Dual Pair Correspondence and Scattering Amplitudes

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2015 — Massive Spinor-Helicity formalism

w/ Conde, Mkrtchyan (2016)

2016

2017

- Any d generalizat.
- (A)dS generalizat.

(unpublished)

2018

2019 — Dual pair correspondence

w/ Basile, Mkrtchyan, Mojaza (2020)

2020

Spinor-Helicity formalism

- a specific "realizat" of a representation of Poincaré sym

$$P_{\alpha\dot\beta} = \lambda^I_\alpha \tilde\lambda^I_{\dot\beta} , \quad L_{\alpha\beta} = \lambda^I_\alpha \frac{\partial}{\partial\lambda^{\beta I}} , \quad \tilde L_{\dot\alpha\dot\beta} = L_{\alpha\beta}^\dagger$$

⚠ Quadratic in λ and $\frac{\partial}{\partial\lambda}$

⚠ Dual (not little) group $U(N)$: $\lambda^I_\alpha \frac{\partial}{\partial\lambda^J_\alpha} - h.c$

More Generally

Realization of Lie algebra $\mathcal{G}$

as quadratic operator in $\left[\begin{array}{l} x^i, \frac{\partial}{\partial x^i} \\ \text{or} \\ a^i, a^{i\dagger} \end{array} \right.$

$\Rightarrow$ Quadratic oscillator realizat' of $\mathcal{G}$

ex) $SU(2)$ (Schwinger-Jordan)

$$J_+ = a^\dagger b, \quad J_- = b^\dagger a, \quad J_3 = \frac{a^\dagger a - b^\dagger b}{2}$$

commute with

$$N = a^\dagger a + b^\dagger b \rightsquigarrow U(1)$$

The Generality

$x^i,\ \frac{\partial}{\partial x^i}$ $\quad$ "Maximal" Sym $\quad$ $Sp(2N,\mathbb{R})$

or

$a^i,\ a^{i\dagger}\ (i=1,\cdots,N)$ $\Longrightarrow$ $\underbrace{x^i x^j,\ x^i\frac{\partial}{\partial x^j}+\frac{\delta^i_j}{2},\ \frac{\partial}{\partial x^i}\frac{\partial}{\partial x^j}}$

or $\quad$ or

$\vdots$ $\quad$ $\vdots$

* Metaplectic representat'

Any quadratic oscillator realizat' of G

$\Rightarrow\ G\subset Sp(2N,\mathbb{R})$ for some N

Then

$\exists$ Stabiliser $G'\subset Sp(2N,\mathbb{R})$

ALL Reductive Dual Pairs !

	G	G'
1	$U(N_1, N_2)$	$U(M_1, M_2)$
2	$GL(N, \mathbb{R})$	$GL(M, \mathbb{R})$
3	$GL(N, \mathbb{C})$	$GL(M, \mathbb{C})$
4	$GL(N, \mathbb{H})$	$GL(M, \mathbb{H})$
5	$Sp(2N, \mathbb{R})$	$O(p, q)$
6	$Sp(N_1, N_2)$	$O^*(2M)$
7	$Sp(2N, \mathbb{C})$	$O(M, \mathbb{C})$

Dual Pair Correspondence

$$\Downarrow$$

1-1 Correspondence between

G irrep π^i_G and G' irrep $\pi^i_{G'}$

See saw

$$\begin{tikzcd}
\pi^i_G & G \arrow[ddrr, leftrightarrow] & & \tilde{G}' & \pi^a_{\tilde{G}'} \\
& \cup & & \cup & \\
\pi^a_{\tilde{G}} & \tilde{G} \arrow[uurr, leftrightarrow] & & G' & \pi^i_{G'}
\end{tikzcd}$$

But Poincaré $iso(1,3)$ is not in the list

$\Rightarrow$ instead $\left[\begin{array}{l} so(1,4) \simeq Sp(1,1) \\ so(2,3) \simeq Sp(4,\mathbb{R}) \end{array}\right.$ are !

Translation in Poincaré : $P_{\alpha\dot\beta} = \lambda_\alpha^I \tilde\lambda_{\dot\beta}^I$

$\Rightarrow$ Transvection $P_{\alpha\dot\beta} = \lambda_\alpha^I \tilde\lambda_{\dot\beta}^I \boxed{+ \Lambda \frac{\partial}{\partial \lambda^{\alpha I}} \frac{\partial}{\partial \tilde\lambda^{\dot\beta I}}}$

simple modificat.

What does it describe?

Ex. dS_4 with $I=1,2$

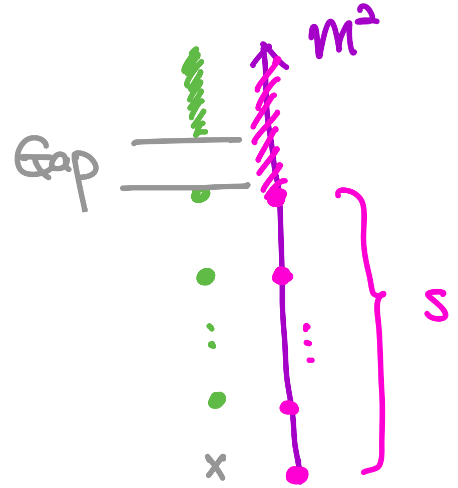
$$\begin{tikzcd} Sp(1,1) \arrow[dr, leftrightarrow] & O^*(4)^2 \simeq \left(SU(2)\times SL(2,\mathbb{R})\right)^2 \arrow[dl, leftrightarrow] \\ Sp(1)\times Sp(1) & O^*(4) \simeq SU(2)\times SL(2,\mathbb{R}) \end{tikzcd}$$

Basically
$$\left[\begin{array}{l} SU(2)^2 \longrightarrow SU(2) \\ SL(2,\mathbb{R})^2 \longrightarrow SL(2,\mathbb{R}) \end{array}\right.$$

In the end.

$$\underline{\underline{Sp(1,1)}} \longleftrightarrow \underline{\underline{SU(2)}} \times \underline{\underline{SL(2,\mathbb{R})}}$$

spin mass



LORENTZ RESTRICTION

dS_4 · · · · · · · Poincaré · · · · · · · AdS_4

$$\begin{array}{ccc} Sp(1,1) & & \boxed{GL(N,\mathbb{C})} \\ \cup & \times & \cup \\ \boxed{GL(2,\mathbb{C})} & & O^*(2N) \end{array} \qquad \vdots \qquad \begin{array}{ccc} Sp(4,\mathbb{R}) & & \boxed{GL(N,\mathbb{C})} \\ \cup & \times & \\ \boxed{GL(2,\mathbb{C})} & & O(p,N-p) \end{array}$$

- $N=1$: Celestial CFT
- $N \geq 2$: Scattering amplitude (Lorentz inv)

$$GL(2,\mathbb{C}) \longleftrightarrow GL(N,\mathbb{C})$$

trivial rep — Most deg principal series rep

Fn on complex Grassmannian $Gr(2,N)$

More familiar context of scattering amplitude

	Lorentz sym	Conformal sym
3d	$SO(1,2) \sim SL(2,\mathbb{R})$	$SO(2,3) \sim Sp(4,\mathbb{R})$
4d	$SO(1,3) \sim SL(2,\mathbb{C})$	$SO(2,4) \sim$ "$Sp(4,\mathbb{C})$" $= U(2,2)$
6d	$SO(1,5) \sim SL(2,\mathbb{H})$	$SO(2,6) \sim$ "$Sp(4,\mathbb{H})$" $= O^*(8)$

$$
\begin{tikzcd}
\text{"}Sp(4,\mathbb{F})\text{"} \arrow[dr, leftrightarrow] & & GL(N,\mathbb{F}) \arrow[dl, leftrightarrow] \\
\cup & & \cup \\
GL(2,\mathbb{F}) & & \text{"}O(N,\mathbb{F})\text{"}
\end{tikzcd}
$$

Fn on $\mathbb{F}$ Grassmannian $Gr(2,N)$

"$O(N,\mathbb{R})$" $= O(p,N-p)$

"$O(N,\mathbb{C})$" $= U(p,N-p)$

"$O(N,\mathbb{H})$" $= Sp(p,N-p)$

p incoming particles

$N-p$ outgoing particles

To Do List

- (A)dS generalizat' of spinor-helicity
- Physical applicat' of dual pair correspondence
    - Scattering amplitude
    - Celestial CFT
- SUSY dual pairs
- Non-compact dual pairs
- Affine extension (string oscillators?)