11D-Supergravity Action & Holography

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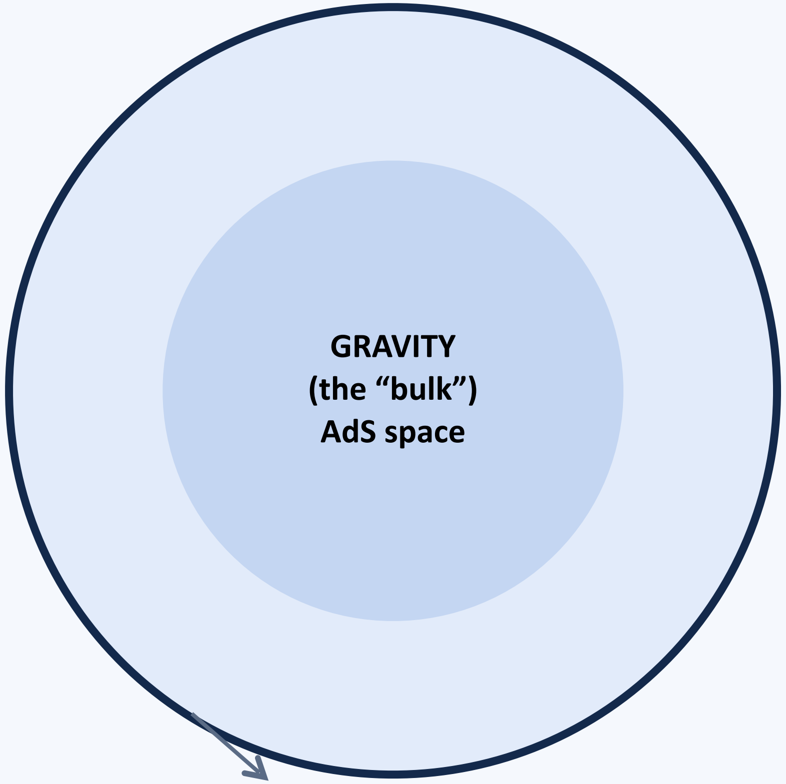
Introduction

The AdS/CFT correspondence [Maldacena ’97] says that a theory of **gravity** living inside a curved “box” (Anti-de Sitter space) is **secretly the same theory** as an ordinary quantum field theory — with no gravity at all — living on the wall of the box.

Like a hologram: all the information about a higher-dimensional gravitating world is stored on a lower-dimensional surface.

It converts impossibly hard questions about quantum gravity into ordinary (if difficult) QFT questions — and vice versa.



Quantum field theory (CFT) lives on the boundary

One equation runs the whole talk

$$\boxed{Z_{\text{gravity}}\,[\,\phi \to J \text{ at } \partial\text{AdS}\,] = Z_{\text{CFT}}[J] \equiv \langle\, e^{\int d^dx\, J\cdot\mathcal{O}} \,\rangle_{\text{CFT}}}$$

When gravity is weak (large N), the LHS can be approximated semi-classically: $e^{-S_{\text{on-shell}}}$

$$S_{\text{on-shell.}} = F_{\text{CFT}}$$

Left side: on-shell action S

Take the classical gravity action, plug in a solution of the equations of motion, integrate. A "classical" calculation.

Right side: free energy F

A genuinely quantum QFT quantity, F = −log Z. Usually very hard — but sometimes computable exactly (supersymmetric localization).

Matching these two numbers is a precision test of holography.

M-theory in three bullets

An 11-dimensional theory

The conjectured parent theory that unifies all five 10-dimensional string theories.

Low energies: 11D supergravity

Only two bosonic fields: the metric g and a 3-form gauge potential C_3 — think of C_3 as a big sibling of the photon field A_μ, just with more indices.

Its charged objects: branes

M2-branes (2+1-dimensional membranes) couple electrically to C_3; M5-branes (5+1-dimensional) couple magnetically. Analogy: C_3 is to M2-branes what the photon is to electrons.

Key point: the standard 11D action contains C_3 but no potential for the M5-branes.

A landmark test: counting with M2-branes

Stack N M2-branes

curve spacetime

Near the stack:

$AdS_4 \times S^7/Z_k$

Boundary theory:
ABJM

3D Chern–Simons-matter CFT
[Aharony, Bergman, Jafferis, Maldacena ’08]

Exact QFT result (localization), large N:

$$F = (\pi\sqrt{2} / 3)\, k^{1/2} N^{3/2}$$

Gravity agrees!

… when computed from the effective 4D gauged supergravity on AdS_4.

But the honest 11D calculation fails

Holography is conjectured in the full 11D supergravity — so the 11D on-shell action itself should reproduce the free energy. Let's check:

$AdS_4 \times S^7/Z_k$

(made by M2-branes — "electric" flux)

11D on-shell action ≠ ABJM free energy

$AdS_7 \times S^4$

(made by M5-branes — "magnetic" flux)

11D on-shell action = Dual 6D CFT free energy

Same theory, same action, same rules. Why does the electric case fail while the magnetic one works?

The culprit: what do you hold fixed?

Recall from classical mechanics: $\delta S = 0$ only gives the equations of motion if the boundary terms in δS vanish. Which boundary data you may hold fixed depends on which boundary term the action carries.

$S[q]$

Natural boundary condition: fix the position q at the endpoints.

$S[q] - (pq)|_{\text{boundary}}$

Add a boundary term → now you must fix the momentum p instead. Same physics, different ensemble of boundary data.

In AdS/CFT, this is not a technicality. The boundary data you hold fixed defines the thermodynamic ensemble of the dual field theory. Choose the wrong boundary term and you compute the free energy of the wrong ensemble — and the numbers won't match.

We were fixing the wrong thing

Recall the standard action contains metric & C_3: $$\delta_{C_3} S = \left(\text{EOM}_{C_3}\right)\delta C_3 + d\left(\cdots \wedge \delta C_3\right)$$

What one fixes:

The Dirichlet condition holds C_3 fixed on the boundary. That fixes the chemical potential for M2-branes:

$$\mu = 2\pi i/(2\pi\ell_p)^3 \int_{\partial M} C_3$$

Grand-canonical ensemble: the number of branes can fluctuate. [van Muiden et al.]

What ABJM actually computes

Localization gives F for $U(N)_k \times U(N)_{-k}$ at a definite rank → fixed brane number N, measured by the flux through the surrounding $Y_7 = S^7/Z_k$:

$$N = 1/(2\pi\ell_p)^6 \int_{Y_7} \Pi_7$$

Canonical ensemble: fixed particle number — a Gauss-law count of M2-brane charge.

Apples vs. oranges. To fix N we must hold the 7-form flux fixed at the boundary, $\Pi_7 = -i\star F_4 + \tfrac{1}{3} C_3 \wedge F_4$. Can Π_7 comes from some 6-form C_6? As its field strength? But, the standard 11D action doesn't even contain C_6!

Why (and how) a six-form must exist

Similar electromagnetism. Electric charges couple to the potential A; to give *magnetic* charges a potential of their own, one can introduce the dual potential $\tilde{A}$, defined through $d\tilde{A} = \star F$. M-theory is the same story, three steps up:

	Maxwell theory (4D)	M-theory (11D)
Electric potential	A_1 (1-form)	C_3 (3-form)
Electric object	electron (point)	M2-brane (membrane)
Magnetic potential	$\tilde{A}_1$, $d\tilde{A} = \star F_2$	C_6 , $\tilde{F}_7 = -i \star F_4$
Magnetic object	monopole (point)	M5-brane
Gauss law counts charge via	$\int \star F_2$ over an S^2	$\int \Pi_7$ over the S^7/Z_k

Why: fixing N is a boundary condition on the dual field — its convenient to introduce C_6 as a field of the action. How? Not straightforward in Pseudo-Action of 11D M-theory. Early work imposed it by hand on the equations of motion [hep-th/9705139]; the PST action derives it →

Step 1 — an action with both potentials

Idea: write a low-energy action containing both C_3 (couples to M2-branes) and C_6 (couples to M5-branes).

The obstacle

To keep C_3 and C_6 as independent — one needs to derive the duality, $\tilde{F}_7 = -i \star F_4$.

The tool: the PST formulation

Pasti, Sorokin & Tonin built a covariant action with one auxiliary scalar field a(x). Its equations of motion (plus a gauge symmetry) enforce the duality automatically — no constraints imposed by hand.

Why it suits us

It is a genuine action — so we can evaluate it on-shell and ask variational-principle questions — and it is consistent in Euclidean signature, which is exactly what free-energy computations need.

The PROPOSAL (Also discussed in a series of works by van Muiden et al in pseudo-action formulation)

Step 2 — add the right boundary term

With both potentials available, we can now do the Legendre transform from the mechanics analogy — swap which quantity is fixed at the boundary:

$$S_{\text{new}} = S_{\text{PST}} - \langle C_3 , \Pi_7 \rangle_{\text{boundary}}$$

Before

C_3 fixed at the boundary → chemical potential μ fixed (grand-canonical, wrong ensemble)

After

Π_7 fixed at the boundary → the number N of M2-branes is fixed (canonical — exactly the ABJM setup)

Bonus — it's simple: once the duality relation holds ("on the duality shell"), the whole correction collapses to a single crisp topological-looking term: $\Delta S = -(i/2\kappa^2) \int F_4 \wedge F_7$

Does it work? Yes — the numbers click into place

Evaluate the 11D action on the whole class of supersymmetric $AdS_4 \times S^7/Z_k$ backgrounds and reduce to 4 dimensions. Compare with minimal 4D gauged supergravity (which is known to reproduce ABJM):

Without the boundary term

$$\int R + 15 - (1/8)\, F^2$$

Wrong coefficients — does not match with the low-d on-shell action.

With the boundary term

$$\int R + 6 - (1/4)\, F^2$$

Exactly minimal 4D gauged supergravity — holography restored.

The result holds for every asymptotically Euclidean-AdS_4 solution that uplifts to M-theory — global $EAdS_4$, AdS_4 black holes, … — recovering $F = (\pi\sqrt{2}/3)\sqrt{k}\, N^{3/2}$ directly from 11 dimensions.

Three things to remember

Boundary terms choose the ensemble

On-shell actions only mean something once you decide what is held fixed at the boundary — in holography, that choice is the thermodynamic ensemble of the dual CFT.

M-theory needs both C_3 and C_6

The PST formulation gives a consistent Euclidean action containing both potentials, with the duality $\tilde{F}_7 = -i\star F_4$ built in.

One boundary term restores holography

Adding $-\langle C_3, \Pi_7\rangle$ fixes the number of M2-branes; the 11D on-shell action then matches the ABJM free energy for a whole class of AdS_4 backgrounds.

Next: establish the correction at the quantum level — a first-principles handle on the M-theory path integral beyond the semiclassical limit.

감사합니다

Thank You

Questions welcome — especially the naive ones.

BACKUP · FOR Q&A

What does the dictionary buy you?

Gravity → QFT · hard quantum-gravity questions become well-posed QFT questions

Black hole microstates

Why is $S = A/4G$? Holographically, an AdS black hole is a thermal state of the CFT — its entropy is a state count. Superconformal indices reproduce the horizon area of SUSY AdS_4 and AdS_5 black holes exactly.

The information paradox

The dual CFT is manifestly unitary, so black hole formation and evaporation cannot destroy information. The sharp question becomes how the bulk encodes this — leading to the recent Page-curve / “island” results.

QFT → Gravity · strong-coupling QFT questions become classical gravity problems

Quark–gluon plasma

Shear viscosity of a strongly coupled plasma = graviton absorption by a black hole: the universal $\eta/s = 1/4\pi$ [Kovtun–Son–Starinets], close to heavy-ion data at RHIC/LHC.

Entanglement entropy

Nearly impossible in an interacting QFT; Ryu–Takayanagi turns it into a minimal-surface (“soap film”) problem in the bulk geometry.

Bonus example from this talk: confinement–deconfinement at strong coupling = the Hawking–Page transition — decided by comparing two on-shell actions, the very tool we used today.