Minimum Mass and Critical Collapse of Primordial Black Holes in Einstein Dilaton Gauss–Bonnet Gravity

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Based on Ongoing Work...

Center for Quantum SpaceTime

Outline

Motivation

Primordial black holes: why they matter

- PBHs form from the collapse of large curvature perturbations re-entering the horizon in the early Universe.
- Mass range: Planck scale to $\sim 10^5\, M_\odot$.
- Candidates for:
  - dark matter,
  - sources of stochastic GW backgrounds

Key point

The low-mass end of the PBH mass function is directly observable and, it's a direct probe of the gravity theory governing the collapse.

Critical collapse: the GR baseline

Naive picture vs. what actually happens

- **Naive expectation:** as you increase the perturbation amplitude past the collapse threshold, the resulting black hole should appear with some small but nonzero mass, which then grows smoothly and substantially as you move further above threshold.
- **Choptuik (1993):** studying the collapse of a scalar field, found that this naive picture is wrong – the mass instead follows a **universal power-law scaling** right at threshold, not a smooth finite jump, making PBH formation a genuine critical phenomenon, much like a phase transition in statistical mechanics.

Scaling law

$$M_{\mathrm{PBH}} \sim (\delta - \delta_c)^{\gamma}$$

δ: perturbation amplitude δ_c: threshold γ: universal exponent (depends only on w)

GR expectation

Arbitrarily small PBHs can form arbitrarily close to threshold $\delta \to \delta_c^+ \Rightarrow M_{\mathrm{PBH}} \to 0$ continuously – **no lower mass bound**. This is exactly what the Gauss–Bonnet correction will challenge.

Why higher curvature terms? The Gauss–Bonnet term

- In 4D, the Gauss–Bonnet invariant

$$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

  is topological $\Rightarrow$ does not affect field equations *unless* coupled to a dynamical field.
- Coupling to a dilaton $\xi(\phi(t,r))$ (Einstein dilaton Gauss–Bonnet, EdGB) is generic in the low-energy effective action of string theory.
- EdGB corrections are well known to reshape **black hole** solutions:
  - scalar hair, modified horizon structure,
  - existence of a **minimum black hole mass / mass gap** in stationary EdGB black holes.

Key question of this work

Does the same curvature–dilaton coupling that bounds *stationary* black hole masses also reshape the *dynamical, critical* collapse that produces PBHs?

Black holes vs. Primordial black holes: role of GB

Stationary EdGB BHs

- Static/spinning solutions with scalar hair
- GB term $\Rightarrow$ modified horizon radius
- Known result: minimum mass / mass gap for existence of hairy solutions

PBH formation (this work)

- Dynamical, spherical gravitational collapse
- GB term $\Rightarrow$ modifies Misner–Sharp mass & anisotropic stress
- **Open question**: does collapse threshold / critical scaling inherit an analogous mass gap?

Conjectured scaling law

$$M_{\mathrm{PBH}} \approx \begin{cases} 0, & \delta < \delta_c \\ M_{\mathrm{gap}} + A(\delta - \delta_c)^{\gamma}, & \delta \gtrsim \delta_c \end{cases}$$

A confirmed M_{gap} would cut off the PBH mass function at low mass.

Note: this is a **formation** *cutoff, distinct from the* **evaporation** *bound ($\sim 10^{15}$ g) below which any PBH would already have evaporated by today.*

Why this matters

- A mass gap M_{gap} would imprint a **cutoff** on the low-mass end of the PBH mass function.
- Directly affects:
  - PBH contribution to dark matter abundance,
  - PBH evaporation constraints (light PBHs),
  - gravitational-wave signatures from PBH mergers.

Our Goal:

Derive the **generalized Misner–Sharp mass** in EdGB gravity and track how curvature–dilaton coupling reshapes collapse dynamics relative to standard GR.

GR Misner–Sharp Framework (Review)

Setup: Spherically symmetric collapse in GR

Metric & matter

$$ds^2 = -A(t,r)^2 dt^2 + B(t,r)^2 dr^2 + R(t,r)^2 d\Omega^2, \qquad T^{\mu\nu} = (p+\rho)u^\mu u^\nu + p\, g^{\mu\nu}$$

Comoving gauge: $u^t = 1/A$, $u^i = 0$. Perfect fluid: $p = w\rho$.

Misner–Sharp mass

$$M(R) := \int_0^R d\tilde{R}\, 4\pi \tilde{R}^2 \rho, \qquad \Gamma = \sqrt{1 + U^2 - \frac{2M}{R}}$$

Generalized Lorentz factor Γ relates M, U, R; also $B = R'/\Gamma$.

(Setup follows the review of Escrivà, Kühnel & Tada, arXiv:2211.05767.)

GR Misner–Sharp equations

$$\dot{U} = -A\left[\frac{w}{1+w}\frac{\Gamma^2\rho'}{\rho R'} + \frac{M}{R^2} + 4\pi R\, w\rho\right] \qquad \dot{R} = A\,U$$

$$\dot{\rho} = -A(1+w)\rho\left[\frac{2U}{R} + \frac{U'}{R'}\right] \qquad \dot{M} = -4\pi A\, w\rho\, UR^2$$

$$A' = -A\,\frac{w}{1+w}\frac{\rho'}{\rho} \qquad M' = 4\pi\rho R^2 R'$$

- Isotropic perfect fluid $\Rightarrow$ **single pressure** $p = w\rho$ enters both radial and time evolution.
- No independent radial/tangential split; $(T^{\mathrm{eff}})^r{}_t$-type flux terms are absent for a perfect fluid.
- These are the equations the EdGB analysis must reduce to in the limit $\xi(\phi) \to 0$.

Compaction function: precise definition & threshold

- To actually locate the threshold, people use the compaction function $C(r,t)$ which is defined as:

$$C(r,t) := 2\,\frac{M(r,t) - M_b(r,t)}{R(r,t)}, \qquad M_b(r,t) := \frac{4\pi}{3}\rho_b R(r,t)^3$$

- Radiation era ($w = 1/3$): $\delta_c \approx 0.50$–0.55, $\gamma \approx 0.36$ (profile-dependent).

Note

This is the GR baseline, and everything from here on is about how the Gauss-Bonnet coupling modifies these numbers and structures.

EdGB Extension: Anisotropic Misner–Sharp Formalism

EdGB action & field equations

Action

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}(\nabla\phi)^2 + \alpha\,\xi(\phi)\,\mathcal{G}\right]$$

$$\Box\phi + \alpha\,\xi_{,\phi}(\phi)\,\mathcal{G} = 0, \qquad G^{(\mathrm{gen})}_{\mu\nu} = \kappa^2\left[T^{(\phi)}_{\mu\nu} + \alpha\,T^{\mathrm{GB}}_{\mu\nu}\right]$$

$$T^{\mathrm{GB}}_{\mu\nu} = 8\nabla^\rho\nabla^\sigma\xi\,R_{\mu\rho\nu\sigma} - 8(\Box\xi)R_{\mu\nu} + 16\nabla^\rho\nabla_{(\mu}\xi\,R_{\nu)\rho} - 8g_{\mu\nu}\nabla^\rho\nabla^\sigma\xi\,R_{\rho\sigma} - 4\left[\nabla_\mu\nabla_\nu\xi - g_{\mu\nu}\Box\xi\right]R$$

- $T^{\mathrm{GB}}_{\mu\nu}$ contains $\nabla\nabla\xi(\phi)$ contracted with the Riemann/Ricci tensors $\to$ genuinely new tensor structures absent in GR.
- Effective stress tensor: $T^{\mathrm{eff}}_{\mu\nu} = T^{(\phi)}_{\mu\nu} + \alpha\,T^{\mathrm{GB}}_{\mu\nu}$.

The Gauss–Bonnet term breaks isotropy. Why?

Define $\rho_{\rm eff} = -(T_{\rm eff})^t{}_t$, $p^r_{\rm eff} = (T_{\rm eff})^r{}_r$, $p^t_{\rm eff} = (T_{\rm eff})^\theta{}_\theta$.

Anistropic contributions in Spherical-Symmetric metric

- **Scalar field** $\phi(t, r)$:

$$p_r - p_t = -\frac{\phi'^2}{B^2}$$

- **Gauss–Bonnet**: key term $\nabla_\mu \nabla_\nu \xi(\phi)$ appears with different coefficients along r and θ,

$$\nabla_r \nabla_r \neq \nabla_\theta \nabla_\theta \quad \Longrightarrow \quad p_r^{(GB)} \neq p_t^{(GB)}$$

- For a **homogeneous** scalar $\phi = \phi(t)$, we have $p^r = p^t$ in GR but anisotropy persists in EdGB gravity:

$$p^r_{\rm eff} - p^t_{\rm eff} = \alpha \times \left[\text{terms in } \phi, A, B, R \text{ and their derivatives}\right] \neq 0$$

since $\Gamma^t_{rr} \sim \dot{B}/B$ while $\Gamma^t_{\theta\theta} \sim R\dot{R}$.

Origin, Key Result & Conclusions

In FRW Case

- Metric:

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2$$

- Symmetry enforces: $\phi = \phi(t)$ and $T^{\mu}{}_{\nu} = \text{diag}(-\rho, p, p, p)$
- Result: No anisotropy ($p_r = p_t$).

- **Key result:**

$$\Delta = p_r - p_t = (\text{scalar gradients}) + (\text{GB terms}) \neq 0 \quad \text{even for } \phi(t)$$

Conclusions:

- FRW: isotropy forbids anisotropy
- Spherical metric: anisotropy allowed, amplified by Gauss–Bonnet
- Sourced by geometry, scalar dynamics, and curvature coupling

Conservation laws

$$\dot{\rho}_{\rm eff} + (\rho_{\rm eff} + p^r_{\rm eff})\frac{\dot{B}}{B} + 2(\rho_{\rm eff} + p^t_{\rm eff})\frac{\dot{R}}{R} = \partial_r (T_{\rm eff})^r{}_t + \left(\frac{A'}{A} + \frac{B'}{B} + 2\frac{R'}{R}\right)(T_{\rm eff})^r{}_t$$

$$(p^r_{\rm eff})' + \left(\frac{A'}{A} + 2\frac{R'}{R}\right)p^r_{\rm eff} - 2\frac{R'}{R}p^t_{\rm eff} + \frac{A'}{A}\rho_{\rm eff} = -\partial_t (T_{\rm eff})^t{}_r - \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + 2\frac{\dot{R}}{R}\right)(T_{\rm eff})^t{}_r$$

Standard GR form, now sourced by the off-diagonal flux. GB modification enters only through A, B, R.

The off-diagonal flux term

$$(T_{\rm eff})^t{}_r = \alpha\left(-\frac{16\dot{R}\dot{R}'\dot{\xi}}{A^4R^2} + \frac{8A'\dot{\xi}}{A^3R^2} + \frac{24\dot{R}^2A'\dot{\xi}}{A^5R^2} + \frac{16\dot{B}\dot{R}R'\dot{\xi}}{A^4BR^2} - \frac{8\dot{\xi}A'R'^2}{A^3B^2R^2}\right)$$

$\propto \alpha\,\dot{\xi}(\phi)$: vanishes as $\xi \to 0$. Sources every equation that follows.

Generalized Misner–Sharp gradient law

Hayward's unified first law:

$$\nabla_a E = A\,\Psi_a + W \nabla_a V, \qquad A = 4\pi R^2,\ \ V = \tfrac{4\pi}{3} R^3$$

gives

$$D_t M = -4\pi R^2 p^r_{\mathrm{eff}}\, D_t R + (T_{\mathrm{eff}})^t{}_r\, D_r R, \qquad D_r M = 4\pi R^2 \rho_{\mathrm{eff}}\, D_r R + (T_{\mathrm{eff}})^r{}_t\, D_t R 2$$

$D_t \equiv A^{-1}\partial_t,\ D_r \equiv B^{-1}\partial_r;\ \ U := D_t R,\ \Gamma := D_r R,\ \Gamma^2 = 1 + U^2 - 2M/R.$

Full generalized Misner–Sharp system

$$\dot{U} = \frac{\Gamma^2}{R'}A' - A\frac{M}{R^2} - 4\pi RA\, p^r_{\rm eff} + \frac{\Gamma}{U}\left(\frac{A}{R} - \frac{RB}{2}\right)(T_{\rm eff})^t{}_r$$

$$\dot{R} = AU \qquad \dot{M} = -4\pi R^2 p^r_{\rm eff}\, A\, U + (T_{\rm eff})^t{}_r\, A\,\Gamma$$

$$\dot{\rho}_{\rm eff} = -\rho_{\rm eff}\left(\frac{\dot{B}}{B} + 2\frac{\dot{R}}{R}\right) - p^r_{\rm eff}\frac{\dot{B}}{B} - 2p^t_{\rm eff}\frac{\dot{R}}{R} + \partial_r(T_{\rm eff})^r{}_t + \left(\frac{A'}{A} + \frac{B'}{B} + 2\frac{R'}{R}\right)(T_{\rm eff})^r{}_t$$

$$M' = 4\pi R^2 \rho_{\rm eff}\, B\,\Gamma + (T_{\rm eff})^r{}_t\, B\, U$$

Reduces to the GR system (slide 8) as $\xi(\phi) \to 0$.

GR vs. EdGB, and the FRW check

	GR	EdGB
Pressure	single $p = w\rho$	$p^r_{\rm eff} \neq p^t_{\rm eff}$
Off-diagonal flux	absent	$\propto \alpha\, \xi'(\phi), \dot{\xi}(\phi)$
$\dot{M}$	$\propto p\, U$	extra flux term

FRW: $\phi = \phi(t)$ only $\Rightarrow (T_{\rm eff})^r{}_t = 0 \Rightarrow M = 4\pi \int_0^r \rho_{\rm eff}\, R^2 R'\, dr$. Anisotropy is sourced entirely by the inhomogeneous perturbation – where PBHs form.

Collapse Threshold in EdGB Gravity

Density contrast & long-wavelength expansion

$$\rho_{\rm eff} = \rho_\phi + \rho_{\rm GB}, \qquad \delta_{\rm eff} = \frac{\rho_{\rm eff} - \rho_b^{\rm eff}}{\rho_b^{\rm eff}}$$

$$\tilde{\rho}_{\rm EdGB} = \underbrace{\frac{3(1+w_{\rm eff})}{5+3w_{\rm eff}}\left[K + \frac{r}{3}K'\right] r_m^2}_{\text{GR piece}} + \underbrace{\Delta^\rho_{\rm GB}}_{\text{new}}$$

Each perturbation variable = GR piece + $\Delta_{\rm GB}$. $f(\phi) \to 0$: reduces to standard cosmological contrast.

Compaction function & collapse criterion

$$C_{\mathrm{EdGB}}(R) = \frac{2\,\delta M(R)}{R} = C_{\mathrm{GR}} + C_{\phi} + C_{\mathrm{GB}}, \qquad \delta_m \equiv C_{\mathrm{EdGB}}(R_m)$$

$$u^{\mu}\nabla_{\mu}u_{\nu} = -\frac{h_{\nu}{}^{\alpha}\nabla_{\alpha}p_r^{\mathrm{eff}}}{\rho_{\mathrm{eff}} + p_r^{\mathrm{eff}}} + F_{\nu}^{\mathrm{GB}}$$

Collapse vs. dispersal: gravity vs. pressure gradient *and* new GB force F_{ν}^{GB}. Critical point: $\ddot{R}|_{R_m} = 0$.

Threshold decomposition & modified scaling

$$\delta_c^{\mathrm{EdGB}} = \delta_c^{\mathrm{GR}} + \Delta_\phi + \Delta_{\mathrm{GB}} \qquad M_{\mathrm{PBH}} = K_{\mathrm{EdGB}}\, M_H \left(\delta_m - \delta_c^{\mathrm{EdGB}}\right)^{\gamma_{\mathrm{EdGB}}}$$

$K_{\mathrm{EdGB}}, \gamma_{\mathrm{EdGB}}$ can depend on $\xi(\phi)$, scalar profile, α, w.

GR limit

$\xi(\phi) \to 0 \Rightarrow \delta_c^{\mathrm{EdGB}} \to \delta_c^{\mathrm{GR}}$: exact recovery of the GR criterion.

Current Status & Outlook

Done

- Generalized Misner–Sharp mass & evolution equations in EdGB gravity, beyond FRW.
- Anisotropic effective stress $p^r_{\rm eff} \neq p^t_{\rm eff}$, sourced by $\xi(\phi)$.
- Off-diagonal flux $(T_{\rm eff})^t{}_r$ derived; vanishes on FRW.
- Exact GR recovery as $\xi(\phi) \to 0$ (matches arXiv:2211.05767).
- Long-wavelength expansion: each variable $=$ GR piece $+\Delta_{\rm GB}$.
- Generalized compaction function & threshold decomposition.

In progress & expected implications

In progress

- Evaluate $\Delta_\phi, \Delta_{\rm GB}$ for representative $\xi(\phi)$.
- Numerically integrate the EdGB system through horizon formation.
- Extract $M_{\rm gap}, K_{\rm EdGB}, \gamma_{\rm EdGB}$.

Implications

$M_{\rm gap} > 0 \Rightarrow$ low-mass cutoff in the PBH mass function – affects dark-matter/evaporation bounds and the low-mass GW spectrum.

Summary

1. GR critical collapse: no minimum PBH mass.
2. EdGB induces anisotropic stress & new energy flux, absent in GR.
3. Generalized Misner–Sharp system derived, with exact GR limit.
4. Conjecture: analogous mass gap $M_{\rm gap}$ in PBH collapse, motivated by stationary EdGB black holes.
5. Numerics for $M_{\rm gap}, K_{\rm EdGB}, \gamma_{\rm EdGB}$: in progress.

References

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Thank you

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