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Fractons from Double Field Theory

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Ergodicity and thermalization

- A foundational idea of statistical physics is ergodicity which means a system explores all its microstates consistent with conservation laws.
- A well-known diagnostic method of quantum chaos is BGS conjecture. Quantum chaotic systems have random matric distribution which is exactly based on the concept of ergodicity.

- If ergodicity is obeyed in a quantum many-body system, we can apply standard statistical physics since the system would be thermalized and reach to be in equilibrium.

- When ergodicity is broken, something interesting happens.
- For example, integrability is an example to break ergodicity. The eigenmode can be very confined in contrast to chaotic system whose eigenmode tends to becomes uniform with increasing mode number. Can we have other options which have finitely many conservation laws?

Fractons ['11 Haah]['15 Vijay, Haah, Fu] [Nandkishore…][Radzihovsky…][Pretko…][Seiberg…]

- Fractons provide a new way to robustly evade equilibration.

- The concept of fracton was firstly discovered in the study of quantum memory by Dr. Jeongwan Haah, and developed by many researchers in condensed matter physics. Those studies were mainly about discrete systems but some quantum field theoretical generalization was also tried. Recently by N. Seiberg and his friends, continuum limit of the original discrete system and its meaning in quantum field theory were intensively explored.

- Defining properties of fracton:

(1) immobility or restricted mobility – not energetical immobility

(2) subsystem symmetry or polynomial shift symmetry[Griffin, Grosvenor, Horava, Yan]

(3) Size dependent groundstate degeneracies ~ exp(L) $\rightarrow \infty$

Fracton field theory: example [Pretko]

- **U(1) Symmetric tensor gauge theory in (2+1)-dimension**

$$H = \int d^2x \left(\frac{1}{2}\tilde{C}^{ijk\ell} E_{ij} E_{k\ell} + \frac{1}{2} B^i B_i + \rho\phi + J^{ij} A_{ij} \right)$$

$$B^i = \epsilon_{jk} \partial^j A^{ki} , \qquad E^{ij}_\sigma = -\partial_t A^{ij} - \partial_i \partial_j \phi$$

$$A_{ij} \rightarrow A_{ij} + \partial_i \partial_j \alpha \, , \qquad \phi \rightarrow \phi + \partial_t \alpha$$

- electric field is expressed in terms of a symmetric rank-2 tensor gauge field as well as the usual scalar potential

- generalized gauge transformation

- scalar potential does not have a conjugate field, but rather acts as a Lagrange multiplier enforcing the scalar Gauss law constraint such as

$$\partial_i \partial_j E^{ij} = \rho$$

Fracton from dipole moment conservation

- Key property of the previous theory: Dipole moment conservation

$$Q = \int d^2x\, \rho = \text{const.}\,, \qquad \mathbf{P} = \int d^2x\, (\rho \mathbf{x}) = \text{const.}$$

- Now, the isolated charge cannot move due to the dipole moment conservation.

- However, a pair of positive charge and negative charge, namely dipole can move.

- If there are quadrupole moment conservation, the four-particle quadrupole can move.

- Similarly, if all multipole moments are separately conserved, there would be no mobile particle. [A. Gromov]

Geometric fracton

- In previous studies, the existence of charge seems to be essential. But, is it really true?
- Regardless of any charge, can we geometrically realize fractons?
- In other words, can the defining properties of fractons be realized geometrically?
- The answer is yes. That's exactly what we achieved. The cost is that we need a new geometry beyond Riemannian geometry.
- For this purpose, we will consider a completely different topic: double field theory.

Double field theory and non-Riemannian geometry I

- Double field theory is a stringy gravity theory.
- DFT has T-duality explicitly and for that, the generalized metric is constructed in terms of metric and antisymmetric B-field . The remaining closed string massless section, dilaton provides a DFT volume element. These fields are mixed in T-duality transformation which are fully generalized to O(D,D) transformation.

- The idea is to double spacetime dimension, to generalized the notion of general covariance in terms of a generalized Lie derivative, and finally to get the undoubled theory by imposing the section condition.

$$\mathcal{H}_{AB} = \begin{pmatrix} \mathcal{H}^{\mu\nu} & \mathcal{H}^{\mu}{}_{\lambda} \\ \mathcal{H}_{\kappa}{}^{\nu} & \mathcal{H}_{\kappa\lambda} \end{pmatrix}$$

$$x^{\mu} = \left(x^{a} \, , \, y^{i} \, , \, \bar{y}^{\bar{\imath}} \right) , \qquad \partial_{\mu} = \left(\partial_{a} \, , \, \partial_{i} \, , \, \bar{\partial}_{\bar{\imath}} \right) ,$$

$$1 \leq a \leq D{-}n{-}\bar{n}, \;\; 1 \leq i \leq n, \;\; 1 \leq \bar{\imath} \leq \bar{n},$$

$$\mathcal{H}^{\mu\nu} = \eta^{ab}\delta_{a}^{\mu}\delta_{b}^{\nu} \, , \quad \mathcal{H}_{\mu\nu} = \delta_{\mu}^{a}\delta_{\nu}^{b}\eta_{ab} \, , \quad \mathcal{H}_{\mu}{}^{\nu} = \delta_{\mu}^{i}\delta_{i}^{\nu} - \delta_{\mu}^{\bar{\imath}}\delta_{\bar{\imath}}^{\nu}$$

- Flat background is not unique!

Double field theory and non-Riemannian geometry II

- Classification of non-Riemannian geometry ['17 Morand, Park]
- This construction includes some well-known examples of non-Riemannian geometry such as Newton-Cartan, Carrolian, non-relativistic string etc.
- Doubled-diffeomorphism invariant particle action

$$S_{\rm particle} = \int {\rm d}\tau \ \tfrac{1}{2}e^{-1}\mathbf{D}_\tau x^A \mathbf{D}_\tau x^B \mathcal{H}_{AB} - \tfrac{1}{2}em^2$$

$$\longrightarrow \quad S^{(n,\bar{n})}_{\rm particle} = \int {\rm d}\tau \ \tfrac{1}{2}e^{-1}\dot{x}^a\dot{x}_a - \tfrac{1}{2}em^2 + \Lambda_i \dot{y}^i + \bar{\Lambda}_{\bar{\imath}}\dot{\bar{y}}^{\bar{\imath}}$$

- We have immobility to non-Riemannian directions.
- DFT flat (n, nbar) background has infinite dimensional isometry ['21 Blair, Oling, Park]

$$\xi^a = w^a{}_b x^b + \zeta^a(y) + \bar{\zeta}^a(\bar{y}) \,, \qquad \lambda_a = \zeta_a(y) - \bar{\zeta}_a(\bar{y}) \,,$$

$$\xi^i = \omega\bar{n}y^i + \zeta^i(y) \,, \qquad \lambda_i = \rho_i(y) \,,$$

$$\bar{\xi}^{\bar{\imath}} = -\omega n \bar{y}^{\bar{\imath}} + \bar{\zeta}^{\bar{\imath}}(\bar{y}) \,, \qquad \bar{\lambda}_{\bar{\imath}} = \bar{\rho}_{\bar{\imath}}(\bar{y}) \,.$$

Double field theory and non-Riemannian geometry III

- Conservative current

$$\begin{aligned}\mathbb{J}^{\mu} \;=\; & (T^{\mu}{}_{a}+\eta_{ab}T^{\mu b})\zeta^{a}(y)+(T^{\mu}{}_{a}-\eta_{ab}T^{\mu b})\bar{\zeta}^{a}(\bar{y}) \\ & +\omega(\bar{n}T^{\mu}{}_{i}y^{i}-nT^{\mu}{}_{\bar{\imath}}\bar{y}^{\bar{\imath}})+T^{\mu}{}_{i}\zeta^{i}(y)+T^{\mu}{}_{\bar{\imath}}\bar{\zeta}^{\bar{\imath}}(\bar{y}) \\ & +T^{\mu i}\rho_{i}(y)+T^{\mu\bar{\imath}}\bar{\rho}_{\bar{\imath}}(\bar{y})+T^{\mu}{}_{a}w^{a}{}_{b}x^{b}\,.\end{aligned}$$

- Power series expansions of the local parameters in non-Riemannian coordinates generate infinitely many higher multipole conservation laws, in general. Complete immobility!
- An exception is (1,1) background where allows only dipole conservation. This seems to be similar with the tensor gauge theory and Seiberg-Shao model:

$$\mathcal{L}=\frac{\mu_0}{2}(\partial_0\phi)^2-\frac{1}{2\mu}(\partial_x\partial_y\phi)^2$$

$$J_0=\mu_0\partial_0\phi\,,\qquad J^{xy}=-\frac{1}{\mu}\,\partial^x\partial^y\phi\,,$$

$$\partial_0 J_0=\partial_x\partial_y J^{xy}\,.$$

- See Kevin's talk for further discussion.

Two bonuses from DFT I

- We found two bonuses by applying DFT to U(1) Maxwell theory.
- To see these, Look at the double Maxwell Lagrangian.

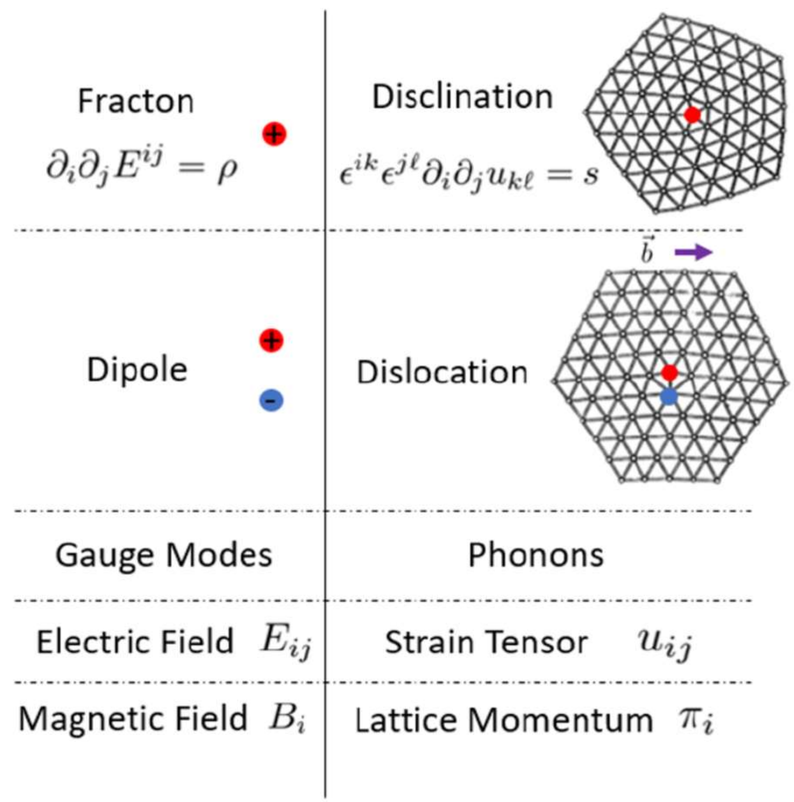
$$L_{\mathrm{YM}} = \mathrm{Tr}\left[P^{AC}\bar{P}^{BD}\mathcal{F}_{AB}\mathcal{F}_{CD}\right]$$

$$\longrightarrow \quad L_0 = -\frac{1}{4}f_{ab}f^{ab} - \frac{1}{4}u_{ab}u^{ab} - f^{-}_{ai}\partial^a\varphi^i + f^{+}_{a\bar{i}}\partial^a\varphi^{\bar{i}} - 2\partial_i\varphi^{\bar{i}}\partial_{\bar{i}}\varphi^i$$

- Fracton-elasiticity duality
  ['18 Pretko, Radzilhovsky]

$$S = \int d^2x dt\, \frac{1}{2}\left[(\partial_t u^i)^2 - C^{ijk\ell}u_{ij}u_{k\ell}\right]$$

Low energy phonon action can be transformed to the U(1) tensor gauge theory through Hubbard-Stratonovich transformation.



Two bonuses from DFT II

- Charged particle minimally coupled to the doubled Maxwell

$$\sum_{\alpha}\int \mathrm{d}\tau\ \tfrac{1}{2}e^{-1}\mathbf{D}_\tau x^A_\alpha \mathbf{D}_\tau x^B_\alpha \mathcal{H}_{AB} - \tfrac{1}{2}em^2 - q\mathbf{D}_\tau x^A_\alpha \mathcal{V}_A$$

$$\longrightarrow \quad S_q = \int \mathrm{d}\tau \left[\begin{array}{l} \frac{1}{2}e^{-1}\dot{x}^a\dot{x}_a - \frac{1}{2}e(m^2 + q^2\varphi^a\varphi_a) - q\dot{x}^\mu A_\mu \\ +\Lambda_i(\dot{y}^i - eq\varphi^i) + \bar{\Lambda}_{\bar{\imath}}(\dot{\bar{y}}^{\bar{\imath}} + eq\varphi^{\bar{\imath}}) \end{array} \right]$$

- The particle mass is increased through coupling to phonon. This phenomenon is well-known in condensed matter physics: Polaron
- Polaron mass formula

$$\begin{aligned} m_{\mathrm{eff.}} &= \sqrt{m^2 + q^2\varphi^a\varphi_a} \\ &= m\Big[1 + \tfrac{1}{2}\big(\tfrac{q}{m}\big)^2\varphi^a\varphi_a - \tfrac{1}{8}\big(\tfrac{q}{m}\big)^4(\varphi^a\varphi_a)^2 + \cdots\Big] \end{aligned}$$

$$m_{\mathrm{known}} \simeq m\Big[1 + \tfrac{1}{6}\alpha_{\mathrm{e-ph}} + 0.0236(\alpha_{\mathrm{e-ph}})^2\Big]$$

Discussion

- We also studied curved spactime generalization and doubled Yang-Mills generalization.

- In both generalizations, we found some interesting properties which are related to fracton physics.

- For example, immobility and multipole symmetry are relaxed, and also some interacting fracton models are naturally obtained.

- We need further studies to analyze those details.