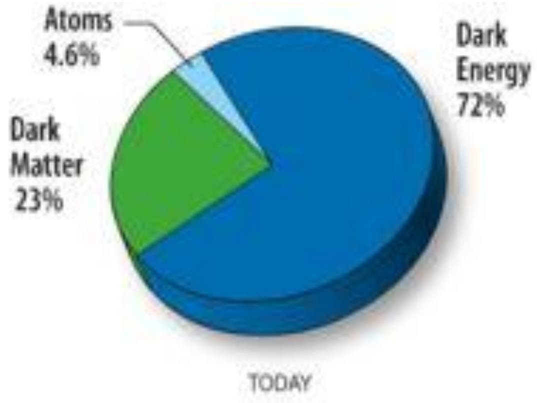
# Complementarity of experiments in the observation of dark matter-nucleon interaction in non-relativistic effective theory

Based on Anja Brenner, Gonzalo Herrera, Alejandro Ibarra, Sunghyun Kang, Stefano Scopel and Gaurav Tomar, Complementarity of experiments in probing the non-relativistic effective theory of dark matter-nucleon interactions, Journal of Cosmology and Astroparticle Physics, Volume 2022, June 2022

Sunghyun Kang

# DM and WIMP

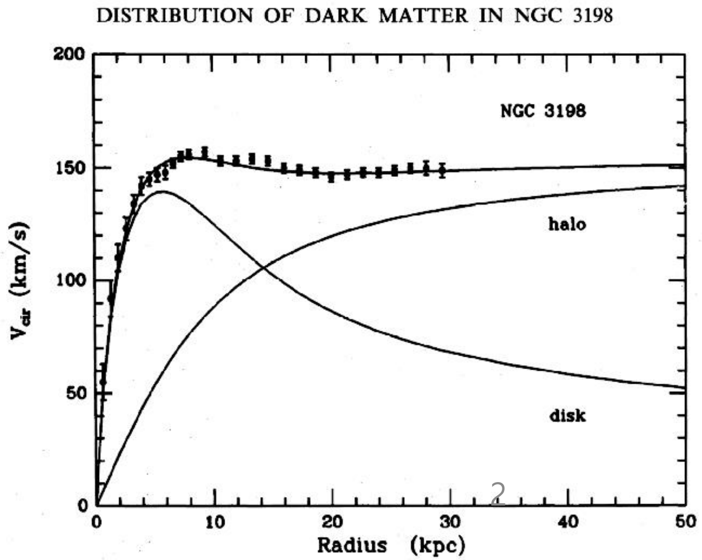



["Content of the Universe - Pie Chart"](#)
*Wilkinson Microwave Anisotropy Probe*.
National Aeronautics and Space Administration. Retrieved 9 January 2018.

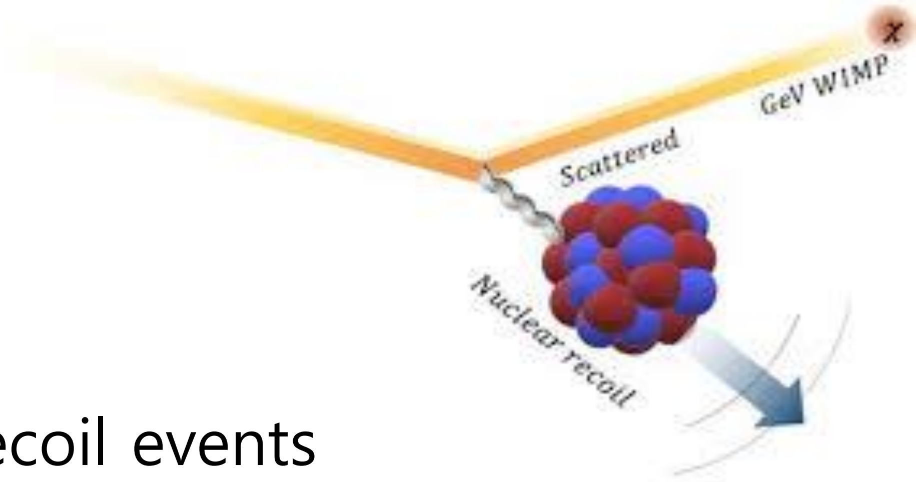
- Many evidences of Dark Matter
  - Rotation curve
  - CMB
  - Lensing

- Many candidates
  - Cold Dark Matter (CDM)
  - Neutrino
  - Weakly Interacting Massive Particle (WIMP)

- **WIMPs** are the most popular Dark Matter candidates
  - few GeV < WIMP mass< few TeV
  - No electric charge, no color
  - Weak-type interactions with ordinary matter keep WIMPs in thermal equilibrium in the early Universe and can provide the correct relic abundance through thermal decoupling ("WIMP miracle")



# Direct detection




- In direct detection, the signals are WIMP-nucleus recoil events following WIMP scattering off target nuclei in underground detectors
- Most models of scattering are driven by new Physics at large scale(≥100GeV)
- Momentum transfer is lower, a few hundred MeV or less.
- No hints from accelerators → phenomenological approach
- NREFT provides a general and efficient way to characterize results with mass of WIMP and coupling constants
- Nuclear response provides compact form of WIMP-nucleus cross section
- Nuclear form factors and new operators → cross section dependent on q and v

# NREFT

- Elastic scattering of a heavy WIMP off a nucleon
- Lagrangian : $\mathcal{L}_{int}(\vec{x}) = c\Psi^*_\chi(\vec{x})\mathcal{O}_\chi\Psi_\chi(\vec{x})\Psi^*_N(\vec{x})\mathcal{O}_\mathcal{N}\Psi_N(\vec{x})$
- Amplitude : $\sum_{i=1}^{\mathcal{N}}\left(c_i^{(n)}\mathcal{O}_i^{(n)} + c_i^{(p)}\mathcal{O}_i^{(p)}\right)$
- Non-relativistic process → include all operators invariant by Galilean transformations
- Building operators using

$$i\frac{\vec{q}}{m_N}, \quad \vec{v}^\perp, \quad \vec{S}_\chi, \quad \vec{S}_N$$

Operators spin up to 1/2

$$\mathcal{O}_1 = 1_\chi 1_N; \quad \mathcal{O}_2 = (v^\perp)^2; \quad \mathcal{O}_3 = i\vec{S}_N\cdot(\frac{\vec{q}}{m_N}\times\vec{v}^\perp)$$

$$\mathcal{O}_4 = \vec{S}_\chi\cdot\vec{S}_N; \quad \mathcal{O}_5 = i\vec{S}_\chi\cdot(\frac{\vec{q}}{m_N}\times\vec{v}^\perp); \quad \mathcal{O}_6 = (\vec{S}_\chi\cdot\frac{\vec{q}}{m_N})(\vec{S}_N\cdot\frac{\vec{q}}{m_N})$$

$$\mathcal{O}_7 = \vec{S}_N\cdot\vec{v}^\perp; \quad \mathcal{O}_8 = \vec{S}_\chi\cdot\vec{v}^\perp; \quad \mathcal{O}_9 = i\vec{S}_\chi\cdot(\vec{S}_N\times\frac{\vec{q}}{m_N})$$

$$\mathcal{O}_{10} = i\vec{S}_N\cdot\frac{\vec{q}}{m_N}; \quad \mathcal{O}_{11} = i\vec{S}_\chi\cdot\frac{\vec{q}}{m_N}; \quad \mathcal{O}_{12} = \vec{S}_\chi\cdot(\vec{S}_N\times\vec{v}^\perp)$$

$$\mathcal{O}_{13} = i(\vec{S}_\chi\cdot\vec{v}^\perp)(\vec{S}_N\cdot\frac{\vec{q}}{m_N}); \quad \mathcal{O}_{14} = i(\vec{S}_\chi\cdot\frac{\vec{q}}{m_N})(\vec{S}_N\cdot\vec{v}^\perp)$$

$$\mathcal{O}_{15} = -(\vec{S}_\chi\cdot\frac{\vec{q}}{m_N})((\vec{S}_N\times\vec{v}^\perp)\cdot\frac{\vec{q}}{m_N}),$$

# NREFT

- Each operators have distinct couplings to proton and neutron

$$\sum_{\alpha=n,p} \sum_{i=1}^{15} c_i^\alpha \mathcal{O}_i^\alpha, \qquad c_2^\alpha \equiv 0.$$

- Equivalent form using isospin

$$\sum_{i=1}^{15} (c_i^0 1 + c_i^1 \tau_3) \mathcal{O}_i = \sum_{\tau=0,1} \sum_{i=1}^{15} c_i^\tau \mathcal{O}_i t^\tau, \qquad c_2^0 = c_2^1 \equiv 0,$$

$$|p\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |n\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad 1 \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \tau_3 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$c_i^0 = \frac{1}{2}(c_i^{\mathrm{p}} + c_i^n) \quad c_i^1 = \frac{1}{2}(c_i^p - c_i^n) \qquad t^0 \equiv 1 \quad t^1 \equiv \tau_3.$$

# Scattering amplitude

- Factorize amplitude into WIMP response functions R and nuclear response functions W:

WIMP response function

Nuclear response function

$$\frac{1}{2j_\chi+1}\frac{1}{2j_N+1}\sum_{\text{spins}}|\mathcal{M}|^2 \equiv \sum_k \sum_{\tau=0,1}\sum_{\tau'=0,1} R_k^{\tau\tau'}\left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}, \{c_i^\tau c_j^{\tau'}\}\right) W_k^{\tau\tau'}(y)$$

- k= $M$, $\Delta$, $\Sigma'$, $\Sigma''$, $\tilde{\Phi}'$, and $\Phi''$
  which transform as vector charge, vector transverse magnetic, axial transverse electric, axial longitudinal, vector transverse electric, and vector longitudinal operators, respectively

W.C. Haxton, N.Anand and A. L.Fitzpatrick, Weakly interacting massive particle-nucleus elastic scattering response, Phys. Rev., 2014
R. Catena and B. Schwabe, Form factors for dark matter captured by the Sun in effective theories, JCAP, 2015

# WIMP & nuclear response functions

## WIMP response functions

$$
\begin{aligned}
R_M^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= c_1^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left[\frac{q^2}{m_N^2} v_T^{\perp 2} c_5^\tau c_5^{\tau'} + v_T^{\perp 2} c_8^\tau c_8^{\tau'} + \frac{q^2}{m_N^2} c_{11}^\tau c_{11}^{\tau'}\right], \\
R_{\Phi''}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \left[\frac{q^2}{4m_N^2} c_3^\tau c_3^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau'}\right)\right]\frac{q^2}{m_N^2}, \\
R_{\Phi'' M}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \left[c_3^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right) c_{11}^{\tau'}\right]\frac{q^2}{m_N^2}, \\
R_{\tilde{\Phi}'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \left[\frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau c_{12}^{\tau'} + \frac{q^2}{m_N^2} c_{13}^\tau c_{13}^{\tau'}\right)\right]\frac{q^2}{m_N^2}, \\
R_{\Sigma''}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{q^2}{4m_N^2} c_{10}^\tau c_{10}^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \right. \\
&\quad \left. \frac{q^2}{m_N^2}(c_4^\tau c_6^{\tau'} + c_6^\tau c_4^{\tau'}) + \frac{q^4}{m_N^4} c_6^\tau c_6^{\tau'} + v_T^{\perp 2} c_{12}^\tau c_{12}^{\tau'} + \frac{q^2}{m_N^2} v_T^{\perp 2} c_{13}^\tau c_{13}^{\tau'}\right], \\
R_{\Sigma'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{1}{8}\left[\frac{q^2}{m_N^2} v_T^{\perp 2} c_3^\tau c_3^{\tau'} + v_T^{\perp 2} c_7^\tau c_7^{\tau'}\right] + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \right. \\
&\quad \left. \frac{q^2}{m_N^2} c_9^\tau c_9^{\tau'} + \frac{v_T^{\perp 2}}{2}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau'}\right) + \frac{q^2}{2m_N^2} v_T^{\perp 2} c_{14}^\tau c_{14}^{\tau'}\right], \\
R_{\Delta}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{j_\chi(j_\chi+1)}{3}\left(\frac{q^2}{m_N^2} c_5^\tau c_5^{\tau'} + c_8^\tau c_8^{\tau'}\right)\frac{q^2}{m_N^2}, \\
R_{\Delta\Sigma'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{j_\chi(j_\chi+1)}{3}\left(c_5^\tau c_4^{\tau'} - c_8^\tau c_9^{\tau'}\right)\frac{q^2}{m_N^2}. \qquad (86)
\end{aligned}
$$

## Nuclear response functions

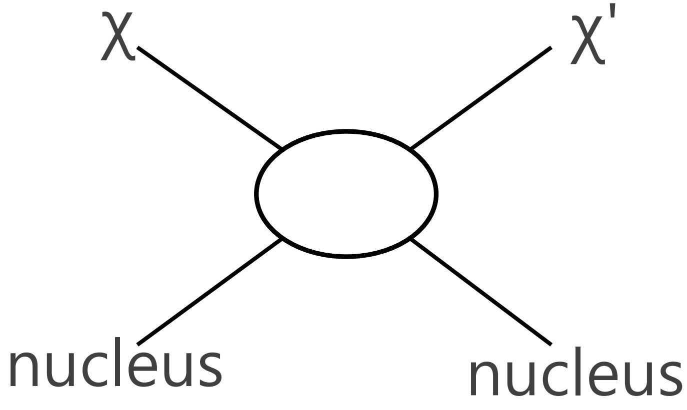
$$
\begin{aligned}
M_{JM;\tau}(q) &\equiv \sum_{i=1}^{A} M_{JM}(q\vec{x}_i)\, t^\tau(i) \\
\Delta_{JM;\tau}(q) &\equiv \sum_{i=1}^{A} \vec{M}_{JJ}^{M}(q\vec{x}_i)\cdot \frac{1}{q}\vec{\nabla}_i\, t^\tau(i) \\
\Sigma'_{JM;\tau}(q) &\equiv -i\sum_{i=1}^{A}\left\{\frac{1}{q}\vec{\nabla}_i \times \vec{M}_{JJ}^{M}(q\vec{x}_i)\right\}\cdot\vec{\sigma}(i)\, t^\tau(i) \\
&= \sum_{i=1}^{A}\left\{-\sqrt{\frac{J}{2J+1}}\,\vec{M}_{JJ+1}^{M}(q\vec{x}_i) + \sqrt{\frac{J+1}{2J+1}}\,\vec{M}_{JJ-1}^{M}(q\vec{x}_i)\right\}\cdot\vec{\sigma}(i)\, t^\tau(i) \\
\Sigma''_{JM;\tau}(q) &\equiv \sum_{i=1}^{A}\left\{\frac{1}{q}\vec{\nabla}_i\, M_{JM}(q\vec{x}_i)\right\}\cdot\vec{\sigma}(i)\, t^\tau(i) \\
&= \sum_{i=1}^{A}\left\{\sqrt{\frac{J+1}{2J+1}}\,\vec{M}_{JJ+1}^{M}(q\vec{x}_i) + \sqrt{\frac{J}{2J+1}}\,\vec{M}_{JJ-1}^{M}(q\vec{x}_i)\right\}\cdot\vec{\sigma}(i)\, t^\tau(i) \\
\tilde{\Phi}'_{JM;\tau}(q) &\equiv \sum_{i=1}^{A}\left[\left(\frac{1}{q}\vec{\nabla}_i \times \vec{M}_{JJ}^{M}(q\vec{x}_i)\right)\cdot\left(\vec{\sigma}(i)\times\frac{1}{q}\vec{\nabla}_i\right) + \frac{1}{2}\vec{M}_{JJ}^{M}(q\vec{x}_i)\cdot\vec{\sigma}(i)\right]\, t^\tau(i) \\
\Phi''_{JM;\tau}(q) &\equiv i\sum_{i=1}^{A}\left(\frac{1}{q}\vec{\nabla}_i M_{JM}(q\vec{x}_i)\right)\cdot\left(\vec{\sigma}(i)\times\frac{1}{q}\vec{\nabla}_i\right)\, t^\tau(i) \qquad (17)
\end{aligned}
$$

# Count rate

- Factorization between particle physics and astrophysics

$$R = \int_{v^*}^{v_{esc}} \mathcal{R}(v)\tilde{\eta}(v)\, dv = \int_{v^*}^{v_{esc}} \mathcal{H}(v) f(v)\, dv$$

$$R = \sigma \times (ParticlePhysics) \times (Astrophysics)$$




- f(v) is the velocity distribution

$$\tilde{\eta}(v) = \frac{\rho_\chi}{m_\chi}\sigma\eta(v), \quad \eta(v) = \int_{v}^{v_{esc}} \frac{f(v')}{v'}\, dv'$$

- For inelastic case, there is minimal bound at fixed $E_R$

$$v^* = \sqrt{2\delta/\mu_{\chi N}} < \text{velocity}(v_{min}) < v_{esc}$$

$$v_{min} = \frac{1}{\sqrt{2 m_N E_R}} \left| \frac{m_N E_R}{\mu_{\chi N}} + \delta \right|$$

# Count rate

- Rate and response function

$$R_{[E_1',E_2']}(t) = \int_{v_T^*}^{\infty} dv \mathcal{H}_{[E_1',E_2']}(v) f(v,t),$$

$$f(v,t) \equiv -v \frac{d}{dv} \eta(v,t),$$

$$\mathcal{H}_{[E_1',E_2']}(v) = \frac{\rho_\chi}{m_\chi} \frac{1}{\pi} \frac{c^2}{v} \sum_T \int_{E_R^{min}(v)}^{E_R^{max}(v)} dE_R \left\{ \mathcal{R}_T^0(E_R) + \mathcal{R}_T^1(E_R) \left( v^2 - v_{T,min}^2(E_R) \right) \right\}$$

- Expand $v\_min$

$$v_{min}(E_R) = \frac{1}{\sqrt{2m_N E_R}} \left| \frac{m_N E_R}{\mu_{\chi N}} + \delta \right|,$$

$$v_{min}(E_R)^2 = \frac{m_N}{2\mu_{\chi N}^2} E_R + \frac{\delta^2}{2m_N} \frac{1}{E_R} + \frac{\delta}{\mu_{\chi N}}$$

$$\bar{\hat{\mathcal{R}}}_{0,1}(E_R) \equiv \int_0^{E_R} dE_R' \hat{\mathcal{R}}_{0,1}(E_R')$$

$$\bar{\hat{\mathcal{R}}}_{1E}(E_R) \equiv \int_0^{E_R} dE_R' E_R' \hat{\mathcal{R}}_1(E_R')$$

$$\bar{\hat{\mathcal{R}}}_{1E^{-1}}(E_R) \equiv \int_0^{E_R} dE_R' \frac{1}{E_R'} \hat{\mathcal{R}}_1(E_R'),$$

# Count rate

- Rate

$$R_{[E_1',E_2']} = N_T M T \frac{\rho_\chi}{m_\chi} \sigma c^2 \sum_{k=1}^{N} \delta\tilde{\eta}^k \times$$

$$\Big\{ \bar{\hat{\mathcal{R}}}_0 \left[E_R^{max}(v_k)\right] - \bar{\hat{\mathcal{R}}}_0 \left[E_R^{min}(v_k)\right] + (v_k^2 - \frac{\delta}{\mu_{\chi N}})(\bar{\hat{\mathcal{R}}}_1 \left[E_R^{max}(v_k)\right] - \bar{\hat{\mathcal{R}}}_1 \left[E_R^{min}(v_k)\right])$$

$$-\frac{m_N}{2\mu_{\chi N}^2}(\bar{\hat{\mathcal{R}}}_{1E} \left[E_R^{max}(v_k)\right] - \bar{\hat{\mathcal{R}}}_{1E} \left[E_R^{min}(v_k)\right])$$

$$-\frac{\delta^2}{2m_N}(\bar{\hat{\mathcal{R}}}_{1E^{-1}} \left[E_R^{max}(v_k)\right] - \bar{\hat{\mathcal{R}}}_{1E^{-1}} \left[E_R^{min}(v_k)\right])\Big\}, \quad (71)$$

- The functions $R_0$, $R_1$, $R_{1E}$ and $R_{1E^{-1}}$ depend on the single argument $E_R$
- Can be tabulated for later interpolation → speed up calculation
- WIMP mass, mass splitting enter only in the argument $E_R$

# Capture rate

- capture rate:

$$C = \sum_T \int_0^{R_\odot} dr\, 4\pi\, r^2\, \eta_T(r)\, \frac{\rho_\chi}{m_\chi} \int_{v \le v^{(\mathrm{Sun})}_{\max,T}(r)} d^3v\, \frac{f(\vec{v})}{v}\, w^2(r) \times \int_{m_\chi v^2/2}^{2\mu_T^2 w^2(r)/m_T} dE_R\, \frac{d\sigma_T}{dE_R}(w(r), E_R)\,.$$

  - $\eta_T$: the number of density of target nucleon T
  - r: distance from the center of the Sun for Standard Solar Model AGSS09ph
  - $v$: DM velocity asymptotically far away from the Sun
  - $v_{esc}(r)$: escape velocity at distance r
  - $w^2(r) = v^2 + v_{esc}^2(r)$
- IceCube:
  - Non-observation of a neutrino excess
  - DM annihilations into $\tau^+\tau^-$ where $m_\chi$ < 100 GeV
  - DM annihilations into $W^+W^-$ where $m_\chi$ > 100 GeV

# Matrix calculation

- Full exploration of large parameter space is needed
- Due to large dimensionality, we used matrix techniques
- 28 couplings strengths $c_i^{\alpha}$, $\alpha$ for proton or neutron, i for 14 operators
  - $c = (c_1^p,\ c_1^n, c_3^p, c_3^n, \dots c_{15}^p, c_{15}^n)$
- Total number of expected events:
  - $N_{\mathrm{E}}^{sig} = c^T N_E c$
  - N: 28*28 real symmetric matrix encoding all the information for experiment E
    - Nuclear response
    - DM velocity distribution
    - Experimental efficiency

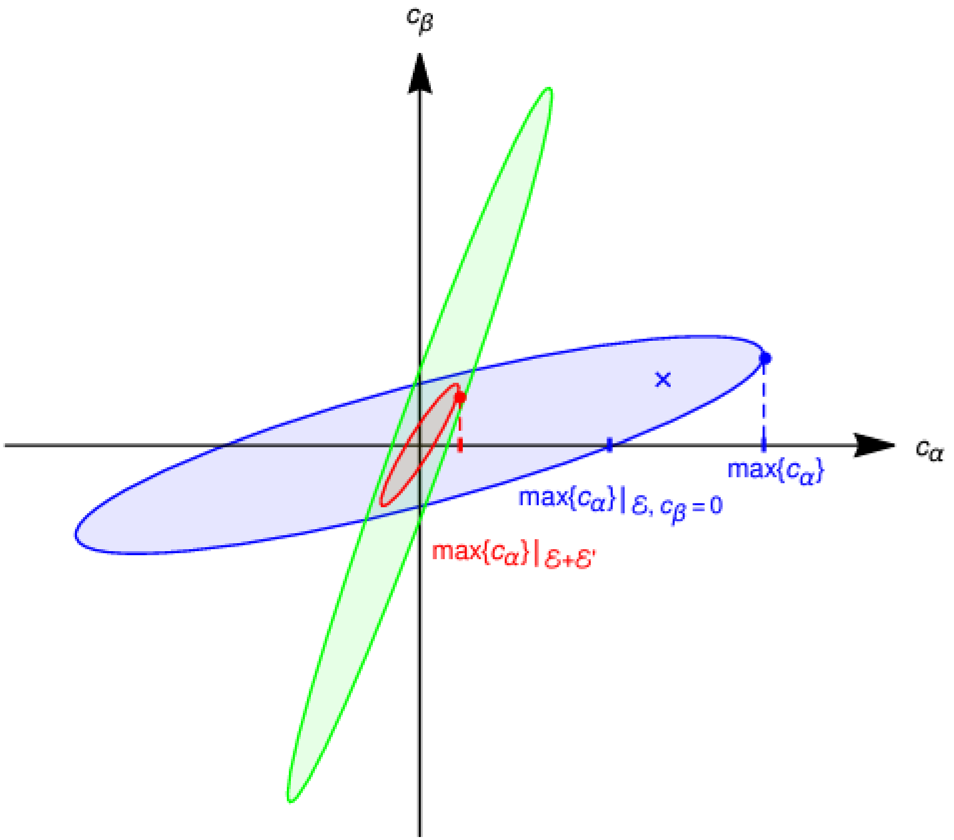
# Matrix calculation

- Single operator at a time to reduce the number of free parameters
- Can not be applied directly
  - Interferences between isospin
  - Interferences among operators

WIMP response functions

$$
\begin{aligned}
R_M^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= c_1^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left[\frac{q^2}{m_N^2} v_T^{\perp 2} c_5^\tau c_5^{\tau'} + v_T^{\perp 2} c_8^\tau c_8^{\tau'} + \frac{q^2}{m_N^2} c_{11}^\tau c_{11}^{\tau'}\right], \\
R_{\Phi''}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \left[\frac{q^2}{4m_N^2} c_3^\tau c_3^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau'}\right)\right]\frac{q^2}{m_N^2}, \\
R_{\Phi'' M}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \left[c_3^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right) c_{11}^{\tau'}\right]\frac{q^2}{m_N^2}, \\
R_{\tilde{\Phi}'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \left[\frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau c_{12}^{\tau'} + \frac{q^2}{m_N^2} c_{13}^\tau c_{13}^{\tau'}\right)\right]\frac{q^2}{m_N^2}, \\
R_{\Sigma''}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{q^2}{4m_N^2} c_{10}^\tau c_{10}^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \right. \\
&\quad \left. \frac{q^2}{m_N^2}(c_4^\tau c_6^{\tau'} + c_6^\tau c_4^{\tau'}) + \frac{q^4}{m_N^4} c_6^\tau c_6^{\tau'} + v_T^{\perp 2} c_{12}^\tau c_{12}^{\tau'} + \frac{q^2}{m_N^2} v_T^{\perp 2} c_{13}^\tau c_{13}^{\tau'}\right], \\
R_{\Sigma'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{1}{8}\left[\frac{q^2}{m_N^2} v_T^{\perp 2} c_3^\tau c_3^{\tau'} + v_T^{\perp 2} c_7^\tau c_7^{\tau'}\right] + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \right. \\
&\quad \left. \frac{q^2}{m_N^2} c_9^\tau c_9^{\tau'} + \frac{v_T^{\perp 2}}{2}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau'}\right) + \frac{q^2}{2m_N^2} v_T^{\perp 2} c_{14}^\tau c_{14}^{\tau'}\right], \\
R_{\Delta}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{j_\chi(j_\chi+1)}{3}\left(\frac{q^2}{m_N^2} c_5^\tau c_5^{\tau'} + c_8^\tau c_8^{\tau'}\right)\frac{q^2}{m_N^2}, \\
R_{\Delta\Sigma'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{j_\chi(j_\chi+1)}{3}\left(c_5^\tau c_4^{\tau'} - c_8^\tau c_9^{\tau'}\right)\frac{q^2}{m_N^2}. \qquad (86)
\end{aligned}
$$

# Matrix calculation

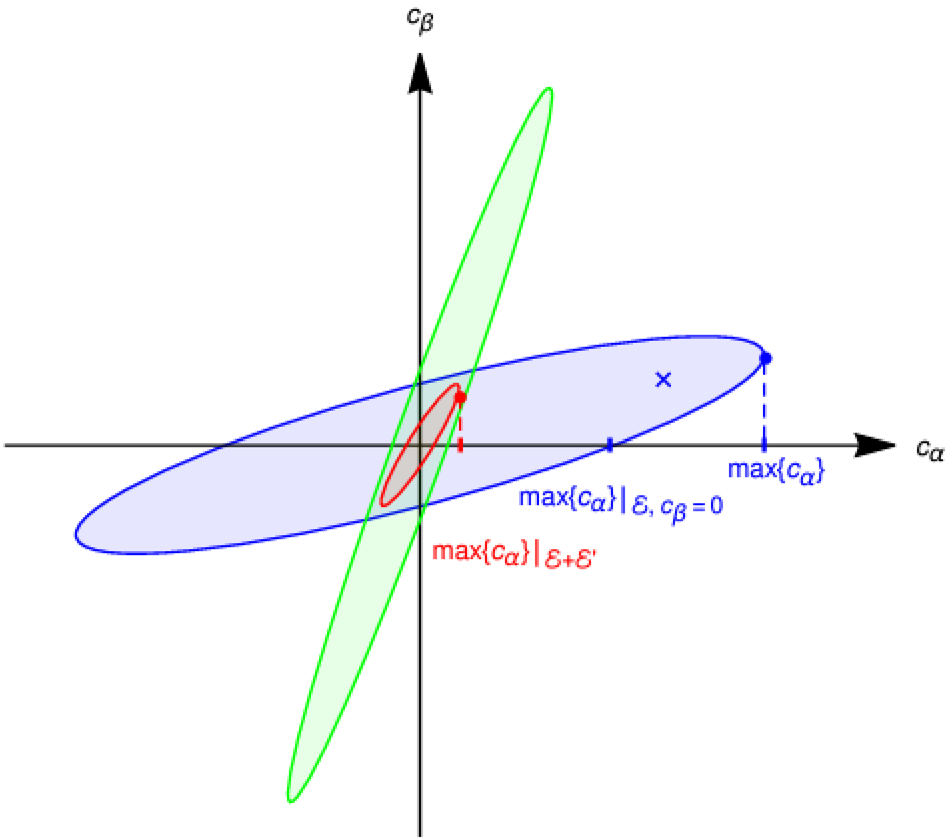



- Blue region: allowed by Experiment E
- Green region: allowed by Experiment E'
- in two-dimensional parameter space spanned by coupling strength $c_\alpha$ and $c_\beta$

- $c_\alpha < \max\{c_\alpha\}$ instead of $c_\alpha < \max\{c_\alpha\}|_{\epsilon,c_\beta=0}$
  - $\max\{c_\alpha\}|_{\epsilon,c_\beta=0}$: model dependent
  - $\max\{c_\alpha\}$: model independent

Anja Brenner, Gonzalo Herrera, Alejandro Ibarra, Sunghyun Kang, Stefano Scopel and Gaurav Tomar, Complementarity of experiments in probing the non-relativistic effective theory of dark matter-nucleon interactions, Journal of Cosmology and Astroparticle Physics, Volume 2022, June 2022

# Matrix calculation




- Major axis might be very huge

- Cross mark: allowed by blue experiment
- Ruled out by green experiment

- Allowed region by combined experiments are smaller
  - $\max\{c_\alpha\}|_{\epsilon+\epsilon'}$ is needed

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# Matrix calculation

- $\chi^2$ distribution:

$$\chi^2_{\mathcal{E}}(\mathbf{c}) = -2\ln\mathcal{L}\big(N^{\rm sig}_{\mathcal{E}}(\mathbf{c})\big)$$

- $\chi^2$ distribution approximated:

$$\chi^2_{\mathcal{E}}(\mathbf{c}) \simeq a_{\mathcal{E}}(N^{\rm sig}_{\mathcal{E}})^2 + b_{\mathcal{E}}N^{\rm sig}_{\mathcal{E}} + c_{\mathcal{E}}$$

- 90% C.L.: $\chi^2_{\mathcal{E}} - \chi^2_{\mathcal{E},\rm min} \leq 2.71$
- minimizing: $\chi^2_{\mathcal{E},\rm min} = c_{\mathcal{E}} - \frac{b^2_{\mathcal{E}}}{4a_{\mathcal{E}}}.$

| experiment | $a_{\mathcal{E}}$ | $b_{\mathcal{E}}$ | $c_{\mathcal{E}}$ |
|---|---|---|---|
| XENON1T | 0.06713 | −1.072 | 8.707 |
| PICO-60 (1st bin) | 0.29010 | −1.728 | 5.440 |
| PICO-60 (2nd bin) | 0 | 2 | 0 |
| IceCube | 0.001046 | 0.01092 | 8.696 |
| DeepCore | 0.002376 | -0.06191 | 8.298 |

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# Lagrange multiplier

- Lagrangian:

$$L = c_\alpha - \lambda\Big[\chi^2_{\mathcal{E}}(\mathbf{c}) - \chi^2_{\mathcal{E},\min} - 2.71\Big]$$

$$\frac{\partial L}{\partial c_\beta}\bigg|_{\mathbf{c}=\mathbf{c}^{\max}} = \delta_{\beta\alpha} - 2\lambda\Big[2a_{\mathcal{E}} N^{\text{sig}}_{\mathcal{E}}(\mathbf{c}^{\max}) + b_{\mathcal{E}}\Big](\mathbb{N}_{\mathcal{E}})_{\beta\gamma} c^{\max}_\gamma = 0\ ,$$

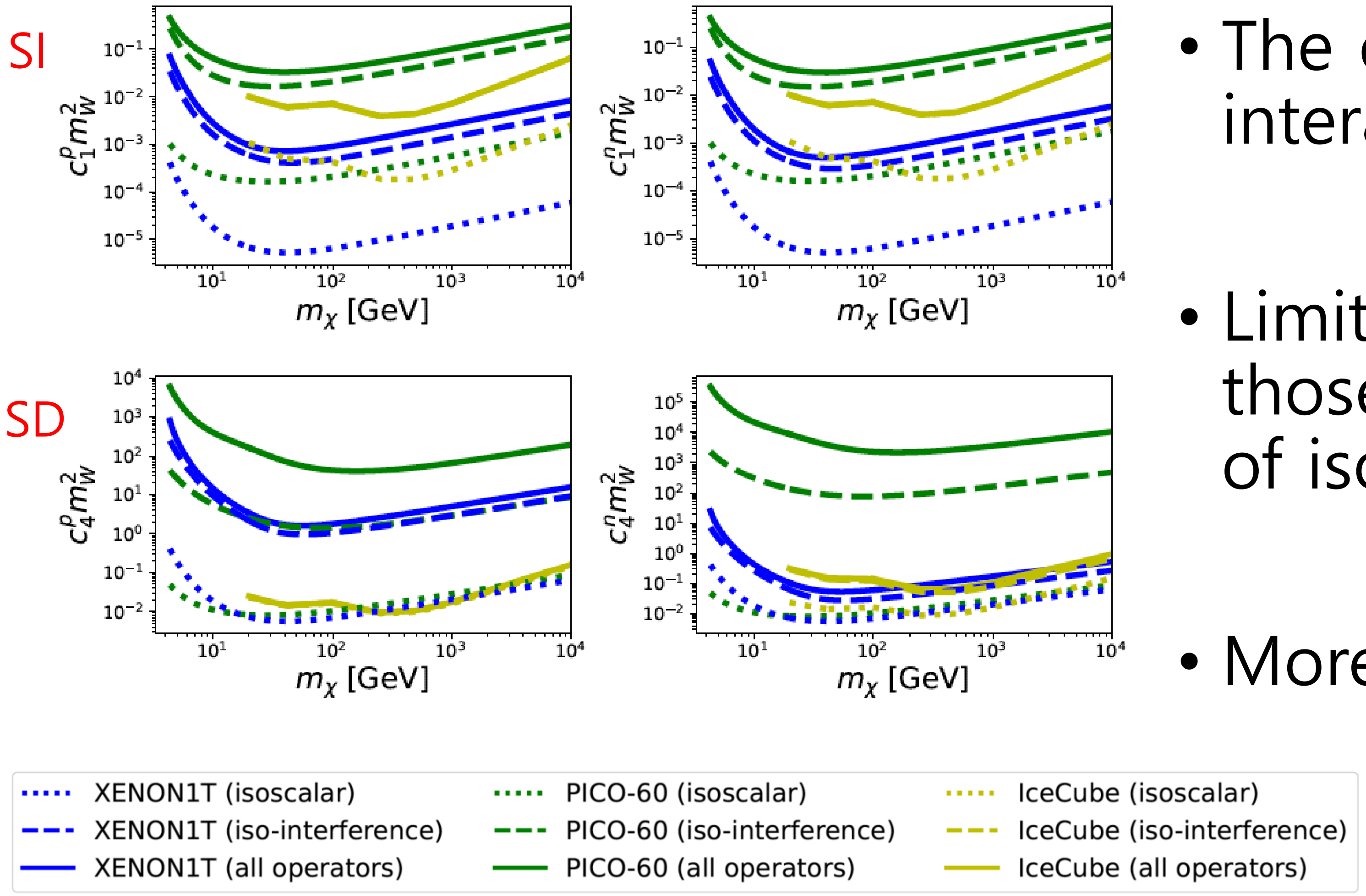
$$\frac{\partial L}{\partial \lambda}\bigg|_{\mathbf{c}=\mathbf{c}^{\max}} = -\Big[a_{\mathcal{E}}\Big(N^{\text{sig}}_{\mathcal{E}}(\mathbf{c}^{\max})\Big)^2 + b_{\mathcal{E}} N^{\text{sig}}_{\mathcal{E}}(\mathbf{c}^{\max}) + c_{\mathcal{E}} - \chi^2_{\mathcal{E},\min} - 2.71\Big] = 0\ .$$

$$c^{\max}_\beta = \frac{1}{2\lambda\Big[2a_{\mathcal{E}} N^{\text{sig}}_{\mathcal{E}}(\mathbf{c}^{\max}) + b_{\mathcal{E}}\Big]}(\mathbb{N}^{-1}_{\mathcal{E}})_{\beta\alpha}.$$

$$N^{\text{sig}}_{\mathcal{E}}(\mathbf{c}^{\max}) = \frac{1}{4\lambda^2\Big[2a_{\mathcal{E}} N^{\text{sig}}_{\mathcal{E}}(\mathbf{c}^{\max}) + b_{\mathcal{E}}\Big]^2}(\mathbb{N}^{-1}_{\mathcal{E}})_{\alpha\alpha}.$$

$$c^{\max}_\alpha = \sqrt{N^{\text{sig}}_{\mathcal{E}}(\mathbf{c}^{\max})(\mathbb{N}^{-1}_{\mathcal{E}})_{\alpha\alpha}}$$

# Interference among operators

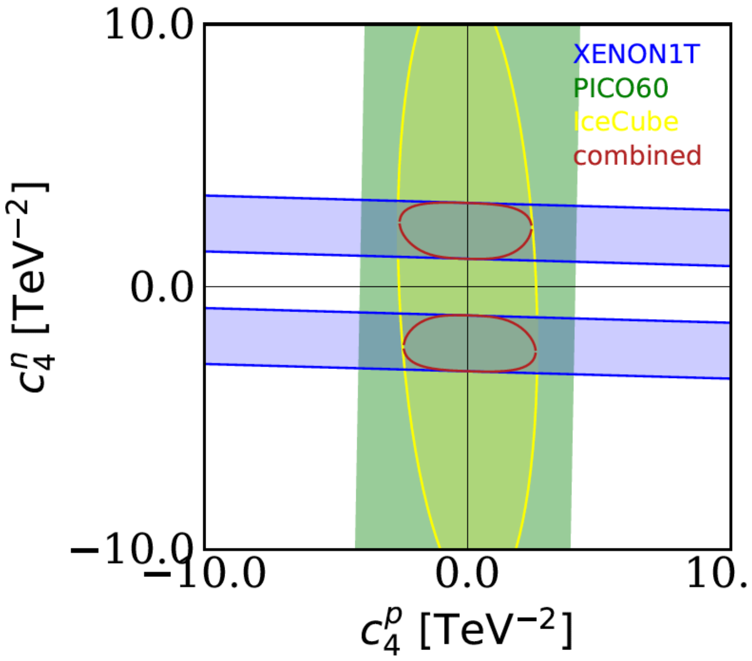



- The effect of isoscalar-isovector interaction is significant
- Limits can be relaxed compared to those obtained under assumption of isoscalar only
- More notable in SD interaction

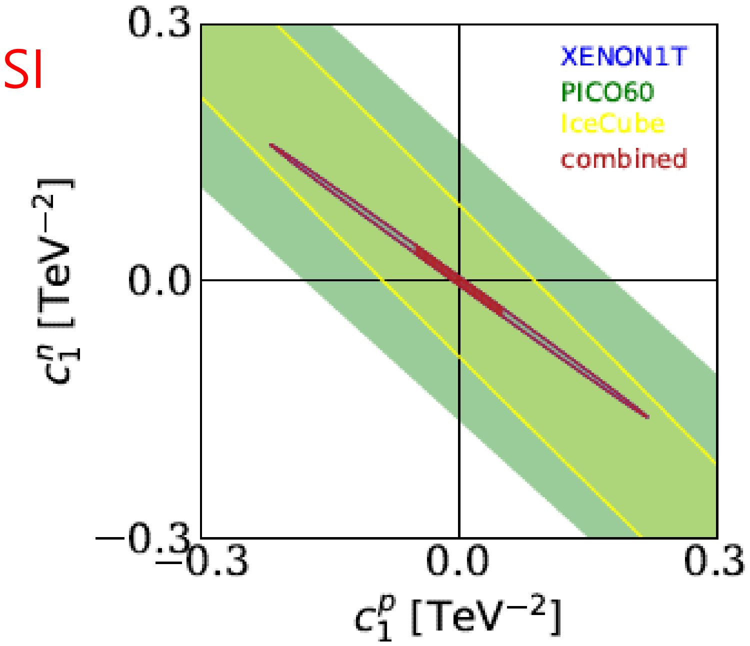
Anja Brenner, Gonzalo Herrera, Alejandro Ibarra, Sunghyun Kang, Stefano Scopel and Gaurav Tomar, Complementarity of experiments in probing the non-relativistic effective theory of dark matter-nucleon interactions, Journal of Cosmology and Astroparticle Physics, Volume 2022, June 2022

# Interference among operators

SD




SI




- XENON 1T & PICO-60 are orthogonal in spin-dependent interaction
- Spin-dependent response functions: $W_{\Sigma''}^{\tau\tau'}$, $W_{\Sigma'}^{\tau\tau'}$
  - Xe: neutron-odd
  - F, H: proton-odd

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# Combining experiments

- New Lagrangian

$$L = c_\alpha - \lambda\left[\chi^2_{\rm tot}(\mathbf{c}) - \chi^2_{\rm tot,min} - 2.71\right]$$

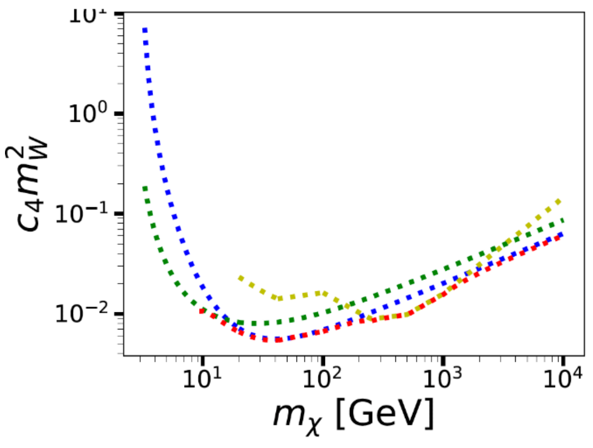
$$\mathbb{X} = \sum_{\mathcal{E}} \left(1 - \frac{N^{\rm obs}_{\mathcal{E}}}{N^{\rm sig}_{\mathcal{E}}(\mathbf{c}^{\rm max}) + N^{\rm bck}_{\mathcal{E}}}\right) \mathbb{N}_{\mathcal{E}}.$$

$$N^{\rm sig}_{\mathcal{E}}(\mathbf{c}^{\rm max}) = \frac{1}{16\lambda^2}\left(\mathbb{X}^{-1}\mathbb{N}_{\mathcal{E}}\mathbb{X}^{-1}\right)_{\alpha\alpha}.$$

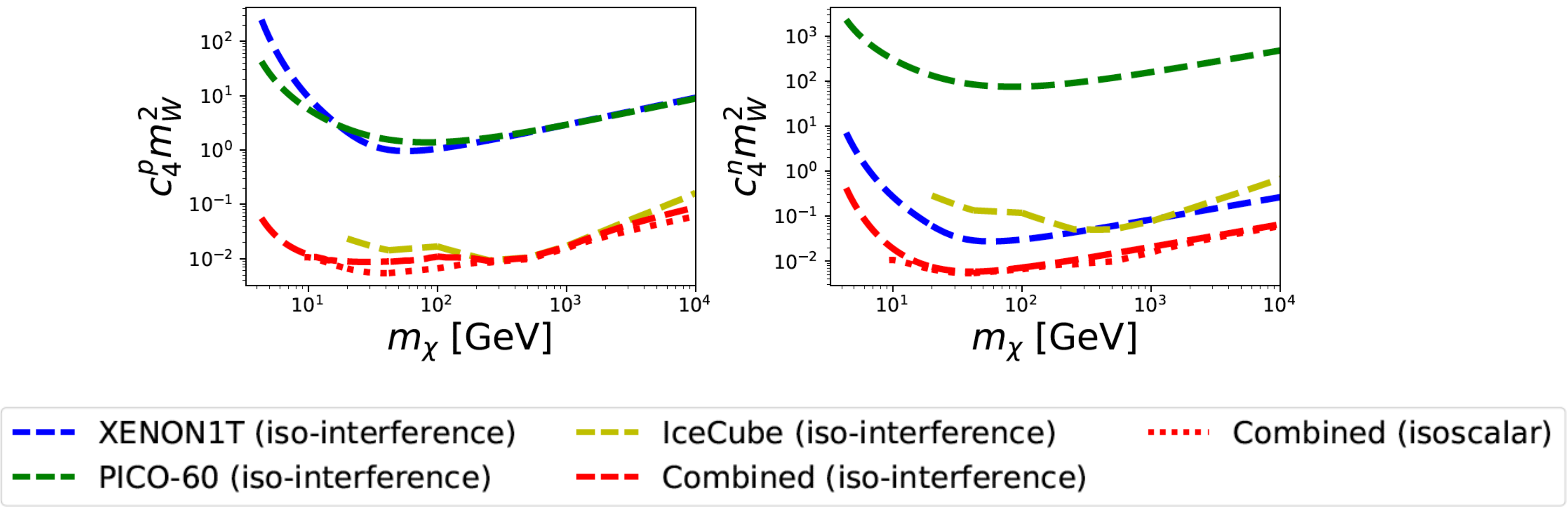
$$c^{\rm max}_\alpha = \frac{1}{4\lambda}(\mathbb{X}^{-1})_{\alpha\alpha}.$$

Anja Brenner, Gonzalo Herrera, Alejandro Ibarra, Sunghyun Kang, Stefano Scopel and Gaurav Tomar, Complementarity of experiments in probing the non-relativistic effective theory of dark matter-nucleon interactions, Journal of Cosmology and Astroparticle Physics, Volume 2022, June 2022

# Combining experiments



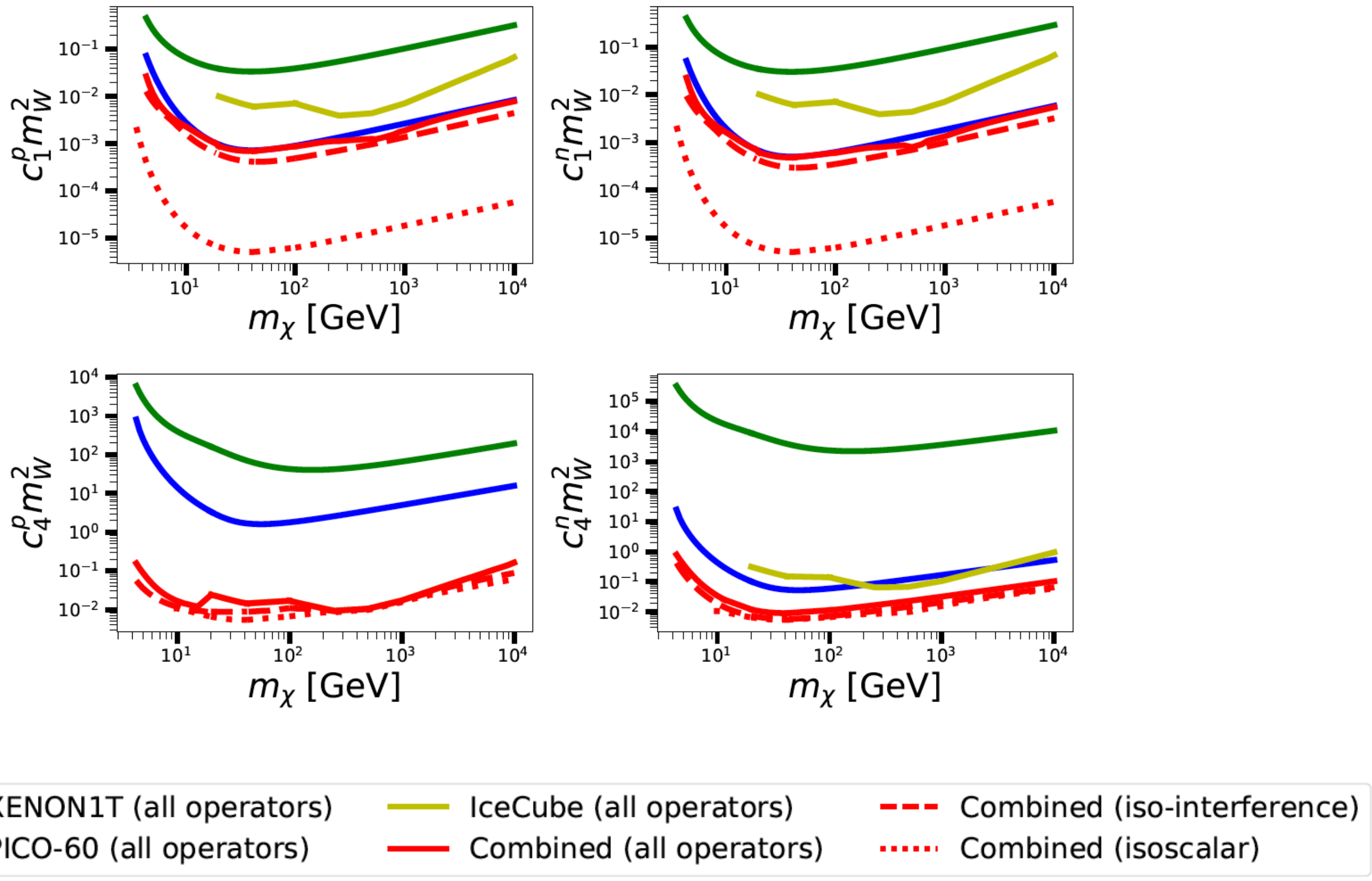

(isoscalar)

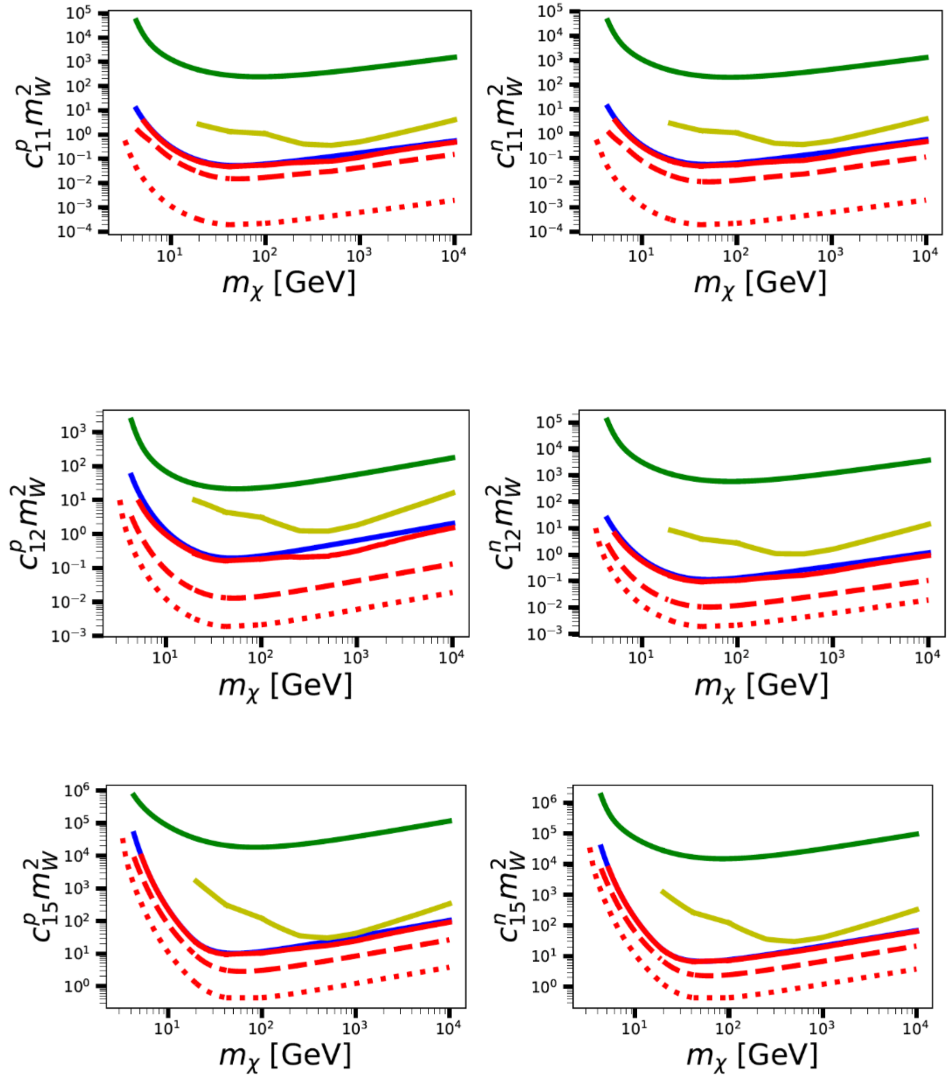



- Isoscalar interaction: no significant difference
- Isoscalar-isovector interference: more pronounced
- Spin-dependent response functions: $W_{\Sigma''}^{\tau\tau'}$, $W_{\Sigma'}^{\tau\tau'}$
  - Xe: neutron-odd
  - F, H: proton-odd

Anja Brenner, Gonzalo Herrera, Alejandro Ibarra, Sunghyun Kang, Stefano Scopel and Gaurav Tomar, Complementarity of experiments in probing the non-relativistic effective theory of dark matter-nucleon interactions, Journal of Cosmology and Astroparticle Physics, Volume 2022, June 2022

# Combining operators




- Operator interference relaxes further

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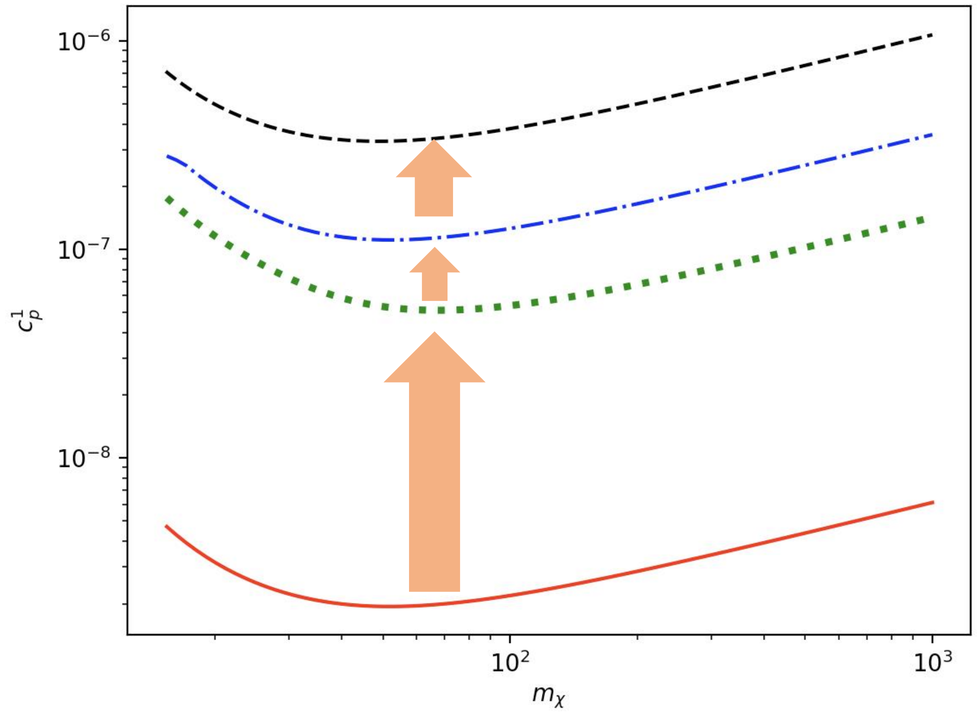
# Linear matrix inequality & Long range interaction

- Extending the number of experiments combined
- Linear matrix inequality
  - $0 < \sum_i \xi_i A_i - x$; positive
  - $\sum_i \xi_i \leq 1$
  - $A_i$: matrix of experiment i

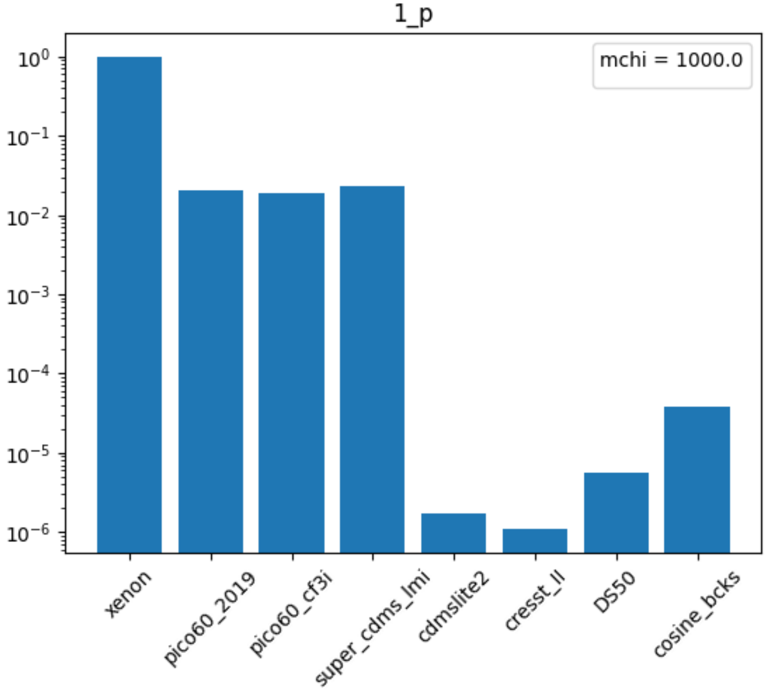



- Long range interaction
  - $\alpha = \frac{g^2}{M^2+q^2}$ : Wilson coefficient
  - g: effective coupling constant
  - M: mediator mass
  - $c_1^p \rightarrow c_1^p + c_1^n \rightarrow c_1^p + c_1^n + c_3^p + c_3^n \rightarrow c_1^{p,n} + c_3^{p,n} + \alpha_1^{p,n} + \alpha_3^{p,n}$

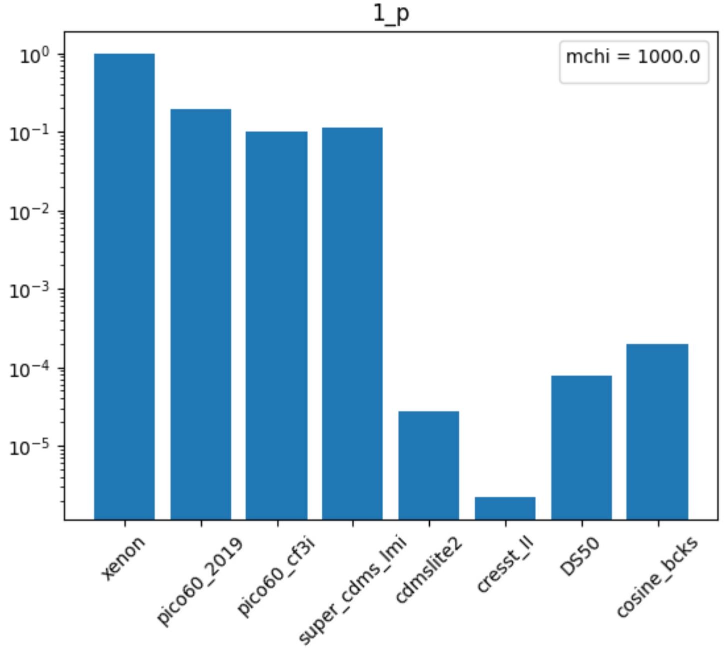
# Normalized rate

- $c^T\, M_E\, c = r_E$ with correct c vector
- $r_E = 1$: saturates the limit

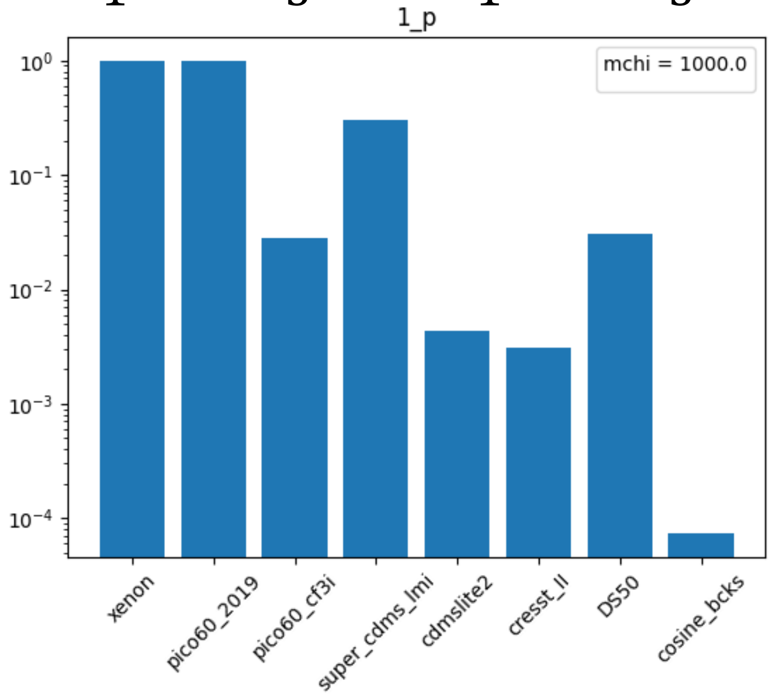
$c_1^p + c_1^n$




$c_1^p + c_1^n + c_3^p + c_3^n$

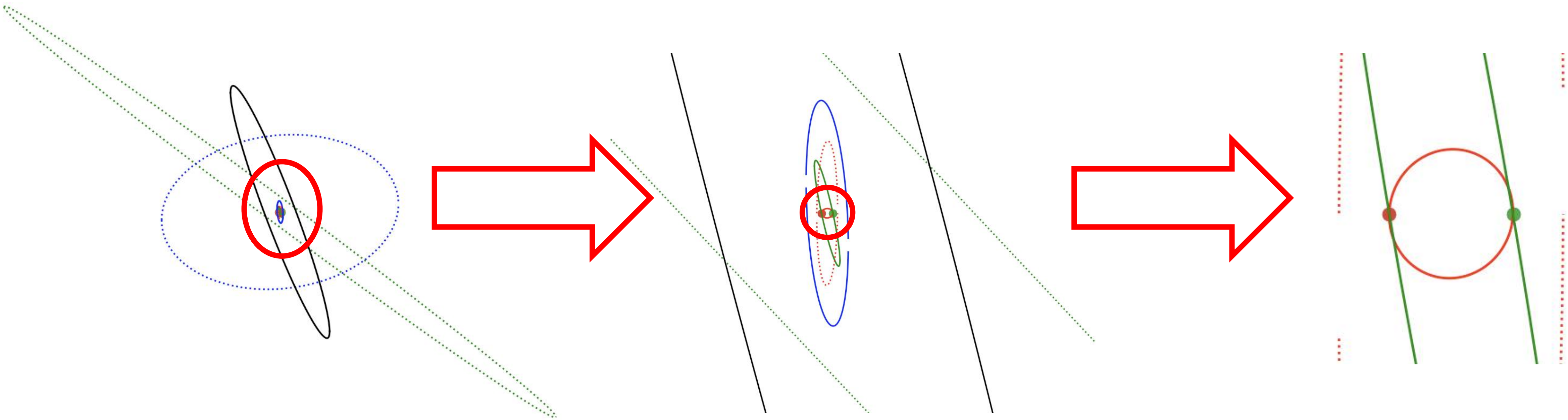



$c_1^{p,n} + c_3^{p,n} + \alpha_1^{p,n} + \alpha_3^{p,n}$



# Ellipsoids

- $c^T M_E c = r_E$ with correct c vector
- $r_E = 1$: saturates the limit
- XENON 1T: red, PICO-60: green



# Conclusions

- Assumption of only isoscalar interaction can provide wrong interpretation of experimental results
- Effect of interference between isoscalar-isovector is significant
- Considering interferences among operators and combining experiments can also provide more accurate interpretation
- Expanding to long range interaction, the most general limits might be obtained