Black hole microstates in gauge theories

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Introduction

To better understand quantum aspect of gravity, we want to put gravity in a box

A natural choice is anti-de Sitter (AdS) spacetime

- solution of Einstein equation with negative cosmological constant

$$ds^2 = -\left(1+\frac{r^2}{\ell^2}\right)dt^2 + \left(1+\frac{r^2}{\ell^2}\right)^{-1}dr^2 + r^2 d\Omega_{d-2}^2$$

- gravitating potential well

$$g_{tt} \approx -(1+2\Phi) \qquad \Rightarrow \qquad \Phi \approx \frac{r^2}{2\ell^2}$$

confining particles toward AdS center

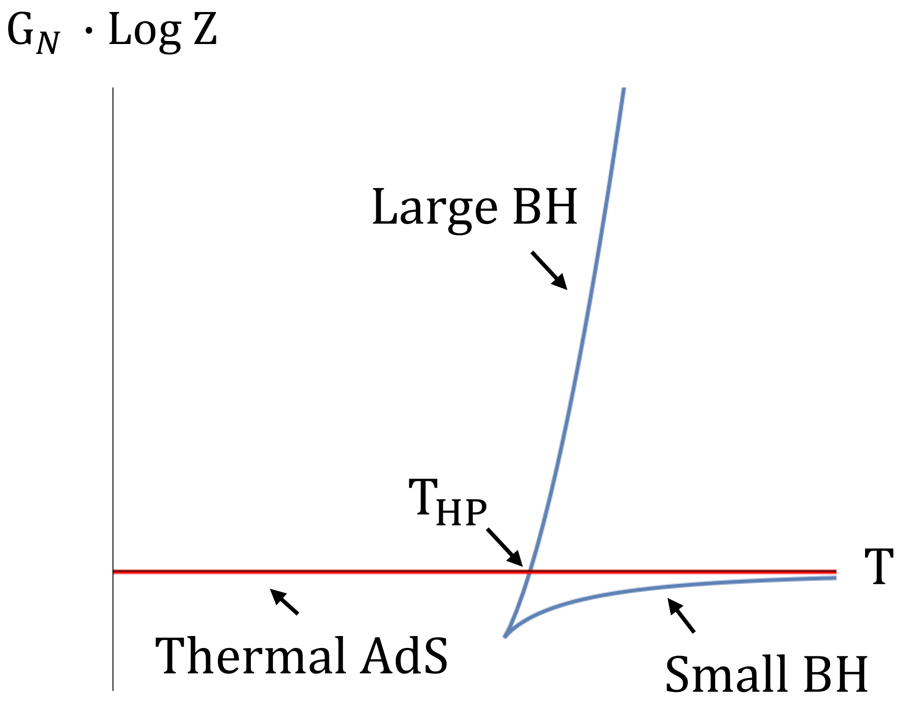
- conformal boundary at asymptotic infinity

Black holes in AdS

Schwarzschild black hole (d=5) $ds^2 = -\left(1+\frac{r^2}{\ell^2}-\frac{\mu}{r^2}\right)dt^2 + \left(1+\frac{r^2}{\ell^2}-\frac{\mu}{r^2}\right)^{-1}dr^2 + r^2 d\Omega_3^2$

- temperature $T = \frac{r_h}{\pi\ell^2} + \frac{1}{2\pi r_h},\ r_h^2 = \frac{-\ell^2+\sqrt{\ell^2(\ell^2+4\mu)}}{2}$

- two branches: small & large black hole



Thermal partition function $Z(\beta) = \int \mathcal{D}g \exp\left[-\frac{1}{G_N}I_{\rm EH}\right]$

- low T : Gas of gravitons $\log Z \sim O(1)$

↑

Hawking-Page transition $\left(T_{HP} = \frac{3}{2\pi\ell}\right)$

↓

- high T : Large AdS black holes $\log Z \sim O(1/G_N)$

AdS/CFT correspondence

Maldacena proposed [Maldacena '97]

boundary of AdS

String theory on $\mathrm{AdS}_5 \times S^5 \leftrightarrow \mathcal{N} = 4$ Super-Yang-Mills theory on $S^3 \times R$

Especially, we can study gravity in AdS by large N, strongly coupled CFT

$$\ell_{\mathrm{AdS}} \gg l_s \quad g_s \to 0 \qquad \leftrightarrow \qquad N \to \infty \quad \lambda \gg 1 \qquad \left(G_N \sim \frac{1}{N^2} \right)$$

Hawking-Page transition can be understood as deconfinement transition in CFT

- low T : Gas of gravitons $\quad \log Z \sim O(1) \quad$ Confinement

Hawking-Page transition $\qquad$ Confinement-deconfinement transition

- high T : Large AdS black holes $\quad \log Z \sim O(1/G_N) \sim O(N^2) \quad$ Deconfined Quark-gluon plasma

Supersymmetric black holes in AdS_5

To be quantitative, we consider supersymmetric setup

Supersymmetric black holes in $\mathrm{AdS}_5 \times S^5$ are known [Gutowski-Reall] [Chong-Cvetic-Lu-Pope] [Kunduri-Luccieti-Reall]

- three electric charges R_1 , R_2 , R_3 & two angular momenta J_1 , J_2
- mass (energy) $E = (R_1 + R_2 + R_3 + J_1 + J_2)$
- preserving two supersymmetry

Dual CFT = $\mathcal{N} = 4$ super-Yang-Mills theory with $SU(N)$ gauge group

We consider the operators preserving two specific supercharges Q , S

$$[Q, O\} = 0 \ \& \ [S, O\} = 0 \qquad \leftrightarrow \qquad [\{Q, S\}, O\} = 0$$

- saturate the BPS bound $E - (R_1 + R_2 + R_3 + J_1 + J_2) = 0$

  which follows from the algebra $2\{Q, S\} = E - (R_1 + R_2 + R_3 + J_1 + J_2) \geq 0$

Cohomology problem

It is hard to study the operators in the strong coupling …

1. we will begin with the weak coupling regime

→ assuming a sort of non-renormalization of spectrum

- entropy of BPS BH are counted from (coupling independent) index

2. we will study supercharge cohomology instead of BPS operators

- due to nilpotency of the supercharge $Q^2 = 0$

$$[\{Q, Q^\dagger\}, O\} = 0 \quad \overset{\text{1-to-1}}{\leftrightarrow} \quad [Q, O\} = 0/(O \sim O + Q - \text{exact})$$

Harmonic forms $\qquad$ Q-cohomology classes

Cohomology problem

In the free limit $g_{\rm YM} \to 0$, BPS operators can be constructed by free BPS letters

$$\bar{\phi}^m \,, \psi_{m+} \,, \bar{\lambda}_{\dot{\alpha}} , f_{++} \equiv F_{1+i2,3+i4} , \quad \text{and} \quad \partial_{+\dot{+}} \equiv \partial_1 - i\partial_2 , \partial_{+\dot{-}} \equiv \partial_3 - i\partial_4$$

i.e. all gauge invariant operators are BPS → Too many !

1-loop order correction to anomalous dimension $\sim O(g_{YM}^2)$

At $g_{\rm YM} \ll 1$, half-loop Q action on elementary fields are

$$Q\bar{\phi}^m = 0 \,, Q\psi_{m+} = -ig_{YM}\epsilon_{mnp}[\bar{\phi}^n, \bar{\phi}^p] \,, Qf_{++} = -ig_{YM}[\psi_{m+}, \bar{\phi}^m]$$
$$Q\bar{\lambda}_{\dot{\alpha}} = 0 \,, [Q, D_{+\dot{\alpha}}](\cdots) = -ig_{YM}[\bar{\lambda}_{\dot{\alpha}}, (\cdots)\}$$

➔ many free cohomology get lifted at 1-loop

Among free BPS operators which remain BPS at weak coupling?

Graviton cohomology

There is a well-known class of cohomology called graviton
which corresponds to BPS Kaluza-Klein gravitons in $\mathrm{AdS}_5 \times S^5$
- stress-tensor multiplet + KK tower

[Gunaydin, Marcus '84]
[Kim, Romans, van Nieuwenhuizen '85]

Explicitly, graviton multiplet consists of
- chiral primary $\mathrm{tr}(\bar{\phi}^{(i_1} \cdots \bar{\phi}^{i_n)})$ & superconformal descendants
  acting supercharges $Q_+^m, \bar{Q}_{m\dot{\alpha}}$ and BPS derivatives $\partial_{+\dot{\alpha}}$ on chiral primary

Graviton cohomology
- the above single trace gravitons & their multiparticle states
- explains low E spectrum
- Q-closed for any N (often called monotone)

Trace relations

As we grow the energy $E \sim N$, finite N effect called trace relation become important

- relations between single/multi-trace operators (mesons/baryons see the gluons)

e.g. SU(2) SYM $\mathrm{tr}(\bar\phi_1^4) = \frac{1}{2}\mathrm{tr}(\bar\phi_1^2)\mathrm{tr}(\bar\phi_1^2)\ ,\ \ \mathrm{tr}(\bar\phi_1^2)\mathrm{tr}(\bar\phi_2^2) - (\mathrm{tr}(\bar\phi_1\bar\phi_2))^2 \sim Q\mathrm{tr}(\psi_{3+}[\bar\phi_1, \bar\phi_2])$

$\longrightarrow$ reduces the states   c.f. Cayley-Hamilton

In gravity dual, this can be understood as stringy exclusion principle [Maldacena, Strominger '98]

- effect of giant gravitons: D-branes wrapping S^3 which have maximal size

[McGreevy, Susskind, Toumbas '00] [Grisaru, Myers, Tafjord '00]

In interacting supersymmetric theories, trace relations can generate new operators

$$Q\mathcal{O} = (\text{Trace relation}) = 0$$

- explains the rest of the cohomology, which we call non-graviton cohomology
- this phenomena is also known as fortuity [Chang, Lin '24]

Non-graviton cohomology

The very first example is obtained only recently in SU(2) SYM [Choi-Kim-Lee-Park '22]

- fortuity in many other models are studied then after

  [Chang-Chen-Sia-Yang '24] [Chang-Lin-Zhang '25] [Belin-Singh-Vadala-Zaffaroni '25] [Behan-de Gioia '25] [Kim-JL-Lee-Oh '25] [Seunggyu's talk]

In $SU(3)$ SYM, we constructed the lowest energy cohomology

$$
\begin{aligned}
-6\mathcal{O}^3 = {} & 288 v^{(j}{}_a v^{k)a}{}_i \times \epsilon_{c_1c_2(j}\mathrm{tr}\left(\bar\phi^{c_1}\bar\phi^{c_2}\bar\phi^{i}\psi_{k+)}\right) - 72 v^a{}_b v^{bk}{}_a \times \epsilon_{c_1c_2(k}\mathrm{tr}\left(\bar\phi^{c_1}\bar\phi^{c_2}\bar\phi^{d}\psi_{d+)}\right) \\
& + 36\epsilon_{a_1a_2(i}u^{a_1k}v^{a_2}{}_{j)} \times \Big[2\mathrm{tr}\left(\phi^{(i}\phi^c\phi^{j)}\psi_{(c}\psi_{k)}\right) + 2\mathrm{tr}\left(\phi^{(i|}\phi^c\phi^{|j)}\psi_{(c}\psi_{k)}\right) \\
& \qquad + 9\mathrm{tr}\left(\phi^{(i}\phi^j\psi_{(c}\phi^{c)}\psi_{k)}\right) - 6\mathrm{tr}\left(\phi^{(i}\phi^{j)}\psi_{(c}\phi^c\psi_{k)}\right)\Big] \\
& - 9\epsilon_{a_1a_2j}u^{a_1b}v^{a_2}{}_b \times \Big[2\mathrm{tr}\left(\phi^{(j}\phi^c\phi^{d)}\psi_{(c}\psi_{d)}\right) + 2\mathrm{tr}\left(\phi^{(j|}\phi^c\phi^{|d)}\psi_{(c}\psi_{d)}\right) \\
& \qquad + 9\mathrm{tr}\left(\phi^{(j}\phi^d\psi_{(c}\phi^{c)}\psi_{d)}\right) - 6\mathrm{tr}\left(\phi^{(j}\phi^{d)}\psi_{(c}\phi^c\psi_{d)}\right)\Big] \\
& - 20u^{a(i}v^{j)}{}_a \times \epsilon_{a_1a_2a_3}\left[2\mathrm{tr}\left(\psi_{(i}\psi_{j)}\phi^{a_1}\phi^{a_2}\phi^{a_3}\right) + \mathrm{tr}\left(\psi_{(i}\phi^{a_1}\psi_{j)}\phi^{a_2}\phi^{a_3}\right)\right] \\
& - 36u^{a(i}v^{j)}{}_a \times \epsilon_{a_1a_2(i}\left[\mathrm{tr}\left(\psi_{j)}\psi_{a_3}\phi^{a_1}\phi^{a_2}\phi^{a_3}\right) + \mathrm{tr}\left(\psi_{j)}\psi_{a_3}\phi^{a_1}\phi^{a_3}\phi^{a_2}\right) + \mathrm{tr}\left(\psi_{j)}\psi_{a_3}\phi^{a_3}\phi^{a_1}\phi^{a_2}\right)\right] \\
& - 36u^{a(i}v^{j)}{}_a \times \epsilon_{a_1a_2(i}\left[\mathrm{tr}\left(\psi_{j)}\phi^{a_1}\psi_{a_3}\phi^{a_2}\phi^{a_3}\right) + \mathrm{tr}\left(\psi_{j)}\phi^{a_1}\psi_{a_3}\phi^{a_3}\phi^{a_2}\right) + \mathrm{tr}\left(\psi_{j)}\phi^{a_3}\psi_{a_3}\phi^{a_1}\phi^{a_2}\right)\right] \\
& - 36u^{a(i}v^{j)}{}_a \times \epsilon_{a_1a_2(i}\left[\mathrm{tr}\left(\psi_{j)}\phi^{a_1}\phi^{a_2}\psi_{a_3}\phi^{a_3}\right) + \mathrm{tr}\left(\psi_{j)}\phi^{a_1}\phi^{a_3}\psi_{a_3}\phi^{a_2}\right) + \mathrm{tr}\left(\psi_{j)}\phi^{a_3}\phi^{a_1}\psi_{a_3}\phi^{a_2}\right)\right] \\
& - 36u^{a(i}v^{j)}{}_a \times \epsilon_{a_1a_2(i}\left[\mathrm{tr}\left(\psi_{j)}\phi^{a_1}\phi^{a_2}\phi^{a_3}\psi_{a_3}\right) + \mathrm{tr}\left(\psi_{j)}\phi^{a_1}\phi^{a_3}\phi^{a_2}\psi_{a_3}\right) + \mathrm{tr}\left(\psi_{j)}\phi^{a_3}\phi^{a_1}\phi^{a_2}\psi_{a_3}\right)\right] \\
& + 60u^{a(i}v^{j)}{}_a \times \epsilon_{a_1a_2(i}\left[\mathrm{tr}\left(\psi_{j)}\phi^{a_1}\phi^{a_2}\right)\mathrm{tr}\left(\psi_{a_3}\phi^{a_3}\right) + \mathrm{tr}\left(\psi_{j)}\phi^{a_1}\phi^{a_3}\right)\mathrm{tr}\left(\psi_{a_3}\phi^{a_2}\right) + \mathrm{tr}\left(\psi_{j)}\phi^{a_3}\phi^{a_1}\right)\mathrm{tr}\left(\psi_{a_3}\phi^{a_2}\right)\right]
\end{aligned}
$$

$$E = \frac{21}{2}\ , \ R_1 = R_2 = R_3 = \frac{5}{2}\ , \ J_1 = J_2 = \frac{3}{2}$$

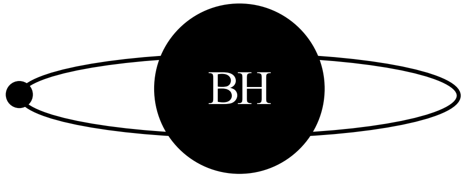
and infinitely many other examples & WIP in SU(4) SYM

Non-graviton cohomology

In $SU(3)$ SYM, we also constructed many other cohomologies

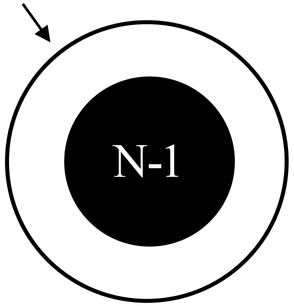
- graviton hair: infinitely many are allowed hair

$$O_{\mathrm{BH}} \cdot \partial_{+\dot{+}}^{n_+} \partial_{+\dot{-}}^{n_-} \mathrm{tr}(\bar{\phi}^2)$$



- dual giant graviton hair

dual giant



dual giant graviton reduce RR flux inside

→ SU(N-1) core BH with dual giant hair

$$(O_3)^{i_1 \cdots i_n} \xrightarrow{SU(2)\times U(1)} \hat{\phi}^{i_1 \cdots i_n} O_2 + \cdots$$

SU(2) threshold

Conclusion

We studied BPS spectrum of 4d N=4 SU(N) SYM

- constructed non-graviton operators for N=3 at weak coupling

Now we can view black holes as an ensemble of microstates

- computing 2pt functions in the black hole background ~ information paradox
  [Ahn, Kim, Lee, Lee, Yoon WIP]
- microscopically understanding the black hole hairs

Increasing N & higher-loop correction

- recently, it has been shown that quantum correction to supercharge can lift the black hole cohomology [Choi, Lee '25] [Kim, Kim, Lee, Park '26]
- it is open question which cohomology survive at higher-loop correction

Thank you

Appendix

Maximal Super-Yang-Mills

The $\mathcal{N}=4$ super-Yang-Mills theory with $SU(N)$ gauge group on R^4

- 3 chiral multiplets $\phi_m\ ,\bar{\phi}^m\ ,\psi_{m\alpha}\ ,\bar{\psi}^m_{\dot{\alpha}}$ $\quad (m=1,2,3)$
- 1 vector multiplet $A_\mu \sim A_{\alpha\dot{\beta}}\ ,\lambda_\alpha\ ,\bar{\lambda}_{\dot{\alpha}}$ $\quad (\alpha,\dot{\alpha}=\pm)$

Operators in R^4

$\updownarrow$

States in $S^3 \times R$

Among 32 supercharges $Q^I_\alpha, \bar{Q}_{I\dot{\alpha}}, S^\alpha_I, \bar{S}^{I\dot{\alpha}}$ $\quad (I=1,2,3,4)$

we choose specific $Q \equiv Q^4_-\ , S \equiv S^-_4$ which BPS operator preserves

(the same SUSY dual black hole preserves)

Such operators are called $\frac{1}{16}$-BPS operators ($\frac{1}{16}=\frac{2}{32}$)

$$[Q,O\}=0\ \&\ [S,O\}=0 \qquad \leftrightarrow \qquad [\{Q,S\},O\}=0$$

- saturate the BPS bound $E-(R_1+R_2+R_3+J_1+J_2)=0$

  which follows from the algebra $2\{Q,S\}=E-(R_1+R_2+R_3+J_1+J_2)\geq 0$

Graviton cohomology

There is a well-known class of cohomology called graviton which corresponds to BPS Kaluza-Klein gravitons in AdS$_5$ $\times S^5$

[Gunaydin, Marcus '84]
[Kim, Romans, van Nieuwenhuizen '85]

- Chiral primary $\mathrm{tr}(\bar{\phi}^{(i_1} \cdots \bar{\phi}^{i_n)})$

  Since $Q\bar{\phi}^m = 0$, Q-closed.
  Due to symmetrization, it is not Q-exact. Recall $Q\psi_{m+} = -ig_{YM}\epsilon_{mnp}[\bar{\phi}^n, \bar{\phi}^p]$

- Superconformal descendants

  By acting supercharges $Q^m_+, \bar{Q}_{m\dot{\alpha}}$ and BPS derivatives $\partial_{+\dot{\alpha}}$ on chiral primary

- Multiplying the above single trace gravitons

- Q-closed for any N

Black hole cohomology

In interacting supersymmetric theories, trace relations can generate new operators

$$Q\mathcal{O} = (\text{Trace relation}) = 0$$

- explains the rest of the cohomology, which we call non-graviton cohomology

These are possible candidate for black hole microstates

- since microstates of black holes are heavy $E \sim 1/G_N \sim N^2$ & deconfined d.o.f.
  ← subjected to trace relations
- we often call them black hole cohomology

This phenomena is also known as fortuity [Chang, Lin '24]

Conclusion

We studied BPS spectrum of 4d N=4 SU(N) SYM

- Computed index of non-gravitons for N=3,4 at weak coupling
- Constructed black hole operators for N=3

So far, we discussed classical cohomology

- recently, it has been shown that quantum correction to supercharge can lift the black hole cohomology [Choi-Lee '25]
- it is open question which cohomology survive at higher-loop correction

Cohomology problem can be studied in various theories

- SYK model [Chang-Chen-Sia-Yang '24] D1-D5 CFT [Chang-Lin-Zhang '25]
  ABJM [Belin-Singh-Vadala-Zaffaroni '25] [Behan-de Gioia '25] ABJ [Kim-JL-Lee-Oh '25] and LS theory → See Seunggyu's talk