Inhomogeneous Mass Deformation of ABJM Theory: Spatial and Null Coordinate-Dependent Mass Function

based on

JHEP 08 (2018) 082, *JHEP* 12 (2019) 153, and ongoing works

CQUeST 2026 Workshop on

Cosmology and Quantum Spacetime

(2026 Jul 20-24)

2026. 07. 22

St. Jones Hotel, Gangneung, Korea

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With

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M-theory ➔ M2 brane ➔ ABJM theory
➔ Mass-deformed ABJM(mABJM) ➔ Inhomogeneously Mass-deformed ABJM(ImABJM)

- M-theory(Mother, Membrane, Matrix, Mystery, …)
  ➔ Strongly coupled string theory in a sense
- Defined in 11 dimensions.
- They are related to each other.

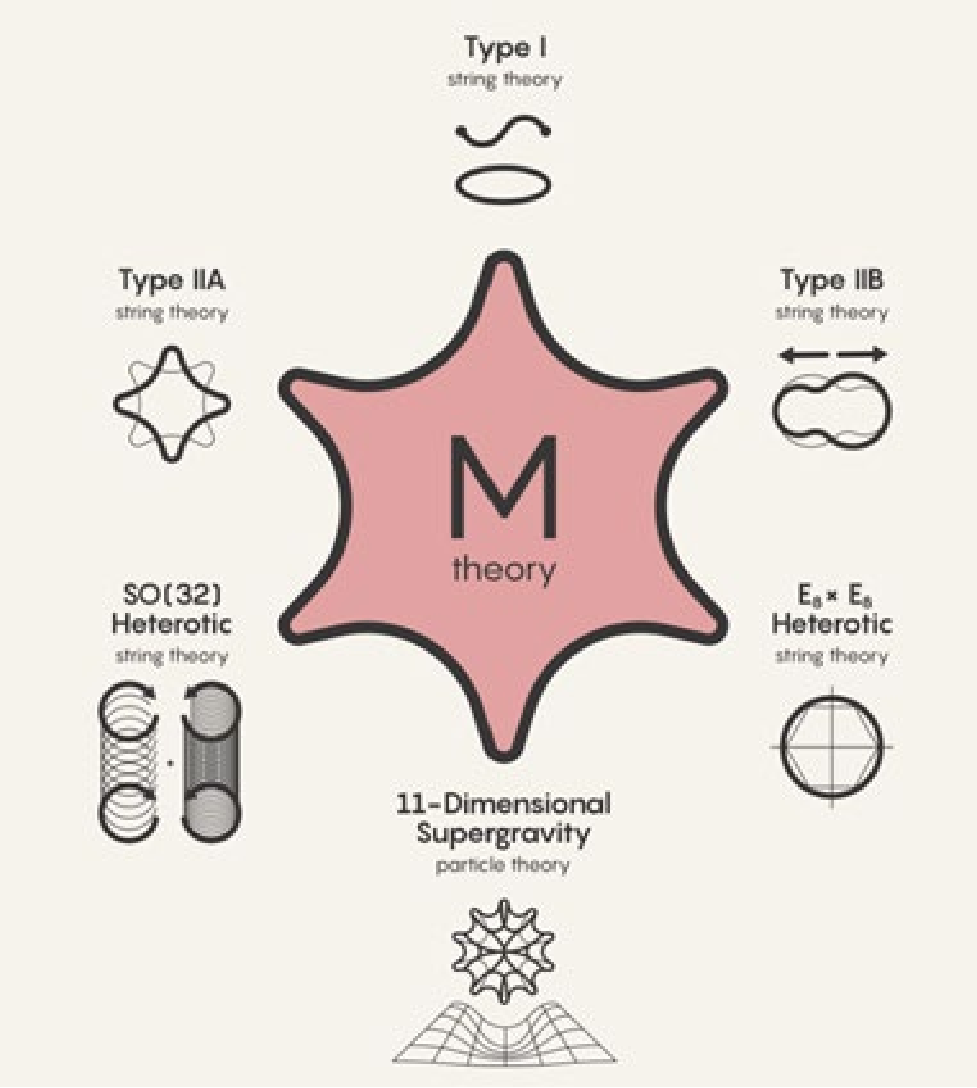
  Ex) By the S1 compactification, M changes to type IIA string theory.

- The low-energy effective theory of M-theory is 11-dimensional supergravity.

$$S_{11} = \frac{1}{2\kappa_{11}^2}\int d^{11}x\sqrt{-g}\left(R - \frac{1}{2\cdot 4!}F_4^2 - \bar{\Psi}_M\Gamma^{MNP}D_N\Psi_P\right)$$

$$-\frac{1}{12\kappa_{11}^2}\int A_3\wedge F_4\wedge F_4$$

$$+\frac{1}{2\kappa_{11}^2}\int d^{11}x\sqrt{-g}\left[\frac{1}{192}\bar{\Psi}_M\left(\Gamma^{MNPQRS}F_{PQRS} + 12g^{M[P}\Gamma^{Q]N}F_{PQRS}\right)\Psi_N\right]$$



M-theory ➔ M2 brane ➔ ABJM theory
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$$S_{11} = \frac{1}{2\kappa_{11}^2}\int d^{11}x\sqrt{-g}\left(R - \frac{1}{2\cdot 4!}F_4^2\right) - \frac{1}{12\kappa_{11}^2}\int A_3 \wedge F_4 \wedge F_4$$

- Therefore, M-theory has 4-form field strength F4.

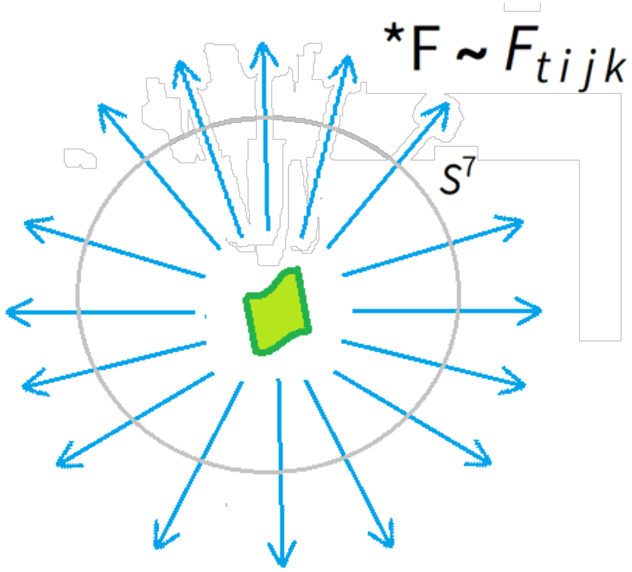
$$Q_{M2} = \int_{S^7} {}^*F$$

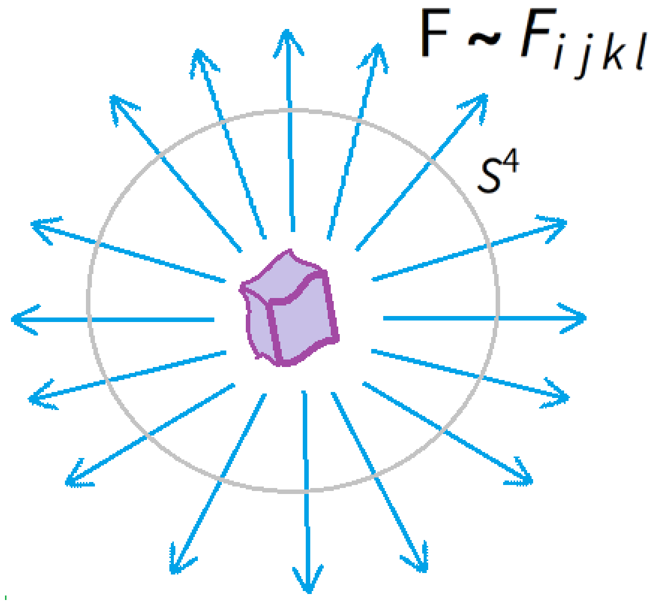
→ (11-1-7-1)-dimensional electric object

$$Q_{M5} = \int_{S^4} F$$

→ (11-1-4-1)-dimensional magnetic object

- They are called M2(Membrane) brane and M5 brane, as is well known.
- The microscopic theory of M2-brane theory is known as the ABJM theory.





N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals

Ofer Aharony (Weizmann Inst.), Oren Bergman (Princeton, Inst. Advanced Study and Technion), Daniel Louis Jafferis (Rutgers U., Piscataway), Juan Maldacena (Princeton, Inst. Advanced Study) (Jun, 2008)

Published in: *JHEP* 10 (2008) 091 • e-Print: 0806.1218 [hep-th]

M-theory ➔ M2 brane ➔ ABJM theory ➔ mABJM ➔ lmABJM

- N=6 ABJM theory

$$S = \int d^3x \left(\mathcal{L}_0 + \mathcal{L}_{\rm CS} - V_{\rm ferm} - V_{\rm bos}\right),$$

$$\mathcal{L}_0 = {\rm tr}\left(-D_\mu Y_A^\dagger D^\mu Y^A + i\psi^{\dagger A}\gamma^\mu D_\mu \psi_A\right),$$

$$\mathcal{L}_{\rm CS} = \frac{k}{4\pi}\epsilon^{\mu\nu\rho}\,{\rm tr}\left(A_\mu\partial_\nu A_\rho + \frac{2i}{3}A_\mu A_\nu A_\rho - \hat{A}_\mu\partial_\nu\hat{A}_\rho - \frac{2i}{3}\hat{A}_\mu\hat{A}_\nu\hat{A}_\rho\right),$$

with complicated interaction terms. $\sim \psi\,\psi\,Y\,Y + Y\,Y\,Y\,Y\,Y\,Y$

- One may deform the theory with mass terms, maintaining supersymmetry.
- Mass-deformed ABJM theory (Supersymmetric but nonconformal)

$$\mathcal{L}_{\rm mABJM} = \mathcal{L}_0 + \mathcal{L}_{\rm CS} - V_{\rm ferm} - V_{\rm bos} - \hat{V}_{\rm ferm} - \hat{V}_{\rm bos}.$$

$$\hat{V}_{\rm ferm} = imM_A{}^B\psi^{\dagger A}\psi_B \quad \hat{V}_{\rm flux} = -\frac{4\pi m}{k}M_B{}^D\left(Y_C^\dagger Y^C Y_D^\dagger Y^B - Y^C Y_C^\dagger Y^B Y_D^\dagger\right),$$

$$\hat{V}_{\rm bos} = \hat{V}_{\rm flux} + \hat{V}_{\rm mass} \qquad \hat{V}_{\rm mass} = m^2 Y_A^\dagger Y^A.$$

M-theory ➔ M2 brane ➔ ABJM theory ➔ mABJM ➔ ImABJM

- Mass-deformed ABJM theory

N=5,6 Superconformal Chern-Simons Theories and M2-branes on Orbifolds #18

Kazuo Hosomichi (Korea Inst. Advanced Study, Seoul), Ki-Myeong Lee (Korea Inst. Advanced Study, Seoul), Sangmin Lee (Seoul Natl. U.), Sungjay Lee (Korea Inst. Advanced Study, Seoul and Seoul Natl. U.), Jaemo Park (POSTECH and Stanford U., Phys. Dept.) (Jun, 2008)

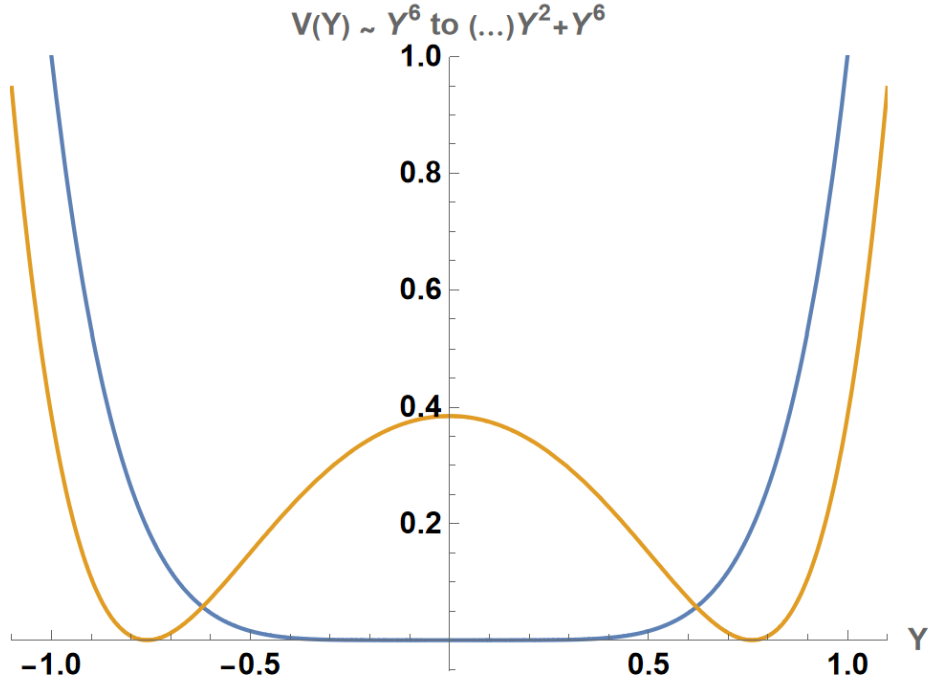
Published in: *JHEP* 09 (2008) 002 • e-Print: 0806.4977 [hep-th]

A Massive Study of M2-brane Proposals #1

Jaume Gomis (Perimeter Inst. Theor. Phys.), Diego Rodriguez-Gomez (Princeton U. and Queen Mary, U. of London), Mark Van Raamsdonk (British Columbia U.), Herman Verlinde (Princeton U.) (Jul, 2008)

Published in: *JHEP* 09 (2008) 113 • e-Print: 0807.1074 [hep-th]

- Roughly speaking, the bosonic potential changes into the following shape qualitatively by adding a mass term.



M-theory ➔ M2 brane ➔ ABJM theory ➔ mABJM ➔ ImABJM

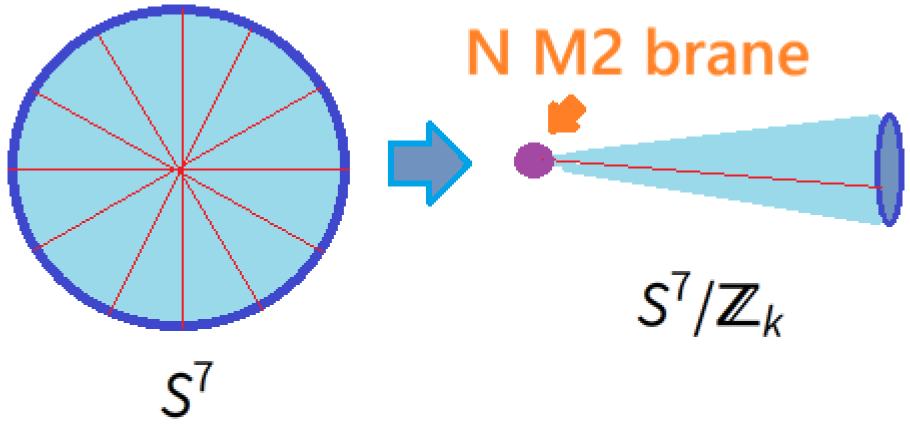
- The gravity dual to the original ABJM theory based on AdS/CFT?
- The supersymmetric(Extremal) black 2-brane in 11-dimensional supergravity is identified N M2-branes

$$Q_{M2} = \int_{S^7} {}^*F$$

➔ Near Horizon limit ➔ AdS4 X S7 ~ Dual to the vacuum of the N membrane theory

- ABJM theory vacuum is dual to $AdS_4 \times S^7/\mathbb{Z}_k$.

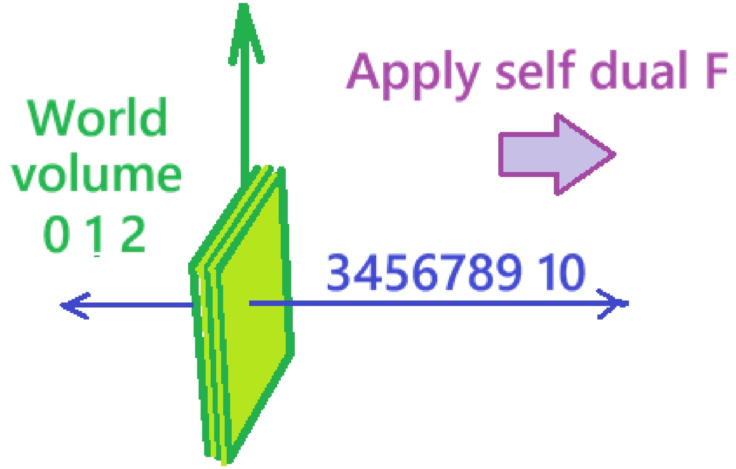
→ Schematically, $S^7/\mathbb{Z}_k$. is regarded as a cone.



- When k=1, the geometry is covered as AdS4 X S7.
  - Many interesting phenomena happen.
  (ex. Susy enhancement....)

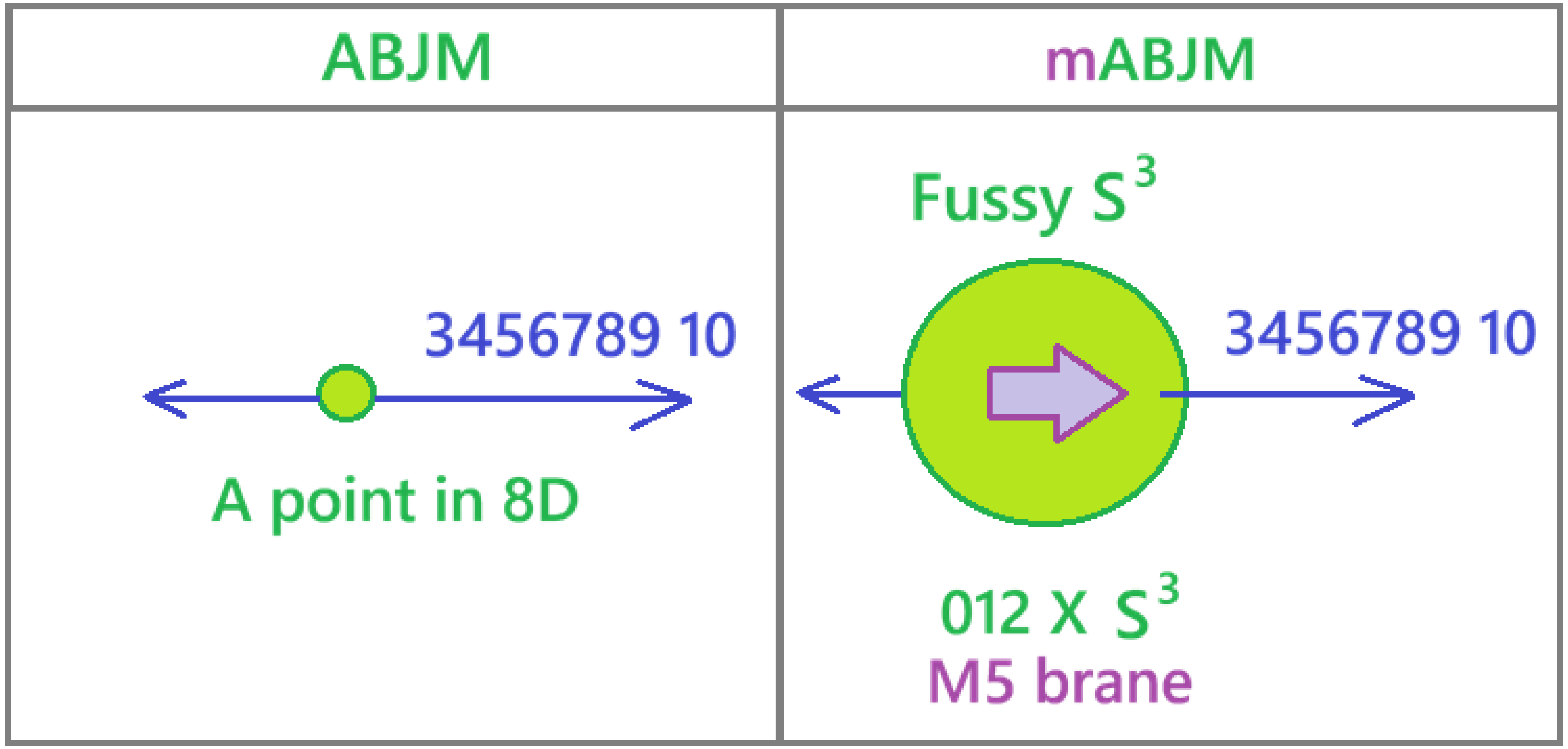
- ABJM theory is the microscopic theory for the N M2 brane.

M-theory ➔ M2 brane ➔ ABJM theory ➔ mABJM ➔ ImABJM



- Mass-deformed ABJM (mABJM) theory?

➔ M2 + Flux = M5 (Myers Effect)



$[\, Y^A , Y^\dagger{}_B \,] = 0$

$G^{BC}_A = \beta^{BC}_A + \delta^{[B}_A \beta^{C]D}_D + m M_A{}^{[B} Y^{C]}$. =0

M-theory ➔ M2 brane ➔ ABJM theory ➔ mABJM ➔ lmABJM

- The vacuum solution of the mABJM theory: GRVV matrices (Infinitely many for N=infinity)

A Massive Study of M2-brane Proposals #1

Jaume Gomis (Perimeter Inst. Theor. Phys.), Diego Rodriguez-Gomez (Princeton U. and Queen Mary, U. of London), Mark Van Raamsdonk (British Columbia U.), Herman Verlinde (Princeton U.) (Jul, 2008)

Published in: *JHEP* 09 (2008) 113 • e-Print: 0807.1074 [hep-th]

- The corresponding gravity dual (Many 11-dimensional Sugra solutions)

Bubbling AdS space and 1/2 BPS geometries #10

Hai Lin (Princeton U.), Oleg Lunin (Princeton, Inst. Advanced Study), Juan Martin Maldacena (Princeton, Inst. Advanced Study) (Sep, 2004)

Published in: *JHEP* 10 (2004) 025 • e-Print: hep-th/0409174 [hep-th]

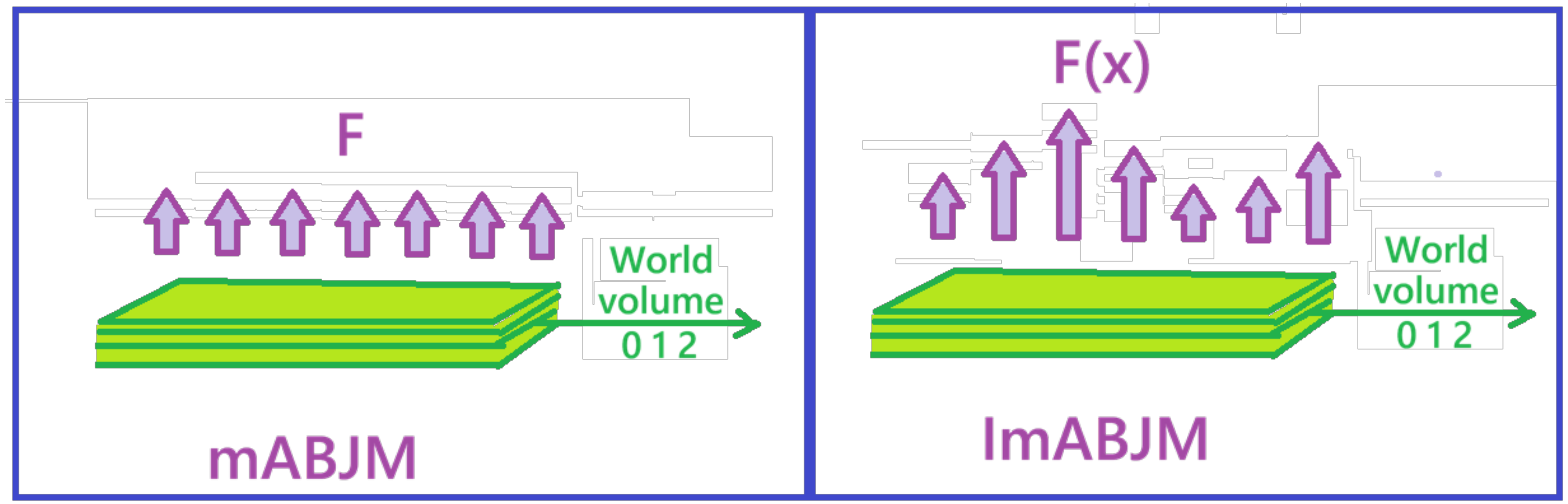
See **2.5 M2 brane theory with a mass deformation**

- The metric has two S^3 structures.

$$ds^2 = |G_{tt}|(-dt^2 + dw_1^2 + dw_2^2) + G_{xx}\left(dx^2 + dy^2\right) + G_{\theta\theta} ds^2_{S^3/\mathbb{Z}_k} + G_{\tilde\theta\tilde\theta} ds^2_{\tilde S^3/\mathbb{Z}_k},$$

M-theory ➔ M2 brane ➔ ABJM theory ➔ mABJM ➔ ImABJM

- Inhomogeneously mass-deformed ABJM (ImABJM) theory
- If the mass parameter has a coordinate dependence, can we construct a supersymmetry theory?
- What is the configuration for the M2 brane and 4-form flux?



- The position-dependent field strength.
- The corresponding supersymmetric theory exists!

M-theory ➔ M2 brane ➔ ABJM theory ➔ mABJM ➔ ImABJM

- Adding the mass terms

Janus ABJM Models with Mass Deformation

Kyung Kiu Kim (Sejong U.), O-Kab Kwon (Sungkyunkwan U.) (Jun 18, 2018)

Published in: *JHEP* 08 (2018) 082 • e-Print: 1806.06963 [hep-th]

$$\Delta\mathcal{L} = m' M_A{}^B \mathrm{Tr}\left(Y^A Y_B^\dagger\right) + m M_A{}^B \mathrm{Tr}\left(i\psi^{\dagger A}\psi_B + \frac{8\pi}{q} Y^C Y_{[C}^\dagger Y^A Y_{B]}^\dagger\right) + m^2 \mathrm{Tr}\left(Y^A Y_A^\dagger\right)$$

- N=3 theory (1/2 susy)

$$\gamma^1\omega_{ab} = -\omega_{ab} \quad \Longleftrightarrow \quad \omega^{ab}\gamma^1 = \omega^{ab},$$
$$\gamma^1\omega_{ai} = \omega_{ai} \quad \Longleftrightarrow \quad \omega^{ai}\gamma^1 = -\omega^{ai},$$

- Why m' ?
  - The origin of m ➔ the 4-form field strength F4 along transverse directions.
  - Bianchi identity - dF = 0
  - m' arises naturally in the M2-brane world volume action.

Aspects of Massive ABJM Models with Inhomogeneous Mass Parameters #1

Kyung Kiu Kim (Sejong U.), Yoonbai Kim (Sungkyunkwan U.), O-Kab Kwon (Sungkyunkwan U.), Chanju Kim (Ewha Women's U., Seoul) (Oct 11, 2019)

Published in: *JHEP* 12 (2019) 153 • e-Print: 1910.05044 [hep-th]

- Vacuum analysis

- N=2,1 with m(x)'s is also possible.

Vacuum structure of ImABJM

Aspects of Massive ABJM Models with Inhomogeneous Mass Parameters #5

Kyung Kiu Kim (Sejong U.), Yoonbai Kim (Sungkyunkwan U.), O-Kab Kwon (Sungkyunkwan U.), Chanju Kim (Ewha Women's U., Seoul) (Oct 11, 2019)

Published in: *JHEP* 12 (2019) 153 • e-Print: 1910.05044 [hep-th]

- BPS equation

$$(i)\ \delta_A^{[B}Y^{C]\prime} + s^{BC}G_A^{BC} = 0,$$

$$(ii)\ \int_{x_L}^{x_R} dx\,\mathcal{K}' = 0,$$

$$\beta_C^{AB} \equiv \frac{2\pi}{k}\left(Y^AY_C^\dagger Y^B - Y^BY_C^\dagger Y^A\right),$$

$$\beta_{BC}^{A} \equiv \frac{2\pi}{k}\left(Y_B^\dagger Y^AY_C^\dagger - Y_C^\dagger Y^AY_B^\dagger\right).$$

$$G_A^{BC} = \beta_A^{BC} + \delta_A^{[B}\beta_D^{C]D} + mM_A{}^{[B}Y^{C]}.$$

$$\mathcal{K} \equiv \mathrm{tr}\left(\frac{1}{2}\beta_a^{ac}Y_c^\dagger + \frac{1}{2}\beta_i^{ik}Y_k^\dagger - \frac{1}{3}\beta_i^{ic}Y_c^\dagger + mM_A{}^BY_B^\dagger Y^A\right).$$

- Non-periodic vacuum – More nontrivial
  - See

Supersymmetric inhomogeneous field theories in 1+1 dimensions

O-Kab Kwon (Sungkyunkwan U.), Chanju Kim (Ewha Women's U., Seoul), Yoonbai Kim (Sungkyunkwan U.) (Oct 25, 2021)

Published in: *JHEP* 01 (2022) 140 • e-Print: 2110.13393 [hep-th]

  - Some vacua carry negative energies.

Examples of vacuum solutions

- Two types of vacua

$$Y^a = P(x)\tilde{Y}^a_D \quad , \quad Y^a = Q(x)\tilde{Y}^i_D \qquad : \text{Diagonal Type},$$

$$Y^a = P(x)\tilde{Y}^a_{\tilde{m}=1} \quad , \quad Y^i = Q(x)\tilde{Y}^i_{\tilde{m}=1} \quad : \text{GRVV Type} \; ,$$

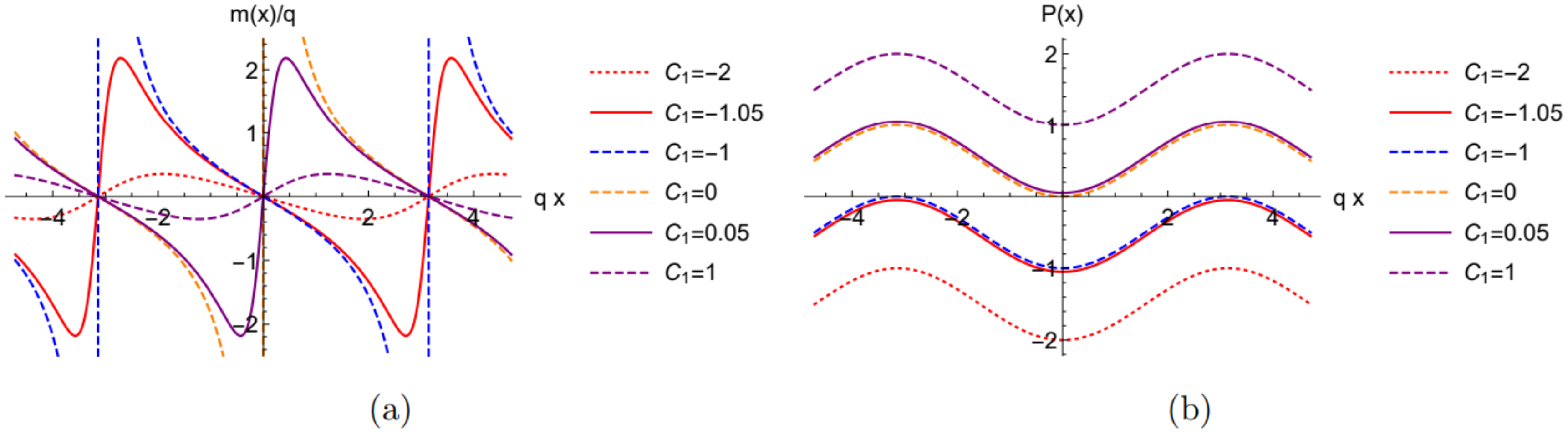


Figure 1: Diagonal Types with $m(x) = \frac{q\sin(qx)}{2\left(C_1+\sin^2\left(\frac{qx}{2}\right)\right)}$.

Vacuum structure of lmABJM

- The general vacuum solutions for a periodic mass function of lmABJM. m (w^1)

$$\text{ABJM type:}\quad Y^a = e^{\Lambda(w^1)}\, {Y_D}^a,\ Y^i = e^{-\Lambda(w^1)}\, {Y_D}^i$$

$$\text{mABJM type:}\ Y^a = e^{\alpha(w^1)}\, {Y_{\mathrm{GRVV}}}^a,\ Y^i = e^{\beta(w^1)}\, {Y_{\mathrm{GRVV}}}^i$$

- Gravity duals are conjectured as

ABJM type

→ 11D SUGRA on S^7 Compactification

mABJM type

→ 11D SUGRA on $S^3 \times \tilde{S}^3$ Compactification

: LLM type solution

Vacuum structure of lmABJM

- The general vacuum solutions for a periodic mass function of lmABJM. $m(w^1)$

$$\text{ABJM type:} \quad Y^a = e^{\Lambda(w^1)}\, Y_D{}^a, \;\; Y^i = e^{-\Lambda(w^1)}\, Y_D{}^i$$

$$\text{mABJM type:} \; Y^a = e^{\alpha(w^1)}\, Y_{\mathrm{GRVV}}{}^a, \;\; Y^i = e^{\beta(w^1)}\, Y_{\mathrm{GRVV}}{}^i$$

- Gravity duals are conjectured as

ABJM type

→ 11D SUGRA on S^7 Compactification

mABJM type

→ 11D SUGRA on $S^3 \times \tilde{S}^3$ Compactification

: LLM type solution

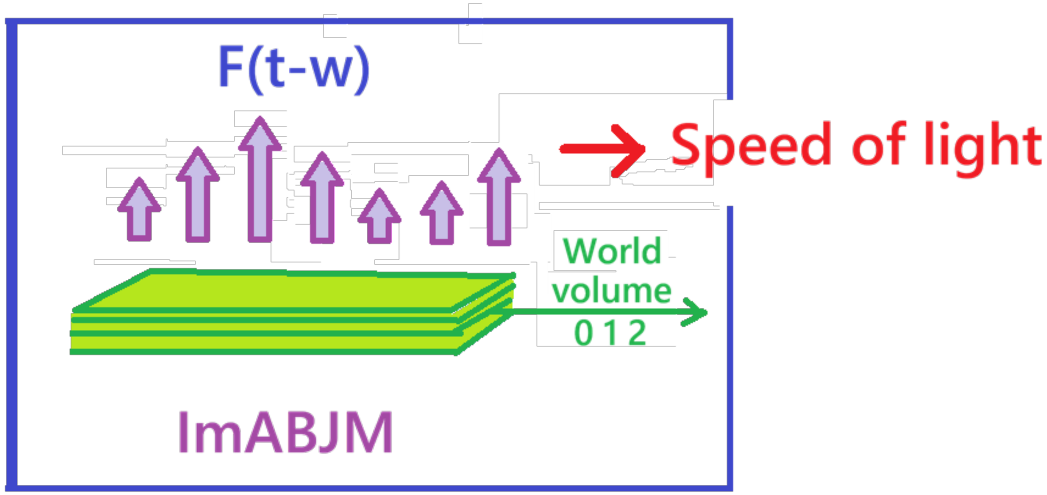
Null coordinate-dependent mass function $m = m(t \pm w_1)$

Janus ABJM Models with Mass Deformation

Kyung Kiu Kim (Sejong U.), O-Kab Kwon (Sungkyunkwan U.) (Jun 18, 2018)

Published in: *JHEP* 08 (2018) 082 • e-Print: 1806.06963 [hep-th]

- The 4-form flux is moving along a world volume direction at the speed of light.



- The additional term to the Lagrangian of the mABJM theory?

$$-im'M^A_B Y^\dagger_C \omega^{BC}\left(\gamma^0 \pm \gamma^1\right)\psi_A + im'M_A{}^B \psi^{\dagger A}\left(\gamma^0 \pm \gamma^1\right)\omega_{BC} Y^C,$$

where $m' \equiv \partial m/\partial t = \partial m/\partial w_1$.

- To make the theory supersymmetric, we impose the following projection

$$\gamma_\pm \omega_{AB} = 0, \text{ where } \gamma_\pm \equiv \gamma^0 \pm \gamma^1.$$

- Then, nothing is added to the original Lagrangian, interestingly.

Gravity duals

- In order to study the strong coupling limit of these theories via the AdS/CFT correspondence, one needs to find the corresponding gravity duals.
- The gravity duals for the lmABJM theory are given as follows:

Susy Q and spatially modulated deformations of ABJM theory

Jerome P. Gauntlett (Imperial Coll., London), Christopher Rosen (Imperial Coll., London) (Aug 7, 2018)

Published in: *JHEP* 10 (2018) 066 • e-Print: 1808.02488 [hep-th]

Spatially modulated and supersymmetric deformations of ABJM theory

Igal Arav (Imperial Coll., London), Jerome P. Gauntlett (Imperial Coll., London), Matthew Roberts (Imperial Coll., London), Christopher Rosen (Imperial Coll., London) (Dec 28, 2018)

Published in: *JHEP* 04 (2019) 099 • e-Print: 1812.11159 [hep-th]

- These are gravity duals for the spatial coordinate-dependent periodic mass function.
  - The authors provide the formulations in N=1 SUGRA and N=8 gauged SUGRA in 4 dimensions.

Gravity duals

- Summary for the BPS configuration
- Bosonic Action

$$S = \int \mathrm{d}^4 x \sqrt{-g}\left(R - \frac{2}{(1-|z|^2)^2}\partial_\mu z \partial^\mu \bar{z} + \frac{2(3-|z|^2)}{1-|z|^2}\right)$$

- Geometry and Complex scalar field

$$ds^2 = H^{-2}\left[-dt^2 + dy^2 + ((\nabla H)^2 - H^2) dw d\bar{w}\right]$$

$$z = -\frac{1}{2\partial_w \ln H} \, .$$

- The BPS equation = A Helmholz equation

$$\nabla^2 H - H = 0$$

- Correspondence between the field theory and gravity

$$S = S_{ABJM} + \int d^3x \left\{ m'(\sigma)\mathcal{O}^{(1)} + m(\sigma)\mathcal{O}^{(2)} + m(\sigma)^2 \mathcal{O}^{(3)} \right\}$$

$$\mathcal{O}^{(1)} = M_A^B \mathrm{Tr}\left(Y^A Y_B^\dagger\right)$$

$$\mathcal{O}^{(2)} = M_A^B \mathrm{Tr}\left(\psi^A \psi_B^\dagger + \frac{8\pi}{k} Y^C Y^\dagger{}_{[C} Y^A Y_{B]}^\dagger\right)$$

$$\mathcal{O}^{(3)} = \mathrm{Tr}\left(Y^A Y_A^\dagger\right) \, .$$

⇨ Not protected via AdS/CFT

Asymptotic behavior of the scalar field

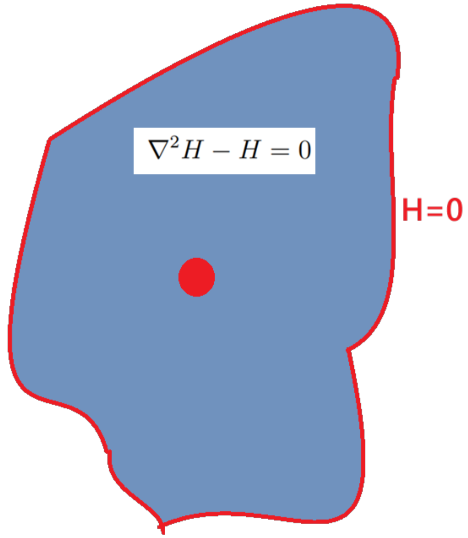
$$z \sim z_1 \beta + z_2 \beta^2 + \dots$$
$$\sim \left(\left\langle \mathcal{O}^{\Delta=1}(\sigma)\right\rangle + i m(\sigma)\right)\beta - \left(m'(\sigma) - i\left\langle \mathcal{O}^{\Delta=2}(\sigma)\right\rangle d\right)\beta^2 + \dots ,$$

Gravity duals

- Various periodic solutions can be obtained!

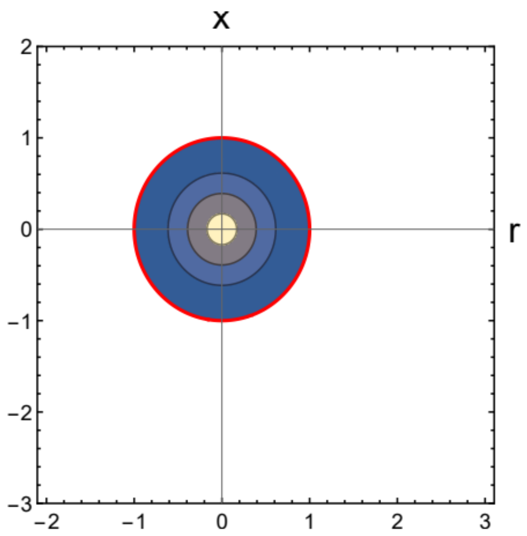
  - Determine the Boundary of AdS4 by H = 0

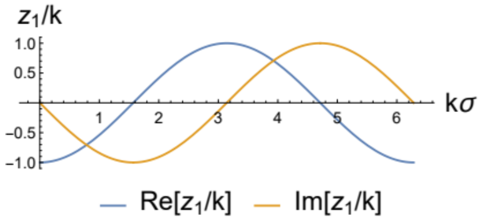
  - Place a delta function source at the origin for an IR AdS4.



- With a suitable coordinate choice, the curve determines the mass function.

  - ex susy Q solution



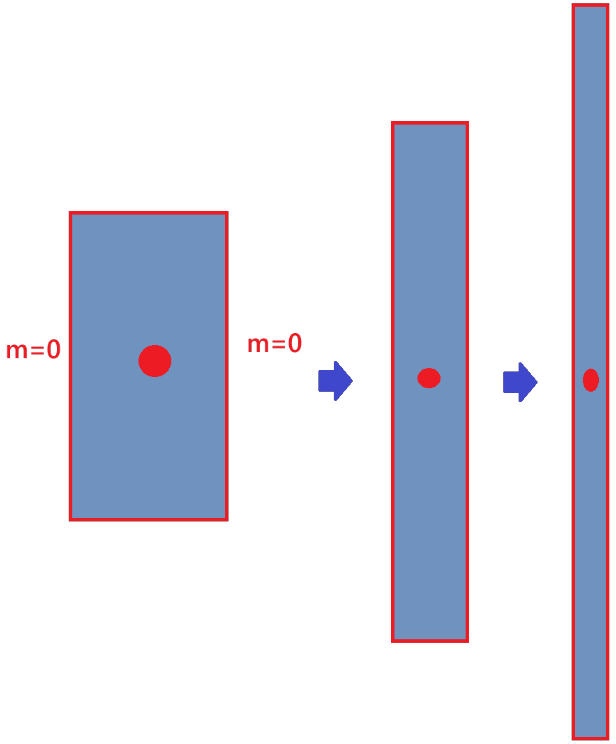


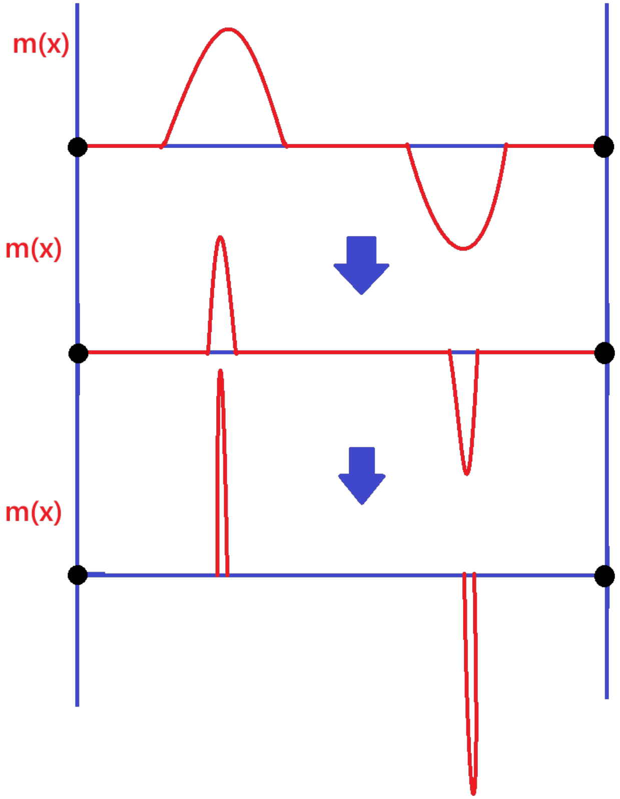
Gravity duals

- Another rectangular shape solution

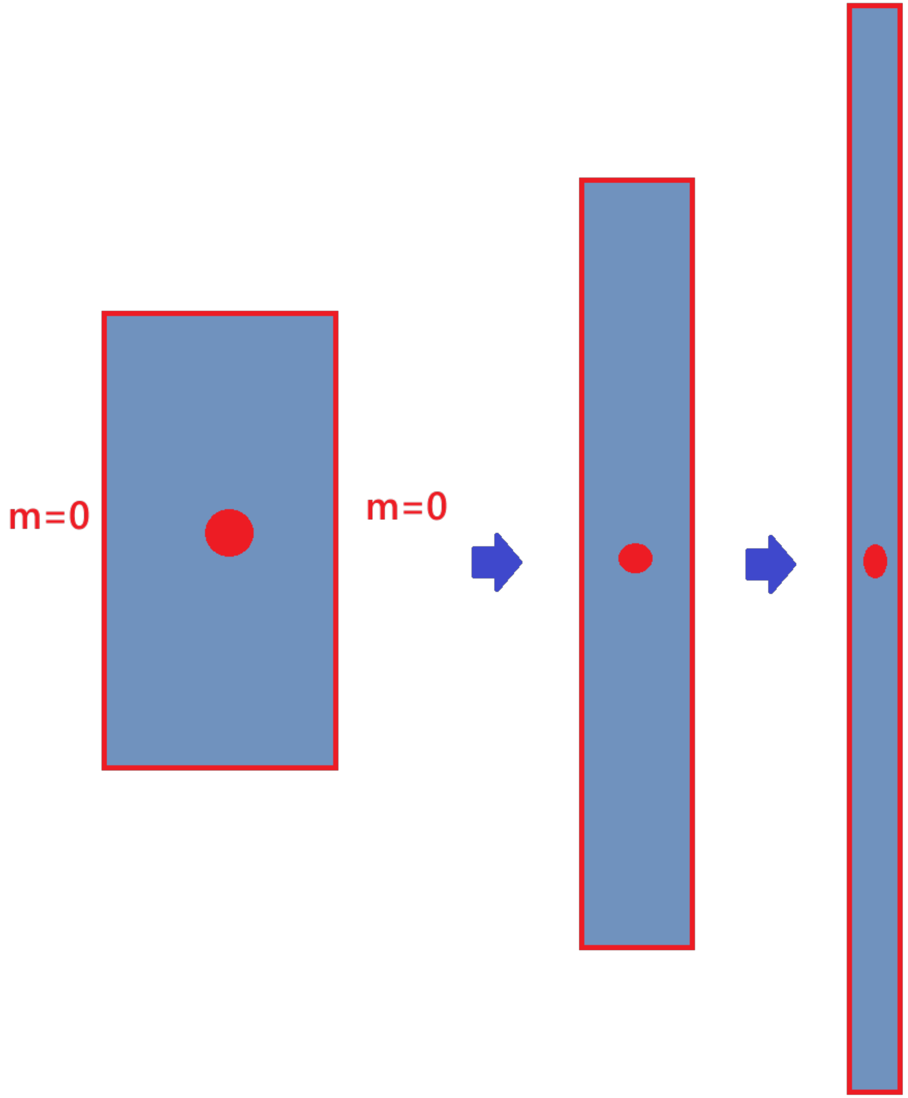
$$G(\mathbf{x};\mathbf{x}') = -\frac{4}{L_1 L_2}\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\frac{\sin\left(\frac{n_1\pi}{L_1}r\right)\sin\left(\frac{n_2\pi}{L_2}x\right)\sin\left(\frac{n_1\pi}{L_1}r'\right)\sin\left(\frac{n_2\pi}{L_2}x'\right)}{1+\left(\frac{n_1\pi}{L_1}\right)^2+\left(\frac{n_2\pi}{L_2}\right)^2}.$$

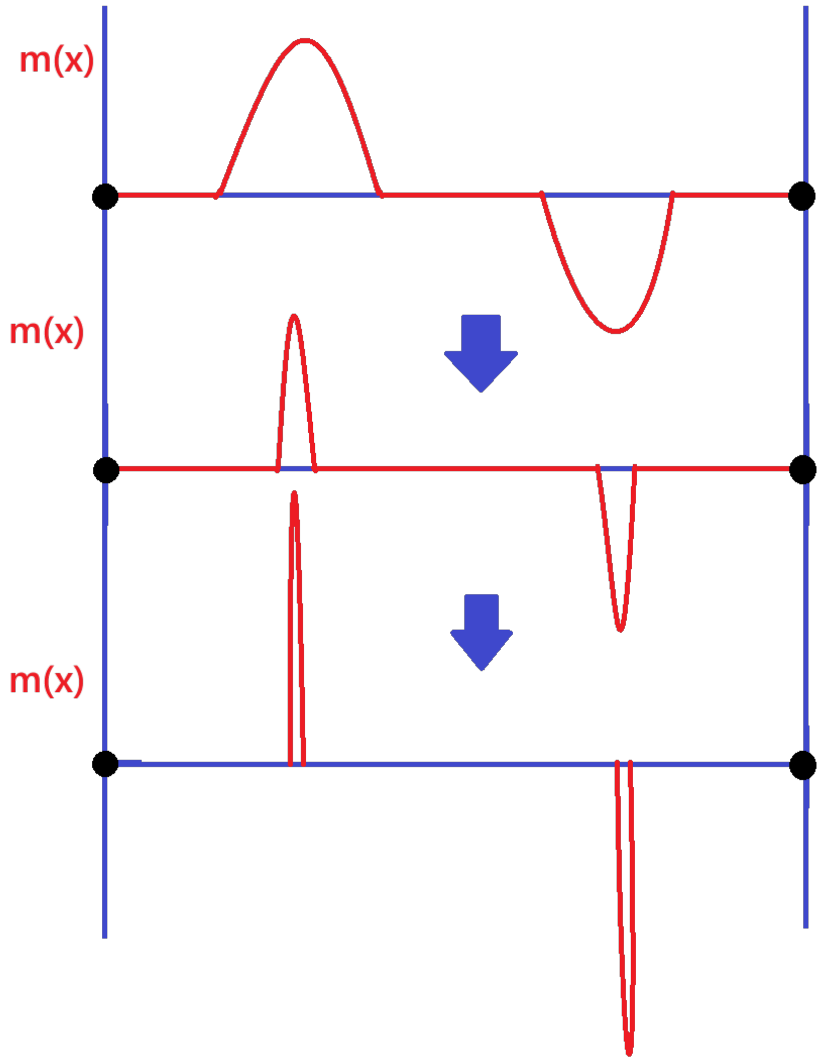
satisfying $(\nabla^2 - 1)G(\mathbf{x};\mathbf{x}') = \delta^{(2)}(\mathbf{x}-\mathbf{x}')$





Long strip limit





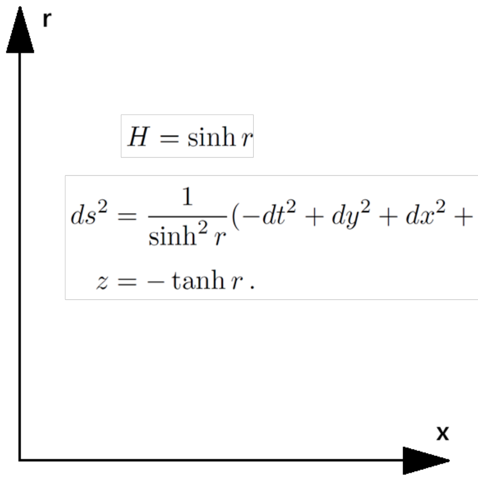
Null-coordinate mass function m(t-x)

- Yonsei University group (Hyun and Song) and I find the gravity dual for this case.

$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{r^2} + \left(\frac{r^2 - \mu(u)^2}{r}\right)^2 \left[-2\mathrm{d}u\mathrm{d}v + \mathrm{d}y^2 + H(r,u)\mathrm{d}u^2\right]$$

$$z(r,u) = e^{\mathrm{i}\theta_0} \frac{2\mu(u)r}{r^2 + \mu(u)^2}.$$

$$H(r,u) = \frac{\mu\ddot{\mu} - 3\dot{\mu}^2}{(r^2 - \mu^2)^2} + \underbrace{C_1(u)}_{0}F(r;\mu) + \underbrace{C_2(u)}_{0}.$$



$$H = \sinh r$$

$$ds^2 = \frac{1}{\sinh^2 r}(-dt^2 + dy^2 + dx^2 + dr^2)$$

$$z = -\tanh r\,.$$

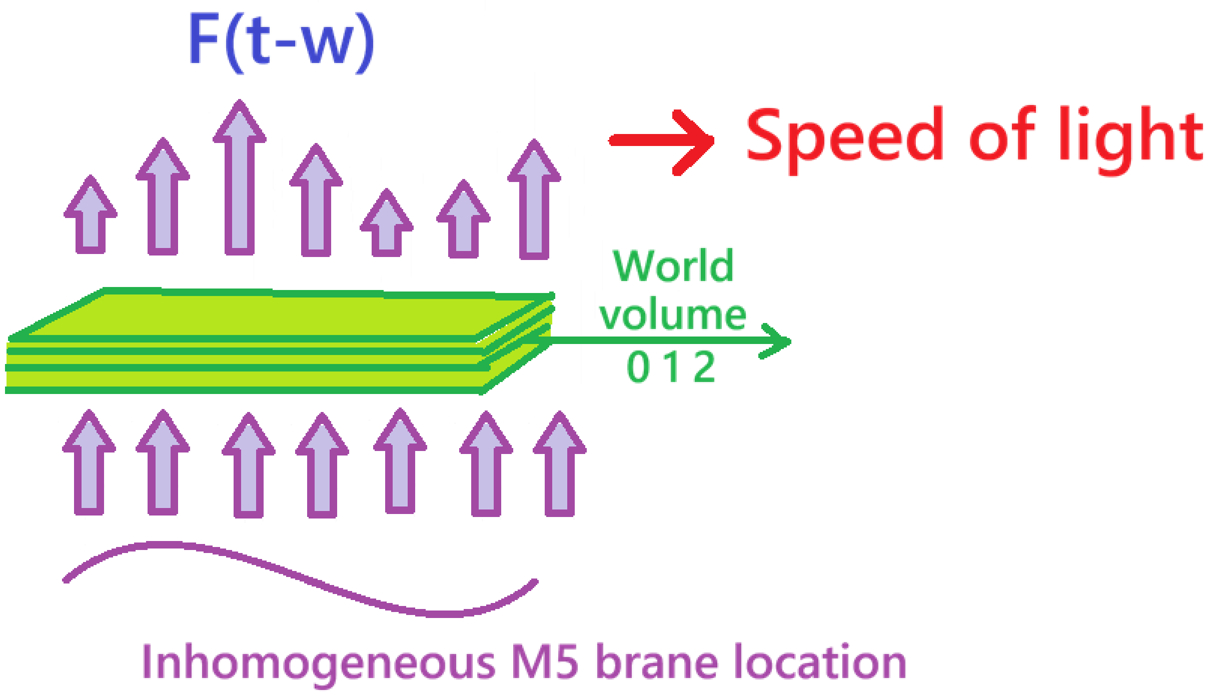
- Constant ₩mu ➔ Dielectric flow solution

C.N. Pope and N.P. Warner, *A Dielectric flow solution with maximal supersymmetry*, *JHEP* **04** (2004) 011 [hep-th/0304132] [INSPIRE].

- At r =₩mu, the curvature is divergent!
  ➔ This singularity is resolved by the location the M5 brane.

Null-coordinate mass function m(t-x)

- We may regard the configuration as follows:



Summary and Potential Applications

- We found some exact supergravity solutions dual to the class of lmABJM theories.
- M(x) solution with the rectangular boundary
- M(t-x) solution is an extension of the dielectric flow solution with the position of the M5 brane.

- Periodic lattice construction and Defect generalization are possible!

  ➔ Condensed matter applications with Chern-Simons Matter
  ➔ BCFT application: Interaction between boundaries and defects

- A class of time-dependent supersymmetric solutions

  ➔ Another interesting branch of Supergravity solutions

Thank you for your attention!