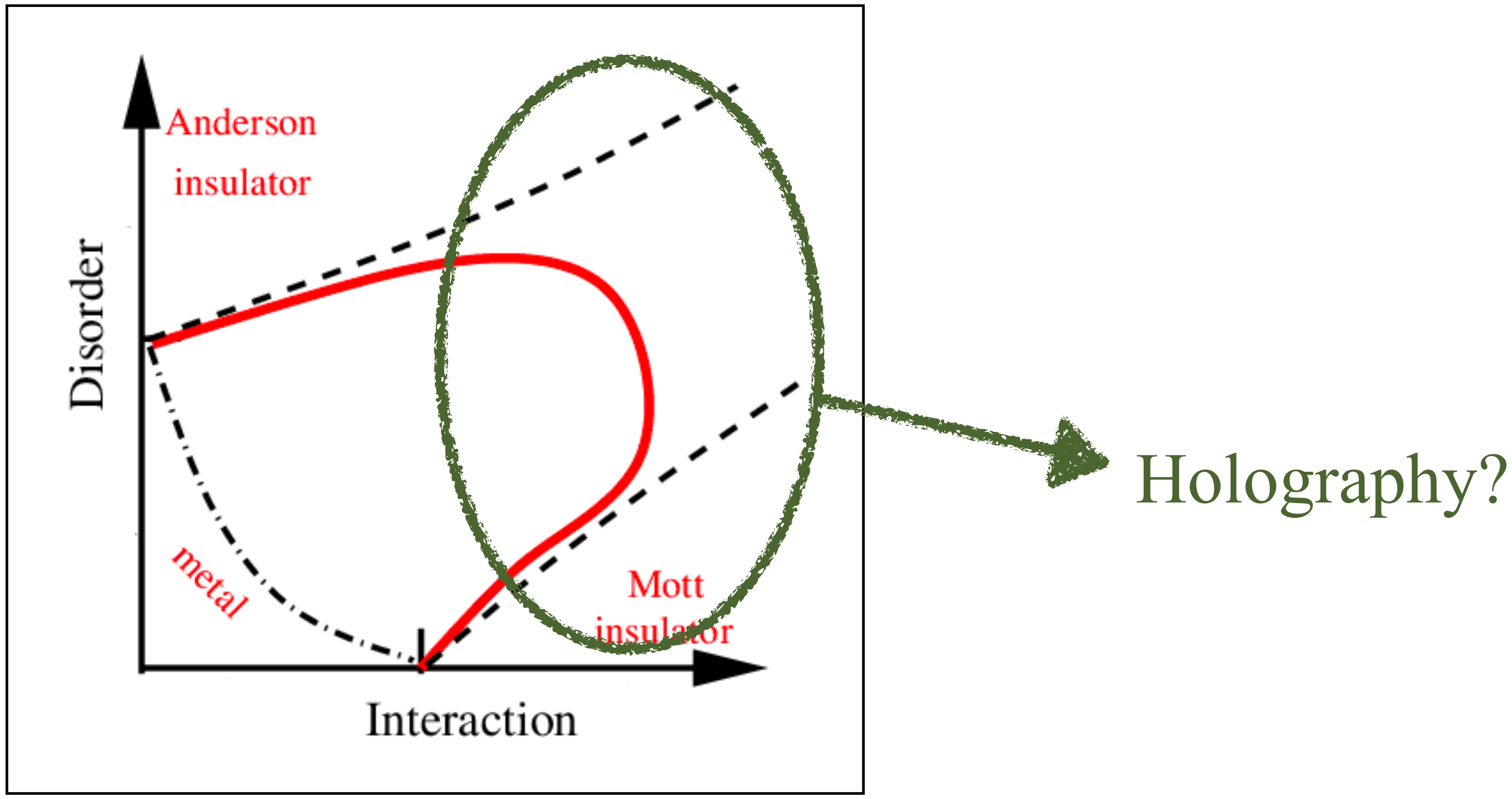

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Introduction

■ Holography(gauge/gravity duality)

- AdS/CFT: Strongly interacting gauge theory in d-dimension can be described by weakly interacting gravity theory in d+1-dimension
- Boundary system ↔ Bulk gravity
- Strongly interacting electron ↔ background geometry
- Temperature ↔ Hawking temperature of black hole
- Conserved charge ↔ U(1) gauge field
- Momentum relaxation ↔ linear axion field
- Operator $\mathcal{O}_\Delta$ ↔ field ϕ_m
- …

Introduction

- Holography(gauge/gravity duality)
  - AdS/CFT: Strongly interacting gauge theory in d-dimension can be described by weakly interacting gravity theory in d+1-dimension
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  - Operator $\mathcal{O}_\Delta \leftrightarrow$ field ϕ_m
  - …
- MIT in holography
  - Helical lattice(Donos, Hartnoll: Nature, 2013)
  - Scalar potential(Refford, Horowitz: PRD, 2014)
  - In massive gravity(Baggioli: PRL, 2015)
  - …

Background Geometry

■ Action in 3+1 dim.

$$S_{tot} = S_0 + S_{int} + S_{bd},$$
$$S_0 = \int d^4x\sqrt{-g}\left(R - 2\Lambda - \frac{1}{2}(\partial\chi)^2 - \frac{1}{4}F^2 - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2\right)$$
$$S_{int} = -\int \sqrt{-g}\frac{\gamma_2}{4}\phi^2 F^2.$$

■ Equations of motion

$$R_{MN} - \frac{1}{2}g_{MN}\mathcal{L} - \frac{1}{2}\partial_M\phi\partial_N\phi - \frac{1}{2}\partial_M\chi\partial_N\chi - \frac{1}{2}\left(1+\gamma_2\phi^2\right)F_{MP}F_M^P = 0$$
$$\nabla^2\phi - \left(m^2 + \frac{1}{2}\gamma_2 F^2\right)\phi = 0$$
$$\nabla_M\left(1+\gamma_2\phi^2\right)F^{MN} = 0,$$

■ Ansaz

$$ds^2 = -U(r)e^{2W(r)-2W(\infty)}dt^2 + \frac{dr^2}{U(r)} + r^2(dx^2+dy^2)$$
$$\chi^I = (\beta x, \beta y), \quad \phi = \phi(r), \quad A = A_t(r)dt.$$

Background Geometry

■ Thermodynamic variables

$$\text{Temperature}: \ T = \frac{U'(r_h)}{4\pi}$$
$$\text{Entropy density}: \ s = 4\pi r_h^2$$
$$\text{Charge density}: \ Q = (1+\gamma_2\phi^2)F^{rt}$$
$$\text{Chemical poential}: \ \mu = A_t(\infty)$$

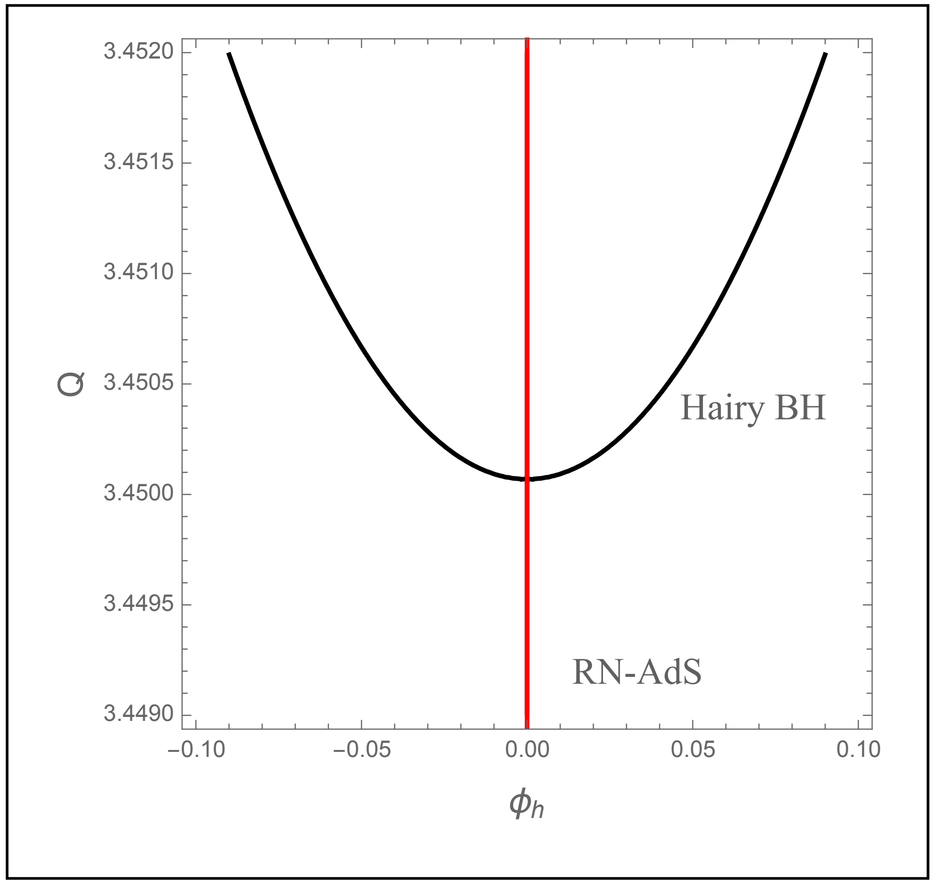
■ Boundary behavior of scalar field

$$\phi(r)|_{r\to\infty} \sim \phi_\infty + \frac{<\mathcal{O}>}{r} + \cdots$$

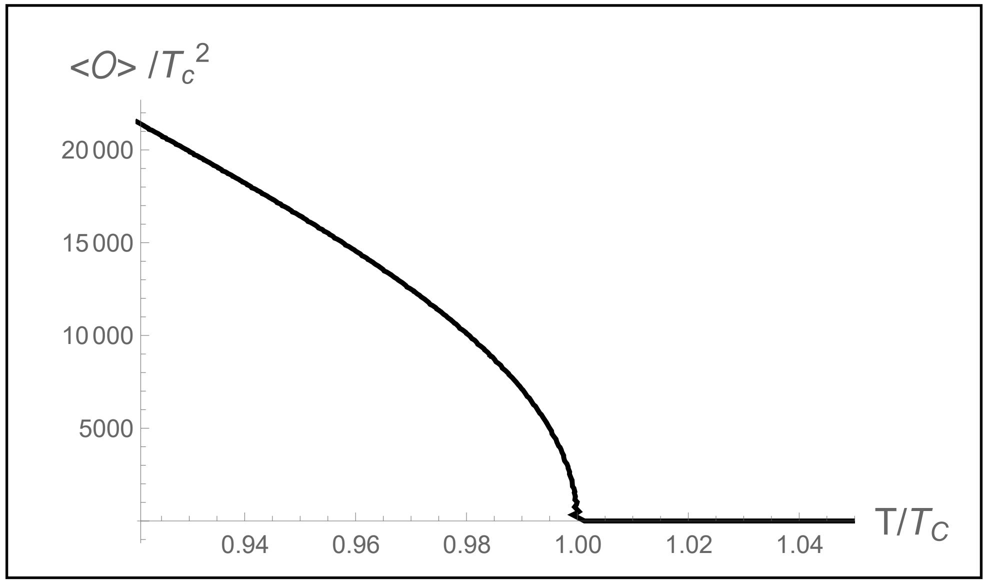
Background Geometry

■ Background Solution($m^2 = -2$)

- Without scalar condensation($\phi = 0$): RN-AdS black hole
- With a scalar condensation($\phi \neq 0$): Hairy black hole



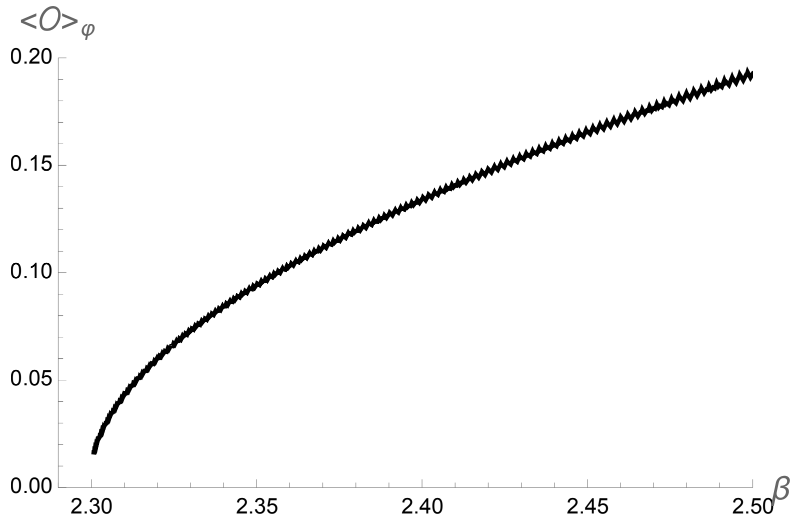
Source free condition



Background Geometry

■ Impurity effect on the scalar condensation

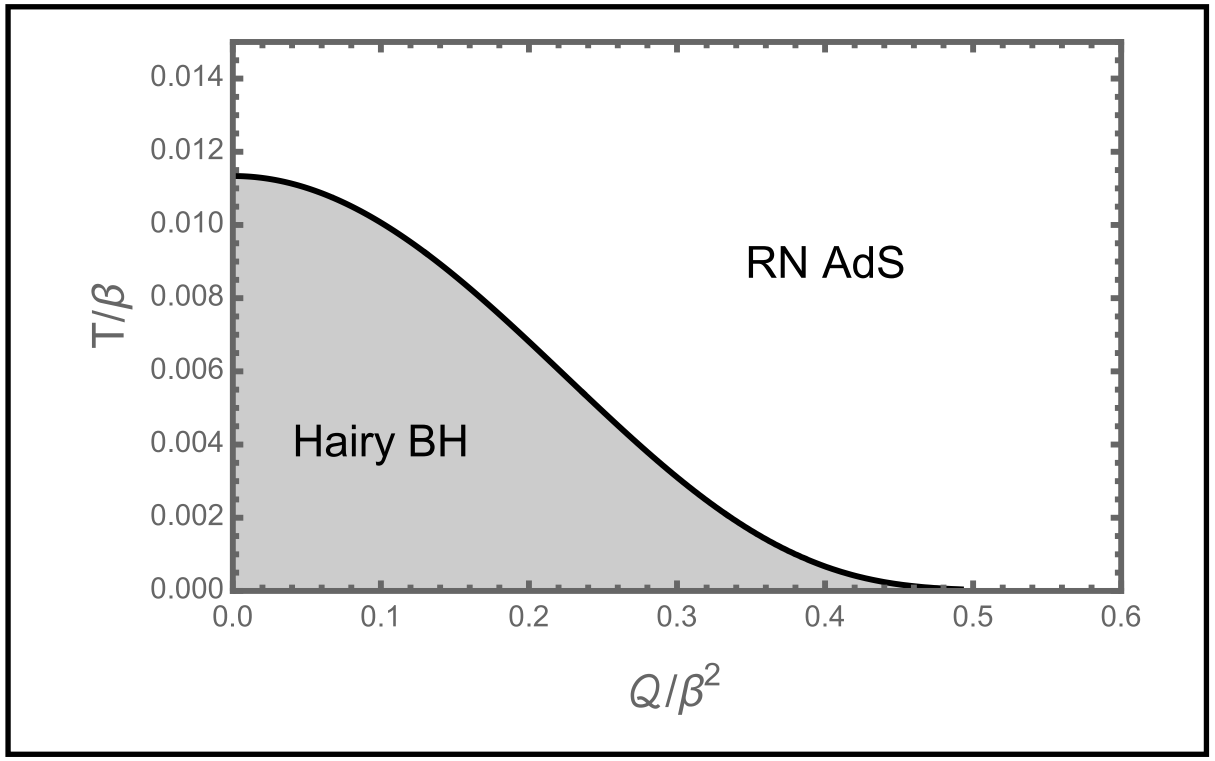
- Impurity enhances scalar condensation through gravity(electron interaction)



(b) $\mathcal{Q} = 0.5$

Background Geometry

■ Phase diagram



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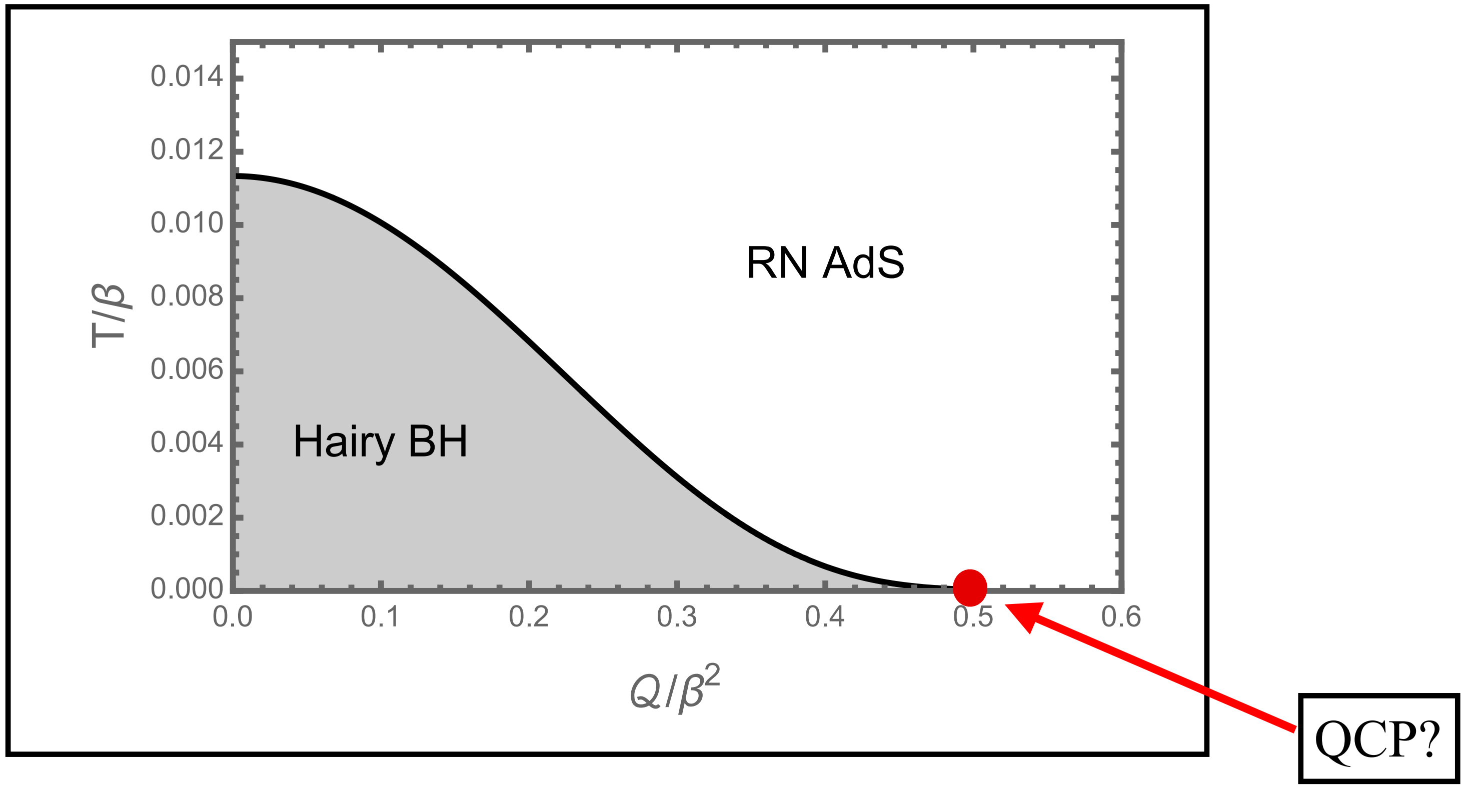
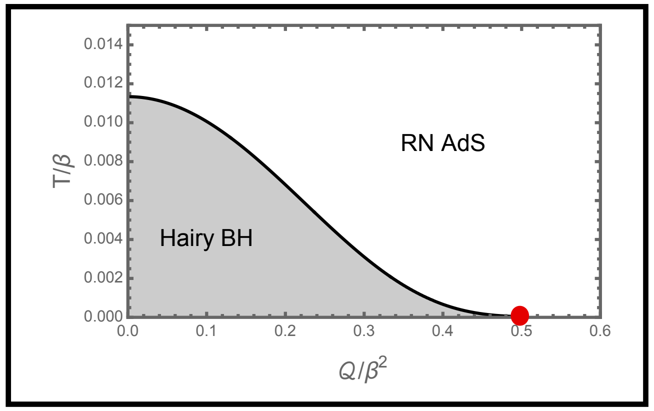
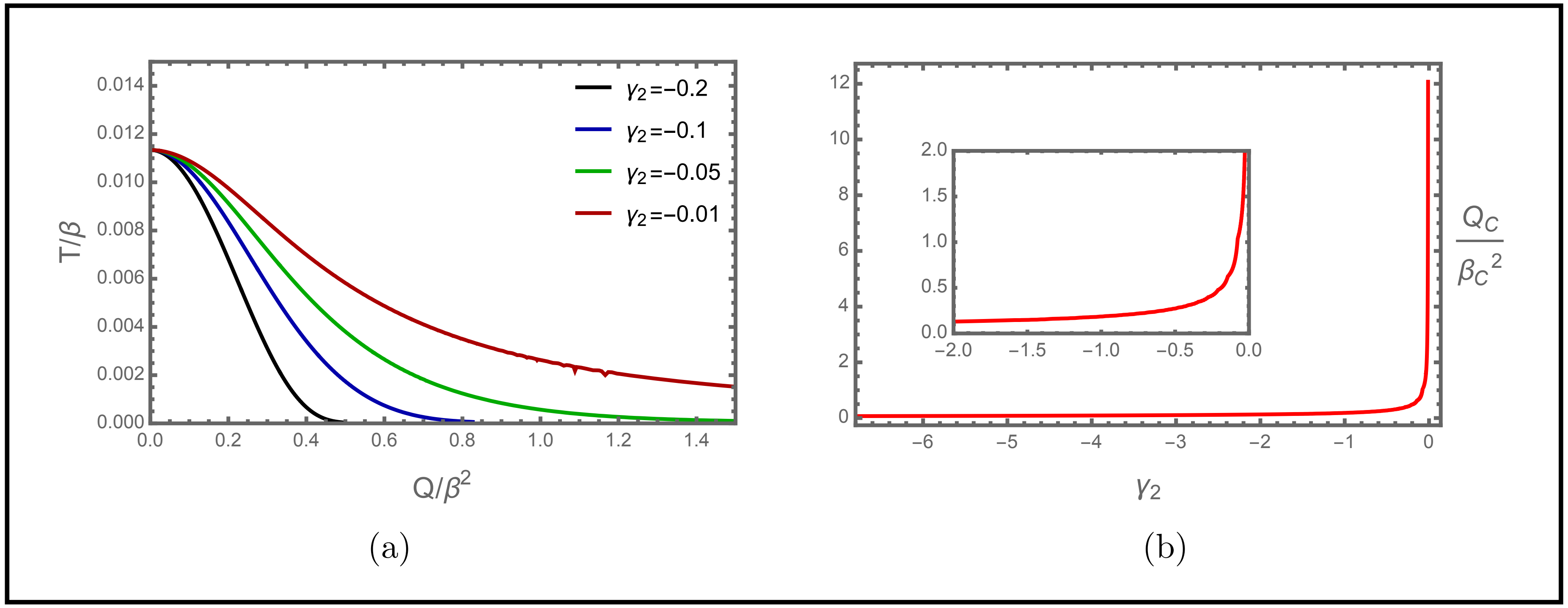


Background Geometry

■ γ_2 dependence of Quantum critical point

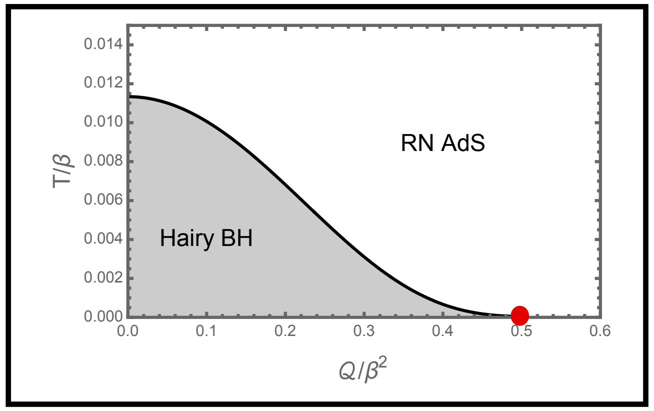




■ Absence of hairy BH solution at large Q/β^2 can be prooven in probe limit

Background Geometry

■ γ_2 dependence of Quantum critical point



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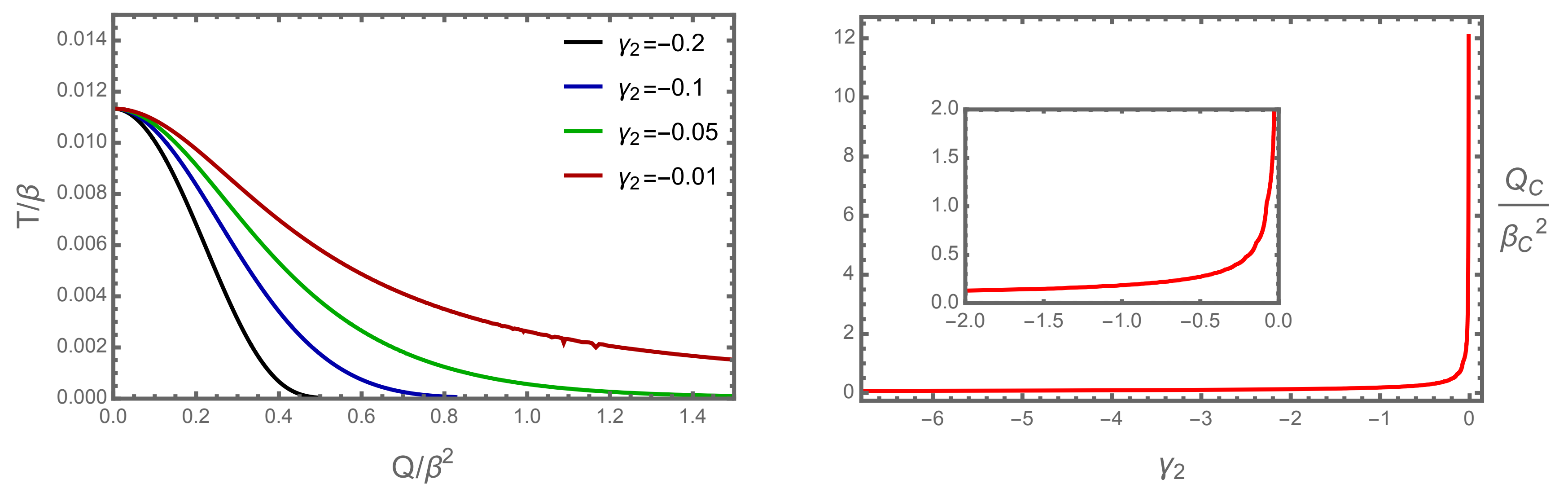

(a) (b)

■ Absence of hairy BH solution at large Q/β^2 can be prooven in probe limit

DC Conductivity

■ Fluctuation around background solution

$$\delta G_{ti} = -t\,U(r)\zeta_i + \delta g_{ti}(r)$$
$$\delta G_{ri} = r^2 \delta g_{ri}$$
$$\delta A_i = t(-E_i + \zeta_i a(r)) + \delta a_i(r)$$

■ Boundary current

$$\begin{aligned}\mathcal{J}^i &= \sqrt{-g}(1+\gamma_2\phi^2)F^{ir}\\ &= -U(r)(1+\gamma_2\phi(r)^2)\delta a_i'(r) - a_t'(r)\delta g_{ti}(r)\end{aligned}$$

■ Fluctuation equation + Regularity condition on the horizon: DC conductivity

$$\sigma_{DC} = (1+\gamma_2\varphi_h^2) + \frac{e^{W(\infty)}\mathcal{Q}^2}{r_h^2\beta^2}$$

DC Conductivity

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$$\sim \frac{\mathcal{J}}{r} + \cdots$$

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DC Conductivity

- Two contributions to DC conductivity

$$\sigma_{DC} = \left(1 + \gamma_2 \phi_h^2\right) + \frac{Q^2}{r_h^2 \beta^2}$$
$$= \sigma_{ccs} + \sigma_{diss}$$

2015: Blake, Donos

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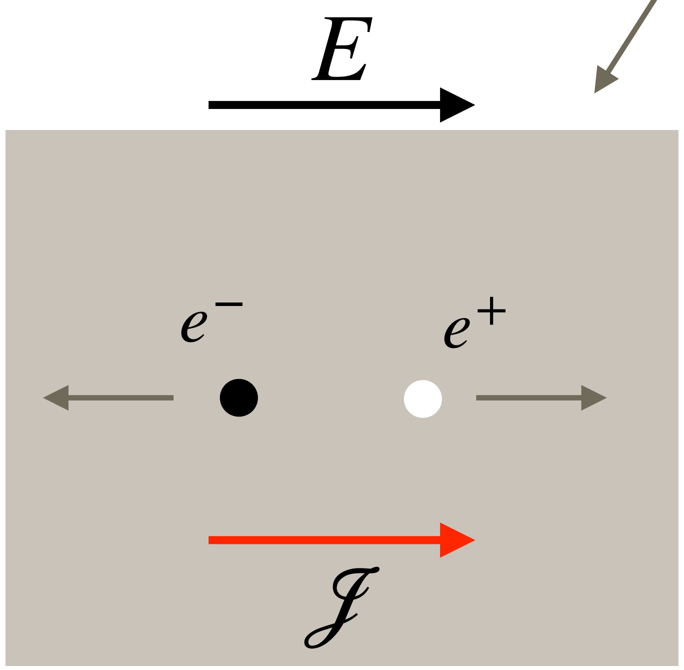

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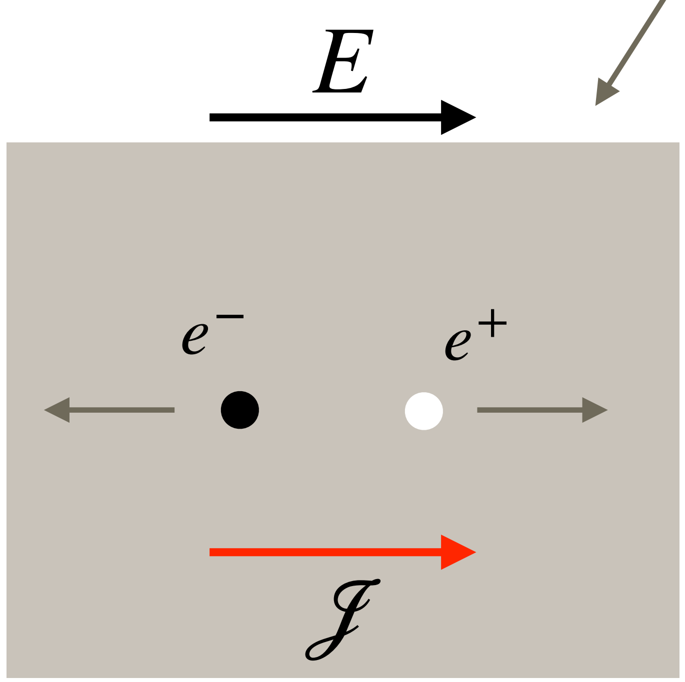

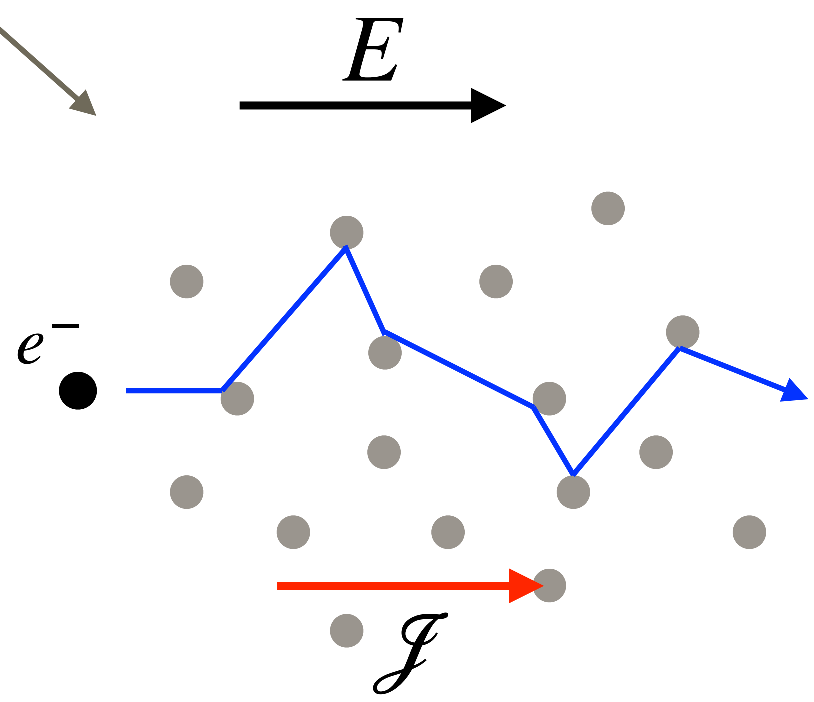

DC Conductivity

- DC conductivity without charge carrier($Q = 0$)

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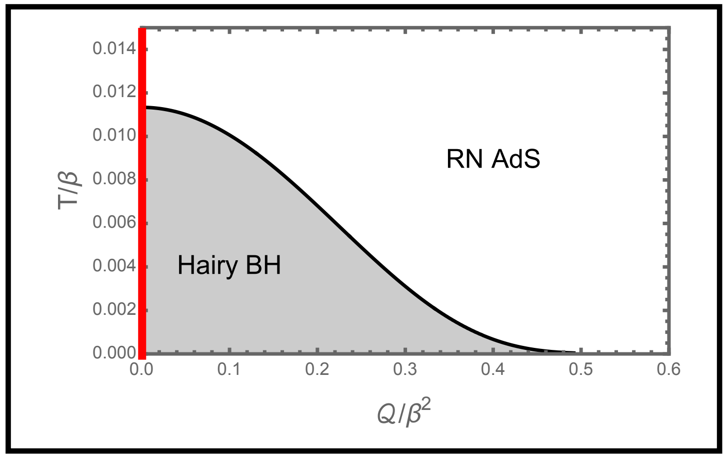

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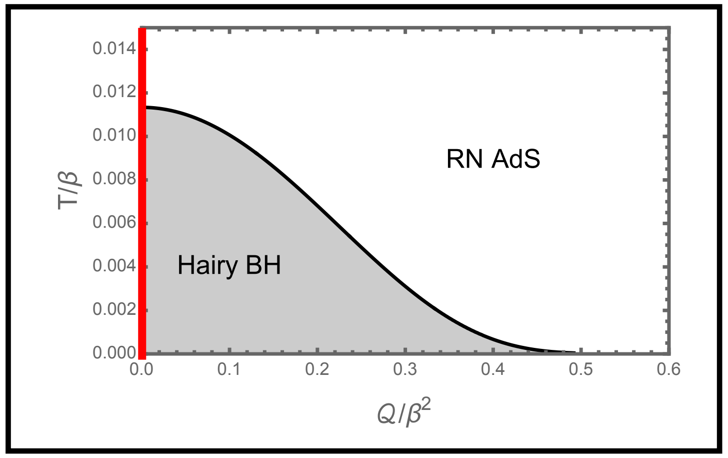
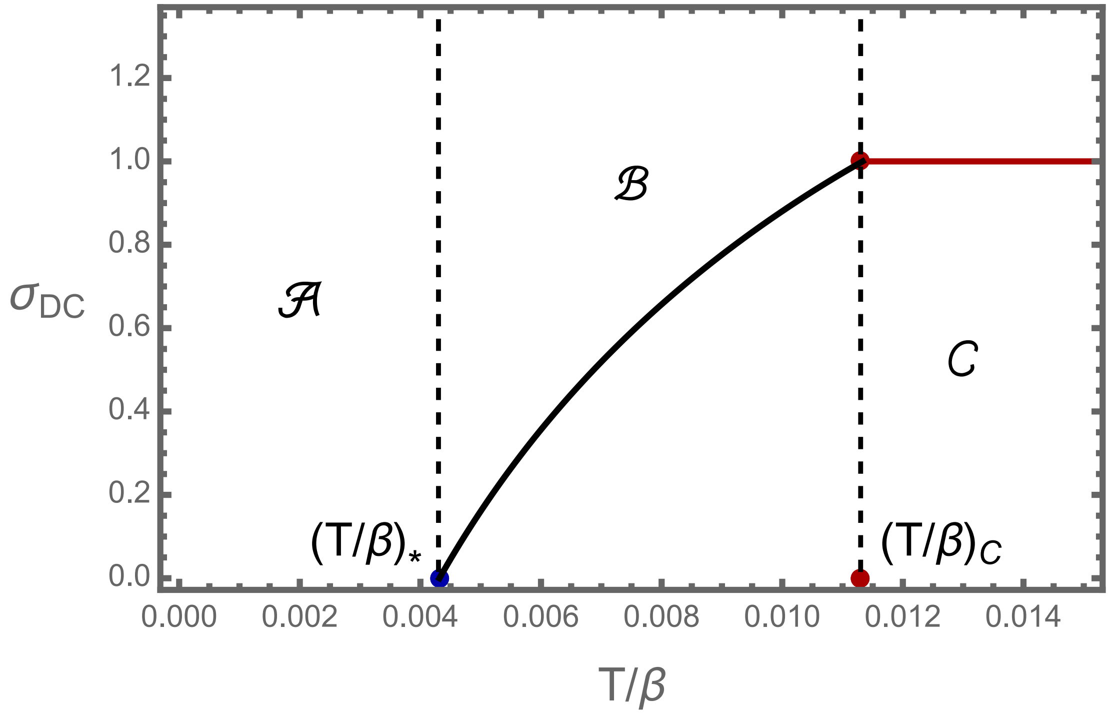


- Temperature dependence of DC conductivity



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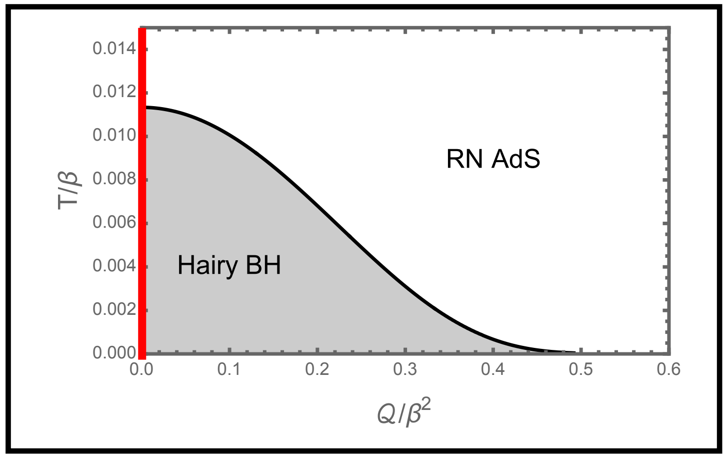
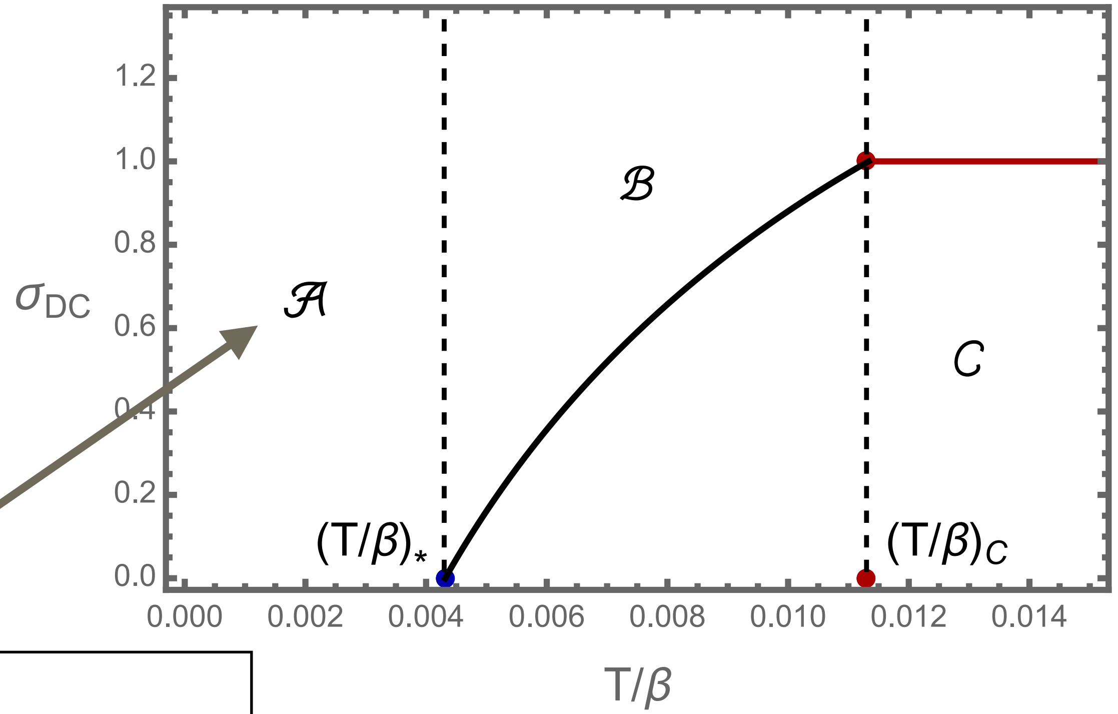


○ Temperature dependence of DC conductivity



$1 + \gamma_2 \phi_h^2 < 0$

Instability of gauge field fluctuation

DC Conductivity

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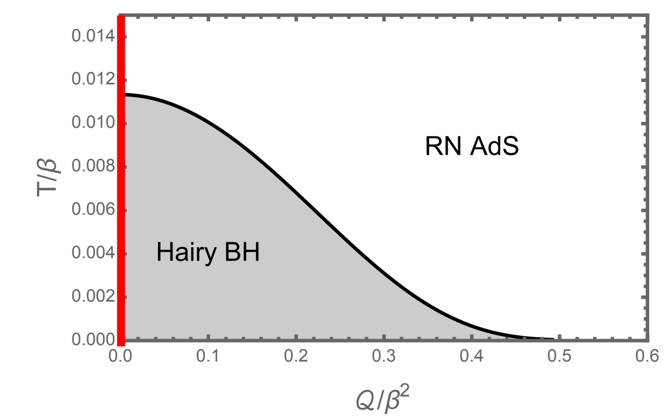

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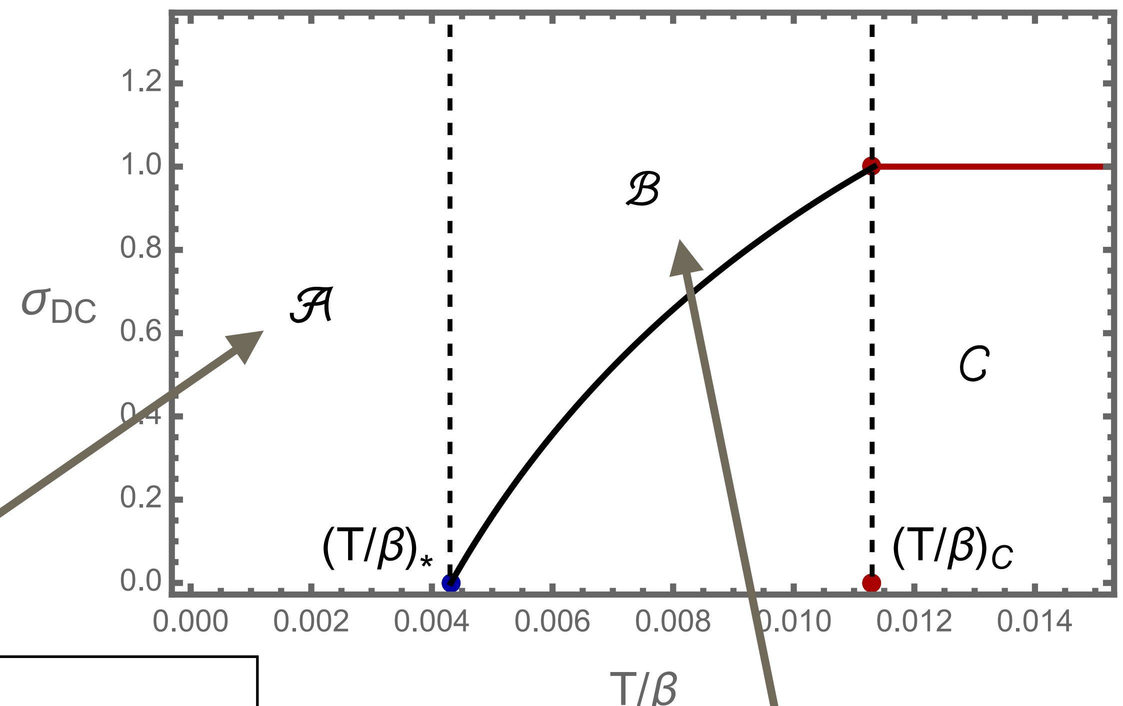

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Instability of gauge field fluctuation

$\frac{\partial \rho}{\partial T} < 0$

Insulating behavior

DC Conductivity

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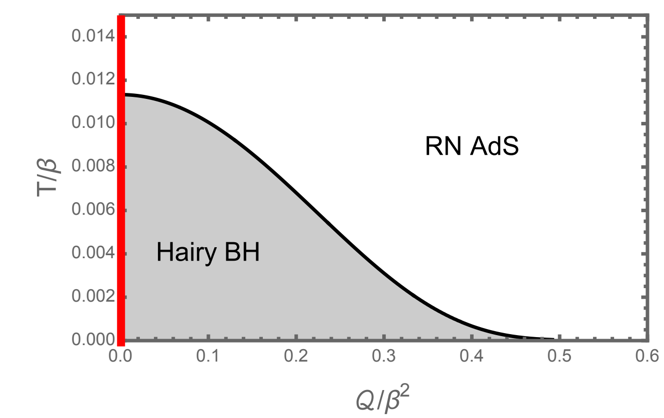

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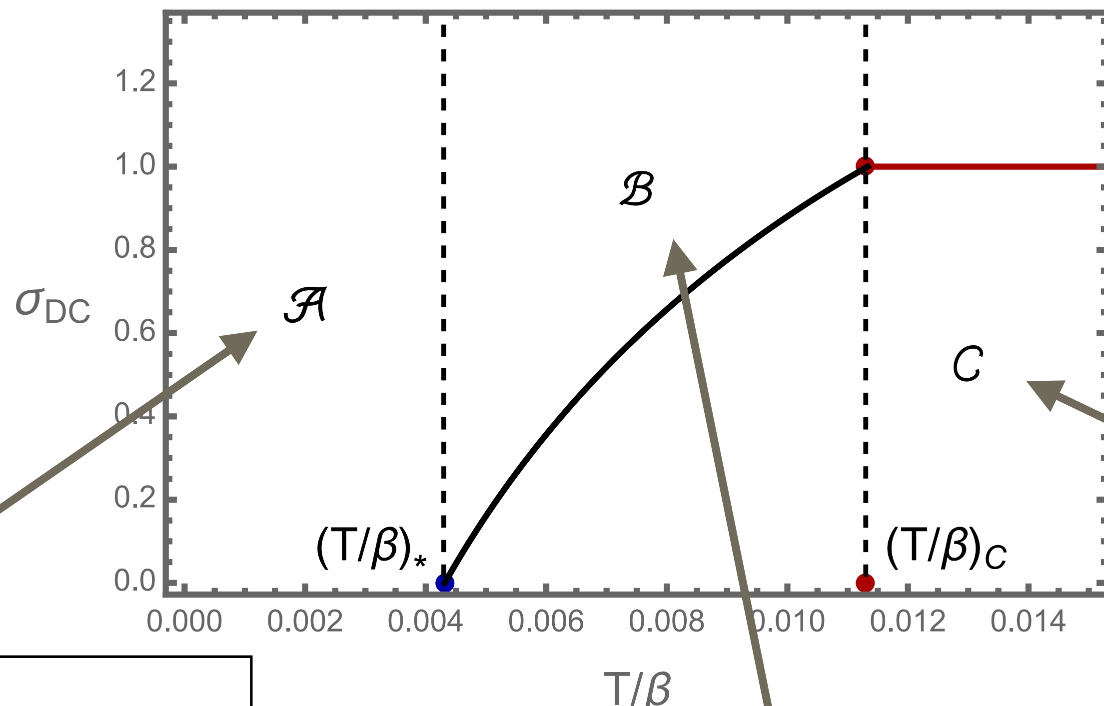

$1 + \gamma_2 \phi_h^2 < 0$

Instability of gauge field fluctuation

$\frac{\partial \rho}{\partial T} < 0$

Insulating behavior

$\frac{\partial \rho}{\partial T} = 0$

Pure ccs.

DC Conductivity

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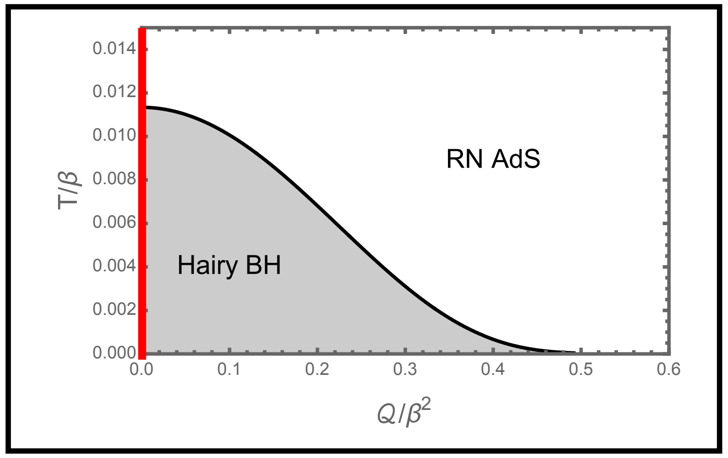
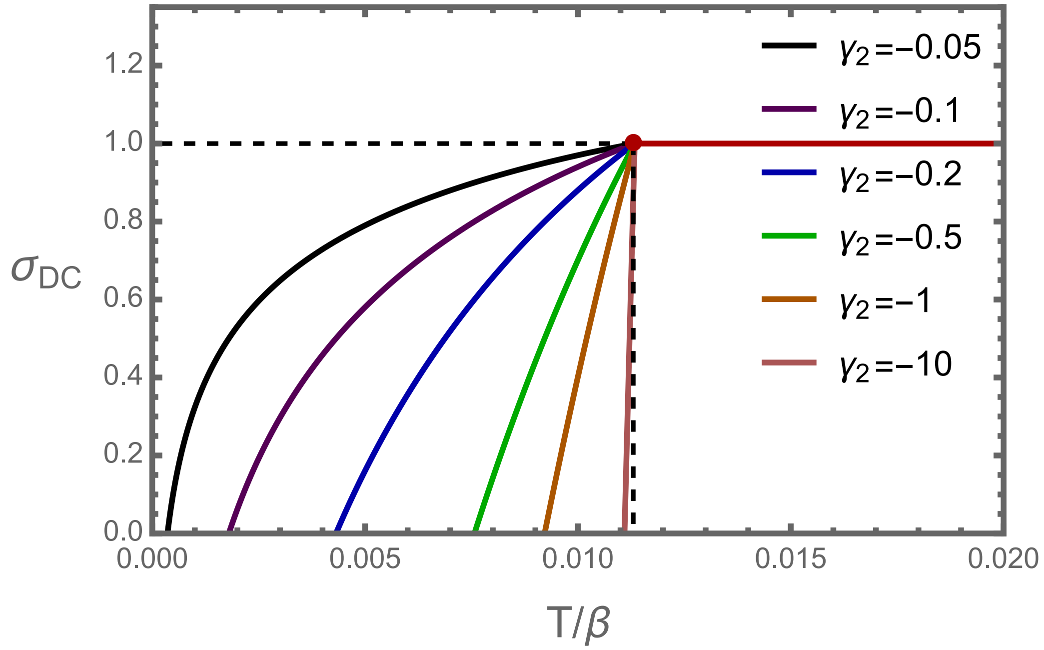


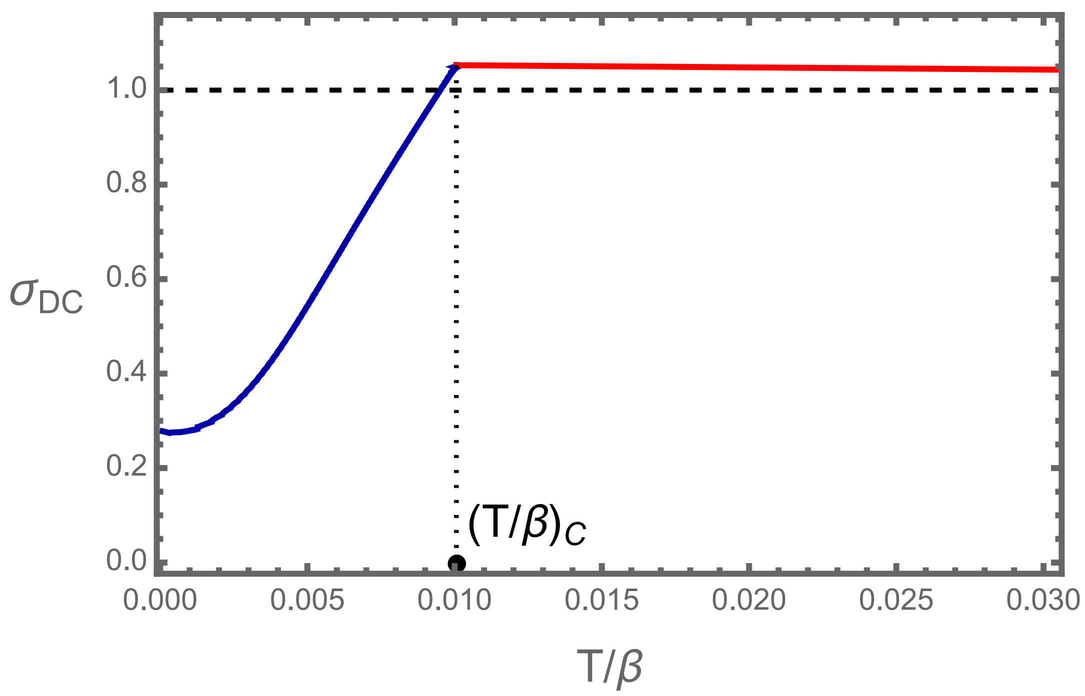
○ γ_2 dependences



DC Conductivity

- DC conductivity with charge carrier($Q \neq 0$)

$$
\begin{aligned}
\sigma_{DC} &= (1+\gamma_2\varphi_h^2) + \frac{e^{W(\infty)}\mathcal{Q}^2}{r_h^2\beta^2} \quad \text{for hairy BH} \\
\sigma_{DC} &= 1 + \frac{\mathcal{Q}^2}{r_h^2\beta^2} \quad \text{for RN AdS BH.}
\end{aligned}
$$



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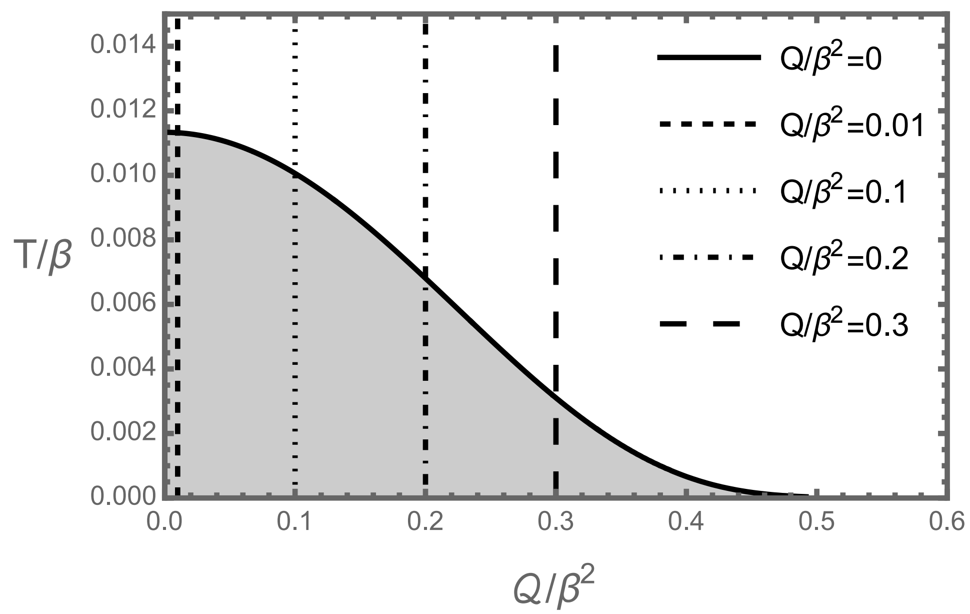
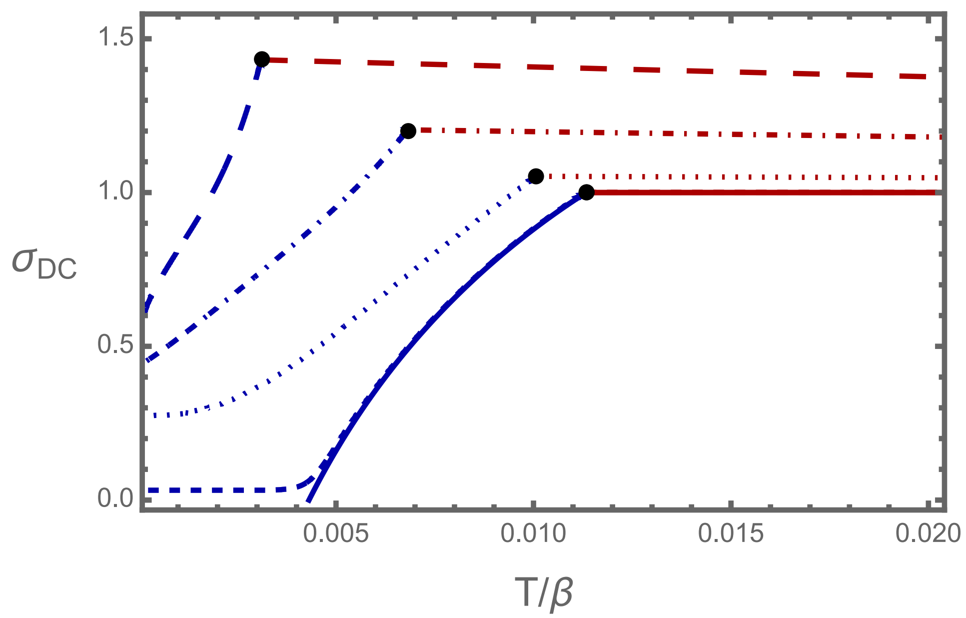


(a)



(b)

DC Conductivity

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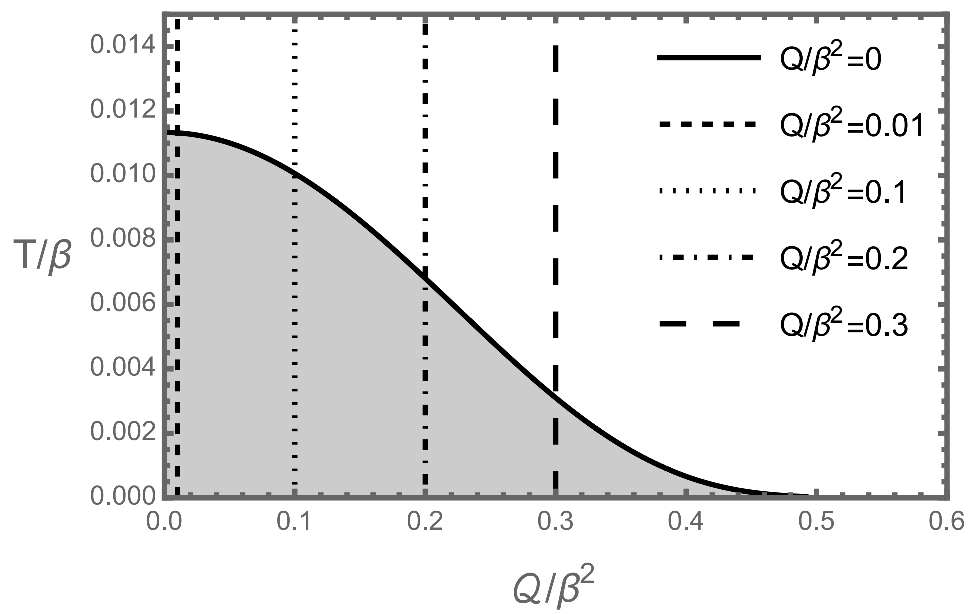

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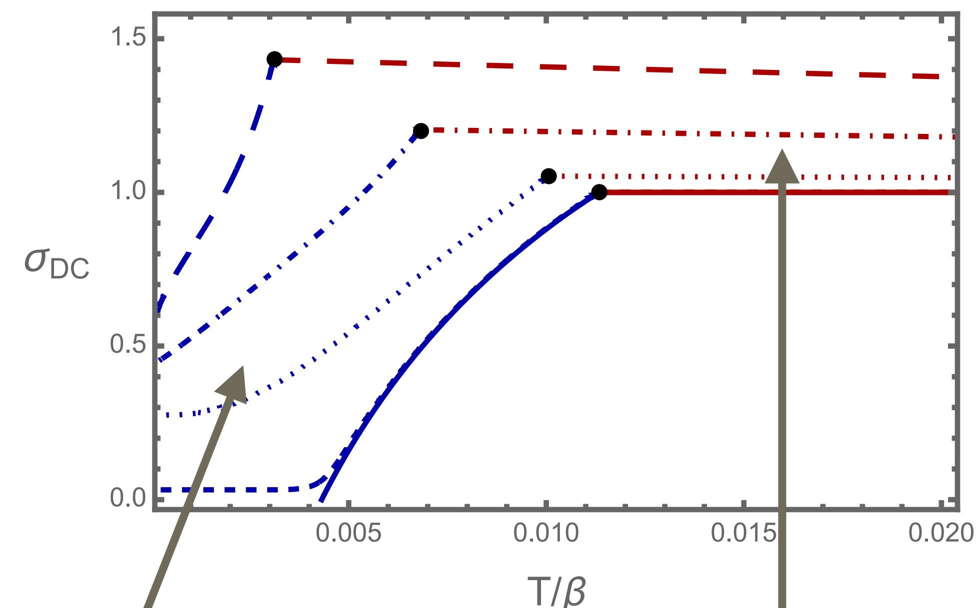


(b)

$\frac{\partial \rho}{\partial T} < 0$

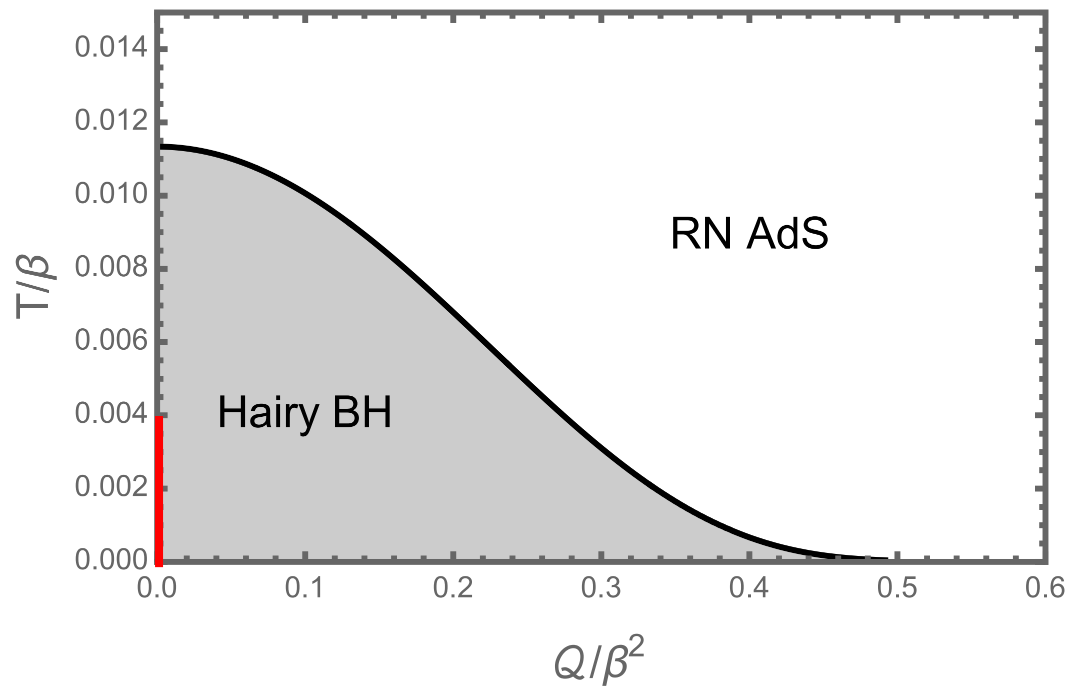
Insulating behavior

$\frac{\partial \rho}{\partial T} > 0$

Metallic behavior

Metal-Insulator Transition

■ Phase diagram



$$\sigma_{DC} = \left(1 + \gamma_2 \phi_h^2\right) + \frac{Q^2}{r_h^2 \beta^2}$$
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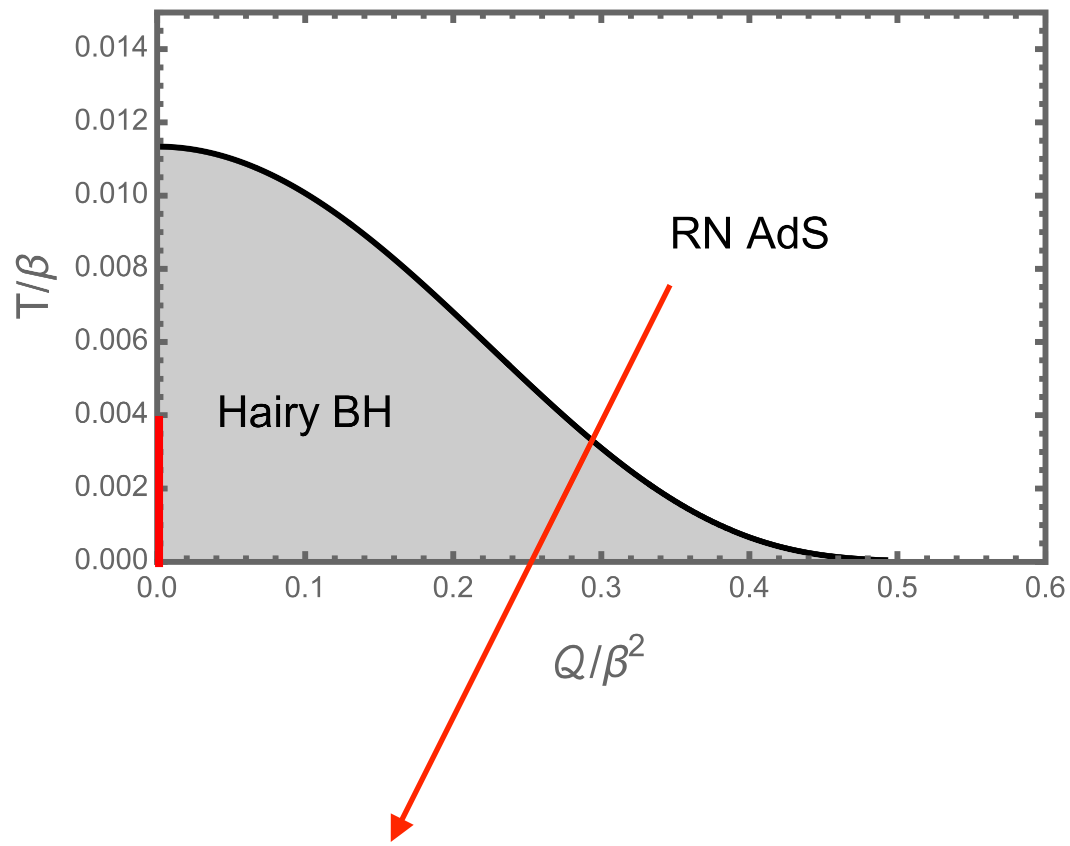
Metal-Insulator Transition

■ Phase diagram



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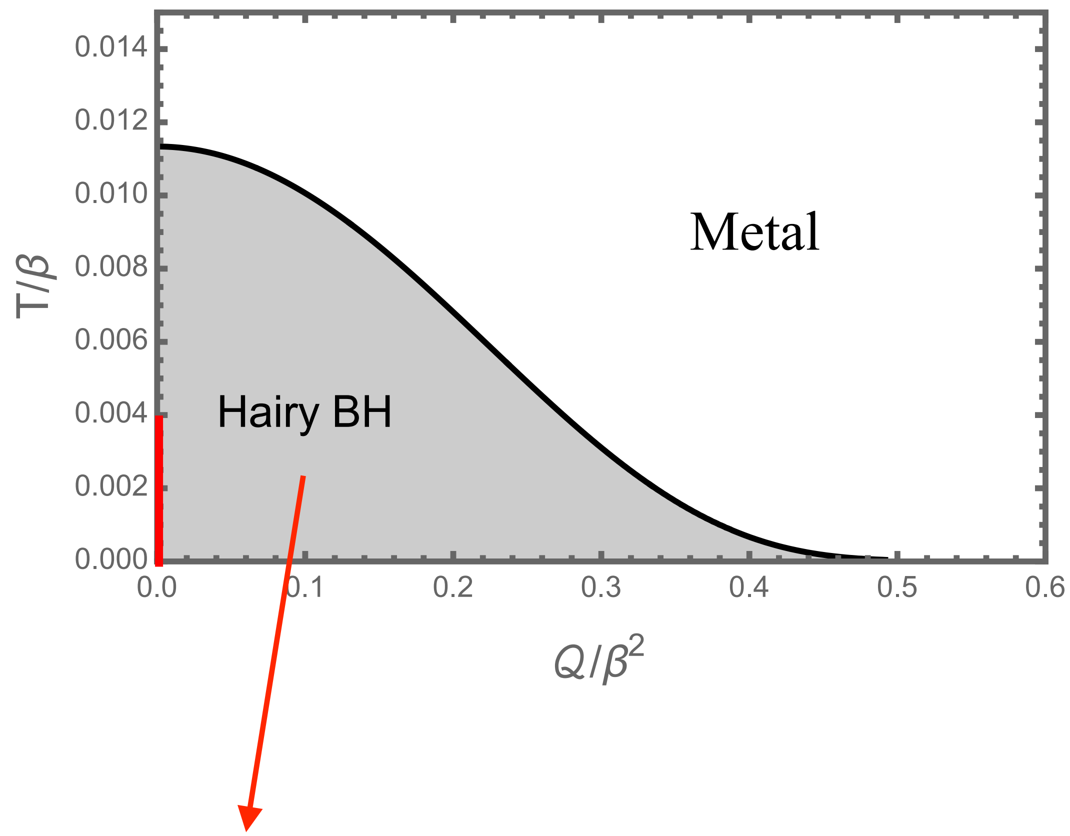
- σ_{diss} dominant
- Drude like behavior
- Resistivity is increasing to T
- Metallic phase

Metal-Insulator Transition

■ Phase diagram

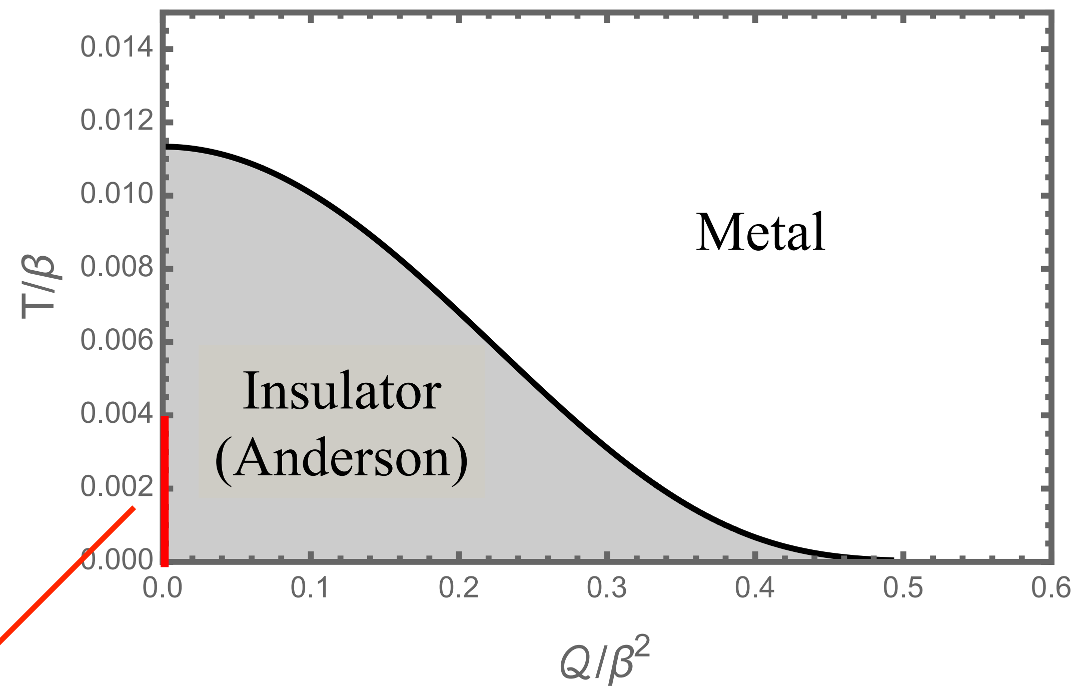


$$\sigma_{DC} = (1 + \gamma_2 \phi_h^2) + \frac{Q^2}{r_h^2 \beta^2} = \sigma_{ccs} + \sigma_{diss}$$

- σ_{ccs} suppression dominant
- Resistivity is decreasing to T
- Impurity induced insulating phase
- 'Anderson insulator'

Metal-Insulator Transition

■ Phase diagram



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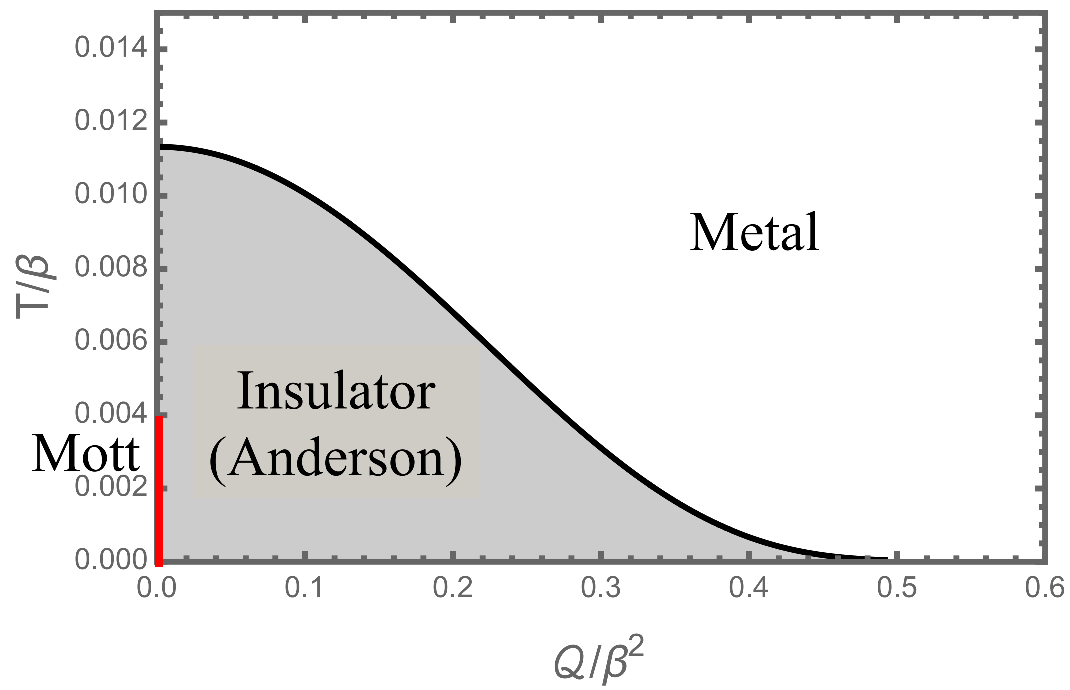
- No black hole solution
- Hawking-Page transition(geometric transition)
- Solitonic(or singular) solution
- Insulating phase
- ‘Mott insulator’

Metal-Insulator Transition

■ Phase diagram

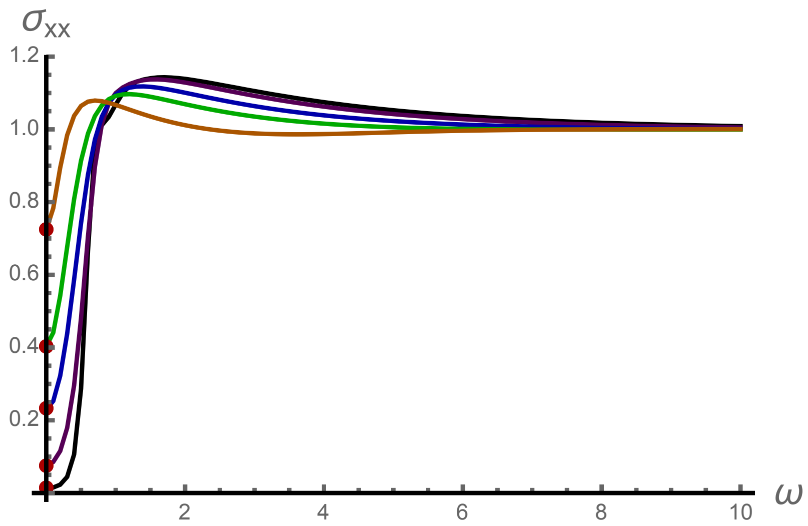


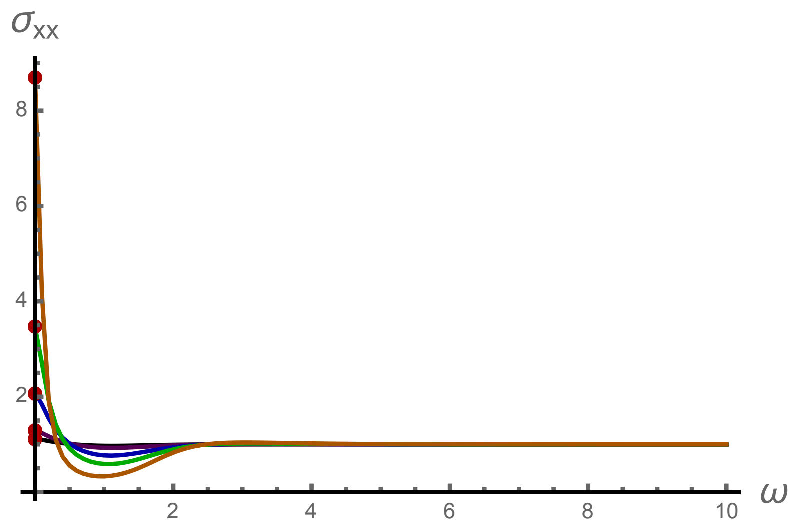
$$\sigma_{DC} = \left(1+\gamma_2\phi_h^2\right) + \frac{Q^2}{r_h^2\beta^2} = \sigma_{ccs} + \sigma_{diss}$$

Summary and Future direction

- We construct a gravity system with scalar-gauge field interaction
  - We find a phase transition between RN AdS black hole and a hairy black hole
  - Impurity enhances scalar condensation
  - Scalar condensation leads to the insulating phase
  - The insulating phase comes from the localization of electron-hole pair creation
  - We realize ‘Anderson insulator’-metal transition in holography

- AC conductivity





Thank you !!