

Large N Universality of 4d $\mathcal{N} = 1$ SCFTs with Simple Gauge Groups

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Motivation & Background

Classification and Large- N Universality

Testing the Weak Gravity Conjecture

Conclusions & Outlook

Motivation & Background

Why classify SCFTs?

Strong-coupling QFT is hard.

- Perturbation theory breaks down. Few exact handles.
- **Supersymmetry** opens an exact window into the strong-coupling regime.
- At an RG fixed point: **Superconformal field theory (SCFT)**.

Why classify SCFTs.

- A direct route to classifying all **gapless phases** of QFT — long-standing and challenging.
- By AdS/CFT, equivalent to charting consistent **AdS₅ quantum gravity theories** $\Rightarrow$ universal SCFT patterns translate into universal quantum-gravity properties, and vice versa.

Today: classify a large set of $\mathcal{N} = 1$ SCFTs $\rightarrow$ test a Swampland conjecture (**Weak Gravity Conjecture**) far beyond the holographic regime.

Status of classification

SUSY	Status
$\mathcal{N} = 4$	All known: $\mathcal{N} = 4$ SYM — fixed by gauge group
$\mathcal{N} = 3$	All known from S-fold constructions; no Lagrangian
$\mathcal{N} = 2$	Rank-1 done; class $\mathcal{S}$; Lagrangian classified
$\mathcal{N} = 1$	Many constructive families: conformal Lagrangians, class $\mathcal{S}_k$, holographic toric quivers. Generic asymptotically free gauge + superpotential deformations: open

Less SUSY $\Rightarrow$ less rigid, richer landscape $\Rightarrow$ classification wide open

$\mathcal{N} = 1$ SCFT: algebra and the role of R -symmetry

Superconformal algebra $SU(2, 2|1)$ extends the super-Poincaré algebra with

dilatation D , special conformal K_μ , conformal supercharges $S, \bar{S}$, $U(1)_R$.

Two key consequences of the superconformal algebra:

- (i) The dimension of a **chiral operator** (annihilated by half of the supercharges) is fixed exactly by R -charge: $\Delta = \frac{3}{2}R$
- (ii) The R -current and stress-energy tensor are **related by SUSY transformations** $\Rightarrow$ central charges a and c are fixed by 't Hooft anomalies of $U(1)_R$.

Once we know the IR R -symmetry, both the BPS spectrum and the central charges follow.

Trace anomaly on a curved background: $\langle T^\mu_\mu \rangle = \frac{c}{16\pi^2} C^2_{\mu\nu\rho\sigma} - \frac{a}{16\pi^2} \tilde{R}^2$

c sets the stress-tensor 2-point function; a decreases monotonically under RG (a -theorem).

In $\mathcal{N} = 1$: since $\mathcal{J}^\mu_R$ and $T^{\mu\nu}$ share a multiplet, (a, c) are completely fixed by $U(1)_R$ 't Hooft anomalies:

$$a = \frac{3}{32} (3 \operatorname{Tr} R^3 - \operatorname{Tr} R), \quad c = \frac{1}{32} (9 \operatorname{Tr} R^3 - 5 \operatorname{Tr} R)$$

$\operatorname{Tr} R^n$ = sum of R^n over all Weyl fermions in the theory (gaugino + each chiral multiplet's fermion).

Problem: the IR superconformal R -symmetry is a non-trivial mixture of UV $U(1)$ s.

Recipe (a -maximization): trial R -symmetry $R_{\text{trial}} = R_0 + \sum_I s_I F_I$; then **locally maximize** $a_{\text{trial}}(s_I)$.

Output: exact R -charges $\Rightarrow$ exact (a, c) at strong coupling — pure arithmetic from the matter content.

“Holographic” CFT $\equiv$ admits a dual weakly-coupled Einstein gravity in AdS_5 .

Necessary CFT-side conditions:

1. Large c — bulk is semiclassical (small $G_N^{(5)} \sim \frac{1}{c}$).
2. Sparse low-lying gauge-invariant operator spectrum.
3. $a \simeq c$ — $a - c$ measures higher-derivative ($R_{\mu\nu\rho\sigma}^2$) corrections.

[Heemskerk, Penedones, Polchinski, Sully; 0907.0151] [Camanho, Edelstein, Maldacena, Zhiboedov; 1407.5597]

Classification and Large- N Universality

Goal. Classify all $\mathcal{N} = 1$ **asymptotically free** gauge theories admitting a large- N limit and flowing to interacting SCFTs.

Assumptions.

1. Classical simple gauge group: $SU(N), SO(N), Sp(N)$.
2. Theory is **asymptotically free at all N** : $b_0 = 3h^\vee - \sum_i T(\mathbf{R}_i) > 0$.
3. Theory flows to an interacting SCFT.
4. No superpotential.
5. Flavor symmetry kept **fixed** as $N \rightarrow \infty$.

Consequences of the assumptions.

- $h^\vee \sim N \Rightarrow$ Dynkin index of matter fields must be $O(1)$ or $O(N)$.
 $\Rightarrow$ only **fundamentals** and **rank-2 tensors** (adj, sym, antisym, conjugates) allowed.
- # of rank-2 tensors bounded: $1 \leq N_{\text{rank-2}} \leq 3$.

Allowed families

Enumerating subject to gauge anomaly cancellation + previous assumptions;
each family parametrized by (N, N_f) :

$$SU(N): \text{20} \quad SO(N): \text{6} \quad Sp(N): \text{9} \quad \Rightarrow \text{35 families total}$$

Example: $SO(N)$.

Type	matter content	conformal window
I	$1 \mathbf{S} + N_f \mathbf{V}$	$0 \leq N_f \leq 2N - 8$
I	$1 \mathbf{Adj} + N_f \mathbf{V}$	$1 \leq N_f \leq 2N - 4$
II	$2 \mathbf{S} + N_f \mathbf{V}$	$0 \leq N_f \leq N - 10$
II	$1 \mathbf{S} + 1 \mathbf{Adj} + N_f \mathbf{V}$	$0 \leq N_f \leq N - 6$
II	$2 \mathbf{Adj} + N_f \mathbf{V}$	$0 \leq N_f < N - 2$
III	$3 \mathbf{Adj}$	—

$\mathbf{S}$: rank-2 symmetric; $\mathbf{Adj}$: adjoint (= antisymmetric for $SO(N)$); $\mathbf{V}$: vector.

Universality $\rightarrow$ three universal types

Within each family, at leading order in large N , all IR R-charges and (a, c) are fixed by the single integer $N_{\text{rank-2}}$:

$$R_{\text{rank-2}} = 1 - \frac{1}{N_{\text{rank-2}}}, \quad R_{\text{fund}} = 1 - \sqrt{\frac{1}{18} + \frac{1}{2N_{\text{rank-2}}^2}}$$

Flavor symmetry kept fixed + asymptotic freedom $\Rightarrow 1 \leq N_{\text{rank-2}} \leq 3 \Rightarrow$ **three universal types**:

Type	$N_{\text{rank-2}}$	$R_{\text{rank-2}}$	R_{fund}	(a, c) at large N	spectrum	holo?
I	1	$O(1/N)$	$\frac{3-\sqrt{5}}{3}$	$O(N), a \neq c$	dense ($\sim 1/N$)	No
II	2	$\frac{1}{2}$	$\frac{12-\sqrt{26}}{12}$	$\frac{27}{128} \dim G, a \simeq c$	sparse	Yes
III	3	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{4} \dim G, a \simeq c$	sparse	Yes

For Type II and III (at least $N_f = 0, 1$), **non-trivial conformal manifolds** emerge upon **relevant deformations**.

Within a given type: different gauge groups, different representation combinations $\rightarrow$ same leading-order (R, a, c) .

Testing the Weak Gravity Conjecture

Weak Gravity Conjecture: from black hole decay to a CFT bound

- **No black hole remnants**: quantum gravity should not admit a large number of exactly stable states.
- An **extremal** black hole is stable unless some particle is **super-extremal** — charge conservation forbids decay otherwise:

$$\text{WGC: } \exists \text{ particle with } \left. \frac{q}{m} \geq \frac{Q}{M} \right|_{\text{ext. BH}} \quad [\text{Arkani-Hamed, Motl, Nicolis, Vafa '06}]$$

- **AdS₅ WGC $\Rightarrow$ CFT₄ bound** [Nakayama, Nomura '15]: extremal AdS-RN black holes must decay $\Rightarrow \exists$ operator charged under flavor $U(1)$ with $(\Delta \simeq Lm)$:

$$\boxed{\frac{q^2}{\Delta^2} \geq \frac{k_F}{12c}} \quad k_F = \text{flavor current 2-pt coefficient}$$

Test against our 35 families: some theories violate the WGC!

Lesson from $\text{AdS}_3/\text{CFT}_2$: there, the WGC *follows* from modular invariance — the same property behind Cardy's formula. The c controlling the **high-energy density of states** sets the WGC bound.

4d analog: Cardy-like limit $\omega_{1,2} \rightarrow 0$ of the $\mathcal{N} = 1$ superconformal index, fixed by anomalies:

$$\log \mathcal{I} \sim \frac{i\pi^3}{6} \frac{\text{Tr } R^3 - \text{Tr } R}{\omega_1 \omega_2} = \frac{16\pi^3 i}{27} \frac{3c - 2a}{\omega_1 \omega_2}$$

$\Rightarrow S(J) \sim (3c - 2a)^{1/3} J^{2/3}$ — reproduces the entropy of large BPS black holes in AdS_5 .

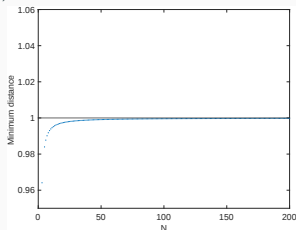
High-energy states are counted by $3c - 2a$, not $c \Rightarrow$ replace $c \rightarrow 3c - 2a$ in the WGC bound:

$$\boxed{\frac{q^2}{\Delta^2} \geq \frac{k_F}{12(3c - 2a)}}$$

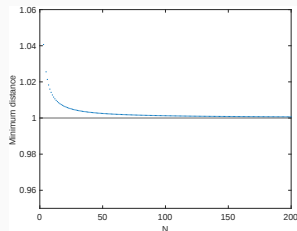
Testing the AdS WGC

Maximal charge-to-mass ratio q/Δ (extremal BH = 1); below 1 $\Rightarrow$ WGC **violated**

e.g. $SU(N)$ with $1\mathbf{S} + 1\bar{\mathbf{S}} + 2\mathbf{A} + 2\bar{\mathbf{A}} + 1(\mathbf{F} + \bar{\mathbf{F}})$:



original WGC: **violated** at all N — even in the Einstein regime



modified WGC: satisfied — asymptotically *saturated*

All 35 families satisfy the modified WGC — even at finite N , some *saturating* the bound

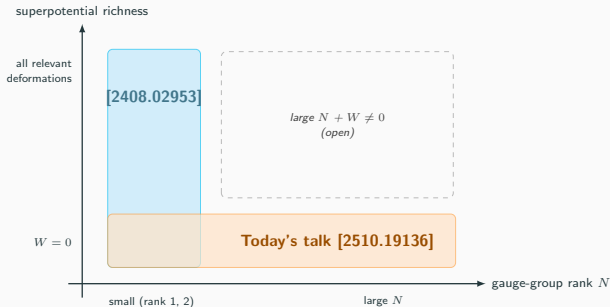
$\Rightarrow$ candidate **universal** property for any consistent 4d SCFT.

Conclusions & Outlook

Summary

1. **Classification:** all $\mathcal{N} = 1$ asymptotically free **simple-gauge** theories admitting a large- N limit, with no superpotential, fixed flavor sym. $\Rightarrow$ **35 families** (SU : 20, SO : 6, Sp : 9).
2. **Large- N universality:** leading R -charges and (a, c) depend **only** on $N_{\text{rank-2}} \in \{1, 2, 3\}$ $\Rightarrow$ **three universal types**; only Type II & III (sparse spectrum, $a \simeq c$) are holographic candidates.
3. **Swampland test:** the original WGC can fail, but the **modified WGC** $\frac{q^2}{\Delta^2} \geq \frac{k_F}{12(3c - 2a)}$ — rooted in the SUSY Cardy formula — holds for **all 35 families**, even at finite N .

Where this sits & open directions



Extending the classification.

- Relax the **fixed flavor symmetry** assumption ($N_f \sim N$).
- Extend to **product gauge groups** (WIP).
- Classify SCFTs that live **only at finite N** .

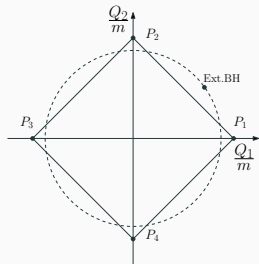
Sharpening the WGC.

- **Derive** the modified WGC.
- Extend the test beyond this classification.

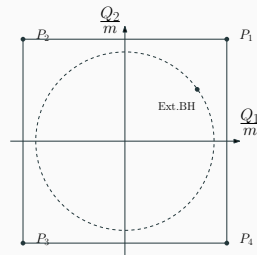
Thank you!

With **multiple** flavor $U(1)$ s, the WGC must hold in *every* charge direction:

- Plot charge-to-dimension vectors $\vec{z}_O = \vec{q}_O / \Delta_O$ of gauge-invariant operators, normalized so that extremal black holes sit on the **unit sphere**.
- Multi-particle states interpolate linearly (Δ 's and q 's add) $\Rightarrow$ decay products span a **convex hull**.



BH outside the hull: **cannot decay**



$\text{hull} \supseteq \text{unit ball}$: consistent

$\text{WGC} \iff \text{hull} \supseteq \text{unit ball} \iff \text{minimum distance from origin to hull} \geq 1$
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Backup: SUSY Cardy formula in detail

Superconformal index (states with $\Delta - J_1 - J_2 - \frac{3}{2}R = 0$):

$$\mathcal{I} = \text{Tr}_{\mathcal{H}} \left[e^{\pi i R} e^{-\omega_1 (J_1 + R/2)} e^{-\omega_2 (J_2 + R/2)} \right]$$

Cardy-like limit $\omega_{1,2} \rightarrow 0$ (saddle point / anomaly-based EFT on S^3):

$$\log \mathcal{I} \sim \text{Tr} R^3 \frac{\delta^3}{6 \omega_1 \omega_2} + \text{Tr} R \frac{\pi^2 \delta}{6 \omega_1 \omega_2}, \quad \delta = \frac{\omega_1 + \omega_2 - 2\pi i}{2}$$

with $\text{Tr} R^3 = \frac{16}{9}(5a - 3c)$, $\text{Tr} R = 16(a - c)$ — the two anomalies combine into $3c - 2a$.

Legendre transform ($\omega_1 = \omega_2$, $J = \frac{J_1 + J_2}{2}$):

$$\text{Re } S = 2^{1/3} 3^{1/2} \pi (3c - 2a)^{1/3} J^{2/3} + \mathcal{O}(J^{1/3})$$

Positive by Hofman–Maldacena $\frac{1}{2} < \frac{a}{c} < \frac{3}{2}$; reproduces the entropy of large BPS AdS_5 black holes [Gutowski, Reall '04; Chong, Cvetič, Lü, Pope '05; Kunduri, Lucietti, Reall '06].

Backup: a -maximization at work — adjoint SQCD

$SU(N)$ with $1 \mathbf{Adj} + N_f (\mathbf{F} + \overline{\mathbf{F}})$:

Fields	$SU(N)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$N_f Q$	$\mathbf{F}$	N	1	R_Q
$N_f \tilde{Q}$	$\overline{\mathbf{F}}$	N	-1	$R_{\tilde{Q}}$
1Φ	$\mathbf{Adj}$	$-N_f$	0	R_Φ

Maximize a_{trial} over mixing with $U(1)_1$; at large N , fixed N_f :

$$R_Q \simeq \frac{3-\sqrt{5}}{3} + \frac{N_f}{2N}, \quad R_\Phi \simeq \frac{\sqrt{5}N_f}{3N}, \quad \frac{a}{c} \rightarrow \frac{10}{11}$$

- $R_\Phi \sim 1/N \Rightarrow \text{Tr } \Phi^n, Q\Phi^m\tilde{Q}$ have arbitrarily small dimensions $\Rightarrow$ **dense spectrum** (Type I).
- Operators hitting the unitarity bound ($\Delta < 1$) **decouple** as free fields $\Rightarrow$ flip them and re-run a -maximization (e.g. $N_f = 1$: $a/c \rightarrow 0.9948$).