Holographic duals of M5-branes on an irregularly punctured sphere

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Outline

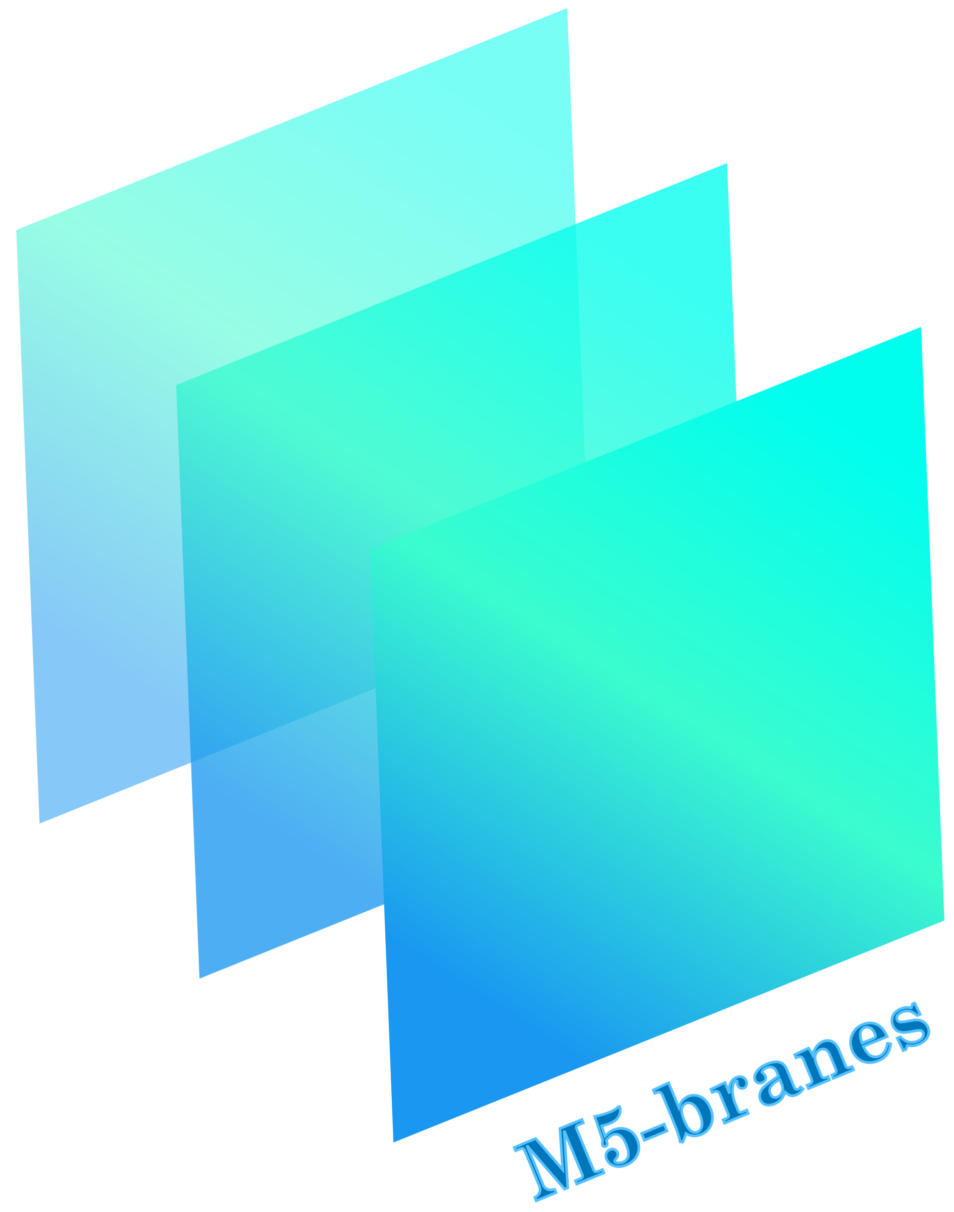
- Introduction : Class S theories, Argyres-Douglas theories, Spindles & Discs
- Holographic duals of M5-branes on a punctured sphere

  (Bah, Bonetti, Minasian, Nardoni 2021)
- Our strategy : Toda system, Electrostatic reformulation
- Generalize the regular puncture
- Match to the dual field theory

Introduction

Class S theory (Gaiotto 2006, Gaiotto, Moore, Neitzke 2009)

- 4d N=2 SCFTs
- Geometric engineered : 6d (2,0) theory compactified on a Riemann surface
- Parent theory with A-type singularity : M5-branes stack
- Lagrangian theory, described by quiver diagram



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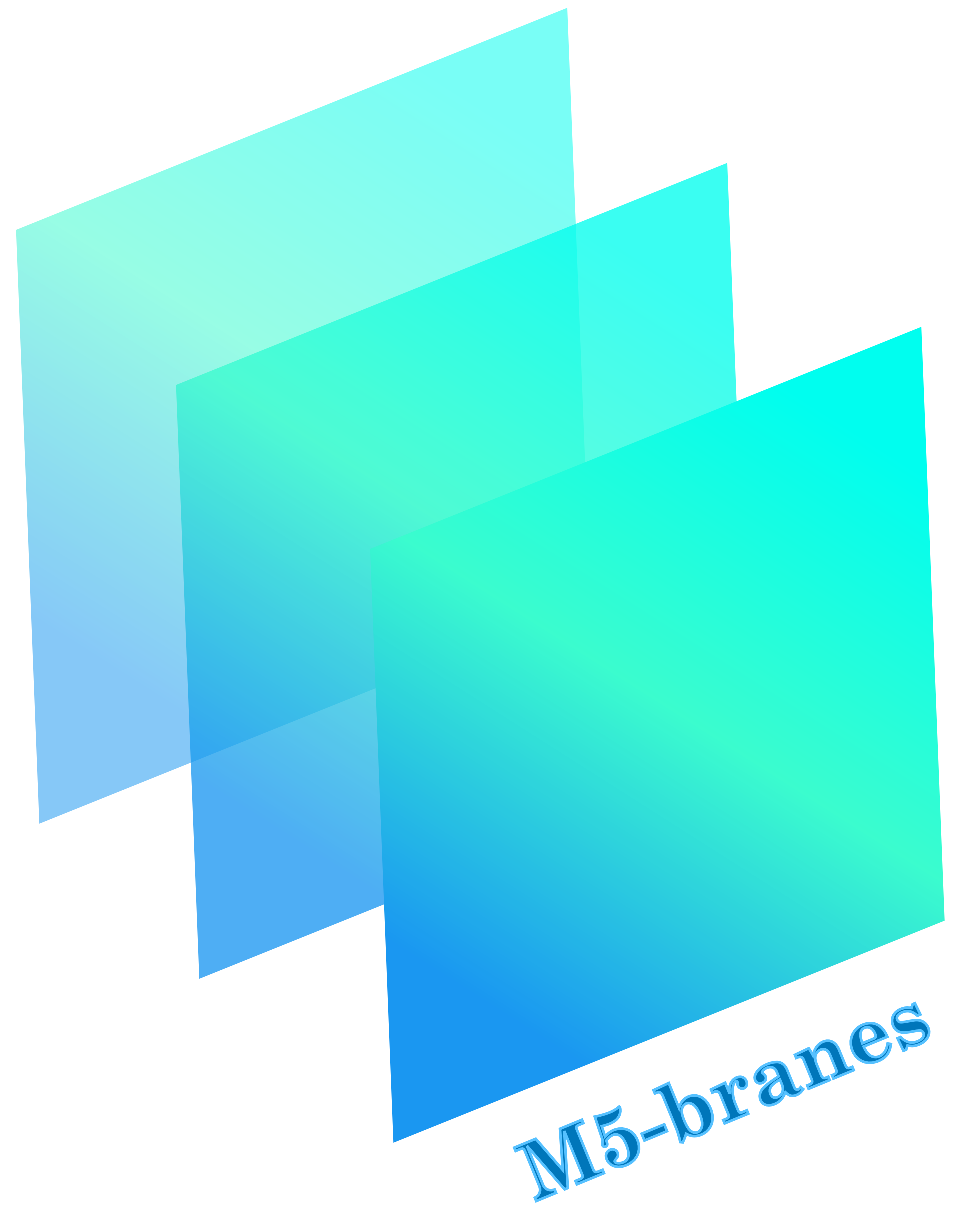

Argyres-Douglas theory (Argyres, Douglas 1995)

- 4d N=2 SCFTs
- Fractional scaling dimensions
- Intrinsically strongly coupled theory
- Non-Lagrangian theory, described by Young diagram and irregular puncture data

Class S theory (Gaiotto 2006, Gaiotto, Moore, Neitzke 2009)

Riemann surface with regular punctures

- 4d N=2 SCFTs
- Geometric engineered : 6d (2,0) theory compactified on a Riemann surface
- Parent theory with A-type singularity : M5-branes stack
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Argyres-Douglas theory (Argyres, Douglas 1995)

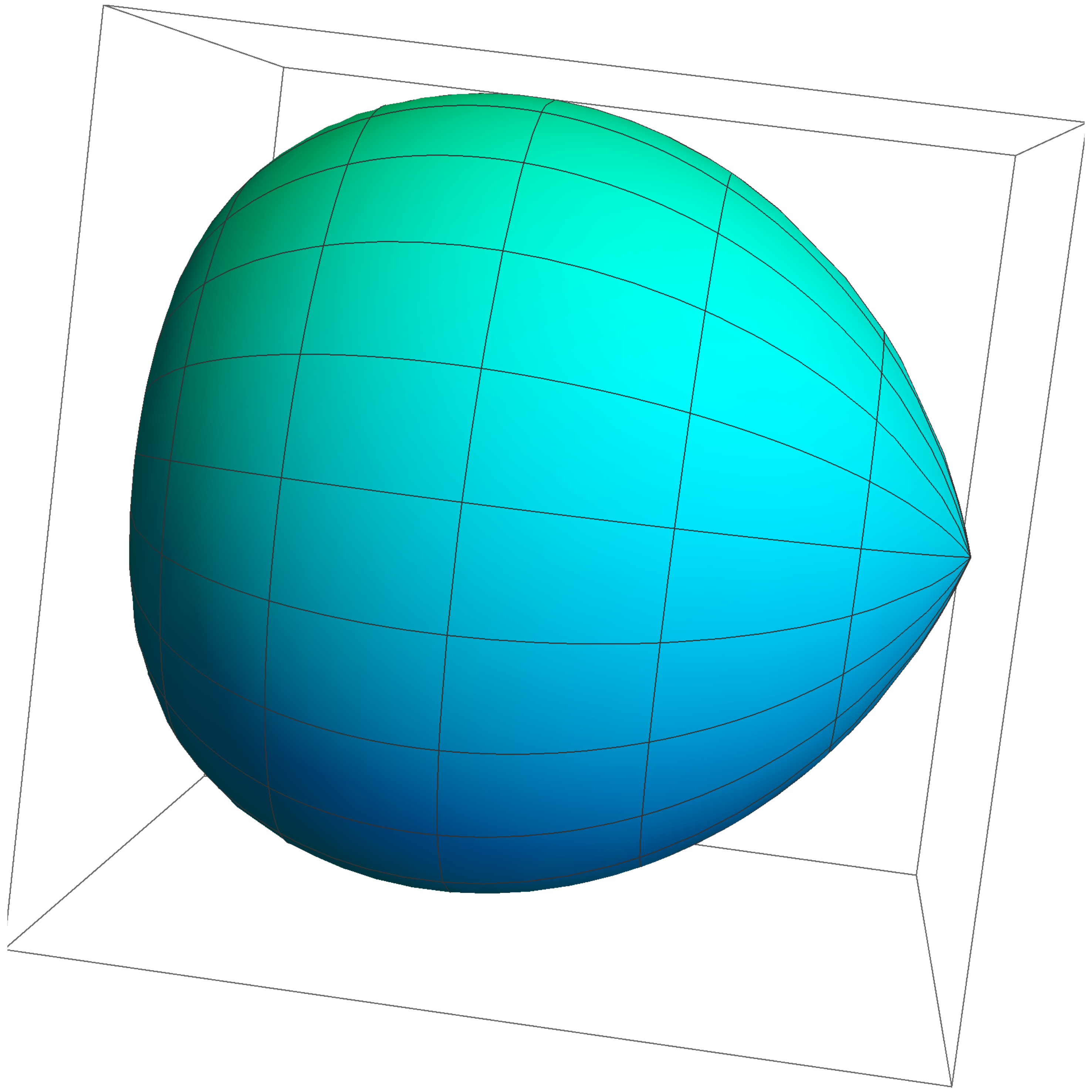
Sphere with irregular punctures

- 4d N=2 SCFTs
- Fractional scaling dimensions
- Intrinsically strongly coupled theory
- Non-Lagrangian theory, described by Young diagram and irregular puncture data

Punctures

(D. Xie 2012)

- Singular solutions $\Phi(z)$ of Hitchin's equation on sphere
  - Regular puncture : simple pole $\Phi(z) \sim \frac{1}{z}$
  - Irregular puncture : higher order pole $\Phi(z) \sim \frac{1}{z^n},\ n > 1$



- An irregular puncture of type I
- Type IV : A regular puncture and an irregular puncture of type I

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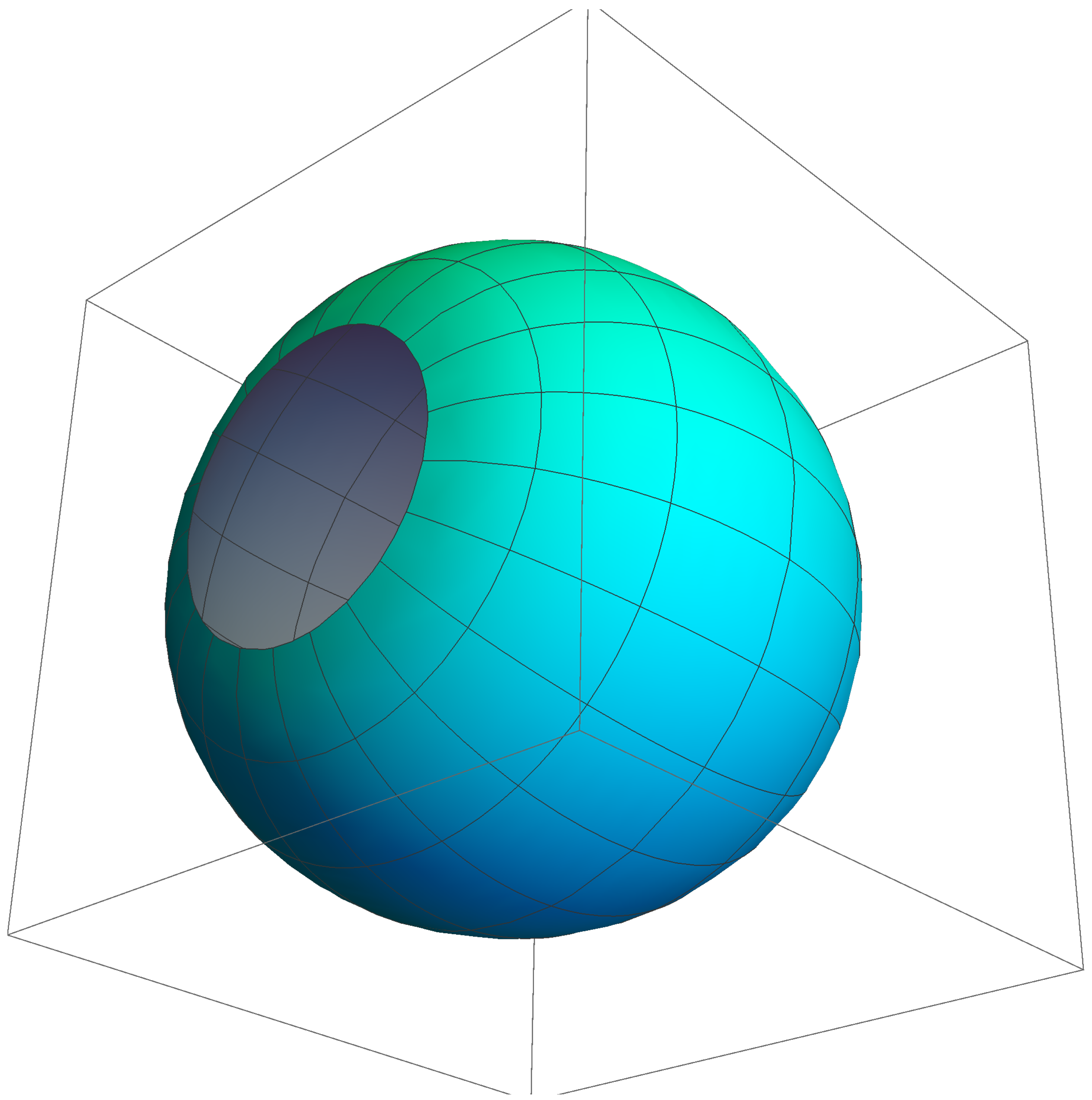


- An irregular puncture of type I
- Type IV : A regular puncture and an irregular puncture of type I

Spindle & Disc

- Holographic dual of punctured sphere

Spindle dual to sphere with 2 regular punctures

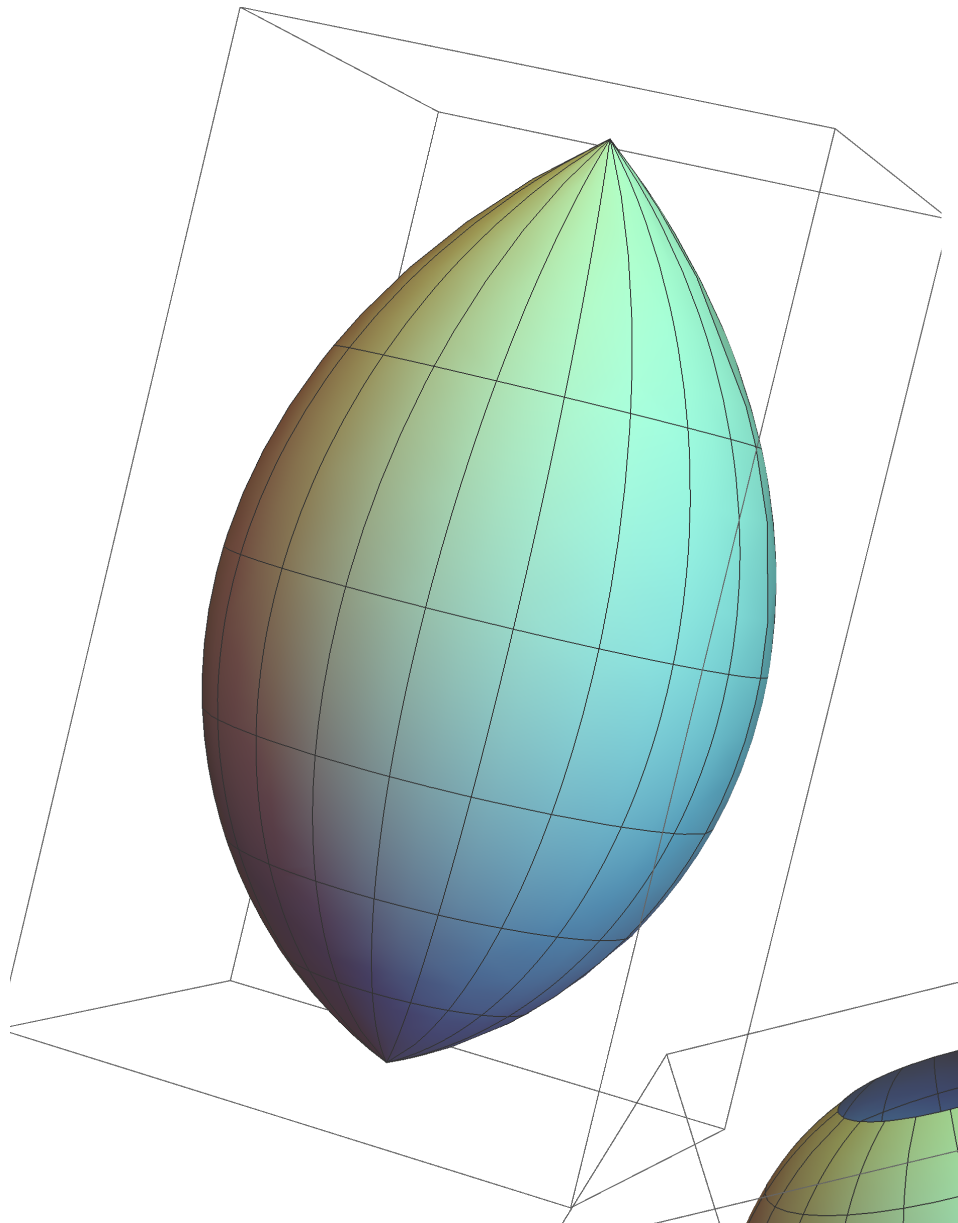
$\mathrm{WCP}^1_{[n_-,n_+]}$: two conical singularities

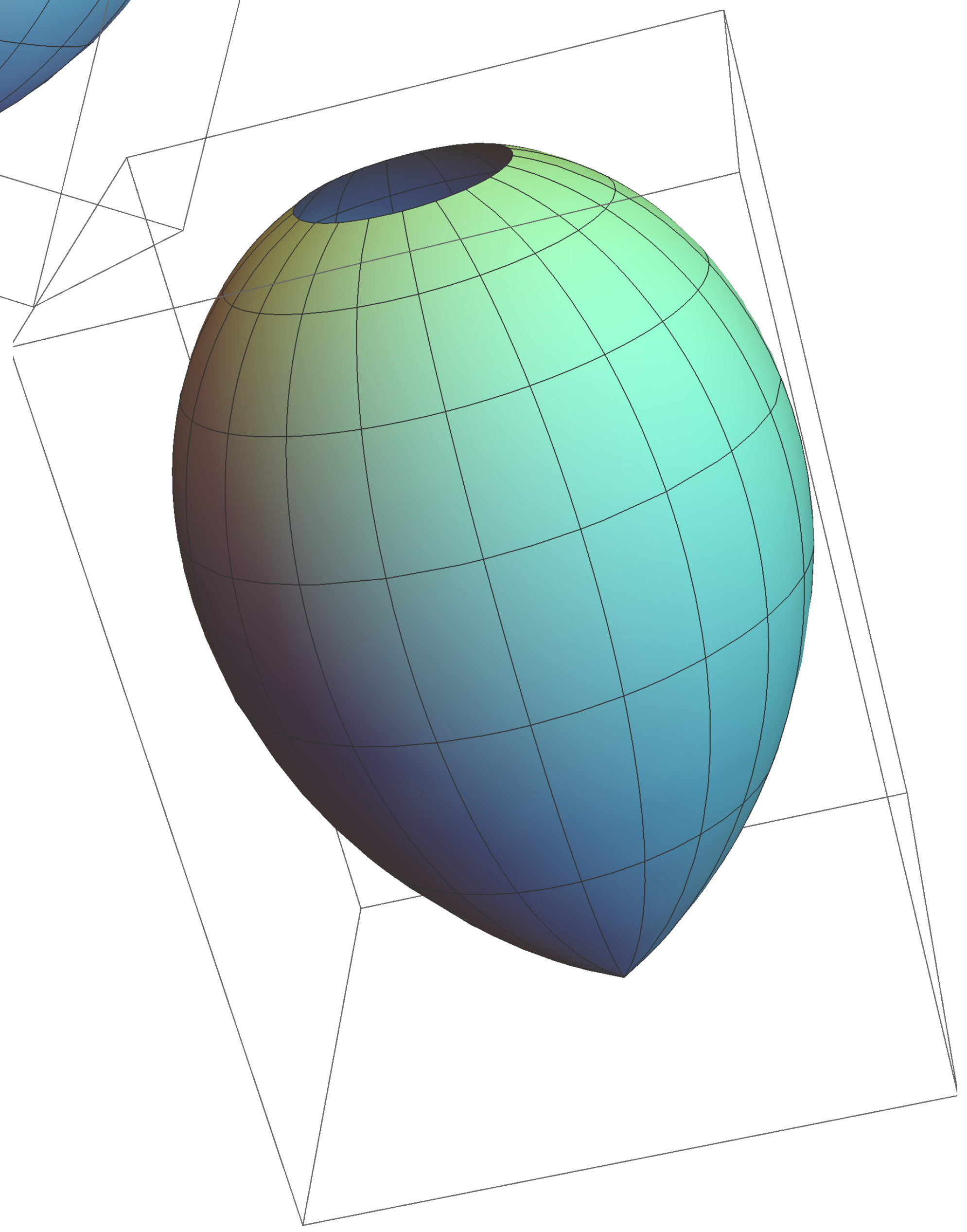
- Uplift : removed in M2, D3

still remain in D4, M5, D2 $\rightarrow$ **physical interpretation**

Disc dual to sphere with a regular and a irregular punctures

conical singularity and physical singularity





Holographic duals of M5-branes on punctured sphere

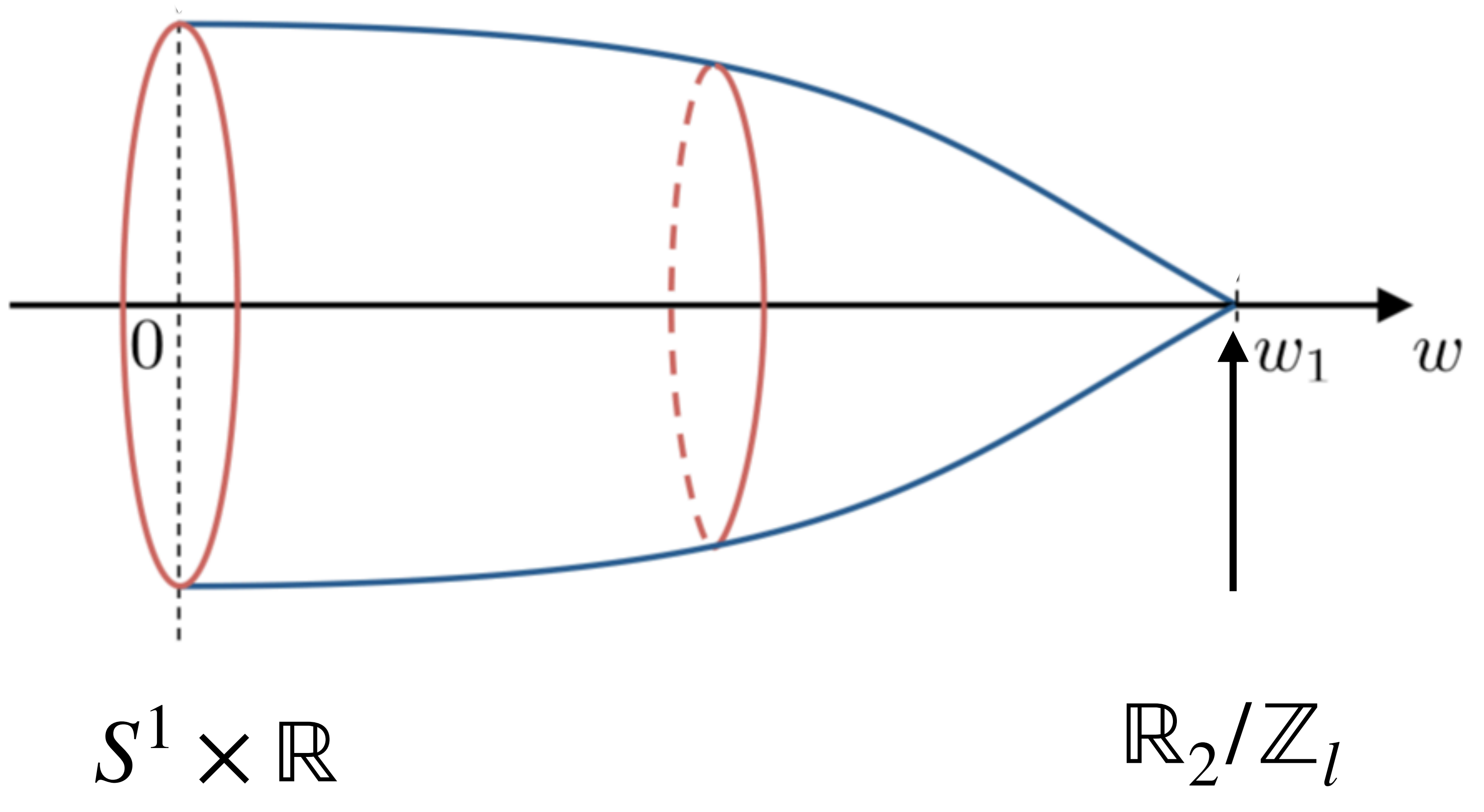
$\mathrm{AdS}_5 \times \Sigma$

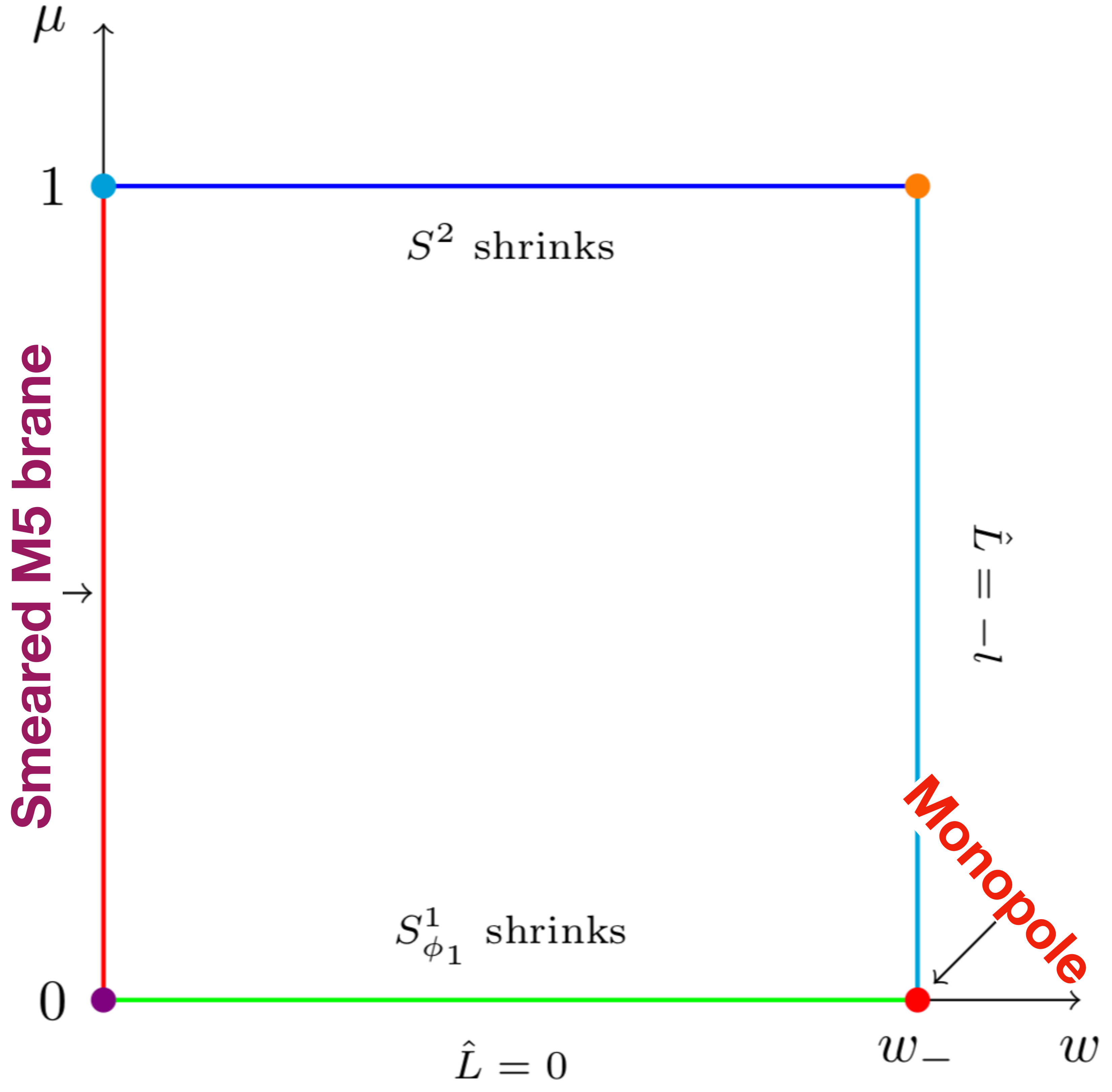
(Bah, Bonetti, Minasian, Nardoni 2021)

$\mathrm{AdS}_5 \times S^2 \times S^1_z \times S^1_\phi$ over $[0, w_-] \times [0,1]$

$$ds_7^2 = \left(wP(w)\right)^{1/5}\left[4ds^2(\mathrm{AdS}_5) + \frac{w}{f(w)}dw^2 + \frac{f(w)}{P(w)}dz^2\right]$$

$$h_1(w) = w^2 - s_1, \quad h_2(w) = w^2, \quad P(w) = h_1(w)h_2(w), \quad f(w) = P(w) - w^3$$





N=2 classification

(Gaiotto, Maldacena 2009)

- Embed into the classification of N=2 preserving AdS_5 solution of 11d supergravity

Toda system $\quad \square_{(x_1,x_2)} D + \partial_y^2 \mathrm{e}^D = 0$

- With extra U(1) isometry, $x_1 + i x_2 = r e^{i\beta}$
- Can perform Bӓcklund transformation

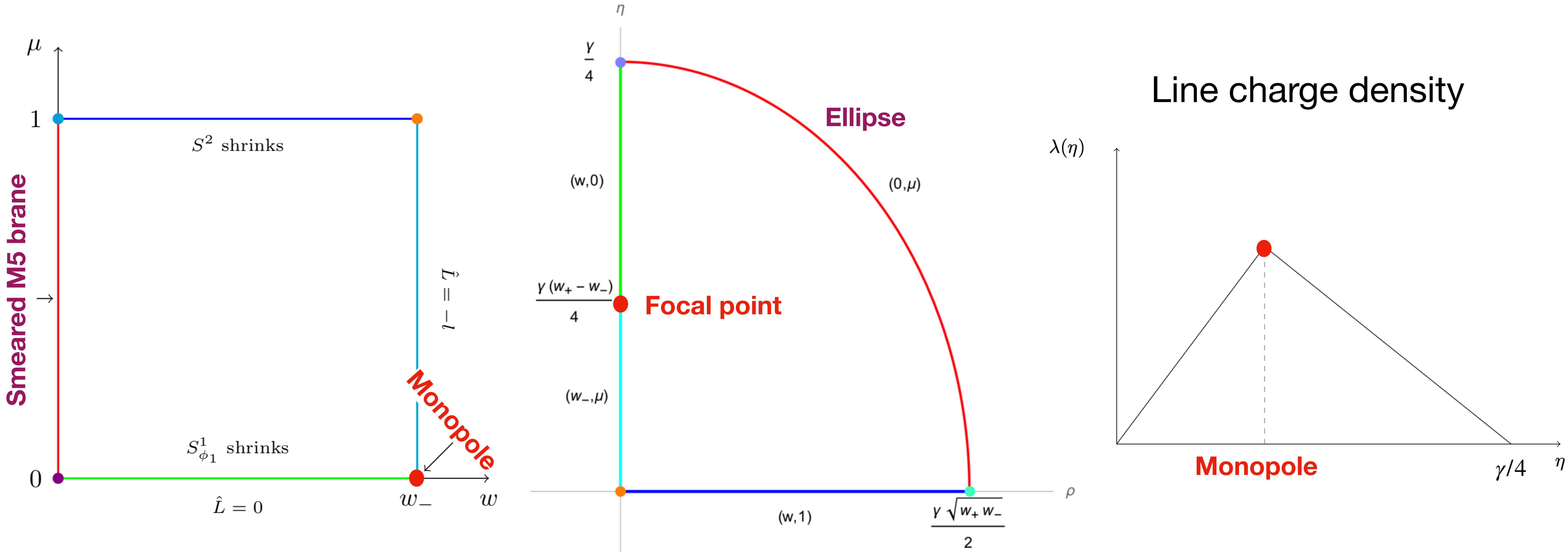
$$r^2 \mathrm{e}^D = \rho^2 \,, \quad y = \rho \partial_\rho V(\rho, \eta) \equiv \dot{V} \,, \quad \log r = \partial_\eta V(\rho, \eta) \equiv V'$$

- End up with the cylindrical Laplace equation

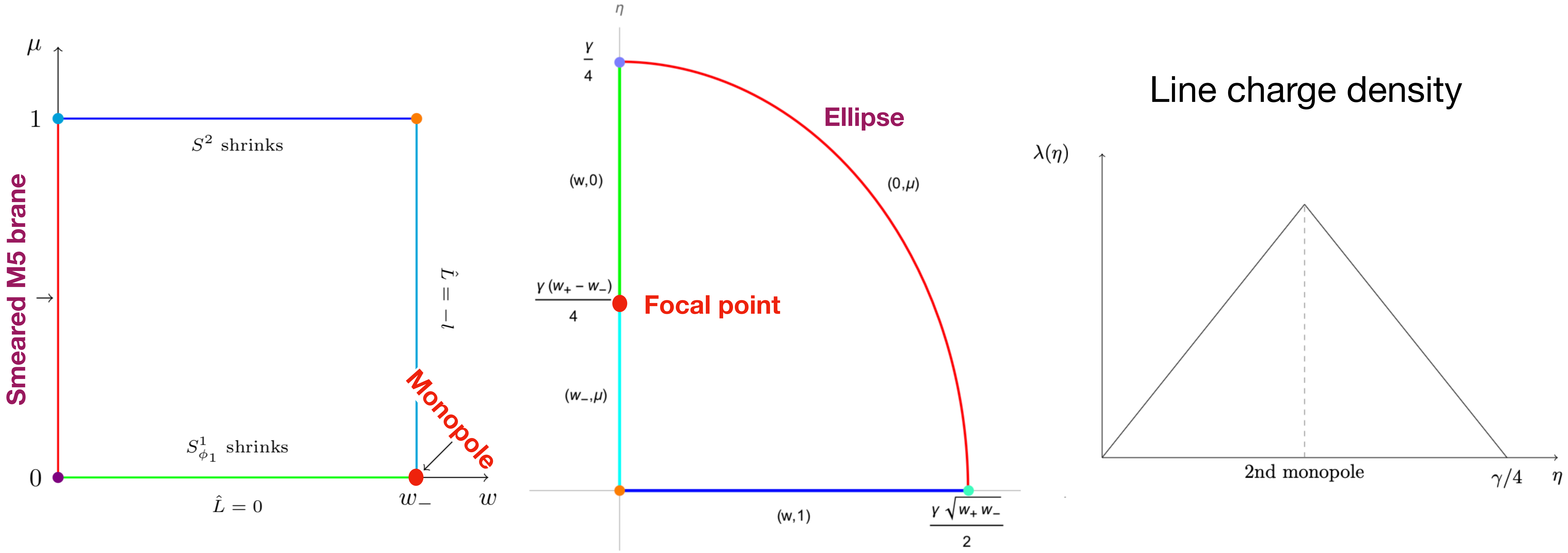
Laplace equation $\quad \ddot{V} + \rho^2 V'' = 0$

- Boundary conditions : $\dot{V} = 0$ along $\eta = 0$

Line charge density : $\lambda(\eta) = y(\rho = 0, \eta)$



μ
1
0
S^2 shrinks
$S^1_{\phi_1}$ shrinks
$\hat{L} = 0$
w_-
w
$l = -\hat{L}$
Smeared M5 brane
Monopole
η
$\frac{\gamma}{4}$
(w,0)
$\frac{\gamma(w_+ - w_-)}{4}$
Focal point
(w_-,μ)
Ellipse
$(0,\mu)$
(w,1)
$\frac{\gamma\sqrt{w_+ w_-}}{2}$
ρ
Line charge density
$\lambda(\eta)$
Monopole
$\gamma/4$
η



Smeared M5 brane
μ
1
0
S^2 shrinks
$S^1_{\phi_1}$ shrinks
$\hat{L} = 0$
$\hat{L} = -l$
Monopole
w_-
w
η
$\frac{\gamma}{4}$
(w,0)
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Focal point
(w_-,μ)
Ellipse
$(0,\mu)$
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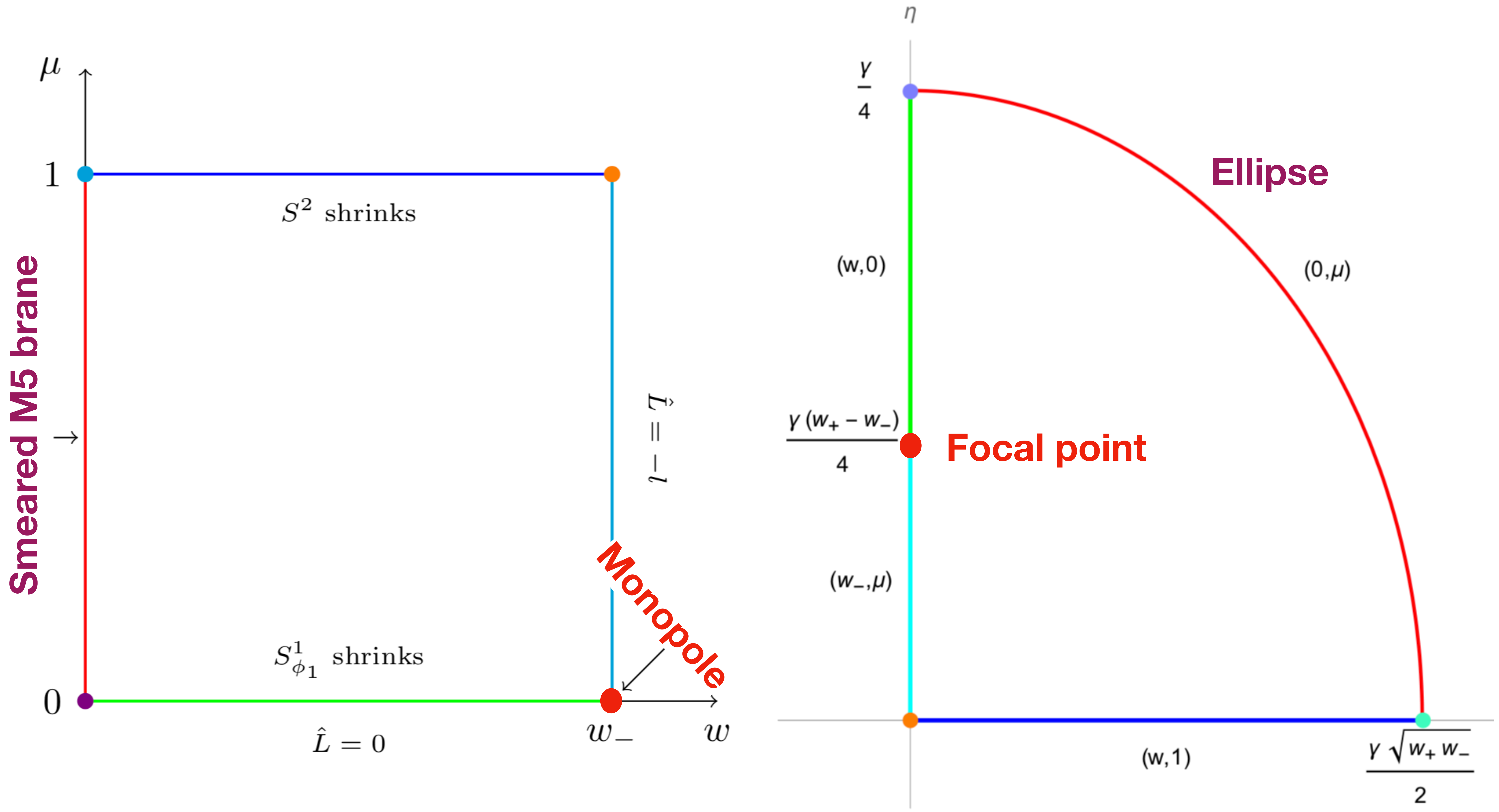
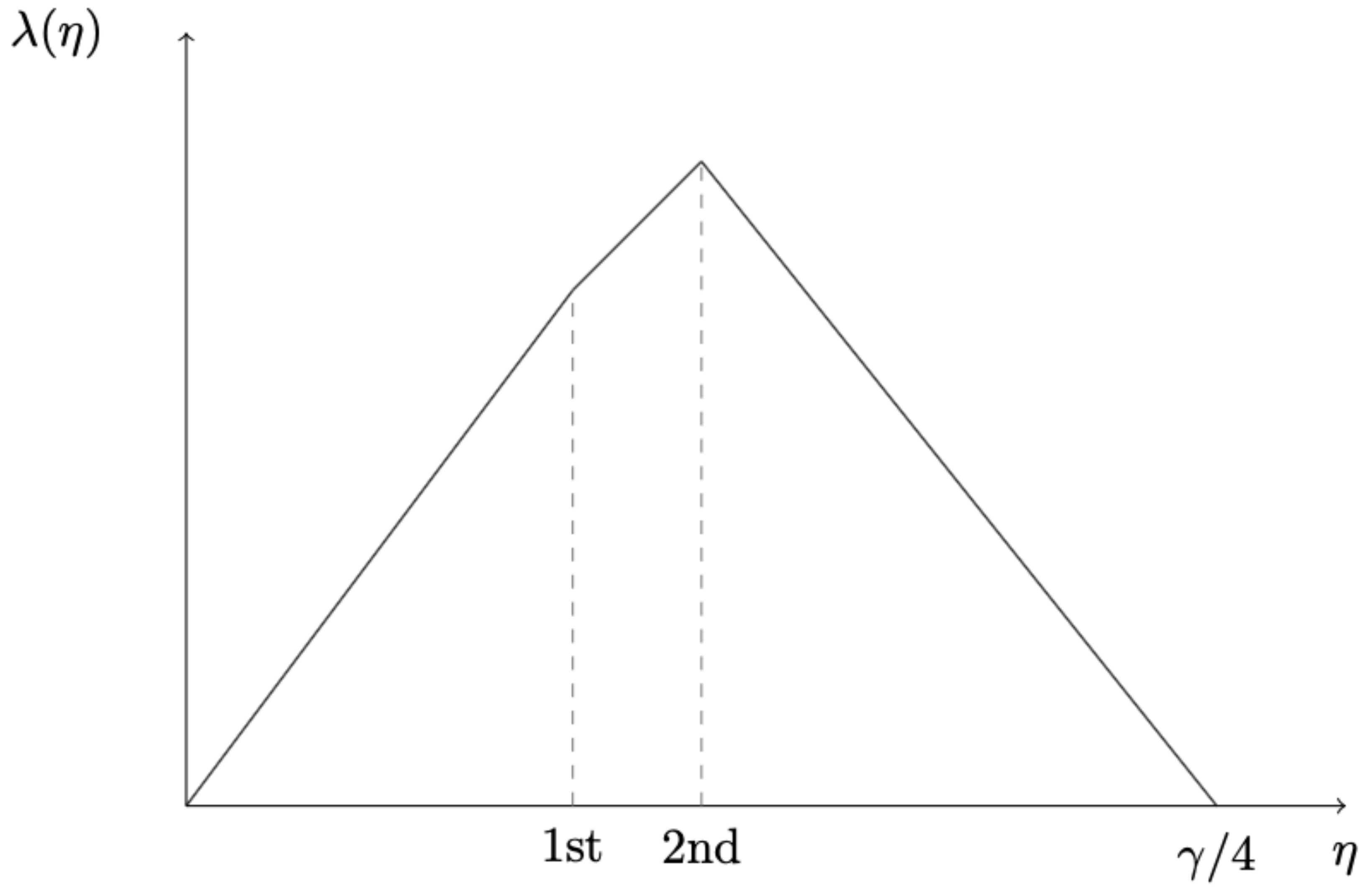
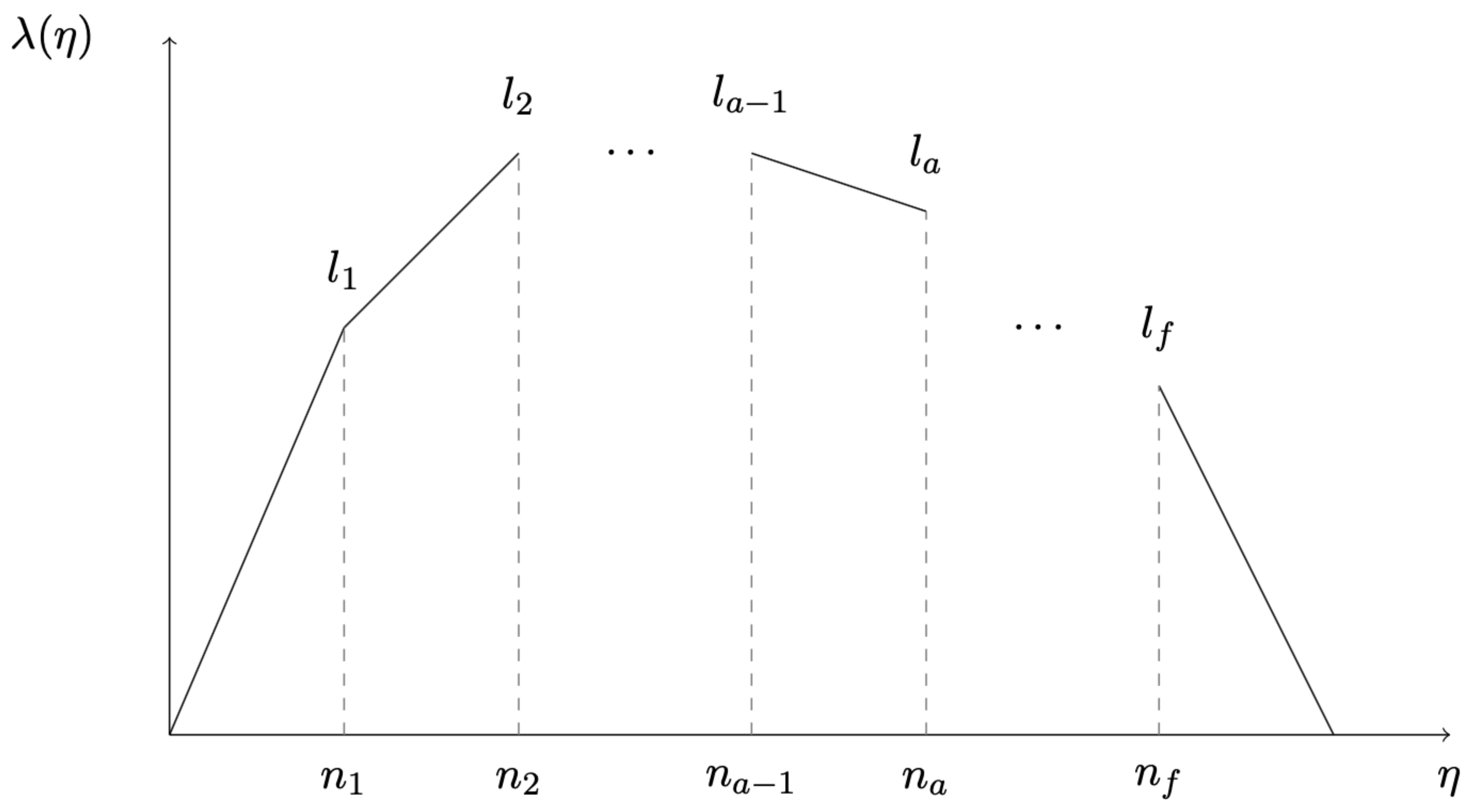
$\frac{\gamma\sqrt{w_+ w_-}}{2}$
Line charge density
$\lambda(\eta)$
2nd monopole
$\gamma/4$
η



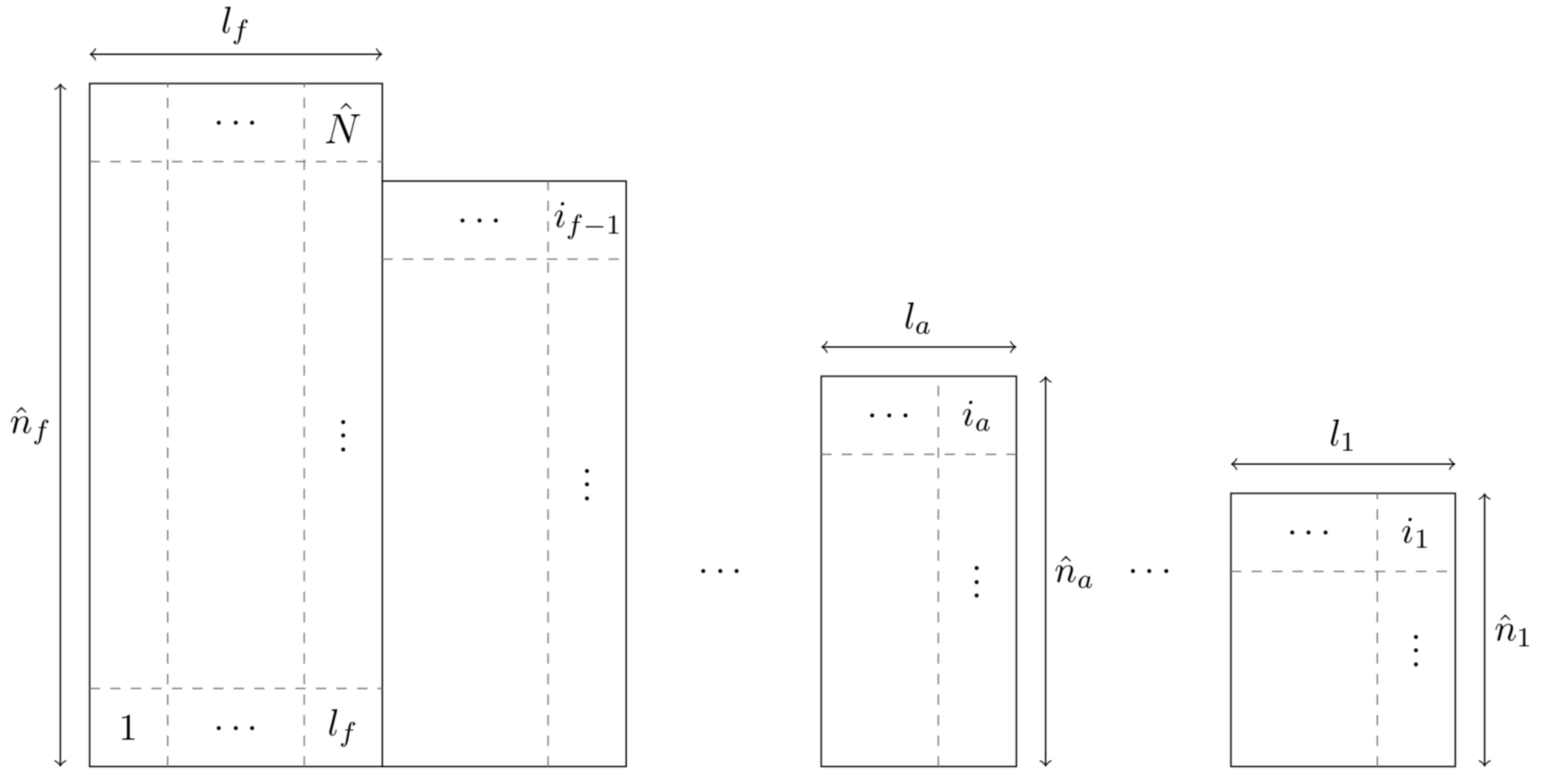
Generalized line charge density



Generalize the regular puncture



Generalized line charge

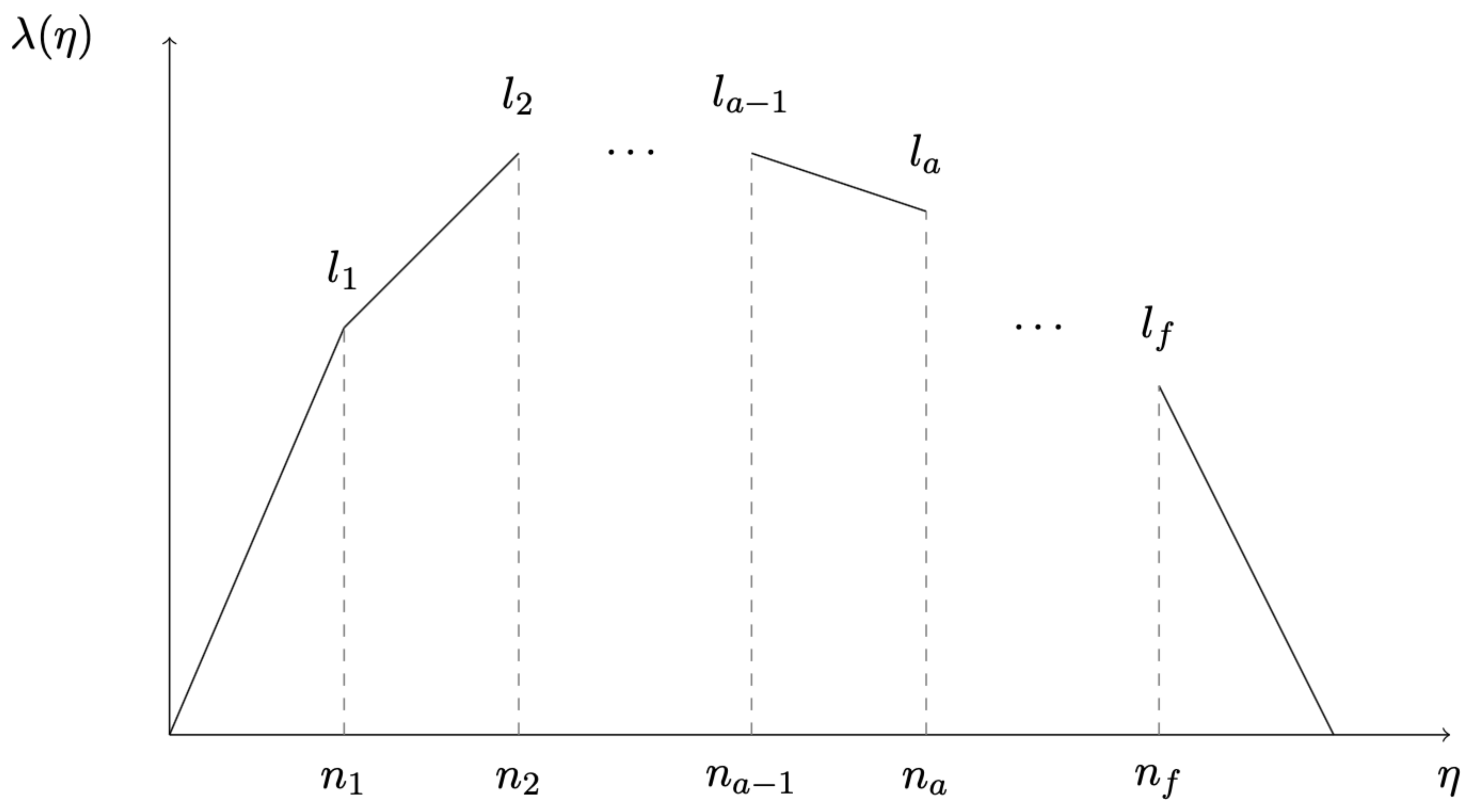


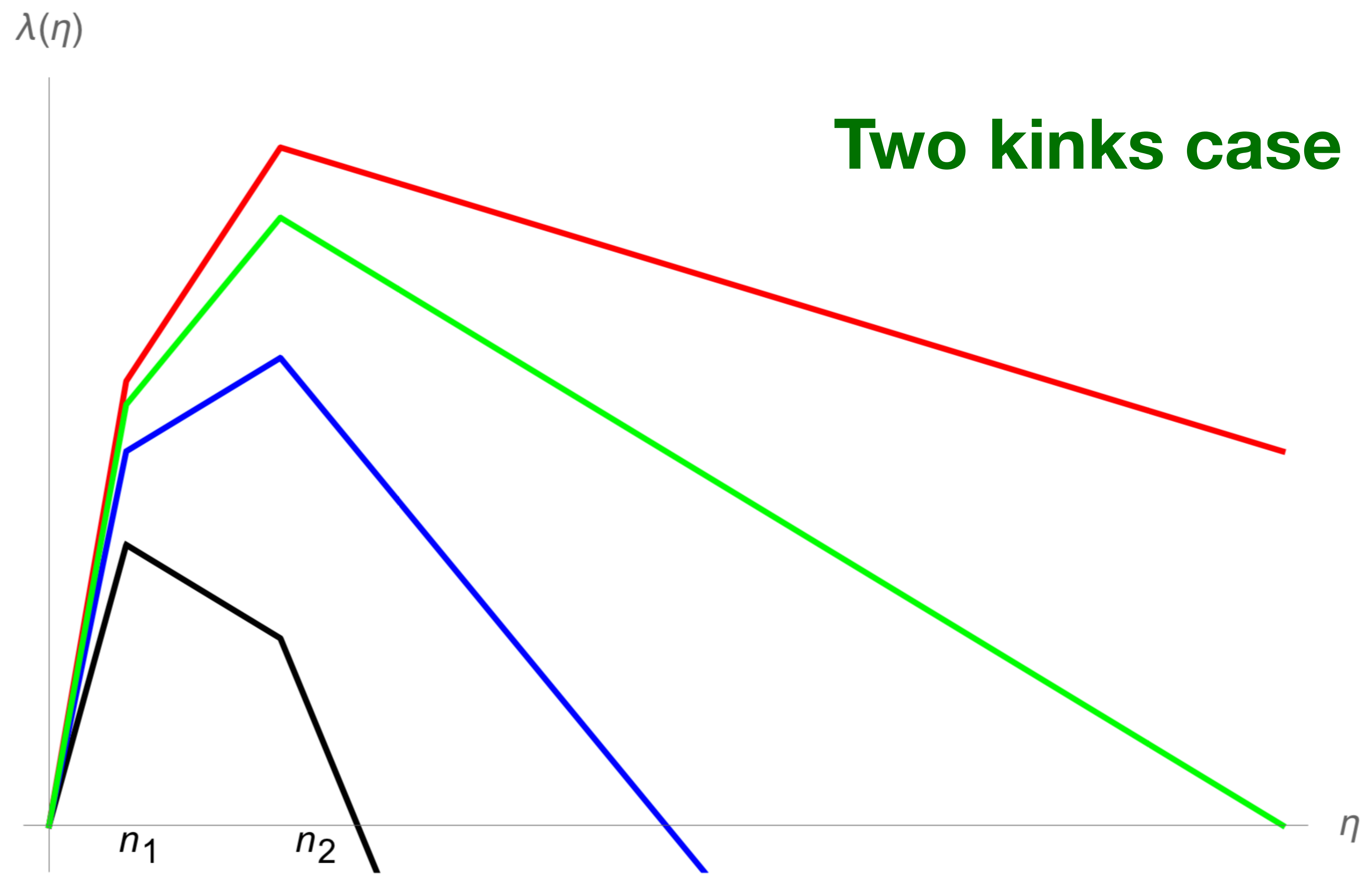
Generalized Young diagram

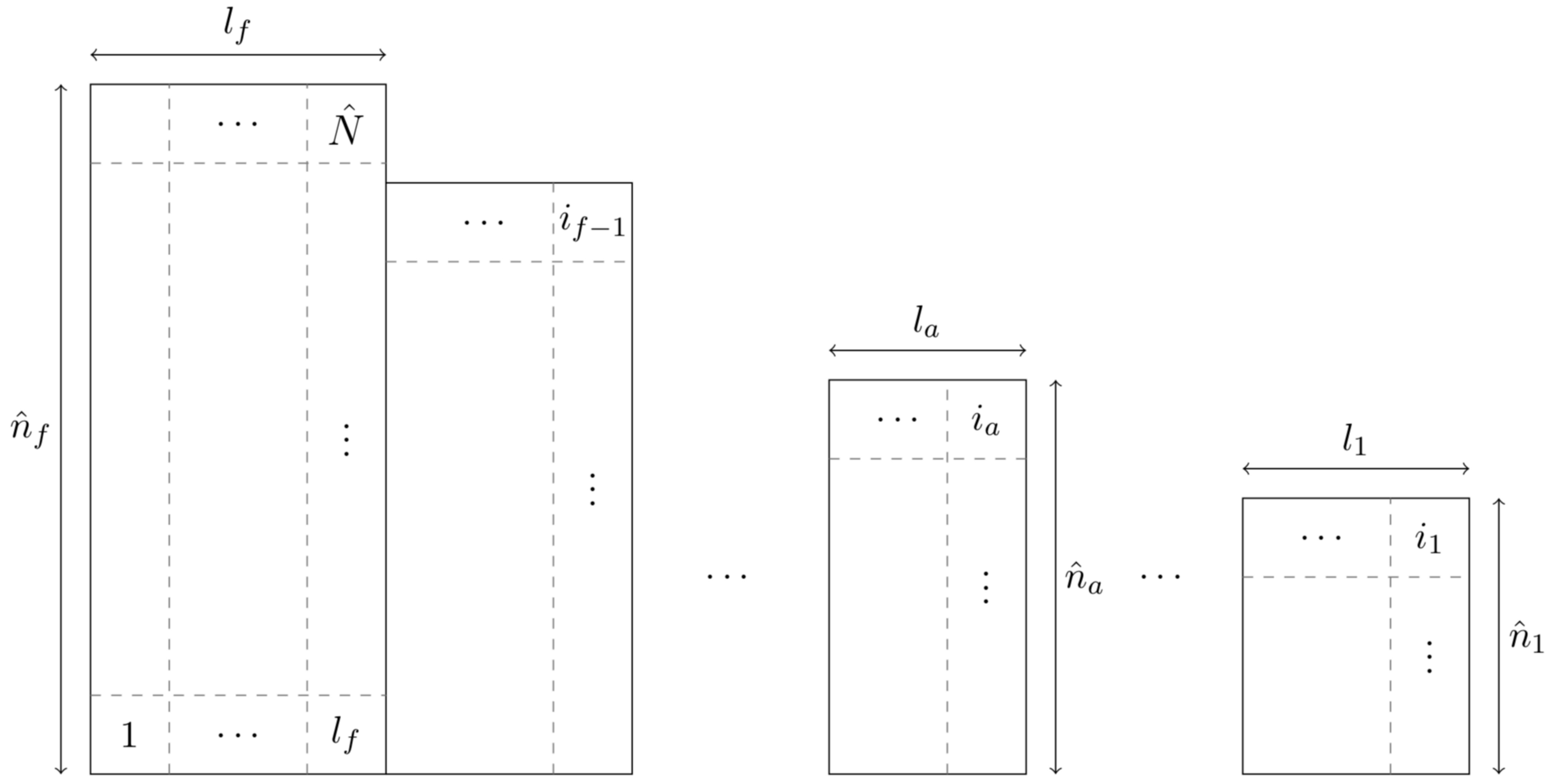
$$\lambda = r_a \eta + m_a \qquad \begin{cases} r_{a-1} - r_a \equiv l_a \in \mathbb{Z}\,, \\ N m_a \equiv M_a \in \mathbb{Z}\,, \\ N n_a \equiv N_a \in \mathbb{Z} \end{cases}$$

[Holographic dictionaries]

$$\hat{k} = N_{f+1} - \hat{N}\,, \quad \hat{N} = \sum_{a=1}^{f} N_a l_a, \quad \hat{n}_a = N_a\,, \quad l_a^{\text{SCFT}} = l_a^{\text{SUGRA}}$$







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Match to the dual field theory

Observables

- From the general line charge density, the central charge is

$$a = N^3 \int_0^{n_{f+1}} \lambda(\eta)^2 \mathrm{d}\eta$$

- Scaling dimensions of BPS probe M2-branes located at the kinks are

$$\Delta(\mathcal{O}_a) = N\lambda(n_a)$$

- The flavour central charges are

$$k_{F_a} = 2N\lambda(n_a)$$

Central charge

- Gravity side computation, using the line charge density

$$a = \frac{N^3}{4}\int_0^{n_3} \lambda(\eta)^2 d\eta = \frac{N^3}{12n_3}\left[n_1^2(n_1-n_3)^2\, l_1^2 + n_1(n_1^2+n_2^2-2n_2n_3)\, l_1 l_2 + n_2^2(n_2-n_3)^2\, l_2^2\right]$$

- For the $(I_{\hat{N},\hat{k}}, Y)$ theory,

$$a = a_Y + \frac{\hat{N}}{\hat{N}+\hat{k}}\frac{6I_{\rho Y} - \hat{N}(\hat{N}^2-1)}{12} + a_{I_{\hat{N},\hat{k}}} \sim a_{\text{leading}} + \mathcal{O}(N^2)$$

$$c = c_Y + \frac{\hat{N}}{\hat{N}+\hat{k}}\frac{6I_{\rho Y} - \hat{N}(\hat{N}^2-1)}{12} + c_{I_{\hat{N},\hat{k}}} \sim c_{\text{leading}} + \mathcal{O}(N^2)$$

$$a_{\text{leading}} = c_{\text{leading}} = \frac{N^3}{12n_3}\left[n_1^2(n_1-n_3)^2\, l_1^2 + n_1(n_1^2+n_2^2-2n_2n_3)\, l_1 l_2 + n_2^2(n_2-n_3)^2\, l_2^2\right]$$

Scaling dimensions

- Scaling dimensions of BPS probe M2-branes, located at the kinks

$$\Delta(\mathcal{O}_1) = n_1(l_1 + l_2) - \frac{n_1}{n_3}(n_1 l_1 + n_2 l_2)$$

$$\Delta(\mathcal{O}_2) = \left(1 - \frac{n_2}{n_3}\right)(n_1 l_1 + n_2 l_2)$$

- Conformal dimensions of BPS operators, corresponding to a'th box of the Young diagram

$$\Delta(\mathcal{O}_a) = i_a - \text{height}(i_a)\frac{\hat{N}}{\hat{k} + \hat{N}}$$

$$\Delta(\mathcal{O}_1) = n_1(l_1 + l_2) - \frac{n_1}{n_3}(n_1 l_1 + n_2 l_2)$$

$$\Delta(\mathcal{O}_2) = n_1 l_1 + n_2 l_2 - \frac{n_2}{n_3}(n_1 l_1 + n_2 l_2)$$

Flavour central charge

- For the flavour groups, arising at the kinks

$$k_{F_1} = 2n_1(l_1 + l_2) - \frac{2n_1}{n_3}(n_1 l_1 + n_2 l_2) = 2\Delta(\mathcal{O}_1)$$

$$k_{F_2} = 2\left(1 - \frac{n_2}{n_3}\right)(n_1 l_1 + n_2 l_2) = 2\Delta(\mathcal{O}_2)$$

- For the a'th non-abelian gauge factor

$$k_{F_a} = 2\Delta(\mathcal{O}_a)$$

Conclusion

- 4d N=2 Argyles-Douglas theory, geometric engineered by wrapping M5-branes on the punctured sphere with a regular and a irregular puncture at the poles
- Holographic dual of the punctured sphere are the warped product of AdS_5 and disc in 7d $U(1)^2$ gauged supergravity
- Generalization to the arbitrary type of a regular puncture, using electrostatics
- Match the observables between supergravity and dual field theory
- Apply similar methods to N=1 theory with either spindle or disc solutions
- Four-dimensional orbifolds in 6d and 7d, dual to 1d or 2d SCFTs

Thank you