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NOTHING IS NOT NOTHING

Quantum black holes: inside and outside

Investigation of the vanishing boundary condition and
the information loss paradox

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IMPORTANCE OF **COHERENT STATES**

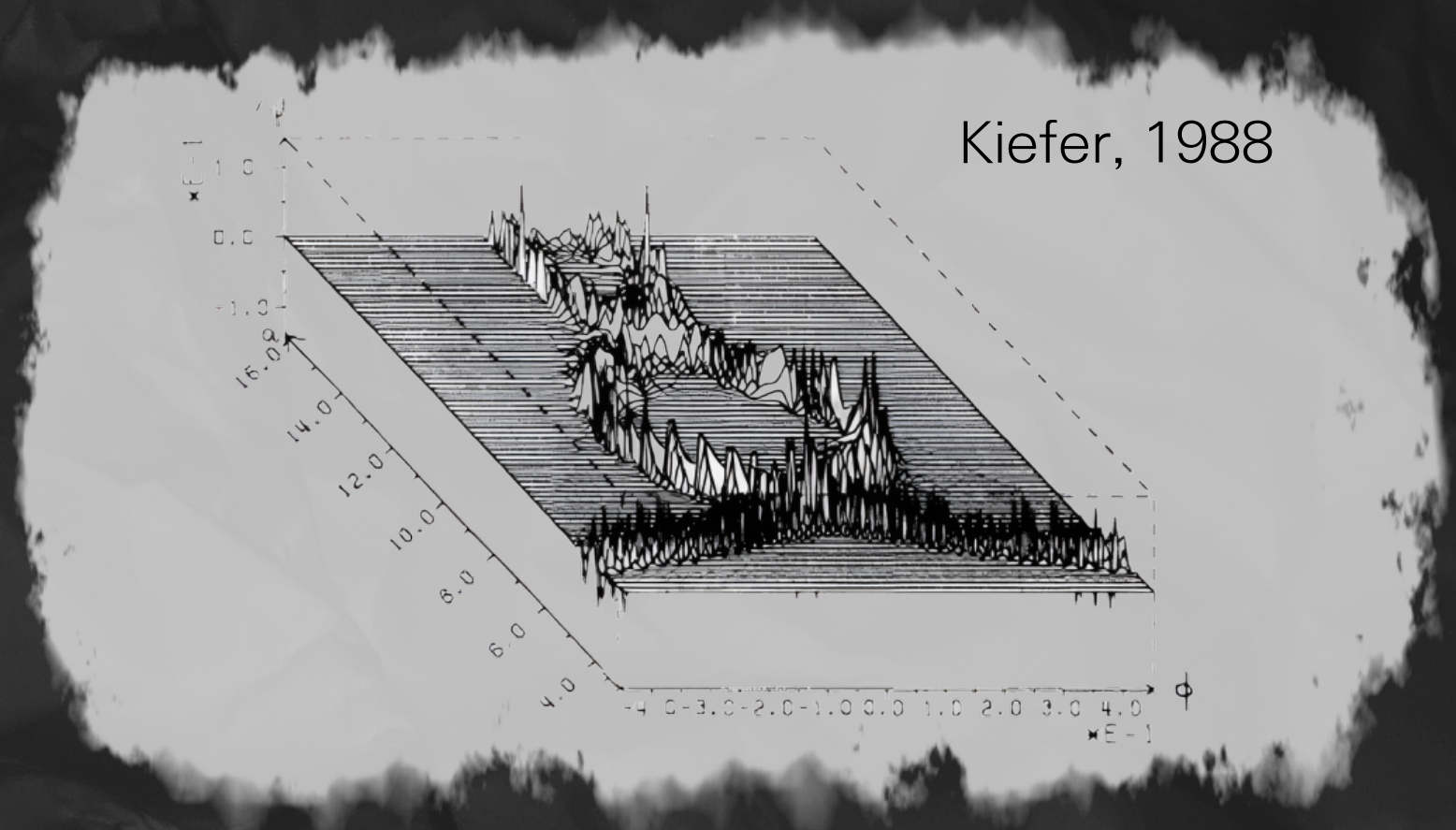
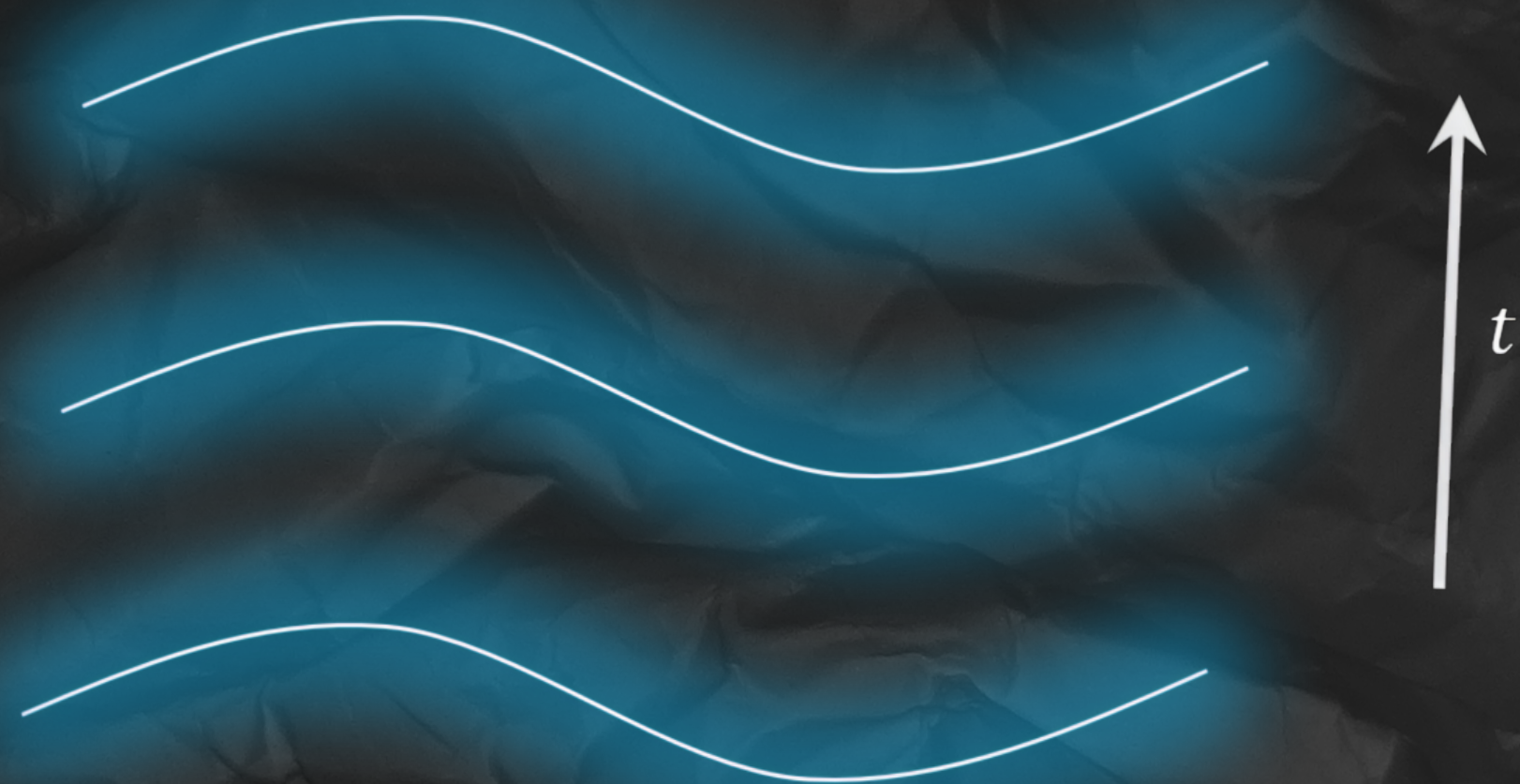
Wang, Sajeendran, DY, Mann and Foo, 2511.20759

Foo, Mann, Sajeendran, Wang and DY, in preparation



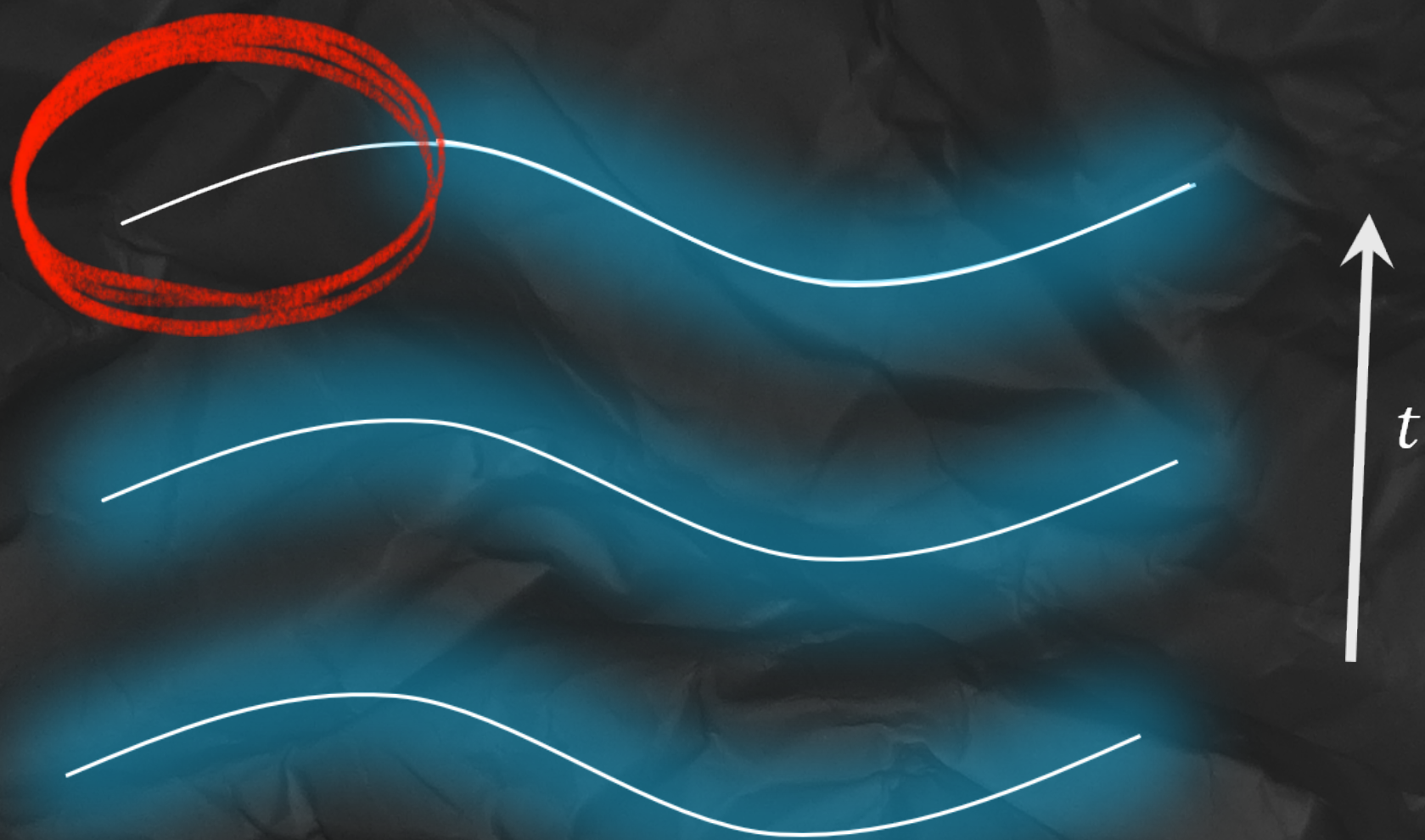
Let us consider the time evolution of hypersurfaces under a specific symmetry.

If the **classical hypersurfaces** are well-defined, then unitary quantum evolution will be well-defined, too.

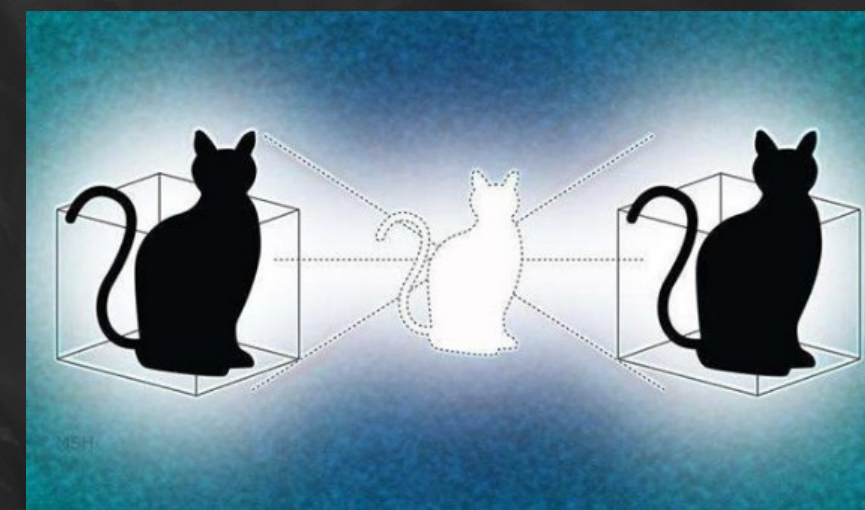
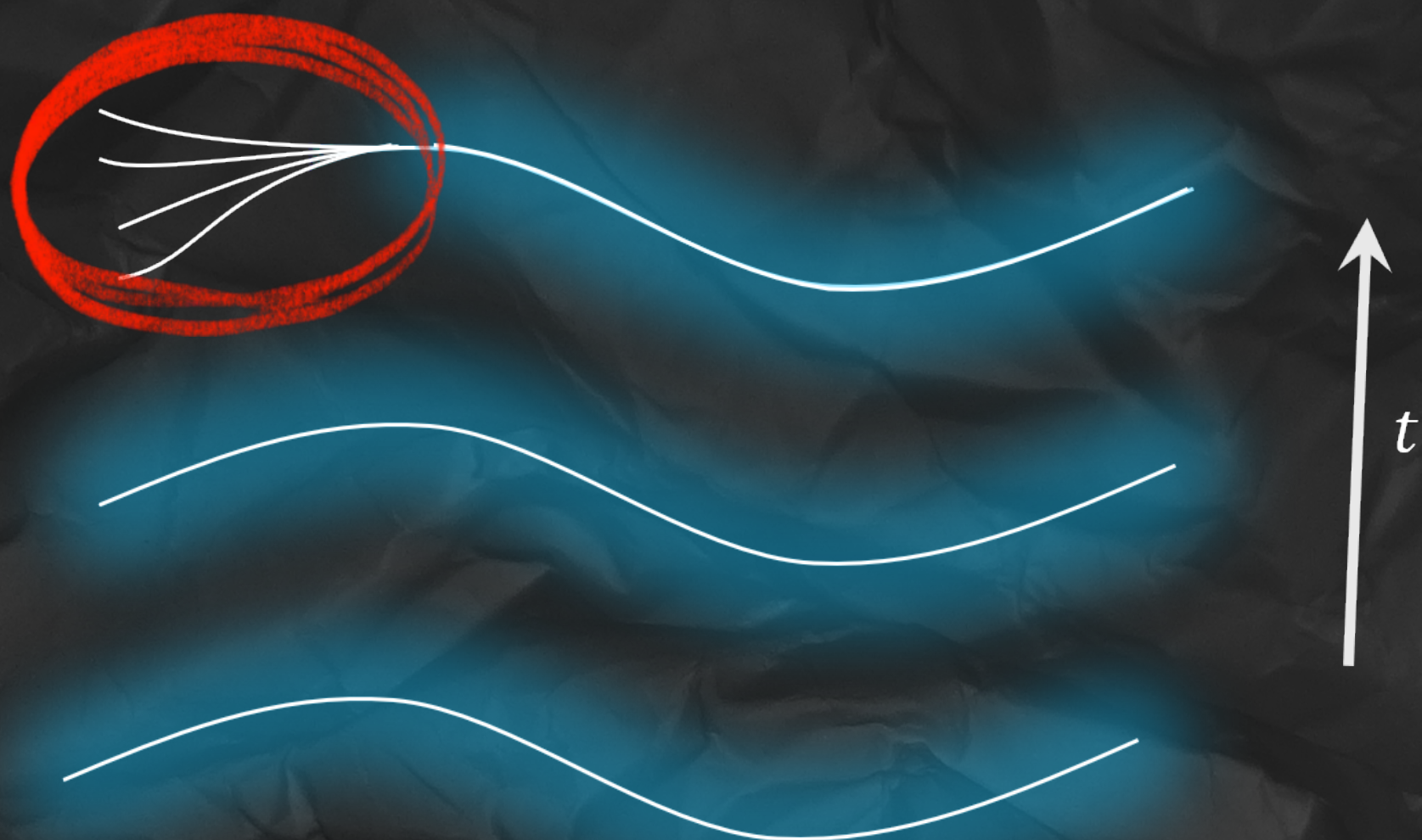


In terms of quantum gravity, the classical hypersurface is a coherent state of the wave function.

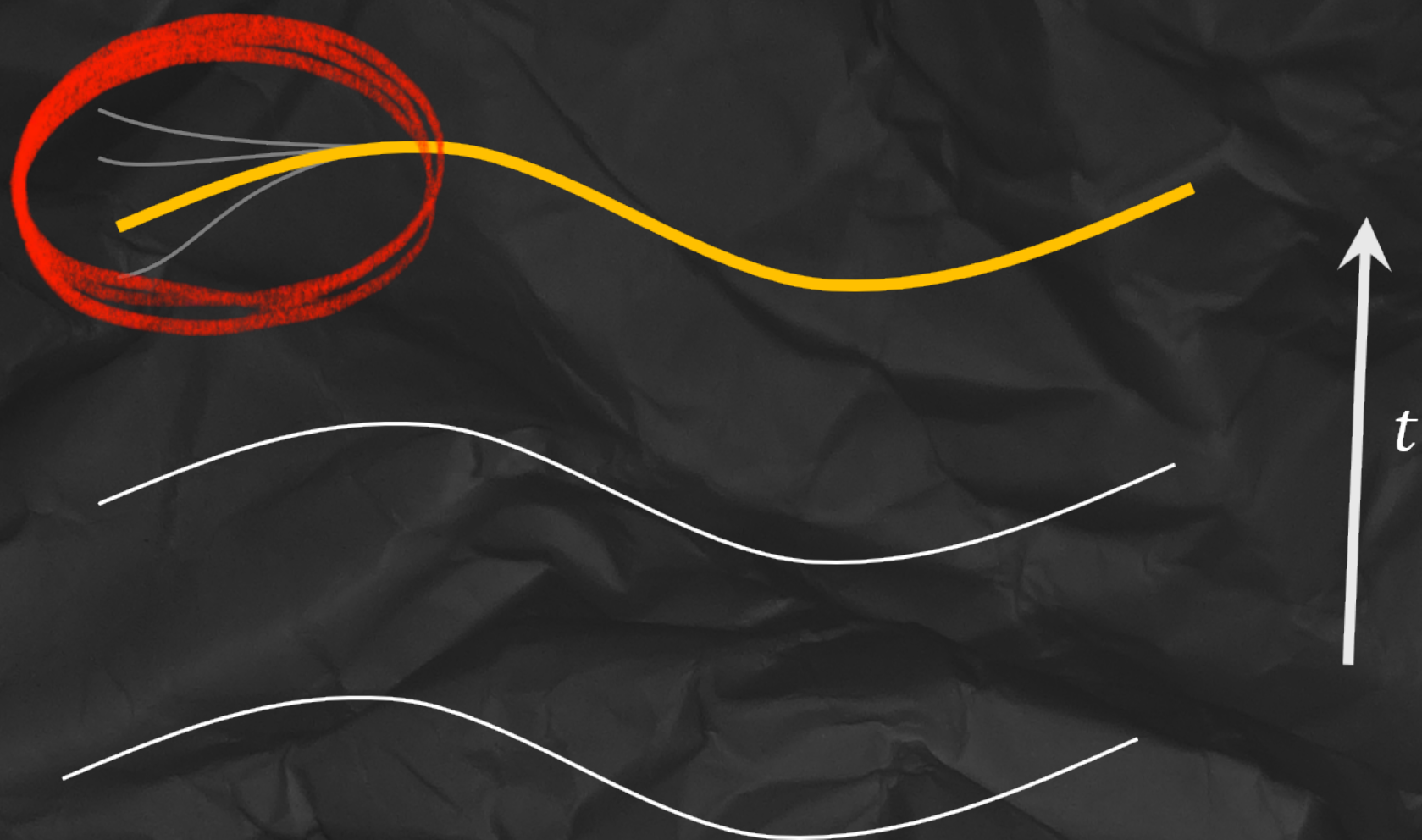
The peak of the coherent state wave packet satisfies the classical equation of motion: the Ehrenfest theorem.



However, what if the coherent-state condition is violated,
for example, because the wave function suddenly vanishes?



This means that the coherent state breaks down:
decoherence. Quantum probability will be dispersed to
different classical hypersurfaces.



In this case, if we force to use the spacetime to satisfy a
specific semiclassical geometry, we already lose unitarity. 

Short summary

- To use a semi-classical hypersurface, we need to require a coherent state.
- If the coherent-state condition breaks down, or decoheres, the wave function becomes a superposition of several semi-classical spacetimes.
- Hence, in this case, if we force to use a specific semi-classical geometry, the time evolution is no longer unitary.

A SIMPLE **TOY MODEL**

Kiefer, *Phys.Rev.D* 38 (1988) 1761–1772

Lin, DY and Stojkovic, 2510.20799

$$S = \int \sqrt{-g} dx^4 \left[\frac{1}{16\pi} (R - 2\Lambda) - \frac{1}{2} (\partial\phi)^2 \right]$$

$$ds^2 = -N^2(t)dt^2 + a^2(t) (dx^2 + dy^2 + dz^2)$$

$$\hat{\mathcal{H}}\psi[\alpha, \varphi] = \left(\frac{\partial^2}{\partial\alpha^2} - \frac{\partial^2}{\partial\varphi^2} + \Lambda e^{24\pi\alpha/\sqrt{3}} \right) \psi[\alpha, \varphi] = 0$$

Let us think of a toy model: **dS with a free scalar field**.

It is not difficult to write down the quantum Hamiltonian equation: the **Wheeler-DeWitt equation**.

$$\psi[\alpha, \varphi] = \int_{-\infty}^{\infty} A e^{-\frac{\sigma^2 k^2}{2}} \left(\frac{H}{8\pi} \right)^{\frac{ik}{4\pi\sqrt{3}}} \Gamma \left[1 - \frac{ik}{4\pi\sqrt{3}} \right] \underbrace{J_{-\frac{ik}{4\sqrt{3}\pi}} \left(\frac{H e^{4\sqrt{3}\pi\alpha}}{4\pi} \right)}_{\text{Bessel function}} e^{ik\varphi} dk$$

classical trajectory

$$\begin{aligned} \varphi = \frac{\phi}{\sqrt{4\pi}} &= \frac{A}{\sqrt{4\pi}} \frac{1}{a^3}, \\ &= -\frac{1}{4\pi\sqrt{3}} \sinh^{-1} \left(\sqrt{\frac{4\pi A^2}{3H^2}} e^{-12\pi\alpha/\sqrt{3}} \right) \end{aligned}$$

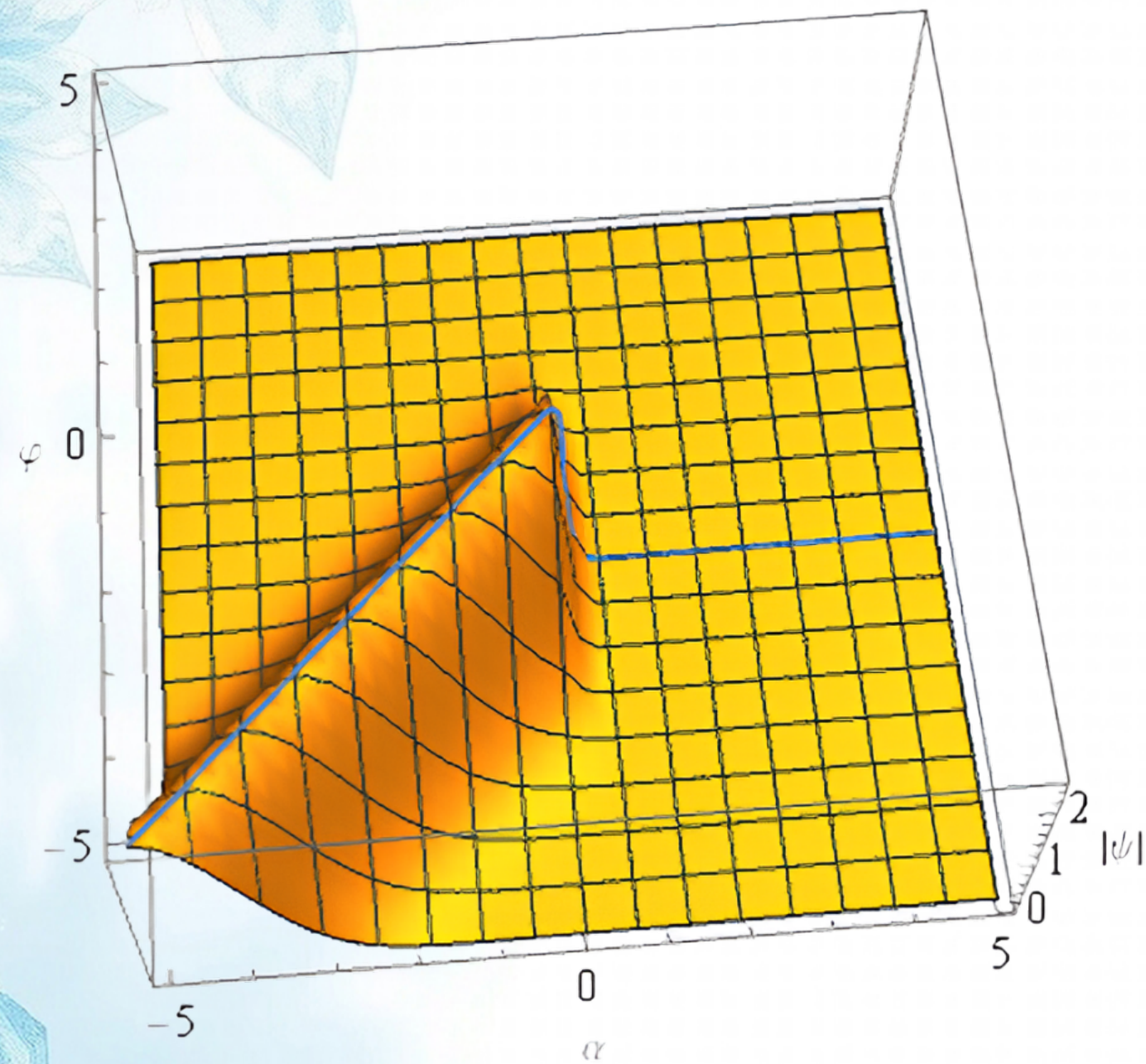
We can exactly solve it and construct a Gaussian wave packet that mimics the **coherent state**. To obtain a bounded wave function, we need to choose a **Bessel function**. 

Bouhmadi-Lopez, Brahma, Chen, Chen and DY, 1911.02129

Brahma, Chen and DY, 2108.05330

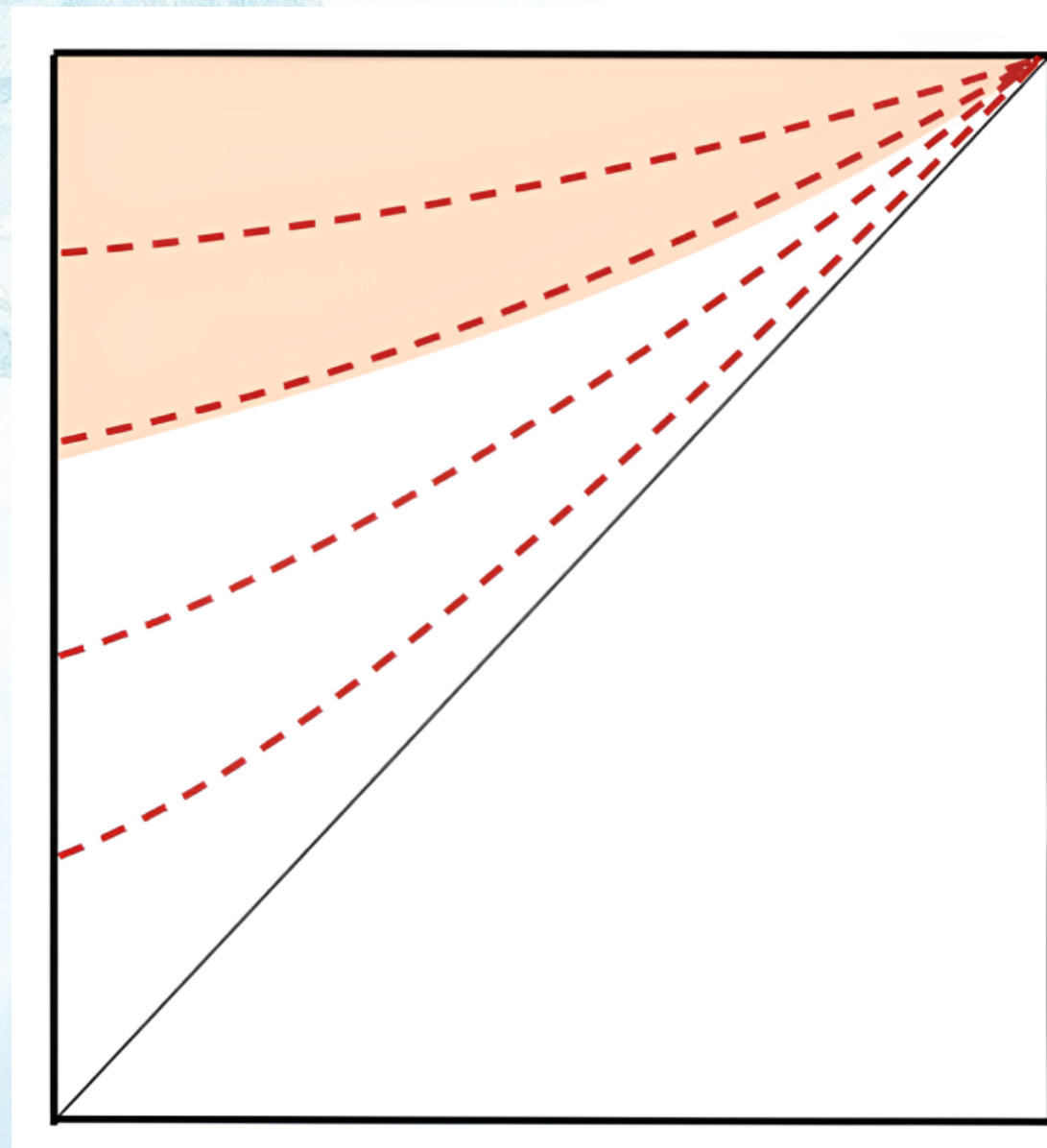
Chien, Tumurtushaa and DY, 2302.00254

Chien, Song, Tumurtushaa and DY, 2509.13483



In the small-scale-factor limit, the coherent state is preserved, while in the large-scale-factor limit, the wave function should vanish. Hence, it seems **nothing**.

However, does this 'nothing' really mean true nothing?



Of course ~~this~~ this cannot be true. It is inflating, and thanks to perturbations, the inhomogeneities should grow. Hence, our original homogeneous metric ansatz is no longer sufficient.

The spacetime becomes a superposition of divergent geometries as it inflates.

If we are forced to use a homogeneous metric ansatz, it is simply a coarse-graining; we lose the unitarity and experience decoherence. This provides a physical meaning of the vanishing wave function, or annihilation-to-nothing.

NOTHING IS NOT NOTHING

Rather, nothing is the **failure** of the present metric ansatz.

Nothing is indeed a **superposition** of diverse metrics.



So far, so good.
However, what about black holes?

In other words, can we assign coherent states for a hypersurface of black holes?

CANONICAL QUANTIZATION OF **SCHWARZSCHILD BLACK HOLES**

Lin, DY and Stojkovic, 2510.20799

$$S = \int d^4x \sqrt{-g} \frac{\mathcal{R}}{16\pi}$$

$$ds^2 = \alpha^2(t, x) (-dt^2 + dx^2) + \frac{\beta^2(t, x)}{\alpha^2(t, x)} d\Omega_2^2$$

Let us write the Lagrangian and the Hamiltonian of a Schwarzschild black hole. To cover both the outside and the inside of the horizon, we need the **Kruskal-like metric ansatz**.

$$L = \int dx \mathcal{L} = \frac{1}{2} \int dx \left(\alpha^2 + \frac{\beta^2}{\alpha^4} (\alpha_{,t}^2 - \alpha_{,x}^2) - \frac{1}{\alpha^2} (\beta_{,t}^2 - \beta_{,x}^2) \right)$$

$$p_\alpha \equiv \frac{\partial \mathcal{L}}{\partial \alpha_{,t}} = \frac{\beta^2 \alpha_{,t}}{\alpha^4},$$

$$p_\beta \equiv \frac{\partial \mathcal{L}}{\partial \beta_{,t}} = -\frac{\beta_{,t}}{\alpha^2}.$$

One can derive the classical Hamiltonian.

$$\begin{aligned} H &\equiv \int dx (p_\alpha \alpha_{,t} + p_\beta \beta_{,t}) - L \\ &= -\frac{1}{2} \int dx \left(\alpha^2 - \frac{\beta^2}{\alpha^4} (\alpha_{,t}^2 + \alpha_{,x}^2) + \frac{1}{\alpha^2} (\beta_{,t}^2 + \beta_{,x}^2) \right) \\ &= \int dx \left(\frac{\alpha^4}{2\beta^2} p_\alpha^2 - \frac{\alpha^2}{2} p_\beta^2 - \frac{1}{2} \left(\alpha^2 - \frac{\beta^2}{\alpha^4} \alpha_{,x}^2 + \frac{\beta_{,x}^2}{\alpha^2} \right) \right) \end{aligned}$$

$$\begin{aligned}
 p_\alpha &\rightarrow \frac{\delta}{i\delta\alpha} & a &\equiv \log \alpha \\
 p_\beta &\rightarrow \frac{\delta}{i\delta\beta} & b &\equiv \log \beta
 \end{aligned}$$

Also, one can write the quantized Hamiltonian constraint, or so-called, the

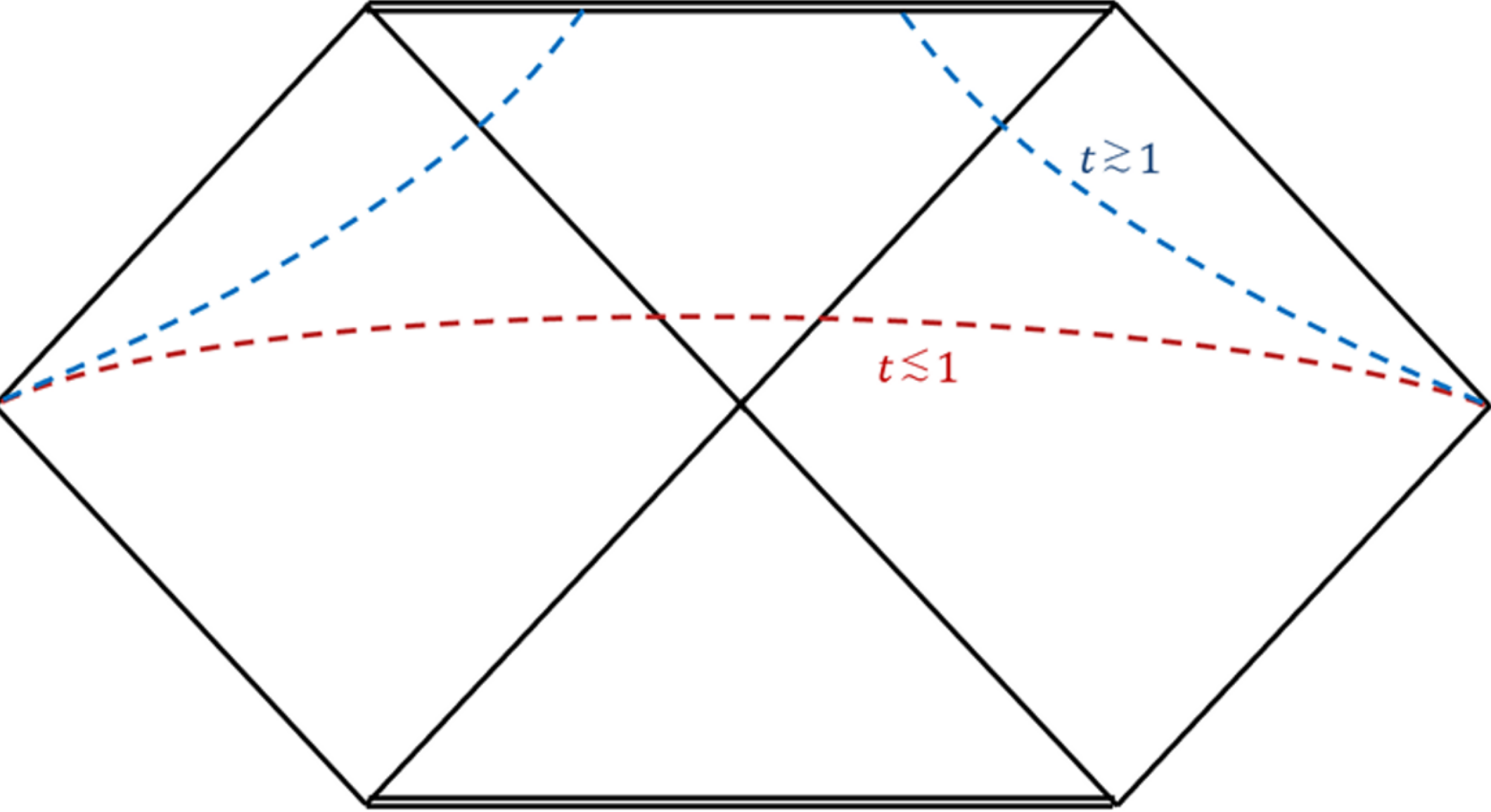
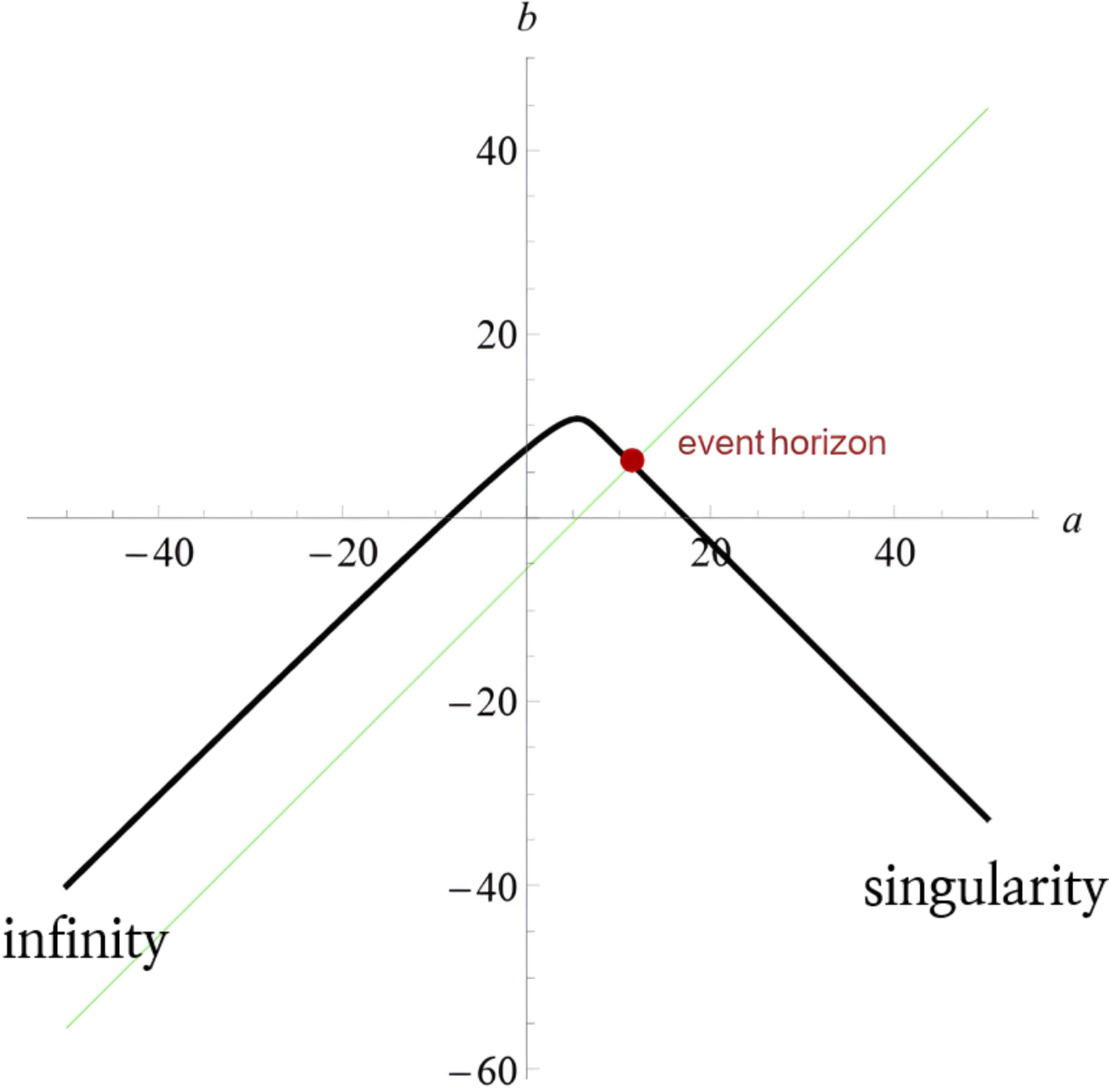
Wheeler-DeWitt equation.

$$\left(-\frac{\delta^2}{\delta a^2} + \frac{\delta^2}{\delta b^2} - \left(e^{2b} - e^{4(b-a)} (a_{,x}^2 - b_{,x}^2) \right) \right) \psi[a, b] = 0$$

$$\mathcal{V}[a, b; x] = e^{2b} - e^{4(b-a)} (a_{,x}^2 - b_{,x}^2)$$

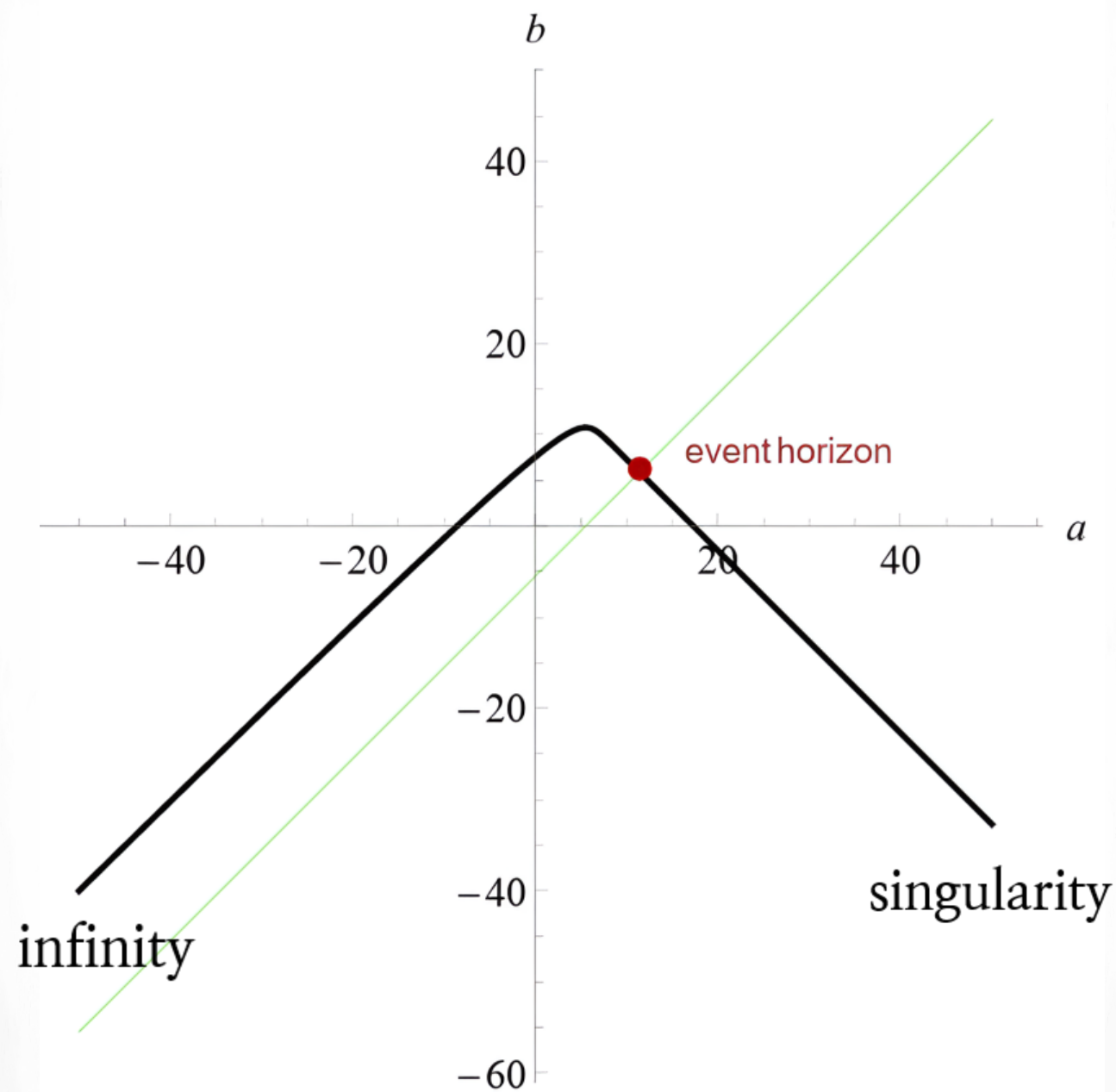
classical trajectory

$$a = \log 32M^3 - b - \frac{e^{b-a}}{2M}$$



$$\mathcal{V}_{\text{eff}}[a, b] = 2e^{2b} - \frac{t_0^2}{32M^3}e^{a+3b} - \frac{1}{2M}e^{3b-a}$$

$$= e^{2b+\log 2} - e^{a+3b+\log \frac{t_0^2}{32M^3}} - e^{3b-a+\log \frac{1}{2M}}$$

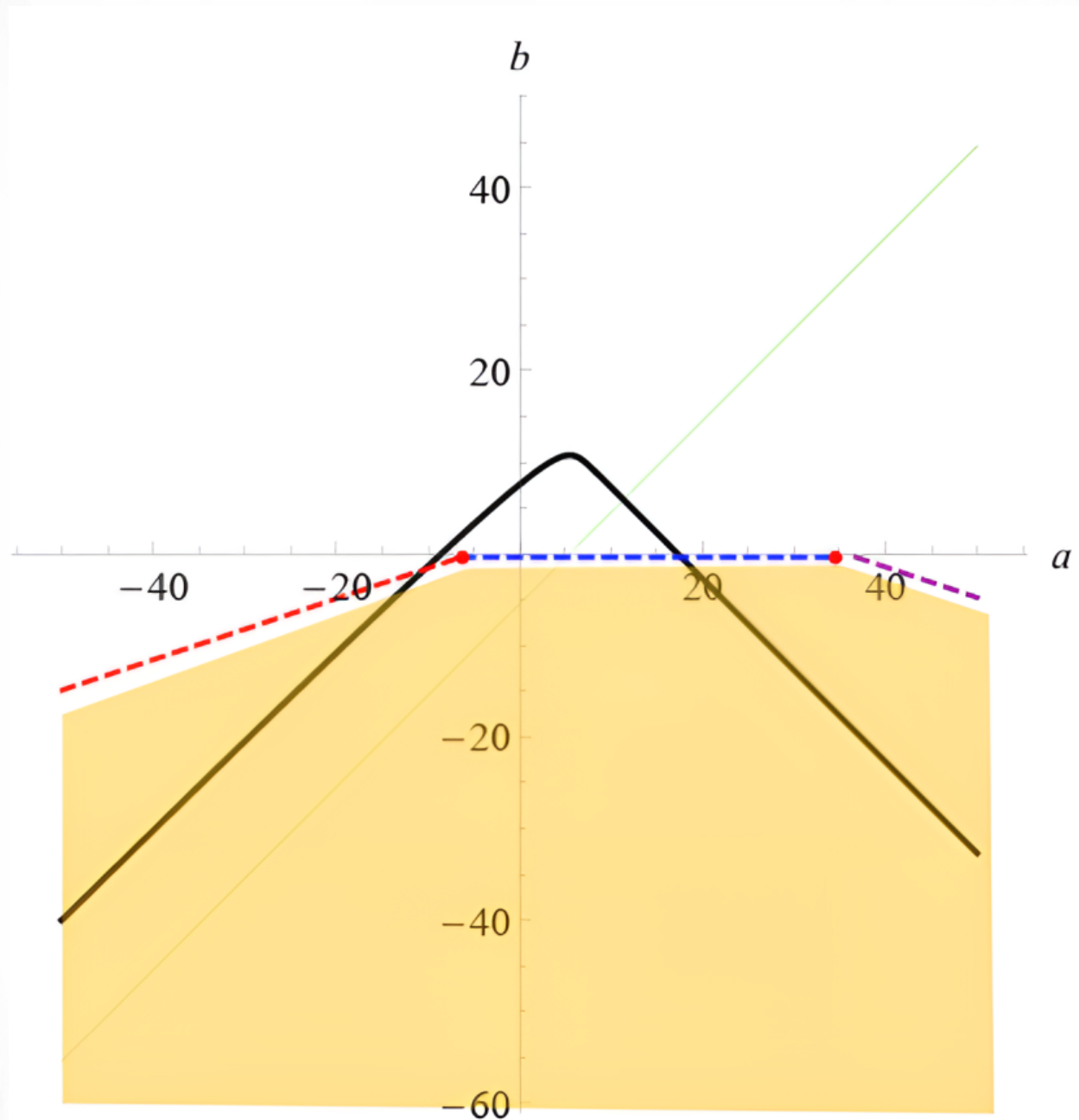


Through the classical contour, one can evaluate the **potential**. One can conclude that the potential near the event horizon is approximately an **exponential barrier**.

The Gaussian wave packet, or the coherent state, can be assigned if we can ignore the potential. Near the potential barrier, we need to provide a boundary condition.

$$\mathcal{V}_{\text{eff}}[a, b] = 2e^{2b} - \frac{t_0^2}{32M^3}e^{a+3b} - \frac{1}{2M}e^{3b-a}$$

$$= e^{2b+\log 2} - e^{a+3b+\log \frac{t_0^2}{32M^3}} - e^{3b-a+\log \frac{1}{2M}}$$



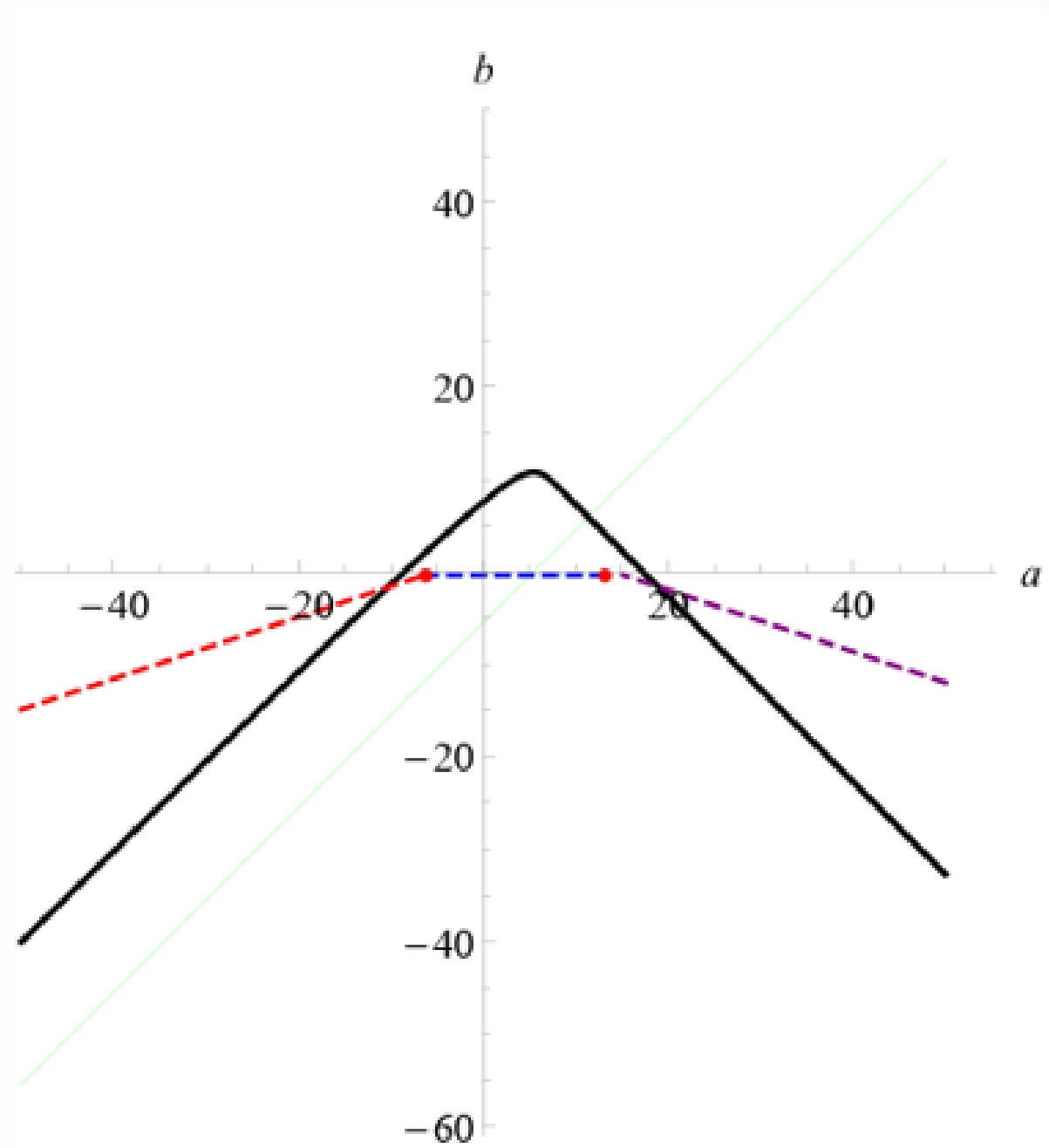
In the yellow-colored region, the potential is negligible. So, the Gaussian wave packet might be well-defined.

However, if the Gaussian wave packet touches the blue boundary, it should be bounced to the off-shell trajectory. Hence, it is not allowed.

This corresponds to the **early-time limit**.

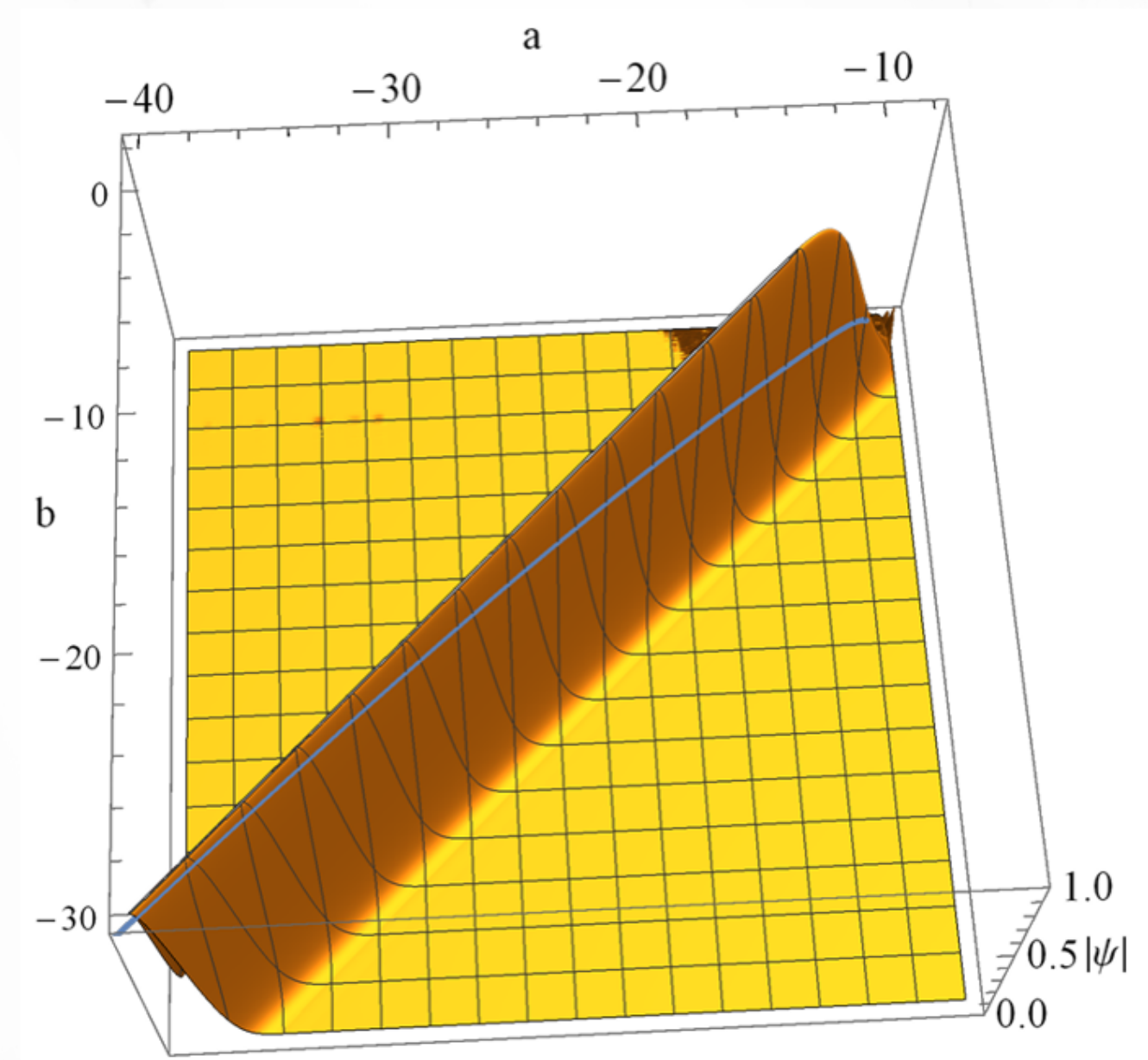
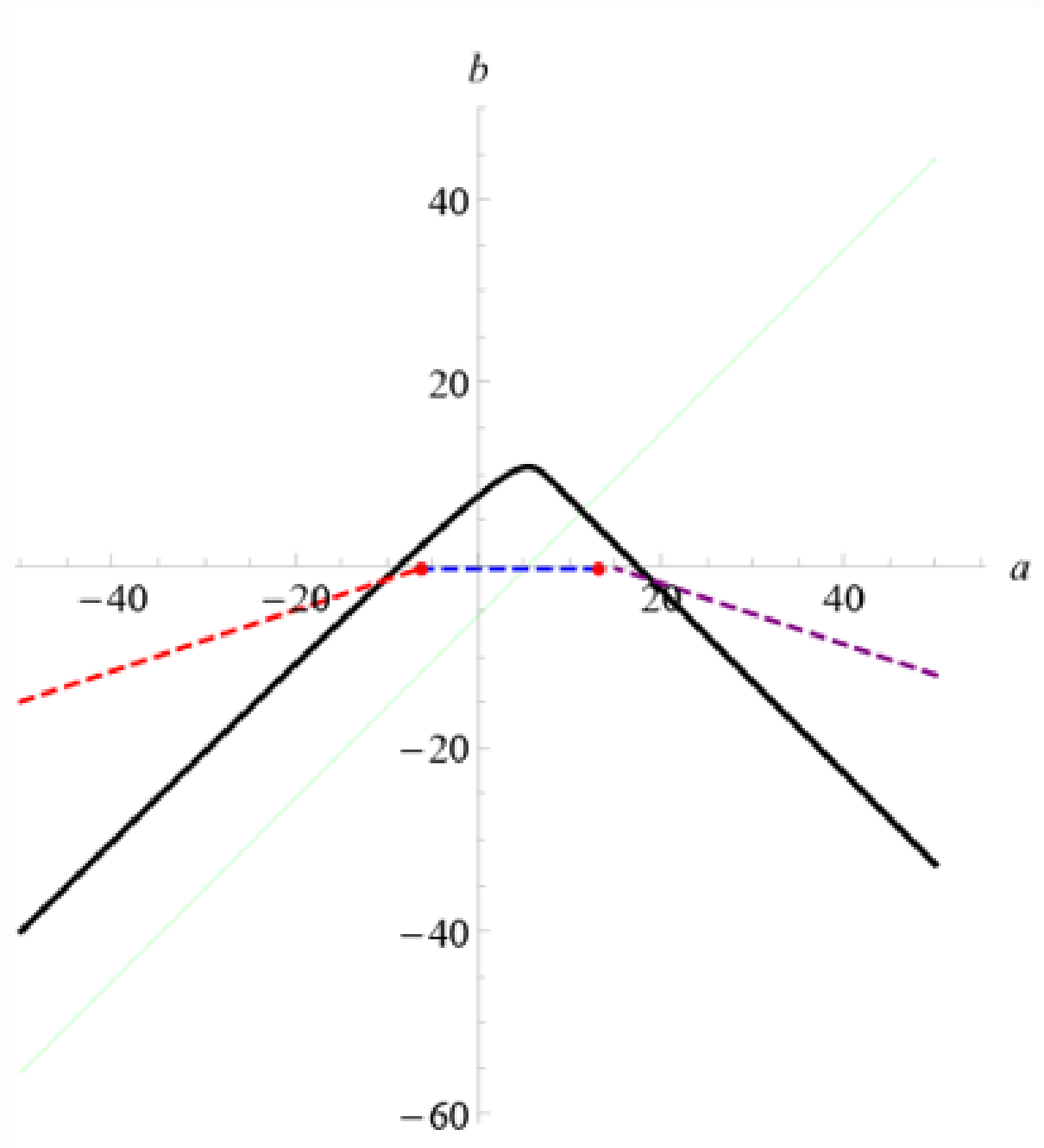
$$\mathcal{V}_{\text{eff}}[a, b] = 2e^{2b} - \frac{t_0^2}{32M^3}e^{a+3b} - \frac{1}{2M}e^{3b-a}$$

$$= e^{2b+\log 2} - e^{a+3b+\log \frac{t_0^2}{32M^3}} - e^{3b-a+\log \frac{1}{2M}}$$

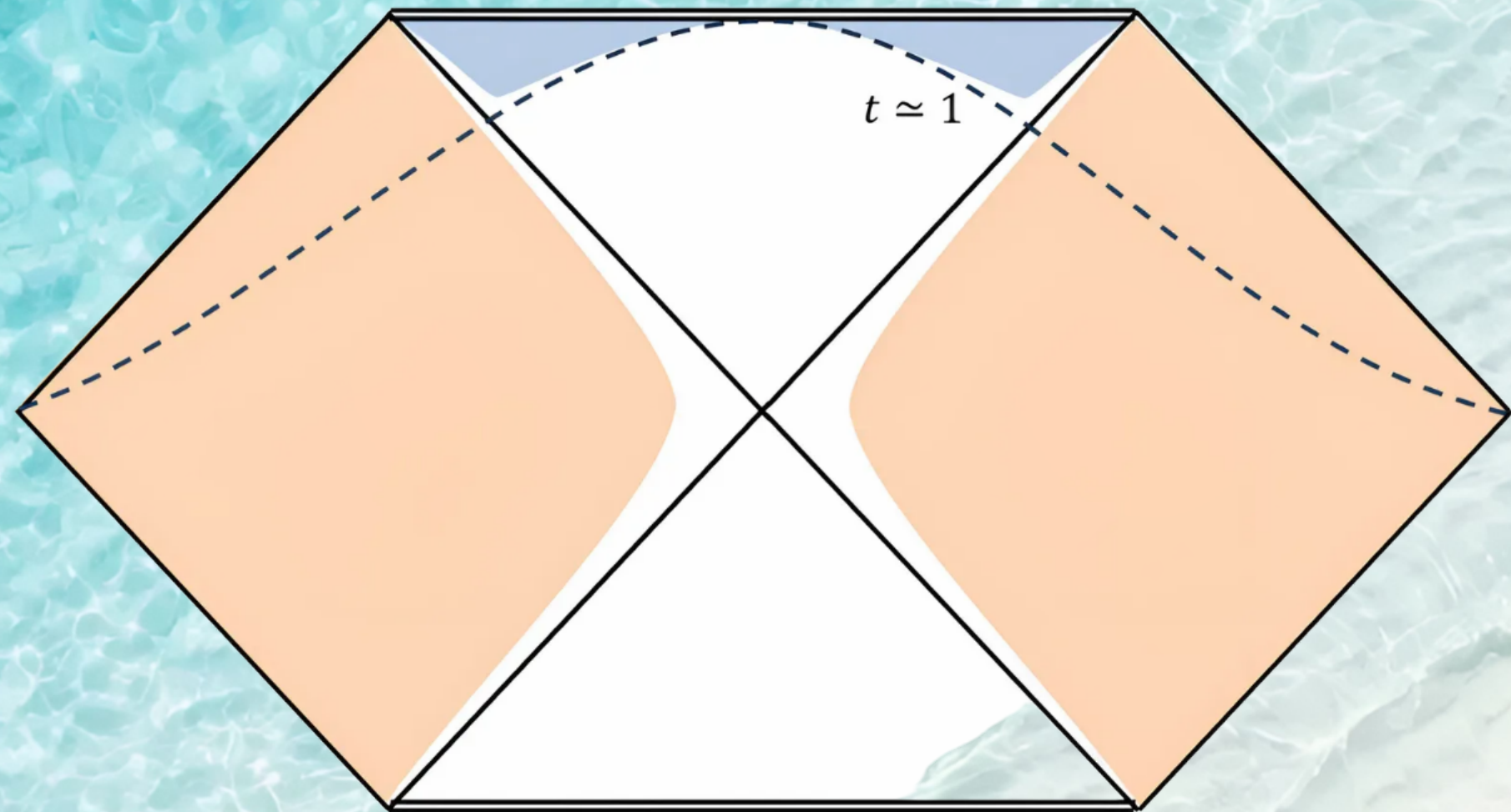


On the other hand, under the **late-time limit**, the wave packet can be annihilated in the potential well.

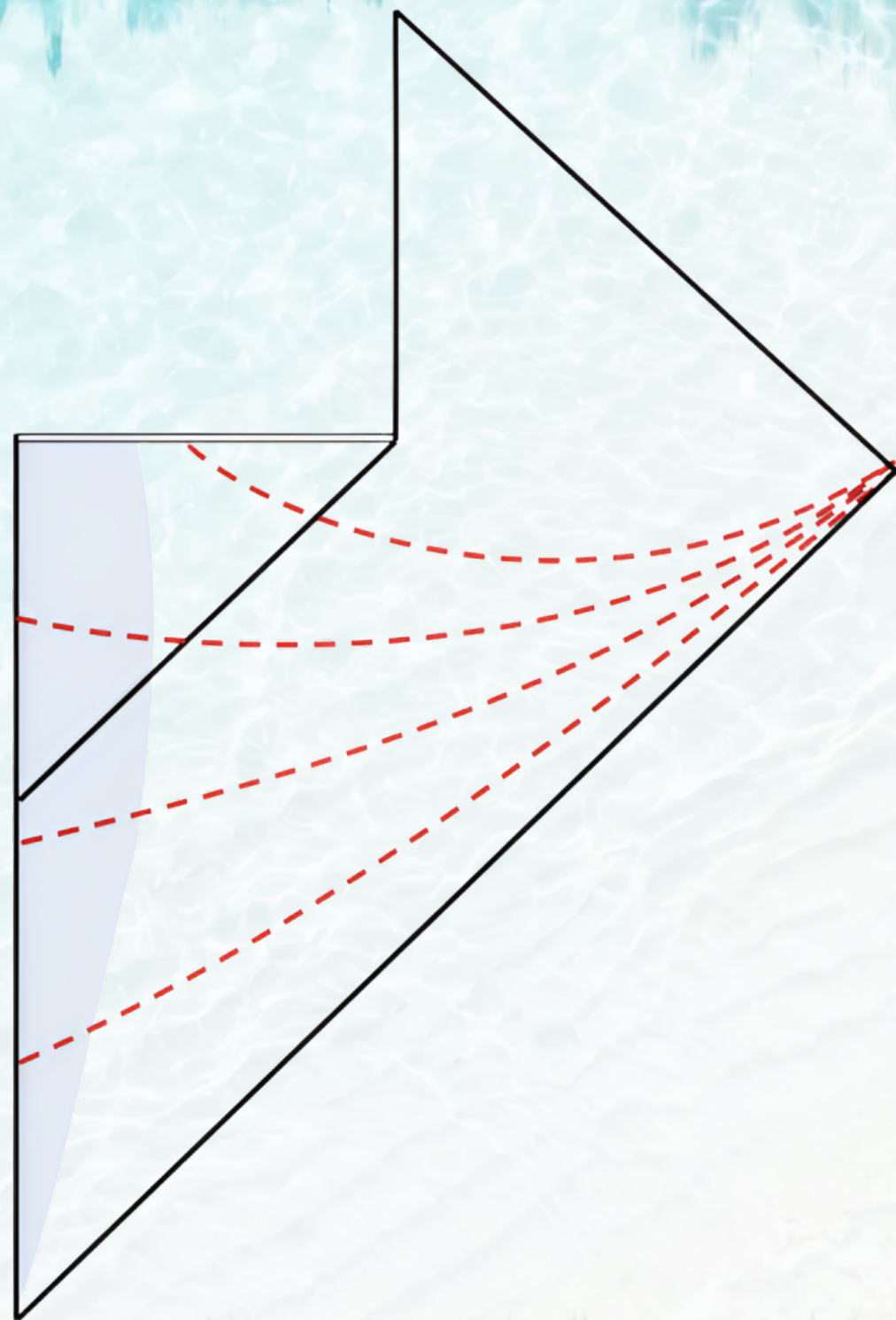
Hence, the Gaussian wave packet is partially allowed for **both inside and outside the horizon**, but not near the horizon.



A numerical example of the wave packet covering outside the horizon. The wave packet is annihilated at slightly outside the horizon.



Therefore, the coherent state can be assigned only to the colored regions. The important point is that **there is no coherent state near the event horizon.**



What does it mean?

If we continuously deform hypersurfaces from the past to the future and a hypersurface crosses the event horizon, **it cannot remain in a coherent state.**

Hence, once we are forced to use such a hypersurface, we already lose unitarity.

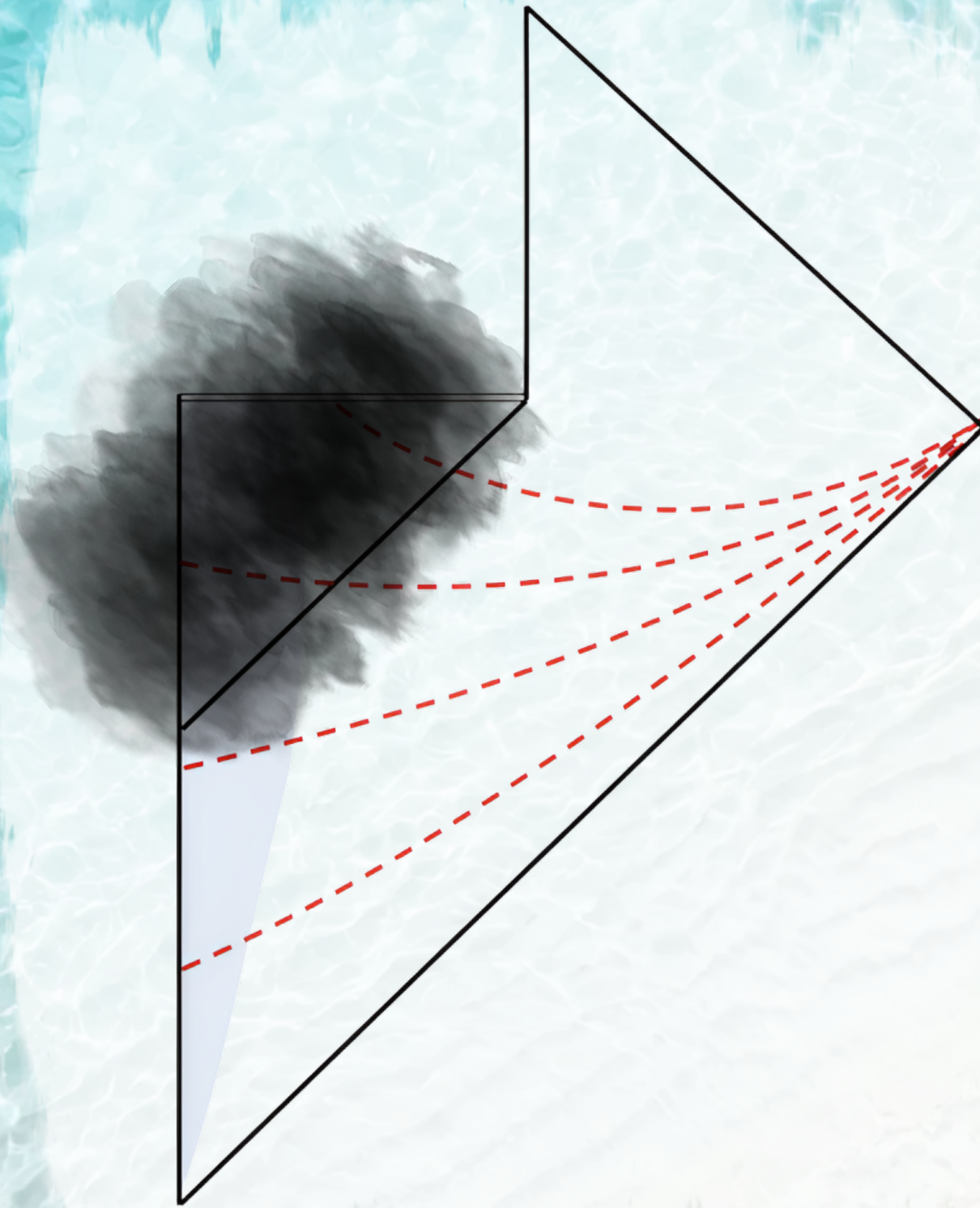
$$\Delta x \Delta p \geq \frac{1}{2} \left(1 + \alpha \ell_P^2 (\Delta p)^2 \right)$$

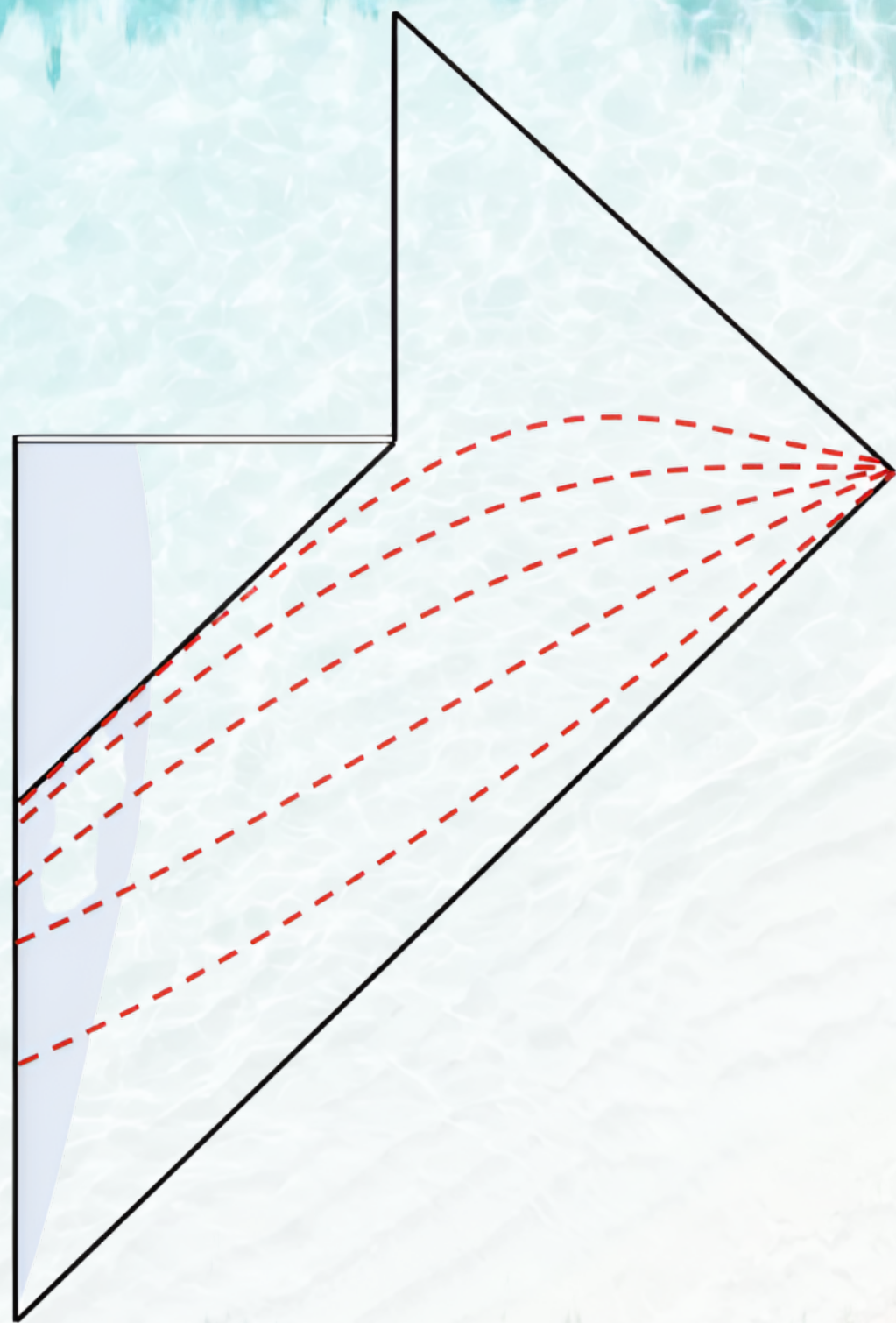
$$\Delta x \propto \Delta p \simeq M$$

Here is the clue to the **in-fall problem**. Regarding the interior of the black hole, the spherically symmetric metric form is not a good choice.

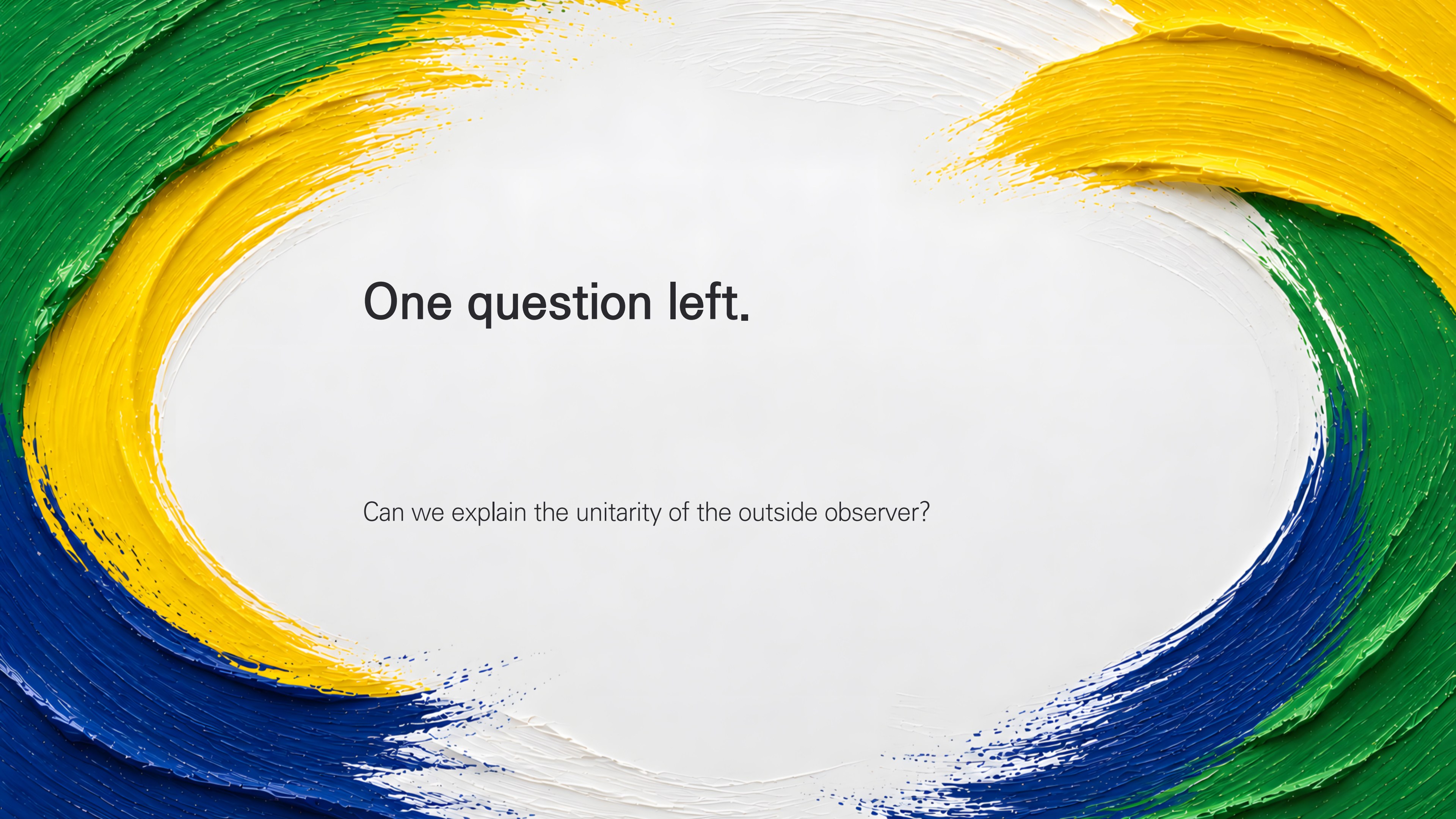
- The **classical** in-falling observer may coarse-grain their geometry, but the observer has already lost unitarity.
- The **unitary** in-falling observer should measure a superposition of many different geometries. Hence, there might be a horizon-scale uncertainty.

Can it be related to the **Generalized Uncertainty Principle**?





However, if we keep the slices **only outside the horizon**, we can maintain the coherent-state condition. Hence, the unitary semi-classical time evolution makes sense.



One question left.

Can we explain the unitarity of the outside observer?

EUCLIDEAN PATH INTEGRAL AND **TOPOLOGY CHANGE**

Sasaki and DY, 1404.1565

Chen, Domenech, Sasaki and DY, 1512.00565

Chen, Domenech, Sasaki and DY, 1704.04020

Chen, Sasaki and DY, 1806.03766

Chen, Sasaki and DY, 2005.07011

Chen, Sasaki, DY and Yoon, 2205.08320

Chen, Sasaki, DY and Yoon, 2206.10251

Chen, Chew, Sasaki and DY, 2311.00311

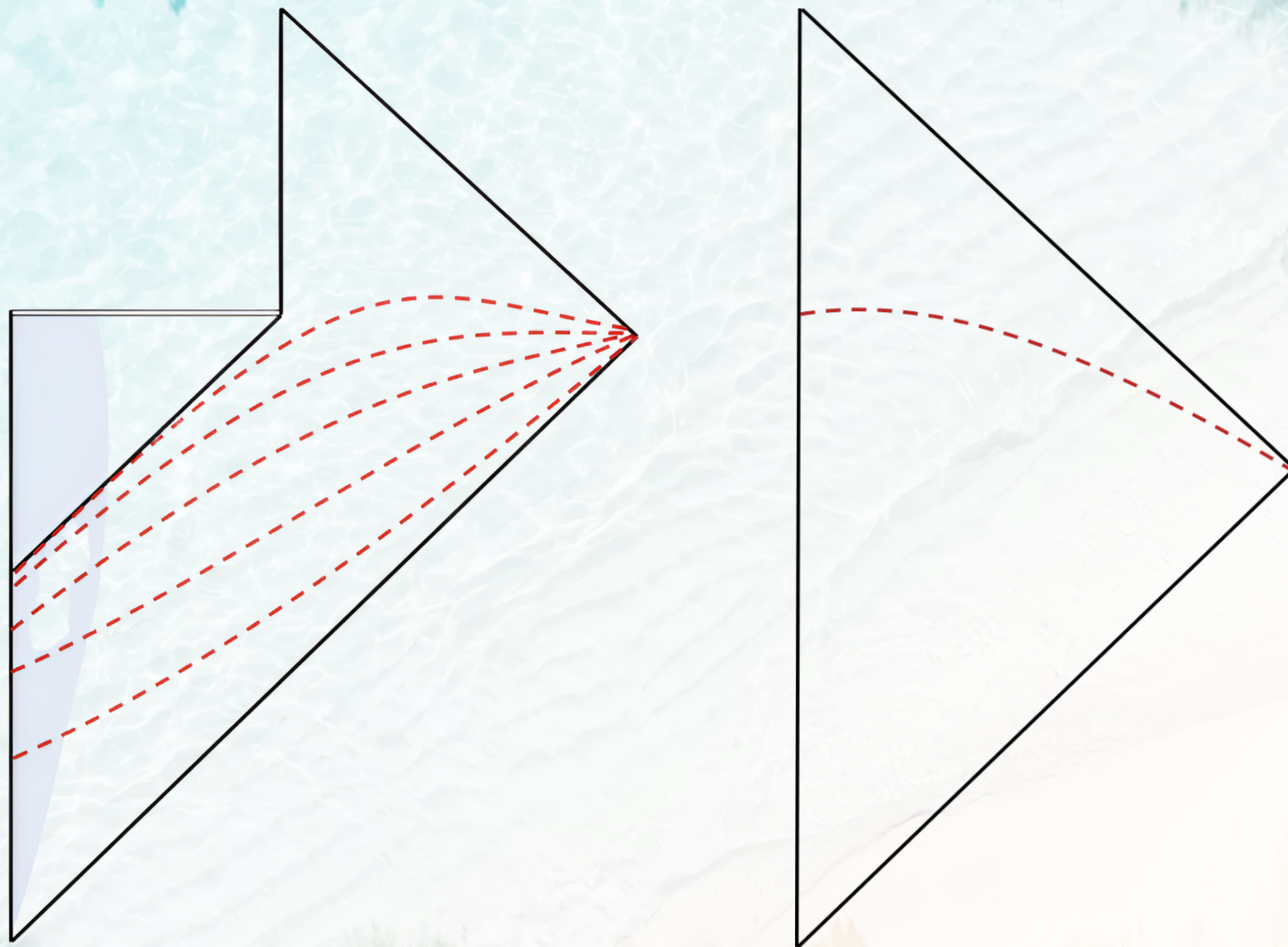
Chen, Lin, Sasaki and DY, in preparation

Importance of topology change

Maldacena, 2003; Hawking, 2005

According to Hawking, the Euclidean Schwarzschild space has a **non-trivial topology**, whereas the Euclidean Minkowski space has **a trivial topology**. Hence, the contribution of the trivial topology is crucial for understanding information recovery.

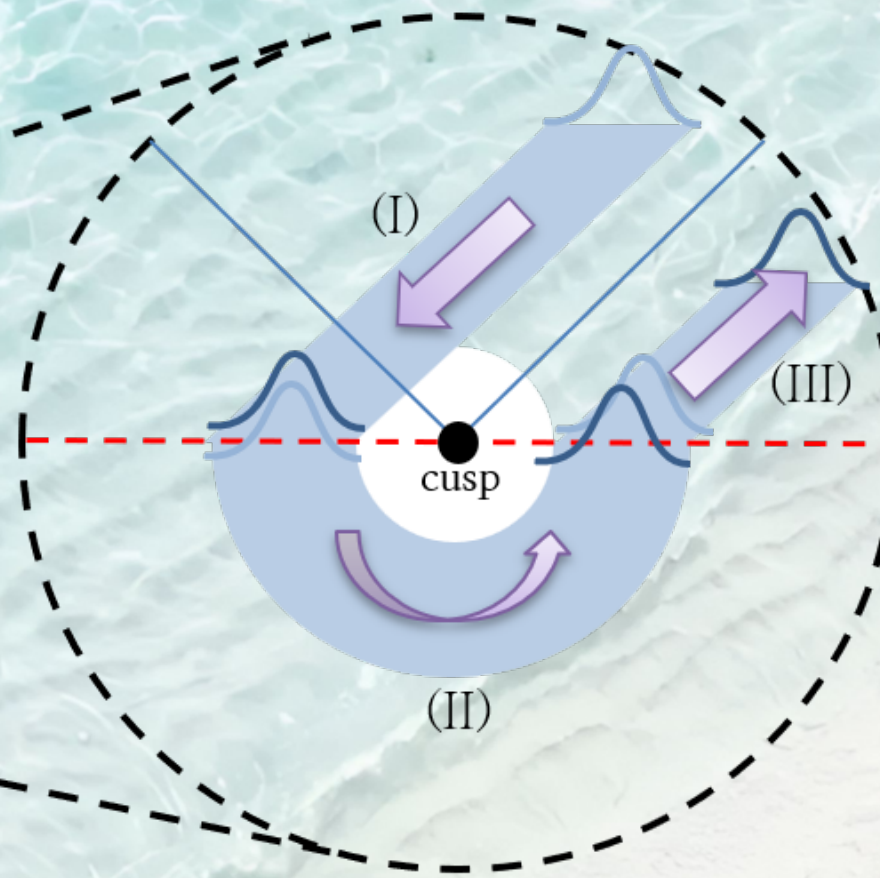
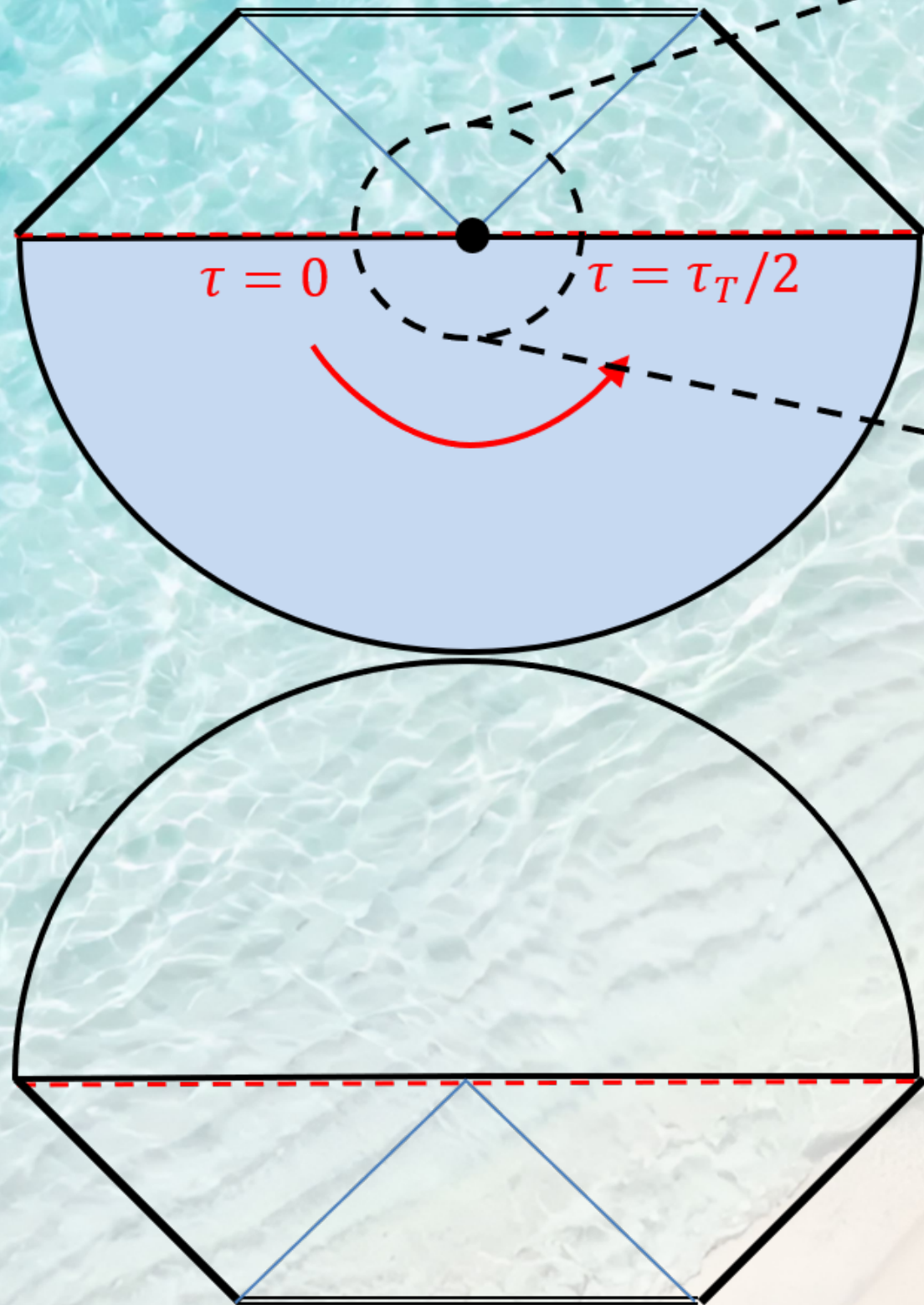
What is the corresponding Lorentzian interpretation?



$$\langle h_f | h_i \rangle = \int_{i \rightarrow f} \mathcal{D}g \mathcal{D}\phi \, e^{-S_E}$$

To explain the topology change from left to right, we can introduce the Euclidean path integral technique.

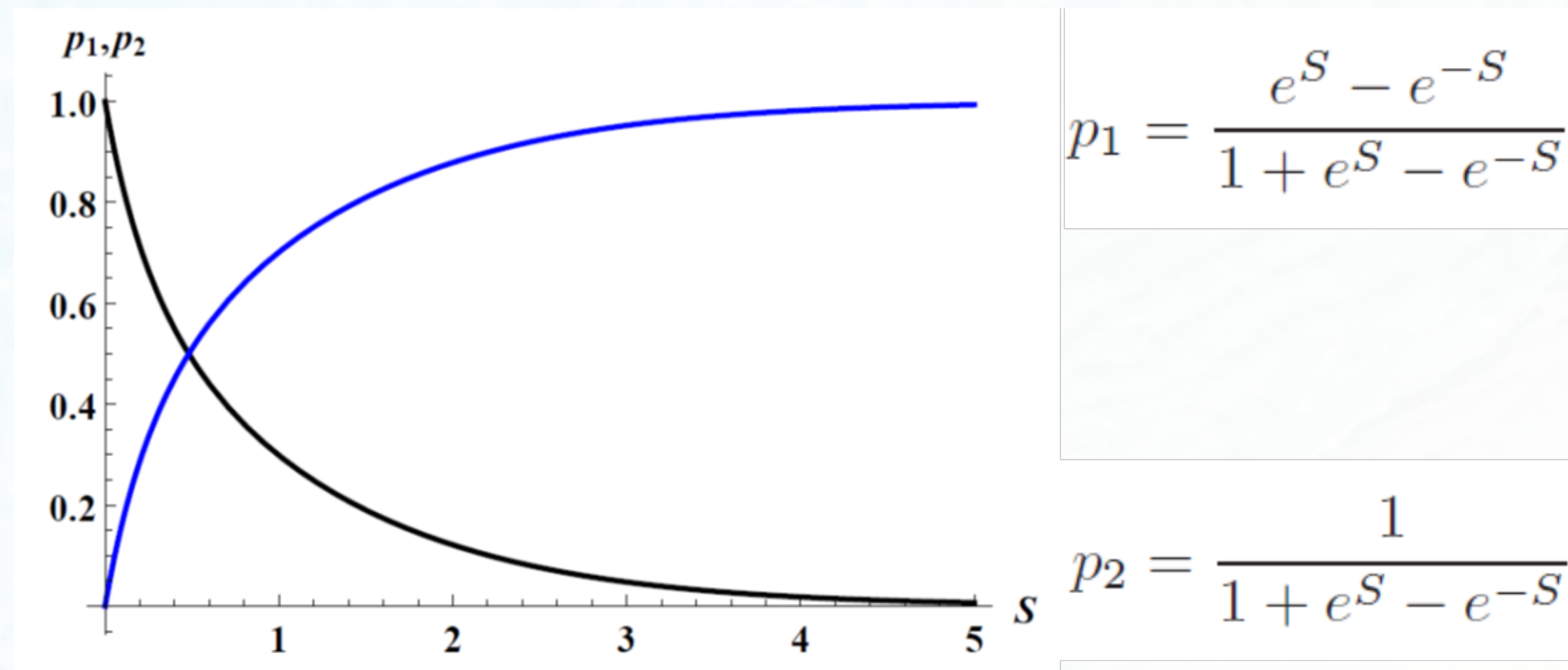
$$\langle f|i\rangle \cong \sum_{i\rightarrow f} e^{-S_E^{\text{on-shell}}}$$



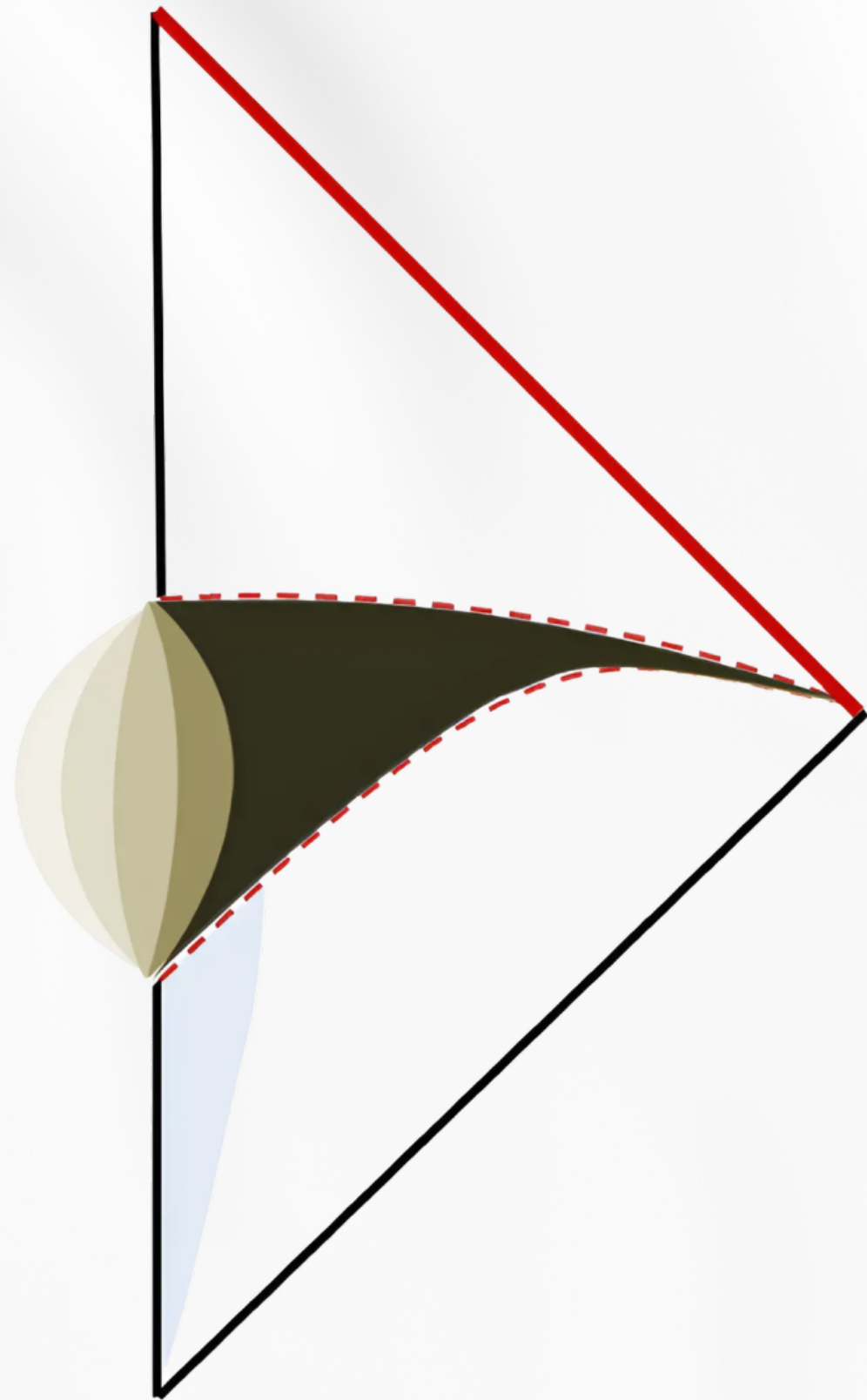
$$\langle h_f|h_i\rangle = \int_{i\rightarrow f} \mathcal{D}g\mathcal{D}\phi e^{-S_E}$$

A free scalar field can account for plenty of instantons; it can even explain **Hawking radiation**.

In addition, there can be an instanton that carries every energy; the probability is exponentially suppressed.



After summing over several Euclidean periods, we obtain the probability between two histories. Eventually, the **topologically trivial geometry will be dominated.**



Final interpretation

- One can follow **unitary semi-classical time evolution outside the horizon** because the coherent state can be obtained from the WDW equation.
- Beyond the horizon, the coherent state cannot be assigned, and our spherical-symmetric ansatz is inadequate; hence, the **spacetime inside the horizon must be a superposition of many different geometries**.
- Eventually, based on the Euclidean path integral, the **topology must be changed**. Thanks to this, all information is spread to infinity.

FUTURE RESEARCH TOPICS

What about the **entropy**? Can the entropy bound be violated?

Can we provide a consistent **Hamiltonian dynamics**?

Can we extend the WDW analysis to **dS/AdS** background?

Do we have further instantons that explain **non-local information transfer**?

Is there a **toy model** that mimics evaporation?

THANK-

YOU