Cosmological Tension and effective field theories

Somyadip Thakur

CQUeST
Department of Physics,Sogang University

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Introduction

- One of the greatest success of field theory is to formulate effective theories from Gravity to particle physics or condensed matter theories. Both classically as well as on microscopic scale.
- One such effective field theory describing the dynamics of the universe is ΛCDM model.
- ΛCDM is phenomenological model in which about 95% of the energy budget of the universe is in the dark sector.
- There are various field theory models to describe the dark matter sector, the remaining 69% in the dark energy (DE) sector is described by the cosmological constant Λ.
- Λ not only suffers from cosmological constant problems, but also a coincidence problem.
- Replace Λ with an Effective Field Theory (EFT) description, typically captured by additional dynamical scalar fields. This has motivated the class of scalar-tensor field theories, which govern the gravity and DE sector of the cosmological models.

The Probe:

- The present local H_0 determinations currently fall in a window between $H_0 \sim 70$ km/s/Mpc (TRGB) and $H_0 \sim 76$ km/s/Mpc (Tully-Fisher).
- BAO data calibrated in an early ΛCDM universe are largely consistent with Planck-ΛCDM, $H_0 \sim 67.5$ km/s/Mpc.
- This biasing of local H0 determinations to larger values can be a game changer for the traditional DE paradigm.
- The bottomline boils down to DE models satisfying the Null Energy Condition, $1 + w_{\rm DE} \geq 0$, which encompass a large class of Effective Field Theories (EFTs) (but, by no means all!), cannot perform better than Λ when it comes to recovering local H_0 determinations.

Conclusion:

- Working in a model independent expansion in powers of redshift z, we show that Brans-Dicke/$f(R)$ and Kinetic Gravity Braiding models within the Horndeski class can lead to marginal and modest increases in H_0, respectively. We confirm that as far as increasing H_0 is concerned, no DE EFT model can outperform the phenomenological two parameter family of the DE models. Evidently, the late universe may no longer be large enough to accommodate H_0, BAO and DE described by EFT.

The EFT's under consideration

The Model:

$$\mathcal{L} = G_2(\phi, X) + G_3(\phi, X)\Box\phi + G_4(\phi)R, \quad X := -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi,$$

Examples

Classifications:

Class	$G_i(\phi, X)$
Quintessence	$G_2 = X - V(\phi)$, $G_3 = 0$, $G_4 = \frac{1}{2}M_{\rm pl}^2$
K-essence	$G_2 = G_2(\phi, X)$, $G_3 = 0$, $G_4 = \frac{1}{2}M_{\rm pl}^2$
Brans-Dicke/$f(R)$ gravity	$G_2 = G_2(\phi, X)$, $G_3 = 0$, $G_4 = G_4(\phi)$
Kinetic Gravity Braiding	$G_2 = G_2(\phi, X)$, $G_3 = G_3(\phi, X)$, $G_4 = \frac{1}{2}M_{\rm pl}^2$

Quintessence & K-essence

- Quintessence models and K-essence models predict *lower* values of H_0 relative to Λ.
- These are at odds with local H_0 determinations. It is fair enough to rule out these models as viable late-time DE EFTs.
- However all of these models can be reconsidered as early dark energy models , but this does not alter the conclusion that, even in such a scenario, Λ is expected to maximise H_0 over the simplest EFTs in the Horndeski class.

Non-minimal coupling

Quintessence sector

$G_2(\phi, X) = X - V(\phi)$, redefine $F(\phi) = 2G_4(\phi)$.

Assumption

$F(\phi)$ is an analytic function

- Evolution in $F(\phi)$ translates into an evolution in Newton's constant G. Currently, observations are largely consistent with no evolution, whether it be constraints in the solar system from lunar laser ranging $\dot{G}/G_0 = (7.1 \pm 7.6) \times 10^{-14}\ \mathrm{yr}^{-1}$, or constraints from the Big Bang nucleosynthesis (BBN), $G_{\mathrm{BBN}}/G_0 = 0.98 \pm 0.06$, where G_0 denotes the value of Newton's constant today.

- The scalar ϕ in redshift z r below $z = 1$ - about its value today ϕ_0, one can also expand the Quintessence potential provided the displacement in the scalar is also small, $|\phi - \phi_0| < 1$.

Scalar Potential

$$\phi = \phi_0 + \alpha z + \beta z^2 + \gamma z^3 + \ldots,$$
$$V(\phi) = V_0 + V_1 (\phi - \phi_0) + V_2 (\phi - \phi_0)^2 + \ldots,$$

Expansion

$$H(z) = H_0(1 + h_1 z + h_2 z^2 + h_3 z^3 + \ldots),$$
$$F(z) = 1 + \frac{F_1}{H_0}(z - \frac{1}{2}(1 + h_1) z^2 + \frac{1}{3}(1 + h_1 + h_1^2 - h_2) z^3$$
$$- \frac{1}{4}(1 + h_1^2 + h_1^3 - h_2 + h_1 - 2h_1 h_2 + h_3) z^4 + \ldots\Big)$$

- Now given the perturbative expansion of the fields we can solve the Friedman equations

Friedmann Eqns

$$3FH^2 = \rho_m + \frac{1}{2}\dot{\phi}^2 + V - 3H\dot{F},$$
$$-2F\dot{H} = \rho_m + \dot{\phi}^2 + \ddot{F} - H\dot{F},$$
$$0 = \ddot{\phi} + 3H\dot{\phi} + \partial_\phi V - 3\left(\dot{H} + 2H^2\right)\partial_\phi F,$$

$$\frac{V_0}{H_0^2} = 3\left(1 - \Omega_m\right) - \frac{1}{2}\alpha^2 - 3\frac{F_1}{H_0},$$
$$\frac{V_1}{H_0^2} = -\frac{1}{2}\alpha^3 - 2\beta + 2\alpha - \frac{3}{2}\alpha\Omega_m + \frac{1}{2\alpha}\frac{F_1}{H_0}\left(12 - 4\alpha^2 - 9\Omega_m\right) - \frac{3}{2\alpha}\frac{F_1^2}{H_0^2},$$
$$h_1 = \frac{1}{2}\alpha^2 + \frac{3}{2}\Omega_m + \frac{1}{2}\frac{F_1}{H_0},$$
$$h_2 = \frac{1}{8}\alpha^4 + \frac{1}{4}\alpha^2 + \alpha\beta + \frac{3}{8}\Omega_m\left(4 - 3\Omega_m\right) - \frac{1}{8}\frac{F_1}{H_0}\left(2 + \alpha^2 + 9\Omega_m\right) - \frac{1}{4}\frac{F_1^2}{H_0^2}$$

DE EOS

$$w_{\mathrm{DE}}(z) = -1 + \frac{\left(\alpha^2 + F_1 H_0^{-1}\right)}{3\Omega_{\phi 0}} + \frac{z}{\Omega_{\phi 0}^2}\left[\frac{\alpha^4}{3}\left(\Omega_{\phi 0} - 1\right) + \frac{\alpha^2}{3}\Omega_{\phi 0}\left(5 - 3\Omega_{\phi 0}\right) + \frac{4}{3}\alpha\beta\Omega_{\phi 0} - \left(\frac{\alpha^2}{6}\left(4 - \Omega_{\phi 0}\right) + \frac{1}{2}\Omega_{\phi 0}\left(1 - \Omega_{\phi 0}\right)\right)\frac{F_1}{H_0} - \frac{\left(2 + \Omega_{\phi 0}\right)}{6}\frac{F_1^2}{H_0^2}\right] + O\left(z^2\right)$$

where $\Omega_{\phi 0} := 1 - \Omega_m$

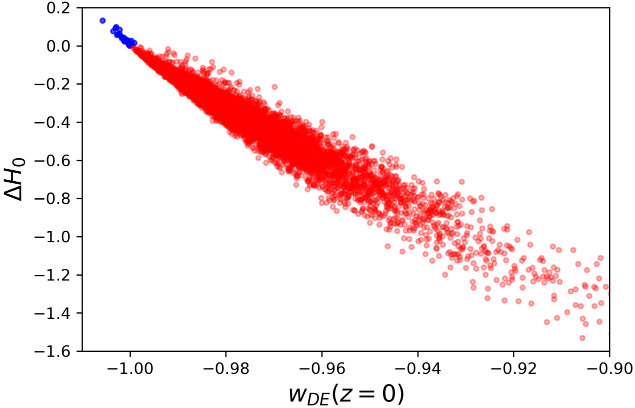

Figure: 9,675 fits to mock data and the resulting ΔH_0 between the non-minimally coupled model and flat ΛCDM versus $w_{\mathrm{DE}}(z=0)$ for the non-minimally coupled model. Blue separates models with $\Delta H_0 > 0$ from models with $\Delta H_0 < 0$ in red.

- One relevant question is whether a non-minimal coupling to gravity can help alleviating H_0 tension?
- The non-minimal coupling $F(\phi)$ varies linearly with cosmic time subject to a recent BBN constraint .
- The slope F_1 contributes to the Hubble parameter in a conflicted manner; if F_1 flattens the slope of the Hubble parameter at leading order in redshift z, it increases the slope at subleading order. For this reason, one would expect increases in H_0 due to linear evolution of $F(\phi)$ with cosmic time to be marginal, and this is indeed what we found.
- Finally, we note that increasing H_0 requires $\dot{F} > 0$ ($F_1 < 0$), which implies the Newton's constant must decrease in the late universe .

Kinetic Gravity Braiding

KGB

$\mathcal{L} = \frac{1}{2}R + K(\phi, X) + G(\phi, X)\Box\phi$

- Note that the above Lagrangian gives a family of KGB models with different choices of K and G.
- For simplicity we choose the following parametrization

KGB Model

$$K = X - V(\phi)$$

$$G(\phi, X) = g_1 X + g_2\left(\phi - \phi_0\right) X + g_3 X^2, \text{ gi's constant parameters.}$$

- The EOM may be expressed as,

$$3H^2 = \rho_m + \frac{1}{2}\dot{\phi}^2 + V(\phi) - \dot{\phi}^2\left(3H\dot{\phi}G_{,X} - G_{,\phi}\right),$$

$$-2\dot{H} = \rho_m + \dot{\phi}^2\left(1 + 2G_{,\phi} + G_{,X}(\ddot{\phi} - 3H\dot{\phi})\right)$$

- For any given $G(\phi, X)$ the KGB class of Horndeski theories is free of instabilities if the following conditions are met

Stability Constraints:

$$
\begin{aligned}
1 + 2G_{,\phi} - H^2(1+z)^2 \left(\phi'\right)^2 G_{,\phi X} - 2H(1+z)\left[H'(1+z)\phi' + H\phi' + H(1+z)\phi''\right] G_{,X} \\
+4H^2(1+z)\phi' G_{,X} - \tfrac{1}{2}H^4(1+z)^4 \left(\phi'\right)^4 G_{,X}^2 > 0 \\
1 + 2G_{,\phi} + H^2(1+z)^2 \left(\phi'\right)^2 G_{,\phi X} + 6H^2(1+z)\phi' G_{,X} + \tfrac{3}{2}H^4(1+z)^4 \left(\phi'\right)^4 G_{,X}^2 > 0
\end{aligned}
$$

$\mathcal{O}(z)$

- We can solve the dynamics of the model in a perturbative expansion in z and we present few of these solutions

$$V_0 H_0^{-2} = 3\left(1-\Omega_m\right) - \frac{1}{2}\alpha^2\left(1+g_2 H_0^2\alpha^2\right) - 6(1-\Delta)$$

$$h_1 = \frac{1}{2}\alpha^2\left(1+g_2 H_0^2\alpha^2\right) + \frac{3}{2}\Omega_m + \frac{(1-\Delta)}{2\Delta}\left(\alpha^2\left(1+g_2 H_0^2\alpha^2\right) + 4\frac{\beta}{\alpha} + 8 + 3\Omega_m\right)$$

where $\Delta := 1 - \frac{1}{2}g_1 H_0^2\alpha^3 - \frac{1}{2}g_3 H_0^4\alpha^5$.

$\mathcal{O}(z^2)$

$$V_1 H_0^{-2} = -\frac{1}{2}\alpha^3 - 2\beta + 2\alpha - \frac{3}{2}\alpha\Omega_m - \frac{1}{2}g_2\alpha^2 H_0^2\left(3\alpha^3 + \alpha\left(6\Omega_m + 4\right) + 8\beta + 2g_2\alpha^5 H_0^2\right)$$
$$- \frac{(1-\Delta)}{2\alpha^2\Delta}\left[\alpha^3\left(3\Omega_m + 26\right) + 4\alpha^2\beta - 6\alpha\left(26\Delta - 9\Omega_m - 24\right) - 24\beta(\Delta - 3) + 2\alpha^9 g_2^2 H_0^4 + 3\alpha^7 g_2 H_0^2\right.$$
$$\left. + \alpha^5\left[1 + 2g_2 H_0^2\left(3\Omega_m + 17\right)\right] + 8\alpha^4\beta g_2 H_0^2\right]$$

$$h_2 = \frac{1}{8}\alpha^4 + \frac{1}{4}\alpha^2 + \alpha\beta + \frac{3}{8}\Omega_m\left(4 - 3\Omega_m\right) + \frac{g_2 H_0^2\alpha^3}{8}\left[5\alpha^3 + 4g_2 H_0^2\alpha^5 + 20\beta + \alpha\left(14 + 9\Omega_m\right)\right]$$
$$+ \frac{(1-\Delta)}{8\alpha^2\Delta^3}\left[2\alpha^4\left(\Delta^2 + 3\Delta + 6\Omega_m + 16\right) + 4\alpha^3\beta\left(2\Delta^2 + 3\Delta + 4\right) + 32\beta^2\left(-\Delta^2 + \Delta + 1\right)\right.$$
$$+ \alpha^2\left[\Delta^2\left(-9\Omega_m^2 + 12\Omega_m - 80\right) - \Delta\left(9\Omega_m^2 + 30\Omega_m + 16\right) + 2\left(3\Omega_m + 8\right)^2\right]$$
$$+ 4\alpha\left[\beta\left(-16\Delta^2 + 3\Delta\Omega_m + 14\Delta + 12\Omega_m + 32\right) + 6\gamma\Delta^2\right] + 2\alpha^{10}\left(2\Delta^2 + 2\Delta + 1\right) g_2^2 H_0^4$$
$$+ \alpha^8\left(5\Delta^2 + 5\Delta + 4\right) g_2 H_0^2 + 4\alpha^5\beta\left(5\Delta^2 + 6\Delta + 4\right) g_2 H_0^2$$
$$\left. + \alpha^6\left(\Delta^2 + \Delta + 2 + g_2 H_0^2\left(\Delta^2\left(9\Omega_m + 14\right) + \Delta\left(9\Omega_m + 30\right) + 4\left(3\Omega_m + 8\right)\right)\right)\right].$$

DE EOS

$$
\begin{aligned}
w_{\mathrm{DE}} &= -1 + \frac{\dot{\phi}^2 \left(1 + 2G_{,\phi} + [\ddot{\phi} - 3H\dot{\phi}]G_{,X}\right)}{\frac{\dot{\phi}^2}{2} + V - \dot{\phi}^2 \left(3H\dot{\phi}G_{,X} - G_{,\phi}\right)} \\
&= -1 + \frac{1}{3\left(1 - \Omega_m\right)} \left[\alpha^2 \left(1 + g_2 H_0^2 \alpha^2\right) + \frac{(1-\Delta)}{\Delta} \left(\alpha^2 \left(1 + g_2 H_0^2 \alpha^2\right) + 4\frac{\beta}{\alpha} + 8 + 3\Omega_m\right)\right] \\
&\quad + O(z)
\end{aligned}
$$

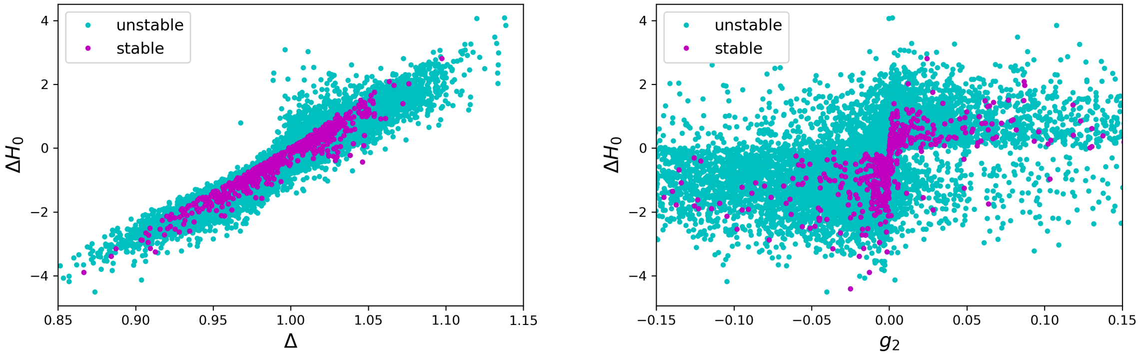

Figure: These plots depict how much one can increase H_0 as we move in the parameter space of perturbative KGB model.

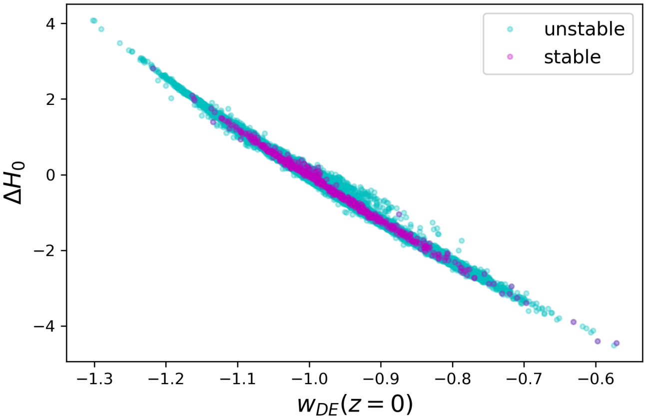

Figure: Increases in H_0 versus $w_{\rm DE}(z=0)$ for the KGB model. The stable configurations in magenta explore a restricted parameter space leading to less pronounced increases in H_0.

- The cosmological data including BAO restricts late universe modifications within Einstein gravity to central values below $H_0 = 70$ km/s/Mpc. Concretely, we found that X-dependent contributions to the braiding function $G(\phi, X)$ alleviate H_0 tension in a meaningful way compared to models with non-minimal coupling.
- This difference can be traced analytically to the fact that any X^n, $n > 0 \in \mathbb{N}$, contribution to $G(\phi, X)$ can coherently flatten $H(z)$ at both leading and subleading order in z for a sizable class of Quintessence models. As we have shown, this leads to tangible increases in H_0 that correlate well with a phantom EoS at $z = 0$, $w_{\mathrm{DE}}(z = 0) < -1$.

Concluding Remarks

- Finally, while still less significant than the H_0 tension, there are other cosmological tensions, viz S_8.
- The lore is that late DE models which alleviate H_0 typically exacerbate S_8. It is reasonable to expect that the S_8 considerations should only strengthen our results here that the DE EFT framework is less likely to hold the answer to the cosmic tensions, but one is always free to venture theoretically beyond EFT.
- In the big picture, if attempts to alter the BAO scale through EDE or equivalent are discredited one is confronted with a Λ that does not appear to admit an EFT description. If confirmed, this in itself is an extremely profound insight into the cosmological constant Λ.

Thank you

Backup

- Cosmological tension and anomalies in anisotropic universe.
- Dynamically generate the anisotropy.
- Bianchi (1&7)classifications

Lagrangian

$$\mathcal{L} = \sqrt{-g}\left[R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\Theta\phi}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}\right] + \mathcal{L}_{\mathrm{PF}}, \tag{1}$$

- We have shown the attractor flow generated flows to table FRW metric. The anisotropy is generated dynamically.
- Expanding in spherical harmonics can generate what cosmologist call the sky map.