Accelerating BHs and AdS/CFT

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General BHs

- Most general black hole solution, explicitly known in Einstein-Maxwell theory?
  - M,Q,J: Mass, Charge, Angular Momentum?
  - 7 parameters: Mass, E&M charge, J, A, NUT, and cc

C-metric

- Revived recent interest, because related to new AdS/CFT duals from branes wrapped on cycles with puncture (can give holographic dual of Argyres-Douglas)

- Has been known for a long time: first discovered by Levi-Civita in 1917

- Called type C, in the classification terminology of Ehlers and Kundt in 1962

- Physical interpretation as a pair of accelerating took a long time: Kinnersley and Walker in 1970

- Modern analysis: Podolsky, Griffiths, Dias, Lemos (2003) etc

dS C-metric

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- A is acceleration parameter, m is mass, q is charge
- Note that F, G are quartic polynomials
- (—+++) signature requires G>0; Consider interval between two roots to give a compact horizon

$$ds^2 = [A(x+y)]^{-2}(-\mathcal{F}dt^2 + \mathcal{F}^{-1}dy^2 + \mathcal{G}^{-1}dx^2 + \mathcal{G}dz^2)\,, \tag{1}$$

where

$$\begin{aligned} \mathcal{F}(y) &= -\left(\frac{1}{\ell^2 A^2} + 1\right) + y^2 - 2mAy^3 + q^2A^2y^4, \\ \mathcal{G}(x) &= 1 - x^2 - 2mAx^3 - q^2A^2x^4\,, \end{aligned} \tag{2}$$

Infinity?

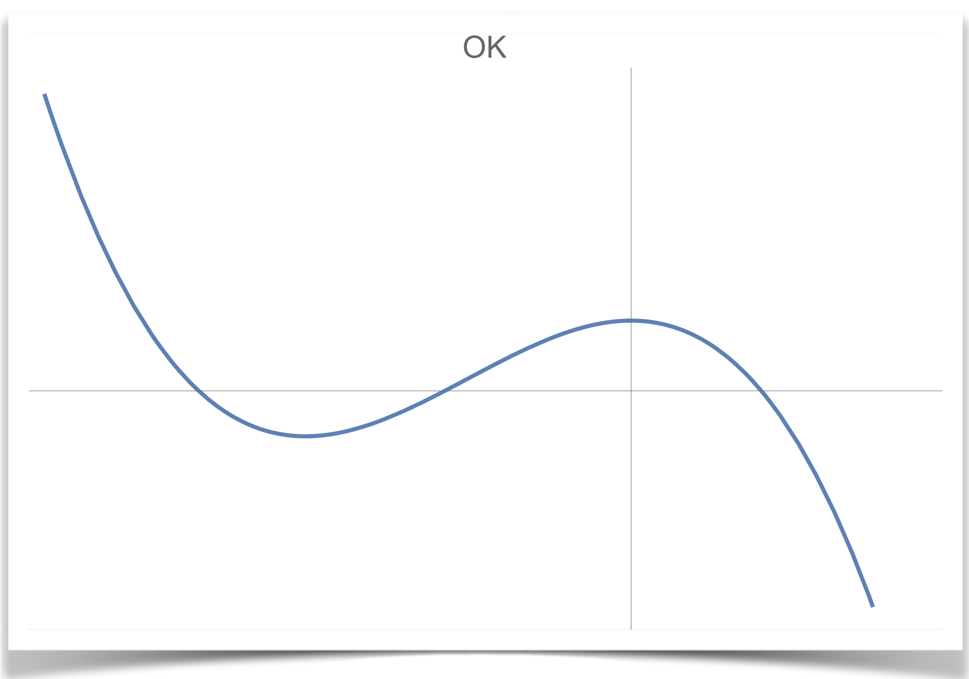
- $r = (A(x+y))^{-1}$ is radial coordinate (A>0)
- Kretschmann scalar

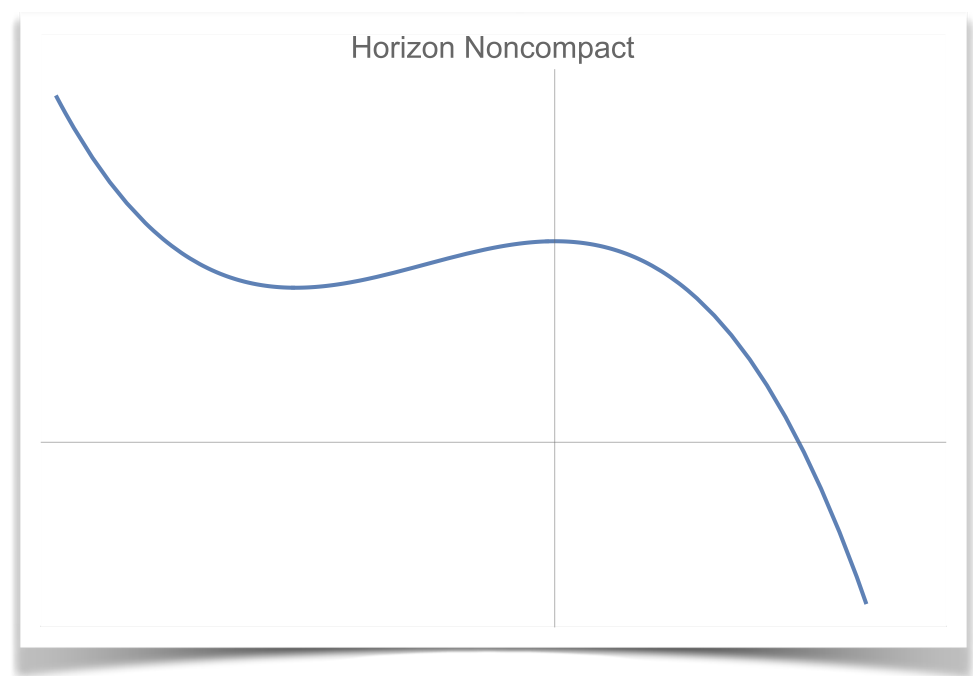
$$R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} = \frac{24}{\ell^2} + \frac{8}{r^8}\Big[6m^2r^2 + 12mq^2(2Axr-1)r + q^4(7 - 24Axr + 24A^2x^2r^2)\Big]. \qquad (7)$$

- Polar angle : $d\theta = G^{-1/2}dx$, involving elliptic integral
- Deficit angle: cannot remove both at N/S by adjusting κ: $\delta = 2\pi(1-(\kappa/2)G'), \phi = z/\kappa$

Uncharged case

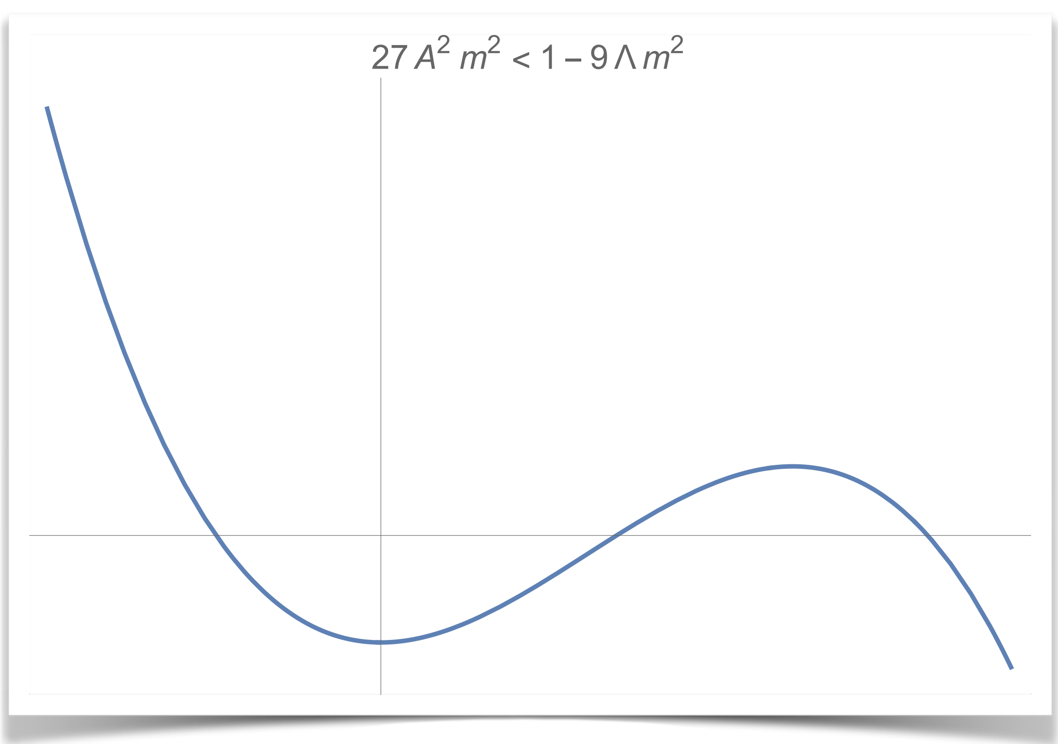
- Let us consider $q = 0$ for simplicity and illustrative purpose
- $G = 1 - x^2 - 2mAx^3, \quad G' = -2x(1 + 3mAx)$
- We want $G(x = -\frac{1}{3mA}) = 1 - \frac{1}{27m^2A^2} < 0; \; x_n, x_s \quad (x_n > x_s)$

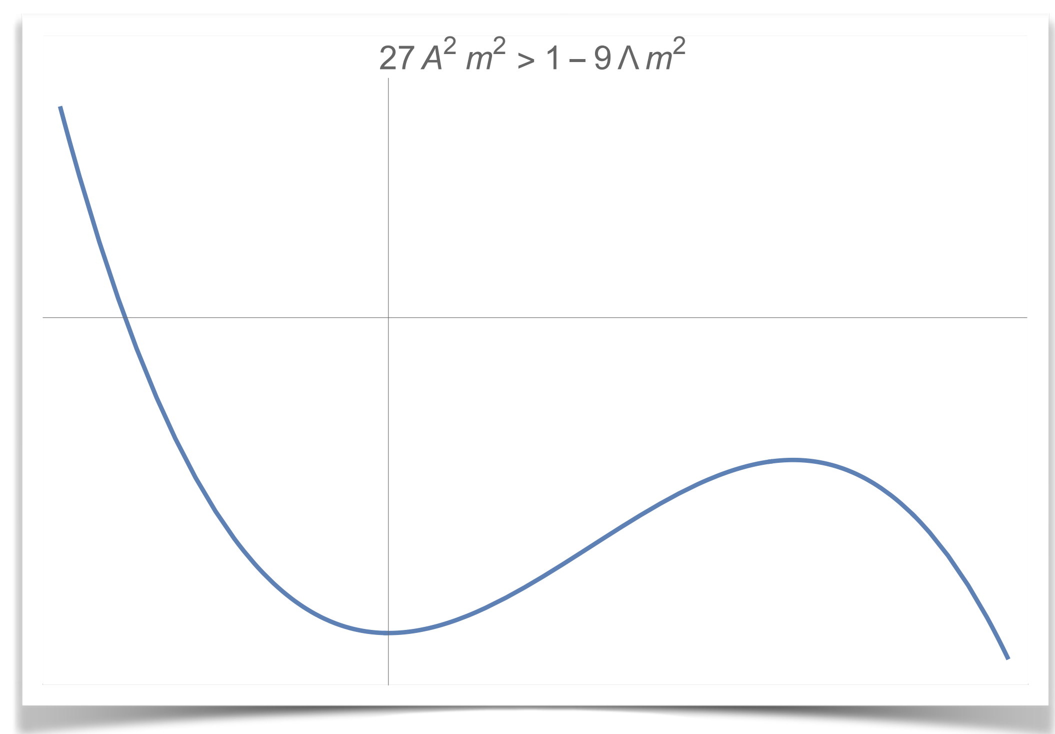




Uncharged case

- $F = -1 - \frac{1}{\ell^2 A^2} + y^2 - 2mAy^3; F' = 2y(1 - 3mAy)$
- $F(y = \frac{1}{3mA}) = -1 - \frac{\Lambda}{3A^2} + \frac{1}{27m^2A^2}$
- Range of y: $y > -x_n$, right panel is non-physical since the singularity is naked





A is acceleration

- Set m=0, q=0 (in fact dS vacuum), and consider transformation:
- The proper acceleration of a particle at $\rho = 0$ is exactly A

$$\tau = \frac{\sqrt{1+\ell^2 A^2}}{A} t\,, \qquad \rho = \frac{\sqrt{1+\ell^2 A^2}}{A}\frac{1}{y}\,,$$
$$\theta = \arccos x\,, \qquad \phi = z\,, \qquad (19)$$

we can rewrite the massless uncharged dS C-metric as

$$ds^2 = \frac{1}{\gamma^2}\left[-(1-\rho^2/\ell^2)d\tau^2 + \frac{d\rho^2}{1-\rho^2/\ell^2} + \rho^2 d\Omega^2\right], (20)$$

with $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ and

$$\gamma = \sqrt{1+\ell^2 A^2} + A\rho\cos\theta\,. \qquad (21)$$

$$|a_4| = \sqrt{a_\mu a^\mu} = \frac{\rho\sqrt{1+\ell^2 A^2} + \ell^2 A}{\ell\sqrt{\ell^2 - \rho^2}}\,.$$

m=0, q=0 is dS

- To see the global structure, we go back to the definition of dS as hyperboloid: $-z_0^2 + z_1^2 + z_2^2 + z_3^2 + z_4^2 = \ell^2$

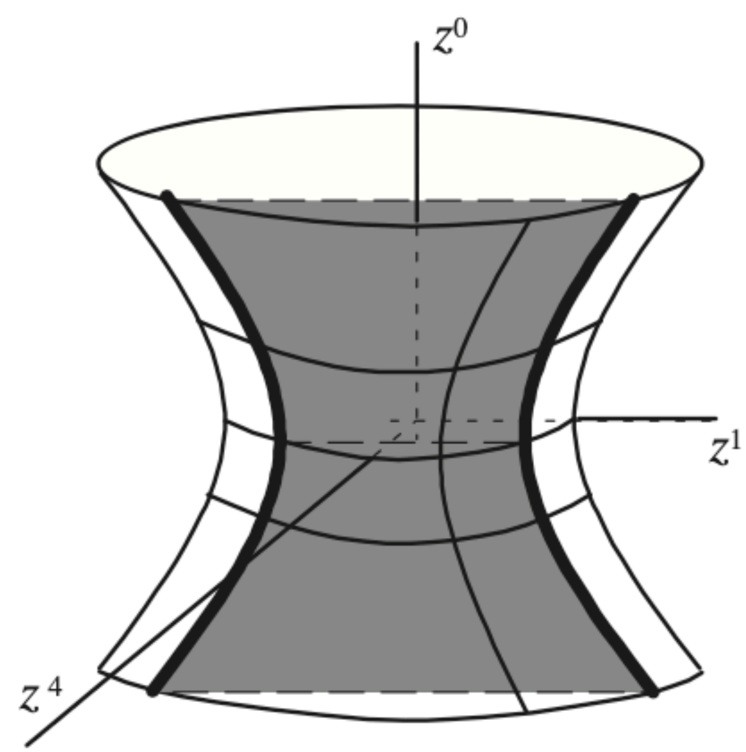
- We obtain the metric (20), using the parametrization below

$$
\begin{aligned}
z^0 &= \gamma^{-1}\sqrt{\ell^2-\rho^2}\,\sinh(\tau/\ell)\,, \qquad & z^2 &= \gamma^{-1}\rho\sin\theta\cos\phi\,,\\
z^1 &= \gamma^{-1}\sqrt{\ell^2-\rho^2}\,\cosh(\tau/\ell)\,, \qquad & z^3 &= \gamma^{-1}\rho\sin\theta\sin\phi\,,\\
z^4 &= \gamma^{-1}[\sqrt{1+\ell^2A^2}\,\rho\cos\theta + \ell^2 A]\,, & & \qquad (25)
\end{aligned}
$$

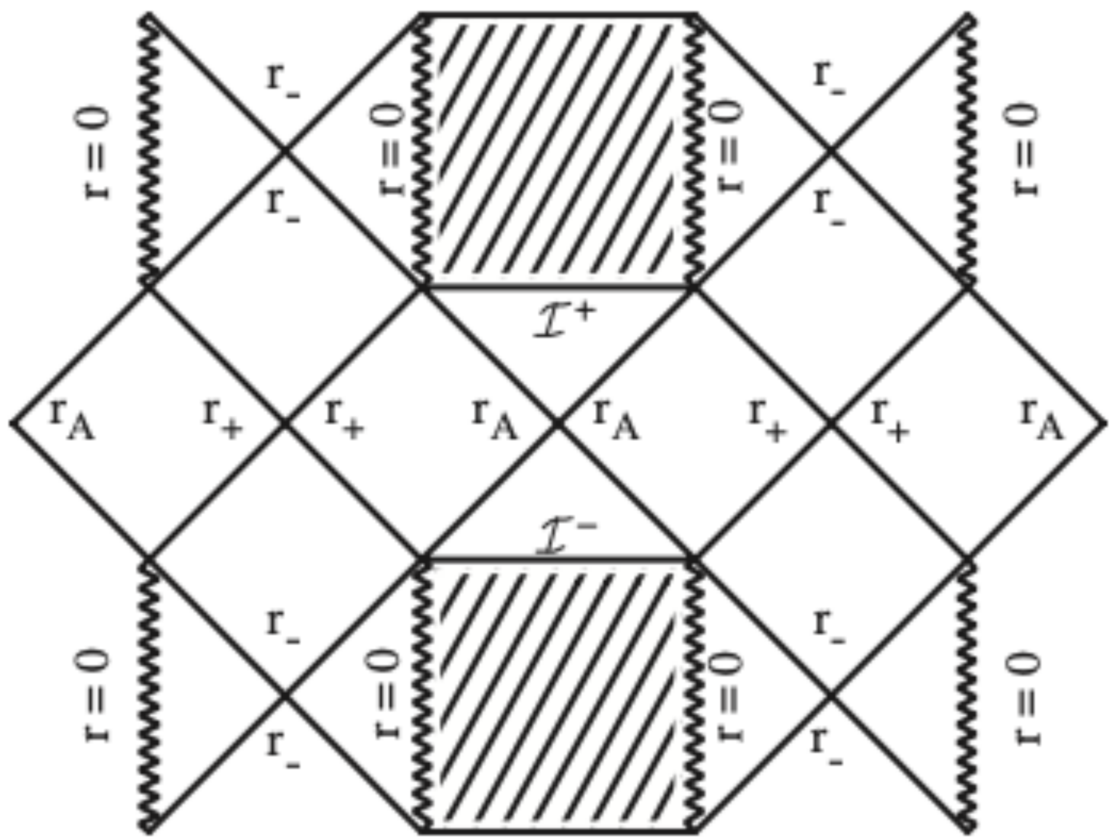
Two BHs

- At the origin $\rho = 0$

$$z^2 = 0\,, \quad z^3 = 0\,, \quad z^4 = \ell^2 A/\sqrt{1+\ell^2 A^2} < \ell \quad \text{and}$$
$$(z^1)^2 - (z^0)^2 = (A^2 + 1/\ell^2)^{-1} \equiv a_5^{-2}\,. \qquad (26)$$



(i) Non-extreme charged Black Hole



AdS C-metric

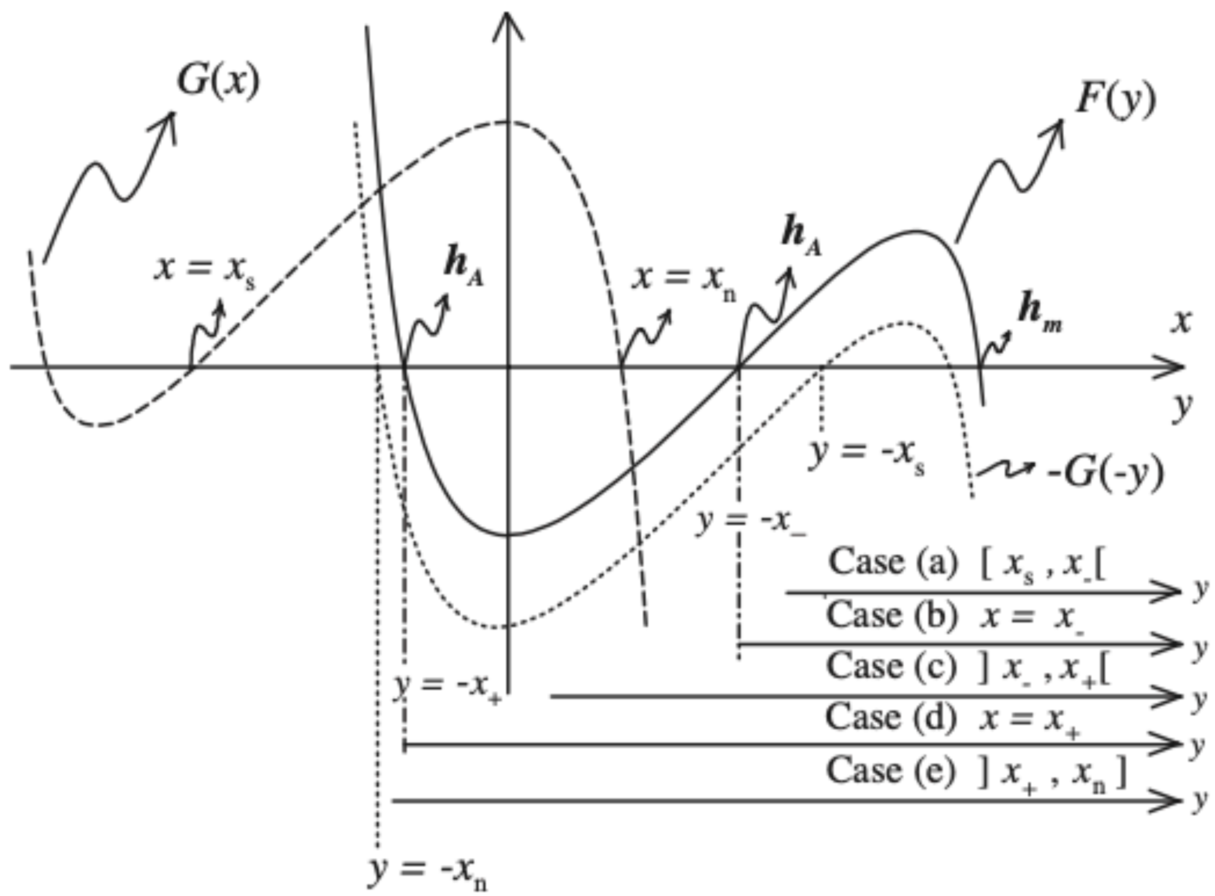
AdS C-metric

- Radial coordinate: $r = (A(x+y))^{-1}$
- Again obtain from positivity of G: $27m^2A^2 < 1$

$$ds^2 = [A(x+y)]^{-2}(-\mathcal{F}dt^2 + \mathcal{F}^{-1}dy^2 + \mathcal{G}^{-1}dx^2 + \mathcal{G}dz^2)\,, \quad (3)$$

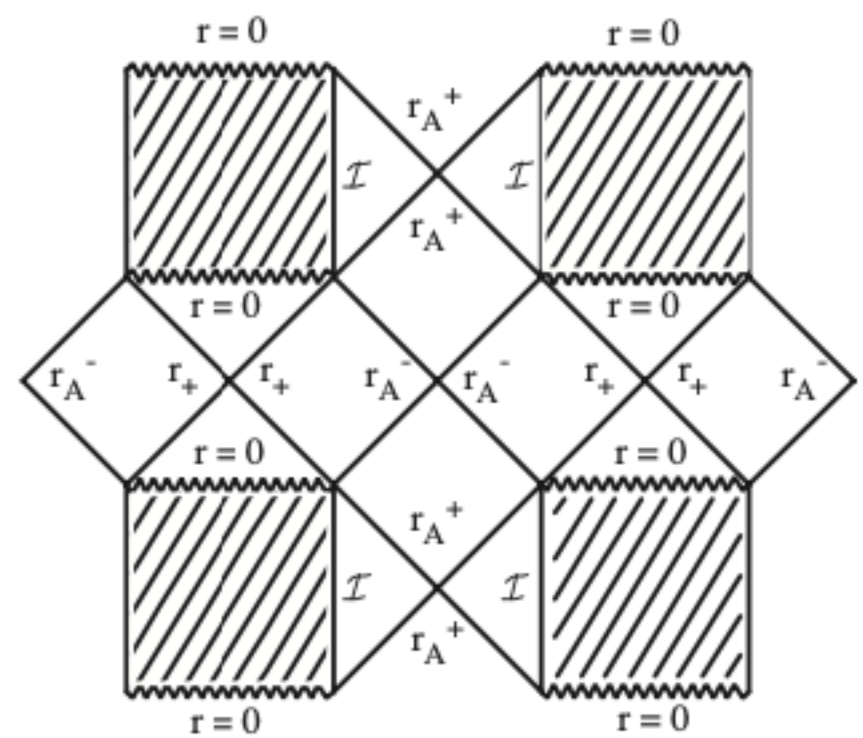
where

$$\mathcal{F}(y) = \left(\frac{1}{\ell^2 A^2} - 1\right) + y^2 - 2mAy^3 + q^2A^2y^4,$$
$$\mathcal{G}(x) = 1 - x^2 - 2mAx^3 - q^2A^2x^4\,, \quad (4)$$


G(x)
F(y)
x = x_s
h_A
x = x_n
h_A
h_m
x
y
y = -x_s
-G(-y)
y = -x_-
Case (a) [x_s , x_-[
Case (b) x = x_-
Case (c)] x_- , x_+[
Case (d) x = x_+
Case (e)] x_+ , x_n]
y = -x_+
y = -x_n

(e)

North


r = 0
r_A^+
I
r_A^-
r_+

Large acceleration

- $A > 1/\ell$: Pair of accelerated BHs
- Acceleration is exactly A at origin

$$\tau = \frac{\sqrt{\ell^2 A^2 - 1}}{A} t \,, \qquad \rho = \frac{\sqrt{\ell^2 A^2 - 1}}{A} \frac{1}{y} \,,$$
$$\theta = \arccos x \,, \qquad \phi = z \,, \tag{32}$$

we can rewrite the massless uncharged AdS C-metric as

$$ds^2 = \frac{1}{\gamma^2}\left[-(1 - \rho^2/\ell^2) d\tau^2 + \frac{d\rho^2}{1 - \rho^2/\ell^2} + \rho^2 d\Omega^2 \right], \tag{33}$$

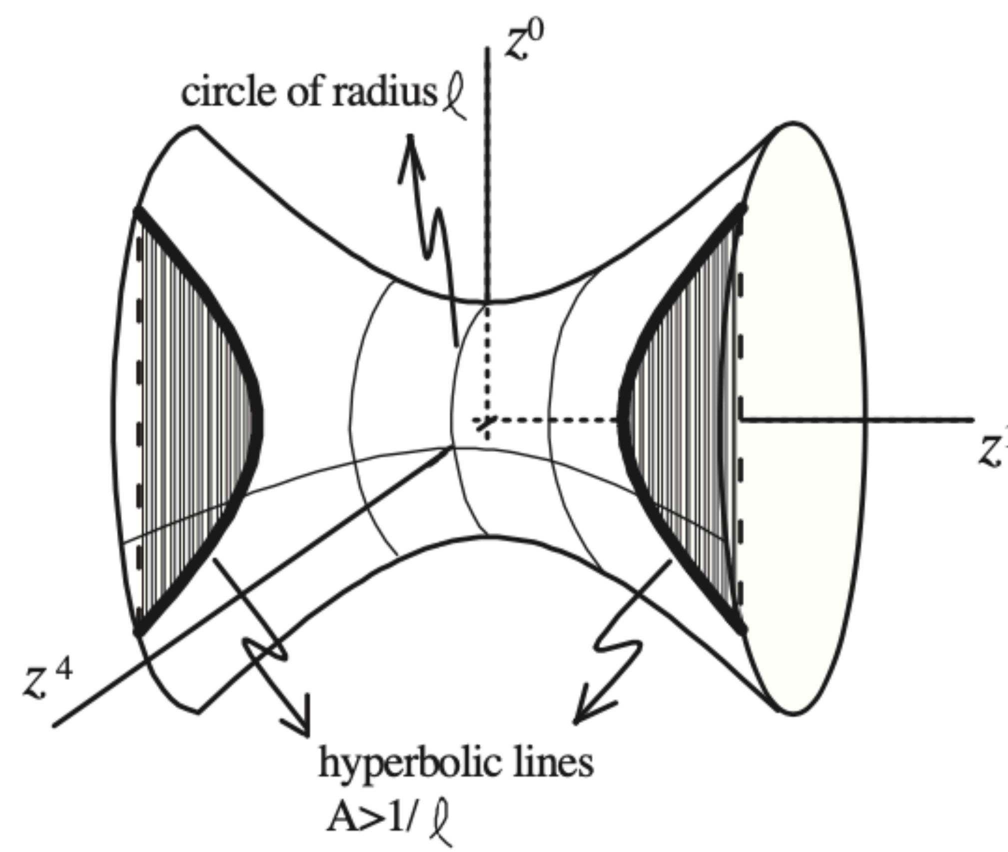
with $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ and

$$\gamma = \sqrt{\ell^2 A^2 - 1} + A\rho\cos\theta \,. \tag{34}$$

As a hyperboloid

- $-z_0^2 + z_1^2 + z_2^2 + z_3^2 - z_4^2 = -\ell^2$

$$z^0 = \gamma^{-1}\sqrt{\ell^2 - \rho^2}\,\sinh(\tau/\ell)\,, \qquad z^2 = \gamma^{-1}\rho\sin\theta\cos\phi\,,$$
$$z^1 = \gamma^{-1}\sqrt{\ell^2 - \rho^2}\,\cosh(\tau/\ell)\,, \qquad z^3 = \gamma^{-1}\rho\sin\theta\sin\phi\,,$$
$$z^4 = \gamma^{-1}[\sqrt{\ell^2 A^2 - 1}\,\rho\cos\theta + \ell^2 A]\,,$$



Small acceleration

- $A \leq 1/\ell$: Single accelerated BH
- Acceleration is exactly A at origin R=0

$$T = \frac{\sqrt{1-\ell^2 A^2}}{A} t \,, \qquad R = \frac{\sqrt{1-\ell^2 A^2}}{A} \frac{1}{y} \,,$$

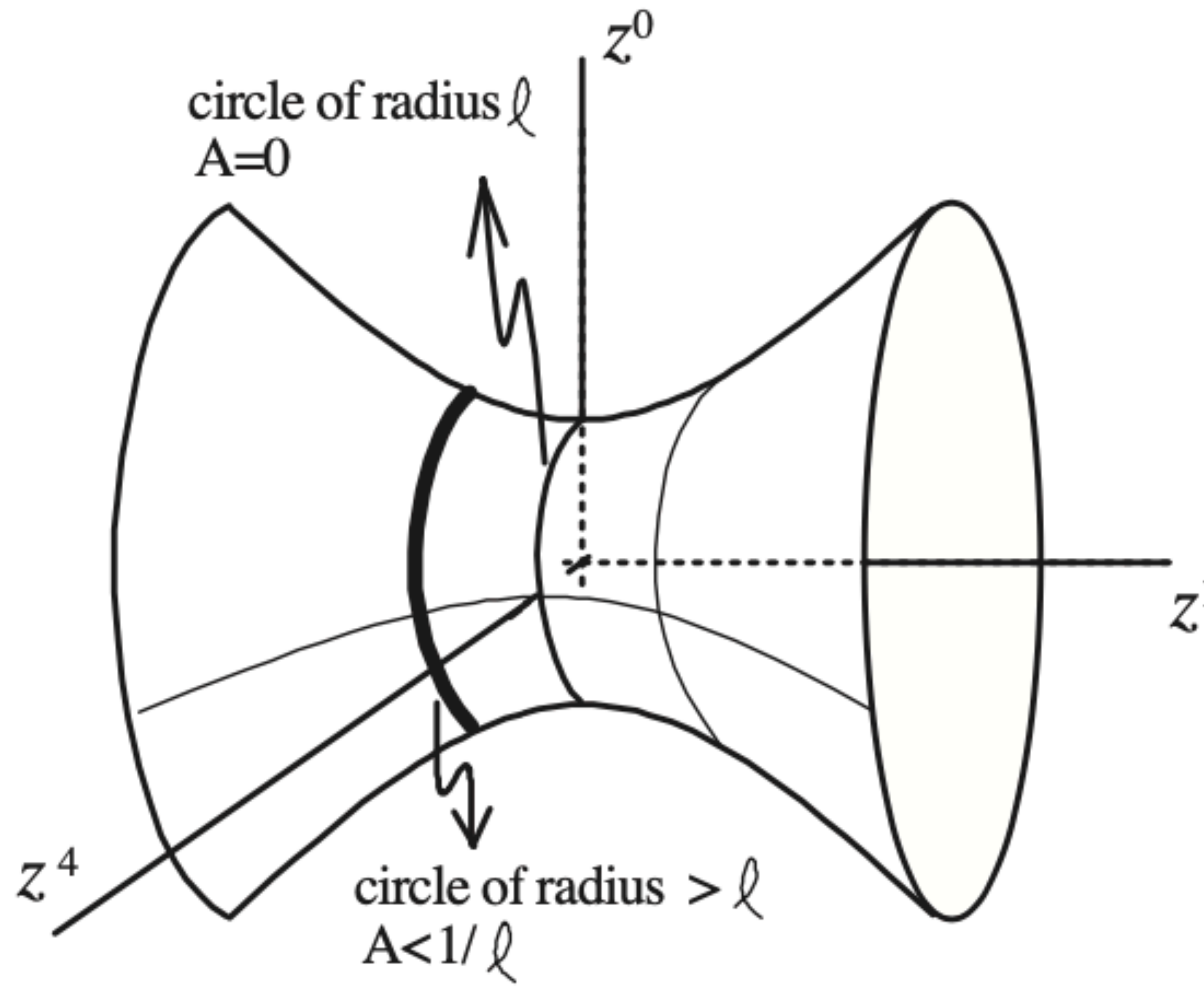
$$\theta = \arccos x \,, \qquad \phi = z \,, \tag{47}$$

we can rewrite the massless uncharged AdS C-metric as

$$ds^2 = \frac{1}{\eta^2} \left[-(1 + R^2/\ell^2) dT^2 + \frac{dR^2}{1 + R^2/\ell^2} + R^2 d\Omega^2 \right], \tag{48}$$

$$\eta^{-1} = \sqrt{1-\ell^2 A^2} + AR\cos\theta$$

$$
\begin{aligned}
z^0 &= \eta^{-1} \sqrt{\ell^2 + R^2}\, \sin(T/\ell)\,, \qquad z^2 = \eta^{-1} R \sin\theta \cos\phi\,, \\
z^4 &= \eta^{-1} \sqrt{\ell^2 + R^2}\, \cos(T/\ell)\,, \qquad z^3 = \eta^{-1} R \sin\theta \sin\phi\,, \\
z^1 &= \eta^{-1} [\sqrt{1 - \ell^2 A^2}\, R \cos\theta - \ell^2 A]\,.
\end{aligned} \tag{50}
$$



Supersymmetric Accelerating AdS BH

- Minimal Gauged Supergravity in D=4
- Plebanski and Demianski solution: “Rotating, charged, and uniformly accelerating mass in GR” (1976)

$$\mathrm{d}s^2 = \frac{1}{H^2}\Big\{ -\frac{Q}{\Sigma}\Big(\frac{1}{\kappa}\mathrm{d}t - a\sin^2\theta\,\mathrm{d}\phi\Big)^2 + \frac{\Sigma}{Q}\,\mathrm{d}r^2 + \frac{\Sigma}{P}\mathrm{d}\theta^2 + \frac{P}{\Sigma}\sin^2\theta\Big(\frac{a}{\kappa}\mathrm{d}t - (r^2+a^2)\mathrm{d}\phi\Big)^2\Big\}$$

where

$$
\begin{aligned}
P(\theta) &= 1 - 2\alpha m \cos\theta + \left(\alpha^2(a^2 + e^2 + g^2) - \frac{a^2}{\ell^2}\right)\cos^2\theta\,, \\
Q(r) &= (r^2 - 2mr + a^2 + e^2 + g^2)(1 - \alpha^2 r^2) + \frac{r^2}{\ell^2}(a^2 + r^2)\,, \\
H(r,\theta) &= 1 - \alpha\, r\cos\theta\,, \\
\Sigma(r,\theta) &= r^2 + a^2\cos^2\theta\,,
\end{aligned}
$$

and the gauge field is given by

$$
\begin{aligned}
A &= -e\frac{r}{\Sigma}\left(\frac{1}{\kappa}\mathrm{d}t - a\sin^2\theta\mathrm{d}\phi\right) + g\frac{\cos\theta}{\Sigma}\left(\frac{a}{\kappa}\mathrm{d}t - (r^2 + a^2)\mathrm{d}\phi\right) \\
&= A_t\,\mathrm{d}t + A_\phi\,\mathrm{d}\phi\,.
\end{aligned}
$$

- parameters: m, e, g, a, α, ℓ and $\Delta\phi$

- Supersymmetry:

$$g = \alpha m\,,$$
$$0 = \alpha^2(e^2+g^2)(\Xi+a^2)-(g-a\alpha e)^2\,.$$

$$\Xi \equiv 1+\alpha^2(a^2+e^2+g^2)-a^2\,.$$

- Gives regular solution in D=11: Ferrero et al 2012.08530

- Extremality: $ag^2(a\alpha e-g)(e+a\alpha g)+\alpha^3e^2(e^2+g^2)^2 = 0$