

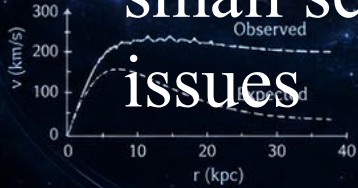
Cosmology of ultralight scalar field dark matter

Jae-Weon Lee (Jungwon University)

DARK MATTER: A KEY TO MANY MYSTERIES

GALAXY PHYSICS

small scale
issues



BLACK HOLE PHYSICS

SMBHs

Illuminating black hole
growth and information
puzzles

COSMOLOGY

H_0
 S_8
 $w(z)$



Explaining cosmic evolution
and resolving tensions

PARTICLE PHYSICS

BSM

ν

Hidden
Sector

Hints of new particles,
forces, and hidden sectors

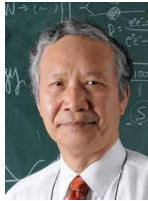
Outline

1. ULDM ($m \sim 10^{-22}$ eV)
and SI-ULDM ($\tilde{m} \equiv \frac{m}{\lambda^{\frac{1}{4}}} \sim 1$ eV)
2. Hubble tension & S_8 tension
3. Neutrino mass and EW scale

Mass scale of dark matter

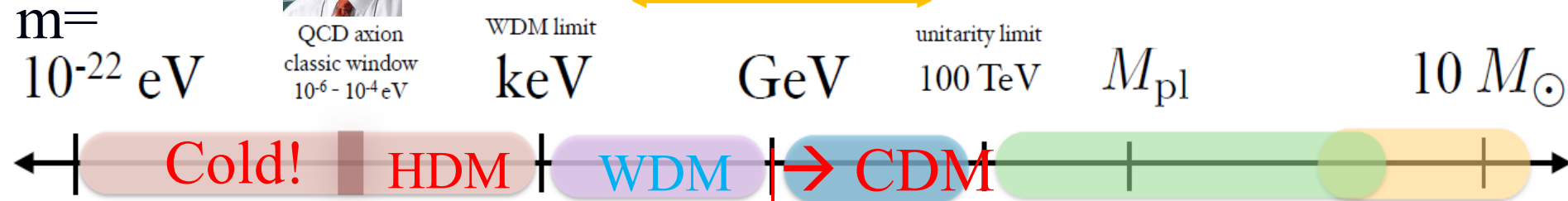
(not to scale)

TASI lectures by Lin arXiv:1904.07915



J. KIM

SIDM



S. Sin

“Ultralight” DM

non-thermal
bosonic fields

“Light” DM

dark sectors
sterile ν
can be thermal

WIMP



Composite DM
(Q-balls, nuggets, etc)

Primordial
black holes
MACHO

Ben. Lee

ULDM = Fuzzy, ULA, BEC, Wave, Scalar Field, ψ , Superfluid, quantum ...

HYU Professor Sin Sang-jin Receives Seongbong Physics Award from the Korean Physical Society

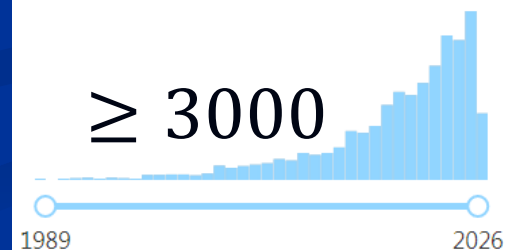
▲ Lee, Hyoung-min translator | ▲ Lee, Hyoung-min translator | ⌚ 입력 2025.05.16 08:59 | 댓글 0

Recognized for pioneering the [quantum dark matter theory](#) and advancing research integrating string theory and condensed matter physics

The Latest News

Date of paper

≥ 3000



Late-time phase transition and the galactic halo as a Bose liquid

Sin1

Sang-Jin Sin

Department of Physics, Hanyang University, Seoul, 133-791, Korea

(Received 28 March 1994)

We consider the ultralight pseudo Nambu-Goldstone boson appearing in the late-time cosmological phase transition theories as a major dark matter candidate. Since it is almost massless, its nature is more wavelike than particlelike. We study quantum mechanically how they form a galactic halo. Three predictions are made: (i) the mass profile $\rho \sim r^{-1.6}$; (ii) there are ripplelike fine structures in the rotation curve; (iii) the rotation velocity multiplied by the ripple's wavelength is galaxy independent.

● Dark matter in halos is in BEC (Bose-Einstein Condensate)

→ single macroscopic wave function $m = 3.3 \times 10^{-23} \text{eV}$

→ DM Halo as a single self-gravitating “giant atom”

● Uncertainty principle prevents collapse

$$\nabla^2 V = 4\pi G(\rho_{\text{dark}} + \rho_{\text{visible}})$$

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi(r)$$



$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + GmM_0 \int_0^r dr' \frac{1}{r'^2} \int_0^{r'} dr'' 4\pi r''^2 (|\psi|^2 + \rho_v)\psi(r).$$

GPE

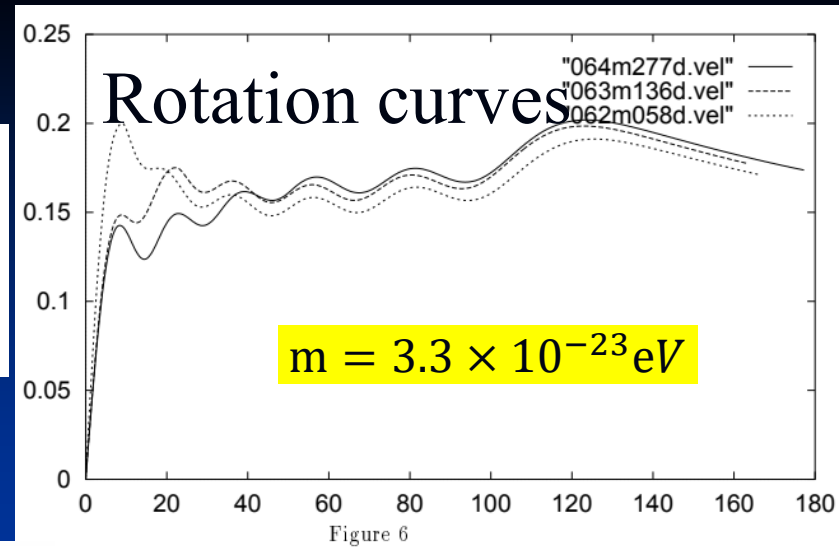
Late-time Phase transition and the Galactic halo as a Bose

Liquid: (II) the Effect of Visible Matter

Sin2

S.U. Ji and S.J. Sin

9409267



Quantum Aspect of Dark Matter and Star Formation in Elliptical Galaxies

JKPS 1995

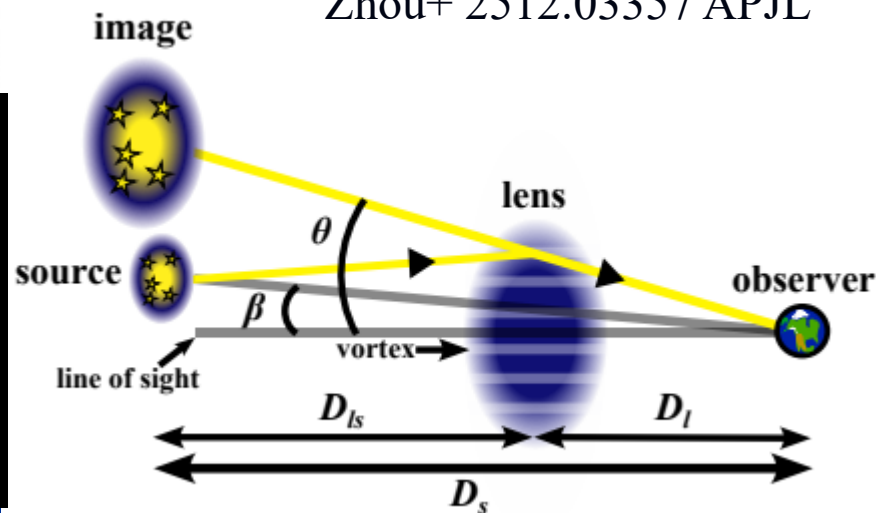
Sin3

Chul H. LEE and Sang J. SIN

Department of Physics, Hanyang University, Seoul 133-791

$$-\frac{1}{\xi} \frac{d}{d\xi} \left(\xi \frac{d\Phi}{d\xi} \right) + \frac{1}{\xi^2} \Phi - \Phi + \Phi^3 = 0.$$

Zhou+ 2512.03357 APJL



Self-Interacting ULDM (SIULDM, SFDM)

Lee and Koh (PRD, hep-ph/9507385)

Galactic DM is **coherent scalar field with self-interaction**:
min. extension of BEC DM

Action

$$S = \int \sqrt{-g} d^4x \left[\frac{-R}{16\pi G} - \frac{g^{\mu\nu}}{2} \phi_{;\mu}^* \phi_{;\nu} - \frac{m^2}{2} |\phi|^2 - \frac{\lambda}{4} |\phi|^4 \right]$$

Energy scale

Metric

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\Omega \text{ Spherical.}$$

Field

$$\phi(r, t) = (4\pi G)^{-\frac{1}{2}} \sigma(r) e^{-i\omega t}$$

$$\tilde{m} \equiv \frac{m}{\lambda^{\frac{1}{4}}} \sim \text{O}(1) \text{eV}$$

Exact
ground
state
(soliton)

$$\sigma_* = \sqrt{\frac{\gamma_0 \text{Sin}(\sqrt{2}r_*)}{\sqrt{2}r_*}}$$

$$r_* = r\Lambda^{-1/2}$$

$$\Lambda \equiv \frac{\lambda m_p^2}{4\pi m^2}, \quad \Lambda \gg 1 \quad (\text{Newtonian \& TF limit})$$

$$RTF \approx \frac{\sqrt{\Lambda}}{m} = \sqrt{\frac{\pi \hbar^3 \lambda}{8cGm^4}} \sim 0.5 \text{kpc} (10\text{eV}/\tilde{m})^2$$

$$\sim \text{kpc} \rightarrow \tilde{m} \sim \text{O}(1) \text{eV}$$

$$\& \text{mass scale } M_{\text{max}} = \sqrt{\Lambda} \frac{m_p^2}{m}$$

- Even tiny self-interaction drastically changes the length & mass scales!
- allows wider range for m to fit galaxy observations
- $\tilde{m}$ determines many physical scales

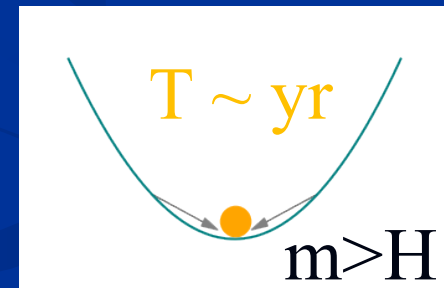
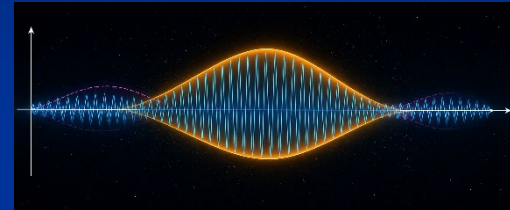
Features of ULDM

$$\phi(t, x) = \frac{1}{\sqrt{2m}} \left[e^{-imt} \psi(t, x) + e^{imt} \psi^*(t, x) \right]$$

fast (bg cosmology), $T \sim \text{yr}$ slow (galaxy) $T \sim \text{Myr}$

for $m \sim 10^{-22} \text{ eV}$

- Typical galaxy size $\sim \lambda_{dB} \sim \text{kpc}$
 → min. & max mass and radius of galaxies
 → min. & max mass of SMBHs
- wave nature → gravitational cooling, interference.
- small dynamical friction
- bg oscillation with $f \sim m \sim \text{nHz}$
 → detection by GW, Atomic clock
- explain DM relic density
 → GUT scale field value (ULA miracle)



→ can explain many mysteries of galaxies

Challenges for Λ CDM

cf) 2105.05208

- *Dark matter is the key to solving some of the Universe's mysteries.*

1. Small scale issues (on galactic scale and below) :

predicts too many small structures not observed

→ Hu+ PRL 2000 (FDM) $m \sim 10^{-22}$ eV (or $\tilde{m} \sim O(1)$ eV for SIULDM)

2. Hubble tension : $5 \sim 7\sigma$ → LJW 2502.11568 with SIULDM for $\tilde{m} \sim 1$ eV

H_0 mismatch between CMB and SN

3. S_8 tension $\sim 2-3\sigma$: → SIULDM $\tilde{m} \sim 1$ eV (SCLee & LJW soon)

$S_8 \equiv \sigma_8 \sqrt{\Omega_m/0.3}$, $z < 2$, Mpc

matter density fluctuation amplitude mismatch between CMB and WL & Cluster

4. DE is not Λ ? $\sim 4\sigma$ (ex, AP test, DESI, SN?) → HDE?

$w_{DE}(z)$ is varying

5. Too early SMBHs ($z \sim 11$) and galaxies ($z \sim 14$?)

Bak & LJW 2601.21676 (→ $m \sim 10^{-22}$ eV or $\tilde{m} \sim 1$ eV?)

6. Li problem, Cosmic birefringence,... etc

Linear pert. Of ULDM

FDM has only 2 parameters m and bg density ρ_0 (or F)
 (+ λ for ϕ^4 self-interacting ULDM)

a =scale factor

Nonrelativistic
 $\longrightarrow$

Madelung
 representation

Density contrast
 (k space)

$$\begin{aligned}
 & i\hbar\left(\frac{\partial\psi}{\partial t} + \frac{3}{2}H\psi\right) = -\frac{\hbar^2}{2ma^2}\Delta\psi + mV\psi + \frac{\lambda|\psi|^2\psi}{2m^2} \quad \text{self-int} \\
 & \text{perturbation with } \psi = \sqrt{\rho}e^{iS}, \quad v \equiv \frac{\hbar}{ma}\nabla S \Rightarrow \text{Sasaki,Mukhanov, Khlopov,Hwang...} \\
 & \begin{cases} \partial_t\rho + 3H\rho + \frac{1}{a}\nabla\cdot(\rho v) = 0 \\ \partial_tv + \frac{1}{a}v\cdot\nabla v + Hv + \frac{1}{\rho a}\nabla p + \frac{1}{a}\nabla V + \frac{\hbar^2}{2m^2a^3}\nabla\left(\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}\right) = 0 \end{cases} \\
 & \text{perturbation } \delta = \delta_k = \delta\rho/\rho_0 \quad \text{Quantum Pressure} \\
 & \Rightarrow \partial_t^2\delta + 2H\partial_t\delta + \left(\left(\frac{\hbar^2k^2}{4m^2a^2} + c_s^2\right)\frac{k^2}{a^2} - 4\pi G\rho_0\right)\delta = 0 \\
 & \quad \text{Hubble drag} \quad \text{gravity}
 \end{aligned}$$

Quantum Jeans length

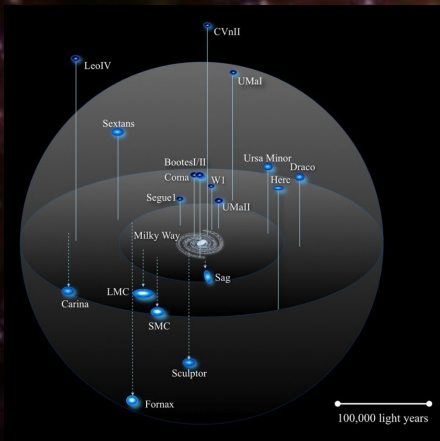
$$\begin{aligned}
 \lambda_{QJ} &= \frac{2\pi}{k_J}a = \pi^{3/4}\hbar^{1/2}(G\rho_0m^2)^{-1/4} \propto 1/\sqrt{mH} \sim 10 \text{ kpc} \\
 &\rightarrow M_J
 \end{aligned}$$

- CDM-like on super-galactic scale (for a small $k < k_J$)
 - Suppress sub-galactic structure (for a large $k > k_J$)
- $\rightarrow$ ULDM is an ideal alternative to CDM

$z = 0.8$

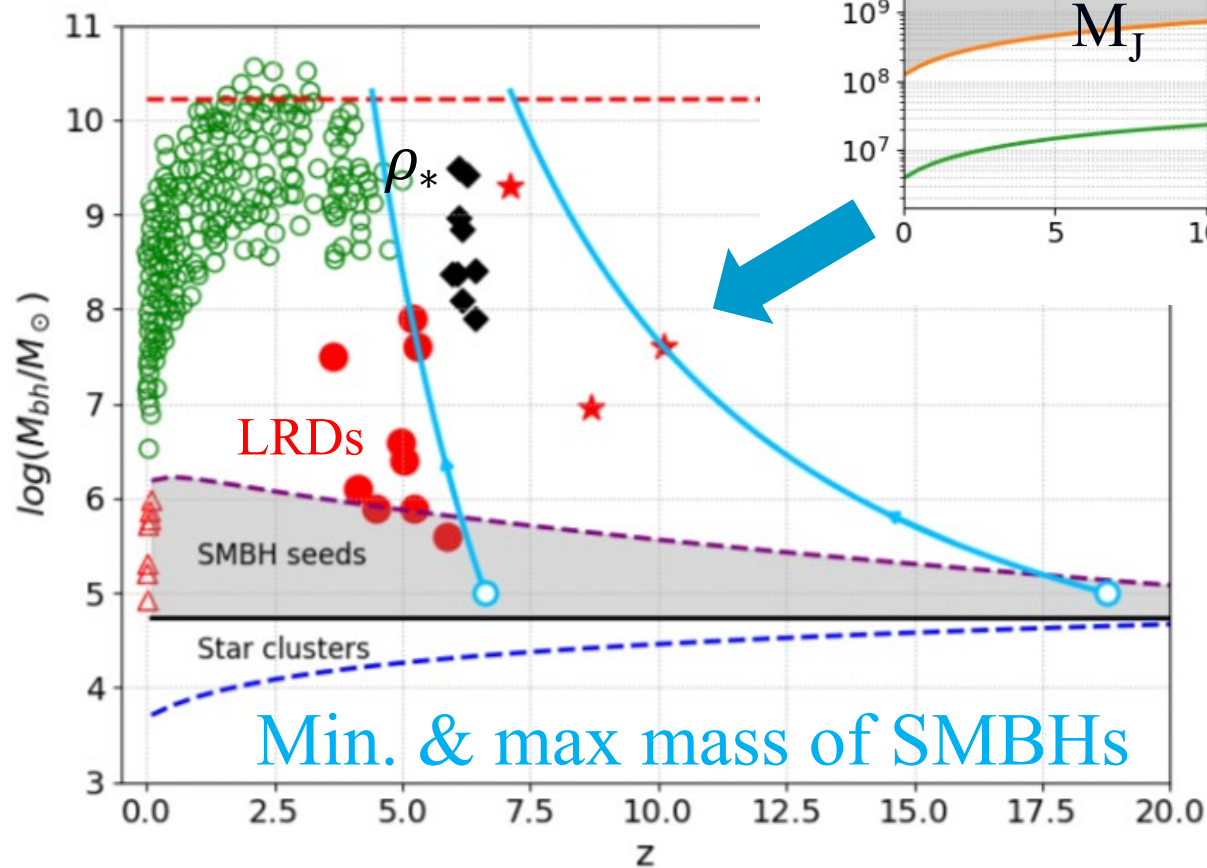
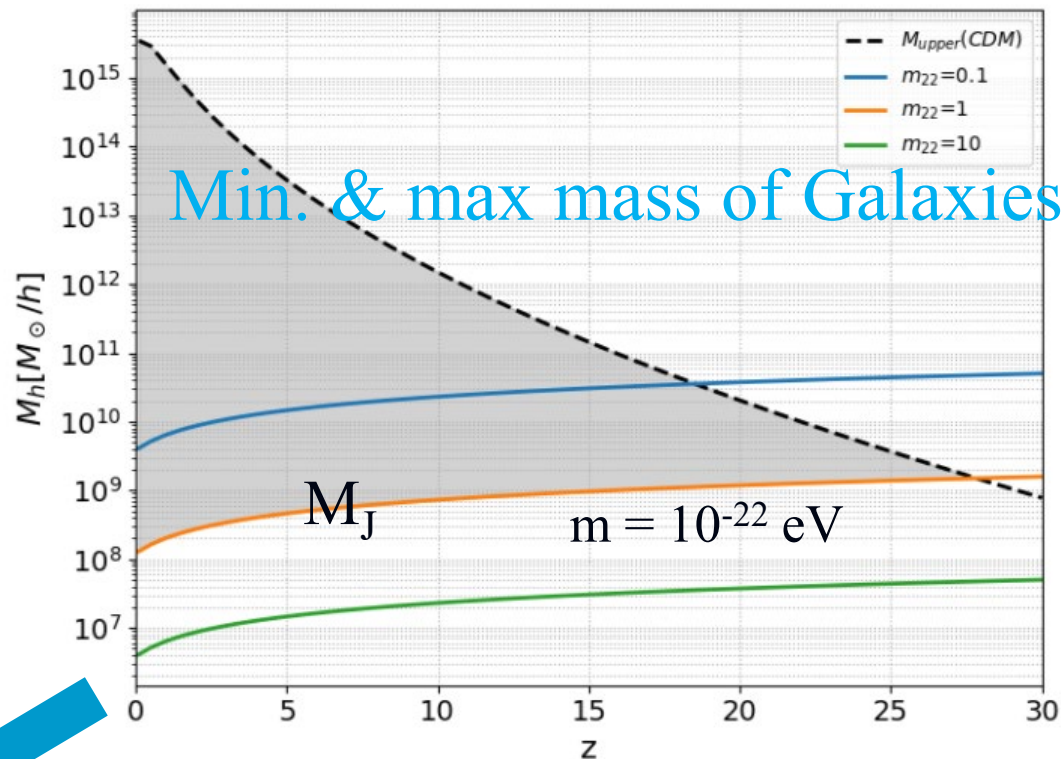
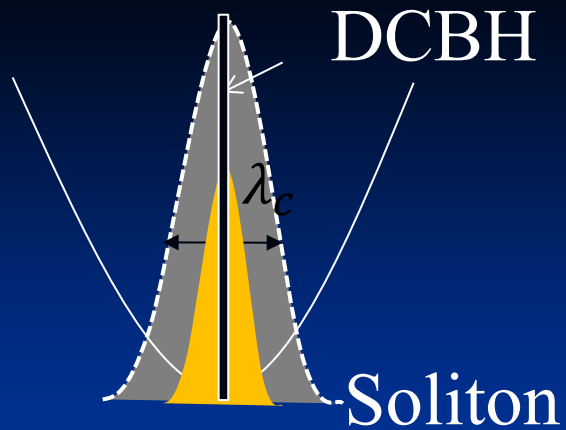
$T = 7.11 \text{ Gyr}$

FDM explains observed galaxies better than CDM.



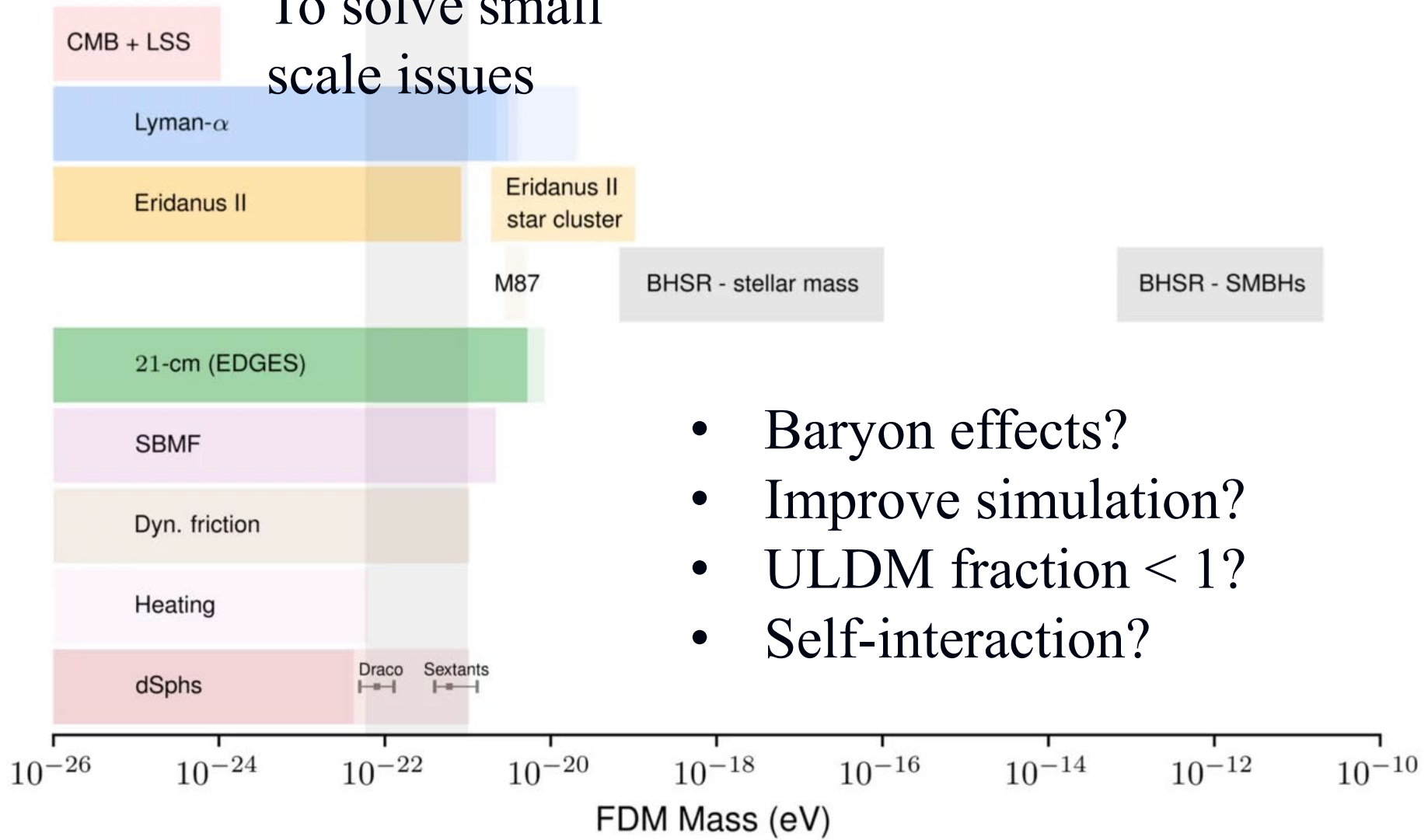
CDM simulation (**Millennium**)

500 kpc



Constraints on FDM (free ULDM) mass

To solve small
scale issues



- Baryon effects?
- Improve simulation?
- ULDM fraction < 1 ?
- Self-interaction?

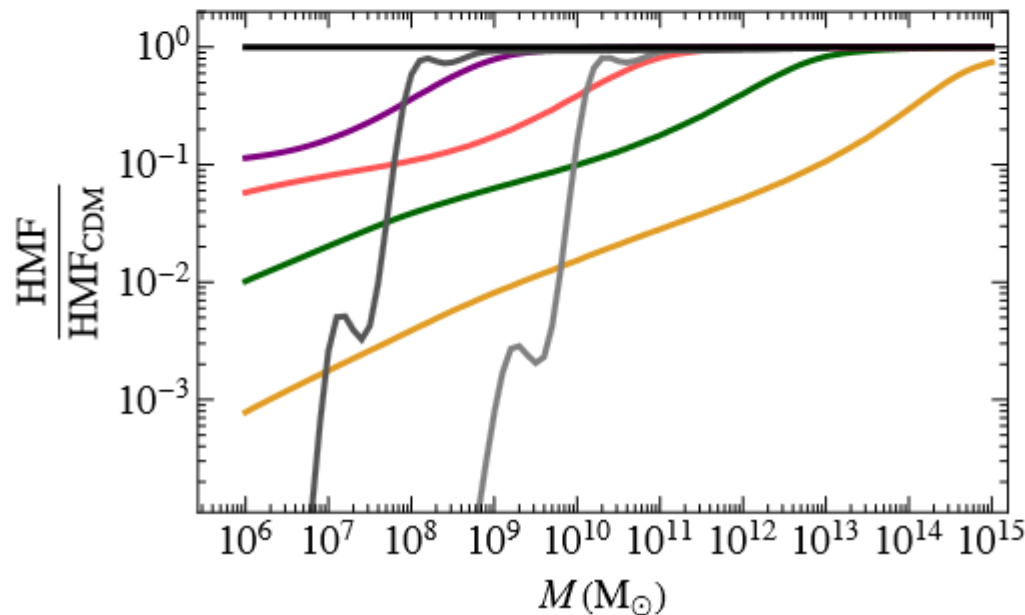
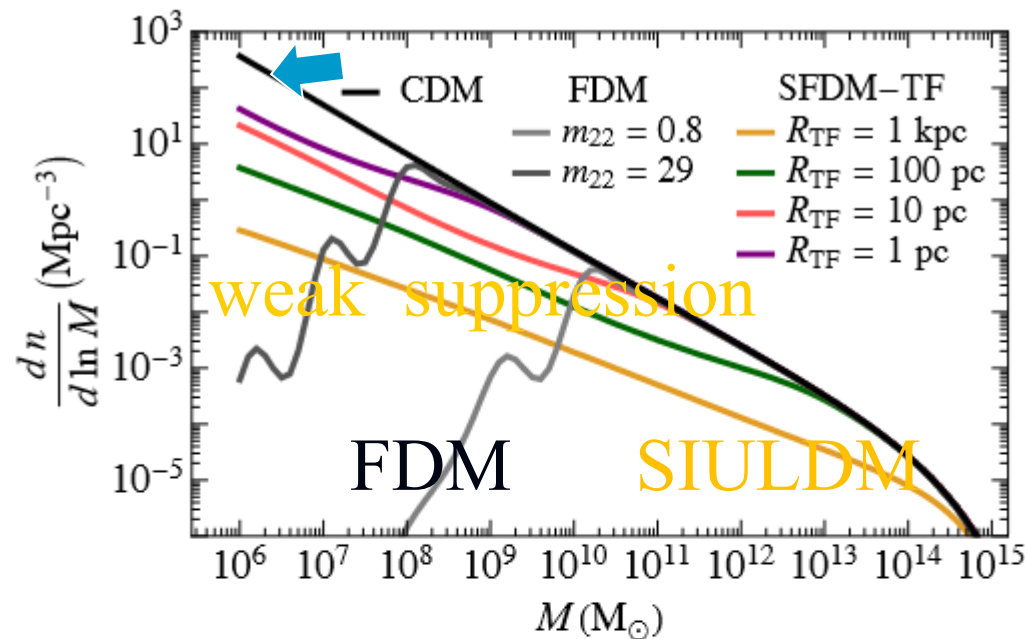
favours $m > 10^{-21}$ eV?

MPA

Merits of studying SIULDM

We can

- allow wider mass range
 - avoid some tensions of free ULDM (FDM)
- study direct, or indirect detection of ULDM
- understand particle models
- minimal extension of FDM
- To make a BEC
- explain other mysteries like Hubble tension & EW scale ...etc



$$\partial_t^2 \delta + 2H \partial_t \delta + \left(\underbrace{\left(\frac{\hbar^2 k^2}{4m^2 a^2} \right)}_{\text{Quantum}} + \underbrace{c_s^2}_{\text{SI}} \right) \frac{k^2}{a^2} - 4\pi G \rho_0 \delta = 0$$

λ_J fixed for SIULDM

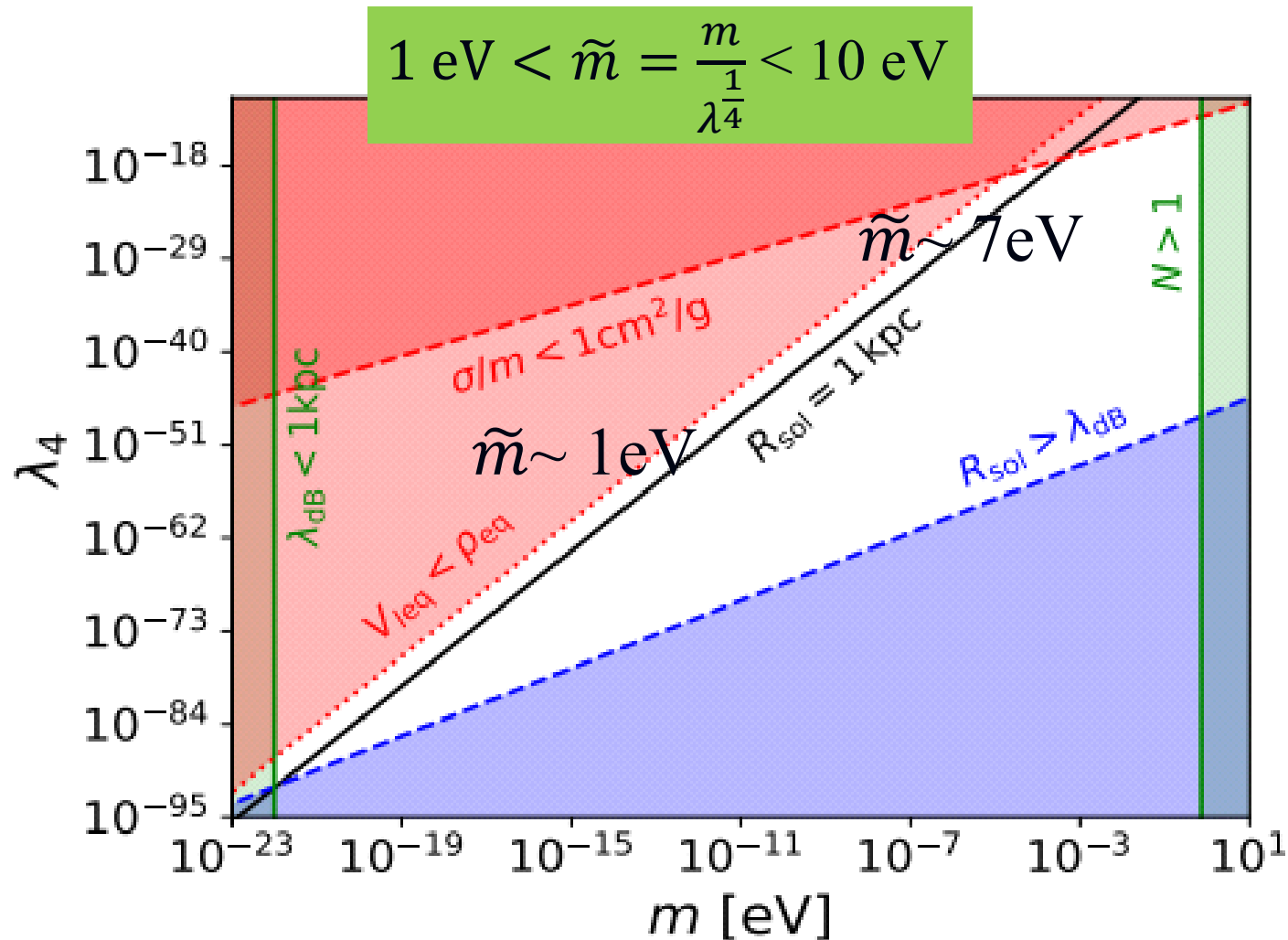
Shrinking Jeans mass

$$M_J \propto a^{-3}$$

→ weak suppression compared to FDM
 → weaken missing satellites problem of CDM *and* FDM?

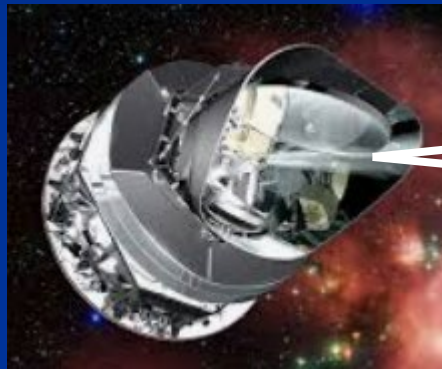
cosmological constraints

Garcia + 2304.10221



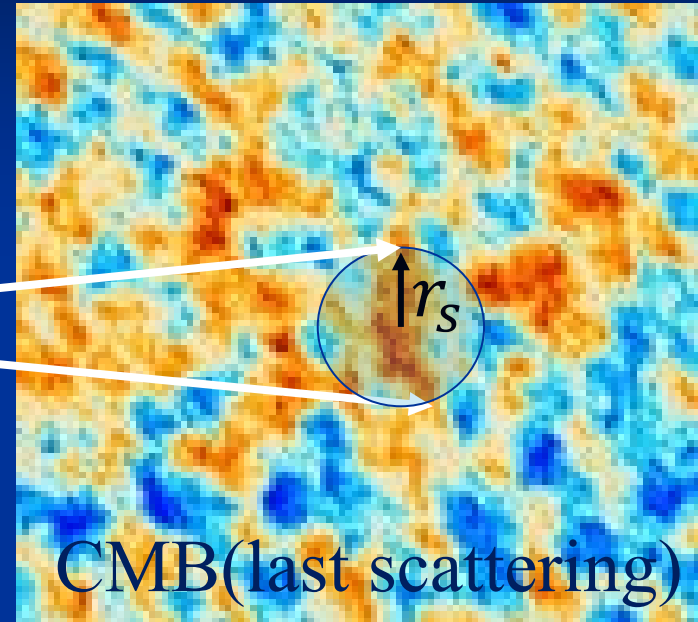
H_0 estimation from CMB

$\theta_s = \frac{r_s}{D_A}$ is a standard ruler with 0.04% precision. (fixed)



$$\theta_s \sim 1^\circ$$

$$D_A$$



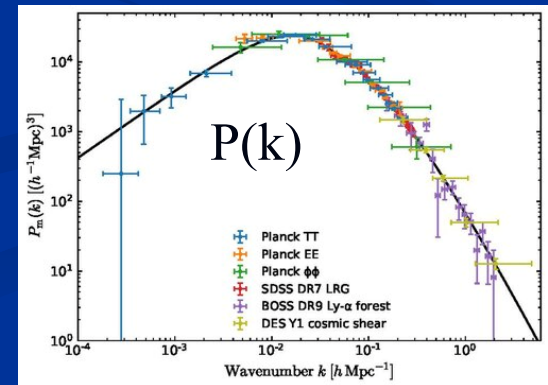
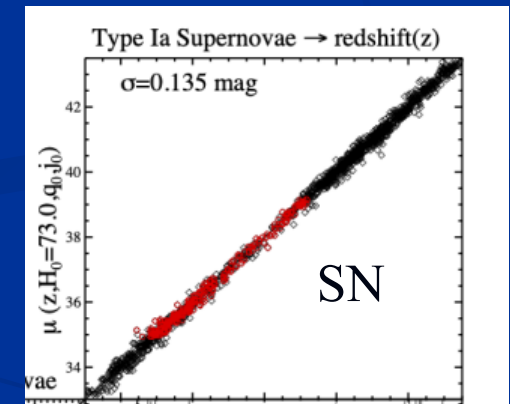
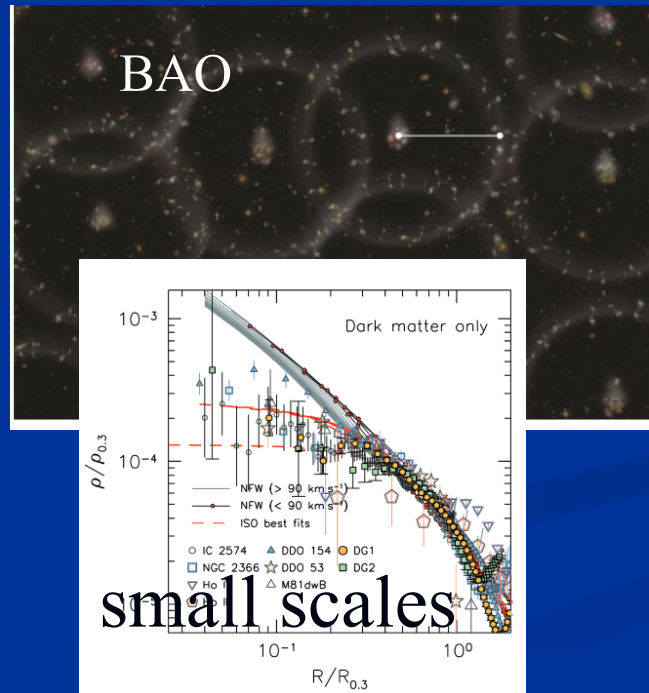
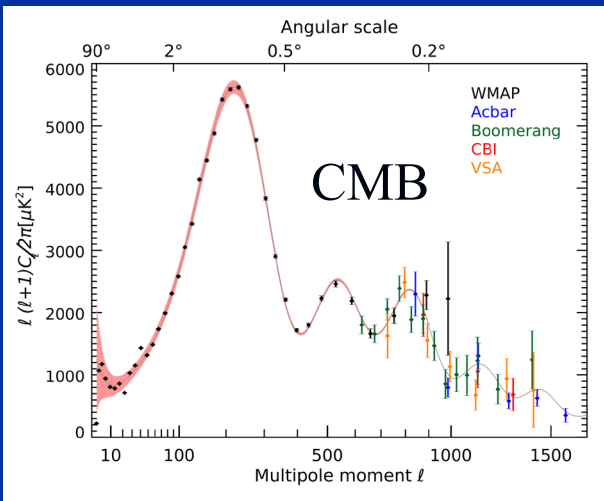
Sound Horizon $r_s = \frac{c}{\sqrt{3}H_{ls}} \int_{z_{ls}}^{\infty} \frac{dz}{[\rho(z)/\rho(z_{ls})]^{1/2}(1+R(z))^{1/2}}$

Angular distance $D_A = \frac{c}{H_0} \int_0^{z_{ls}} \frac{dz}{[\rho(z)/\rho_0]^{1/2}} \sim \frac{1}{H_0}$

r_s & D_A depend on expansion history (matter contents $\rho(z)$)

It's almost impossible to solve H_0 *and* S_8 Tension

- Does H_0 increase?
- Does S_8 decrease?
- Are z_{eq} and the acoustic peak heights, θ_s preserved?
- Are the BAO observables D_M/r_d and D_H/r_d and SN consistent with observations?
- Are the TT, TE, EE, and CMB lensing spectra consistent with observations?
- Is the matter power spectrum $P(k)$ consistent with Lyman- α forest and galaxy cluster observations?
- Does DM explain small scale issues?



$$H_0 = \theta_s H_{ls} \frac{\int_{z_{ls}}^0 \frac{c dz}{[\rho(z)/\rho_0]^{1/2}}}{\int_{\infty}^{z_{ls}} \frac{c_s(z) dz}{[\rho(z)/\rho(z_{ls})]^{1/2}}}$$

late time (decrease $\rho(z)$)

early time (increase $\rho(z)$)

To increase H_0 we can ($\theta_s = \frac{r_s}{D_A}$ fixed)

1) early time solutions (decrease r_s to decrease D_A)

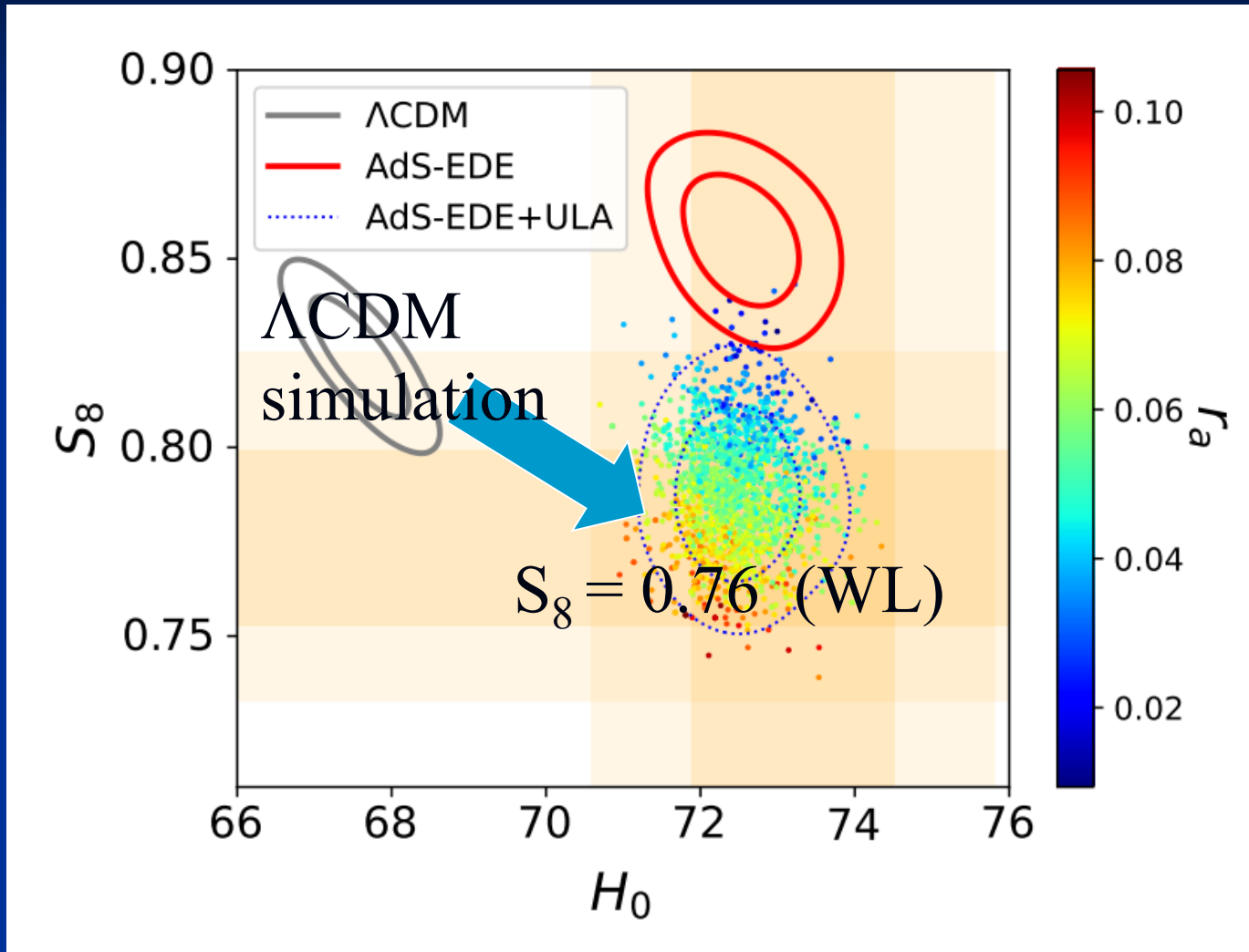
- increase $\rho(z)$ (extra radiation, EDE) **just before ls**
- but need more perturbation (CMB acoustic peak)
- **worsen S_8 tension**, BBN (extra radiation), exotic matter, fine tuning, coincidence?

$$D_A = \frac{c}{H_0} \int_0^{z_{ls}} \frac{dz}{[\rho(z)/\rho_0]^{1/2}}$$

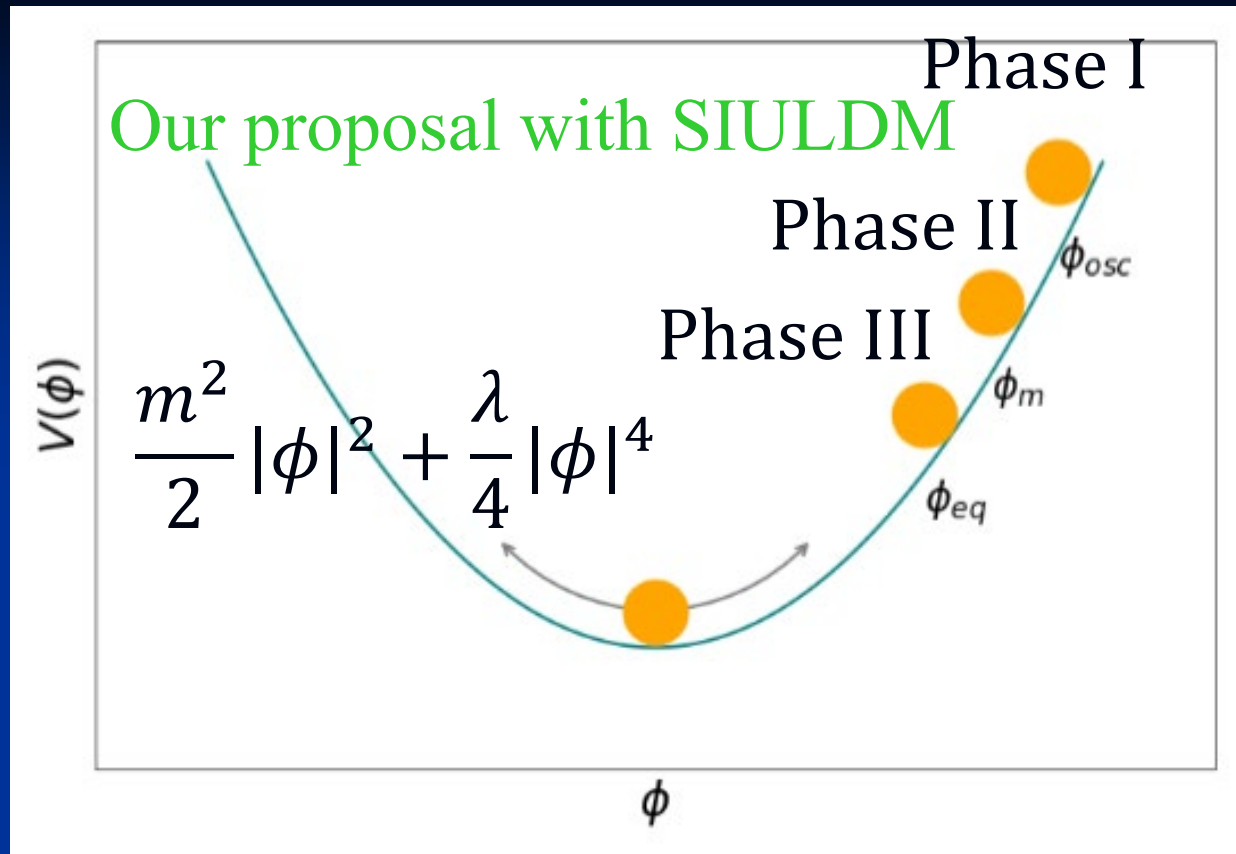
2) late time solutions (r_s & D_A fixed, increase the integral)

- decrease $\rho(z)$ by decaying DM after ls and/or increase (phantom) DE to increase the integral, local void, late neutrino?
- late evolution delicately changes
- **tensions with SN, BAO**; null energy violation; fine tuning ...

Can ULDM solve both H_0 and S_8 ?



→ need exotic DE & ULDM with $m < 10^{-24}$ eV

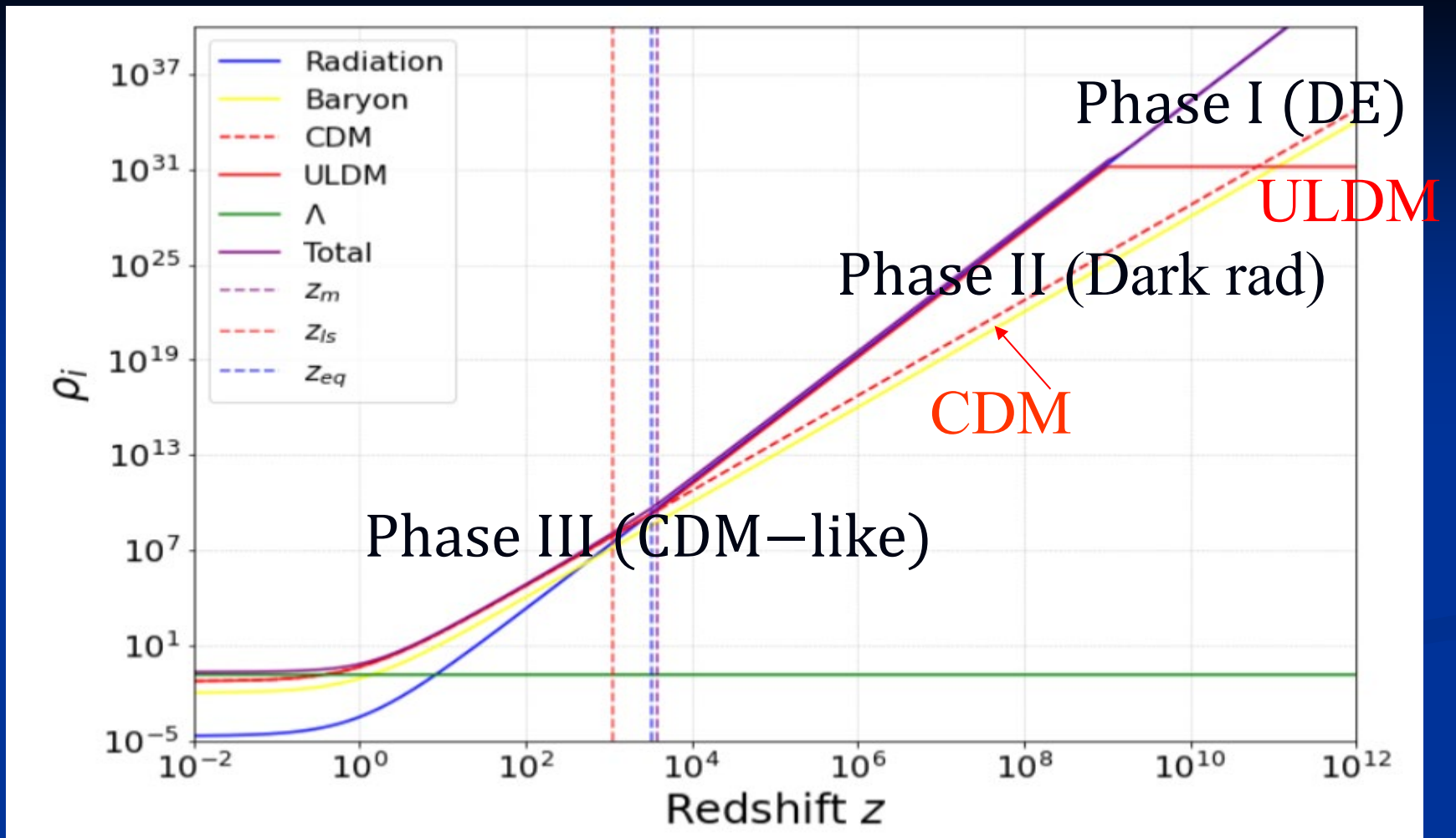


JLee
2502.11568

$V(\phi) \propto \phi^n$ decays with an equation of state $w = \frac{(n-2)}{(n+2)}$ Turner

$\left\{ \begin{array}{l} \text{DE-like (slow-roll) for } z_{osc} < z \text{ (Phase I)} \\ \text{Dark rad-like } (\phi^4 > \phi^2) \text{ or } z_m < z < z_{osc} \text{ (Phase II)} \\ \text{CDM } (\phi^4 < \phi^2) \text{ or } z < z_m \text{ (Phase III)} \end{array} \right.$

SIULDM alone is enough!



- SIULDM acts as a natural extra radiation comp. just before ls
- We don't need exotic matter or fine tuning to solve H_0 tension?

$$\frac{H_0(ULDM)}{H_0(CDM)} \simeq \frac{\int_{z_{ls}}^{\infty} \rho(CDM)(z) \, {}^{-1/2} \frac{dz}{\sqrt{1+R(z)}}}{\int_{z_{ls}}^{\infty} \rho(ULDM)(z) \, {}^{-1/2} \frac{dz}{\sqrt{1+R(z)}}}$$

$$\rho(ULDM)(z) \propto \begin{cases} \sqrt{\omega_r(1+z)^4 + \omega_m(1+z)^3 + \omega_\Lambda} & \text{for } z < z_m \text{ (Phase III)} \\ \sqrt{\omega_r(1+z)^4 + \omega_\phi \frac{(1+z)^4}{1+z_m} + \omega_b(1+z)^3 + \omega_\Lambda} & \text{for } z_m < z < z_{osc} \text{ (Phase II)} \\ \sqrt{\omega_r(1+z)^4 + \omega_b(1+z)^3 + \omega_\phi \frac{(1+z_{osc})^4}{1+z_m} + \omega_\Lambda} & \text{for } z_{osc} < z \text{ (Phase I)} \end{cases}$$

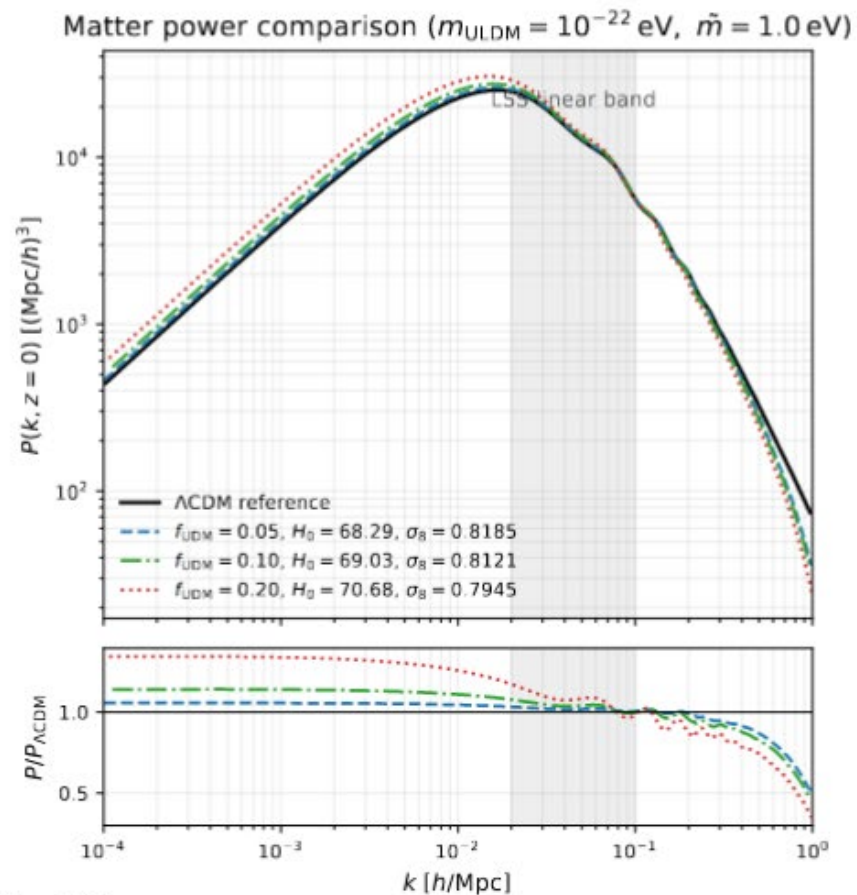
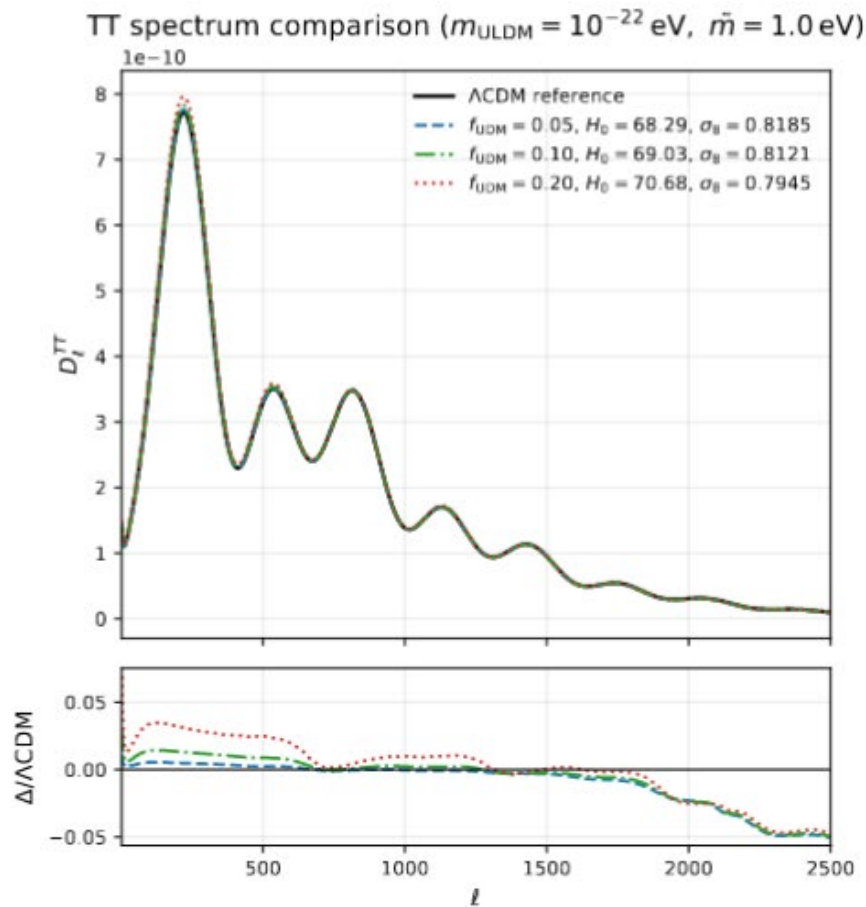
$\omega_i = h^2 \Omega_i$

where $\omega_b = 0.022, \omega_\phi = 0.119, \omega_r = 4.2 \times 10^{-5}, \omega_\Lambda = 0.314$ from Planck's Λ CDM values

Using the densities, we numerically obtain $\frac{H_0(ULDM)}{H_0(CDM)} \simeq 1.049$ for $\tilde{m} = 0.9$

* $\tilde{m}$ is near lower bound, input data assumed Λ CDM, age problem?

Diagnostics with modified Class with perturbation



Successful mixed cases, $\theta_{\text{reinit}} = \text{kg}$

f_{UDM}	$H_0 [\text{km s}^{-1} \text{Mpc}^{-1}]$	σ_8
0.05	68.29	0.8185
0.10	69.03	0.8121
0.20	70.68	0.7945

with SCLee in preparation

Surprisingly, in SIULDM model
Galactic dynamics, H_0 tension, S_8 tension
all indicate similar $\tilde{m} \sim O(1) \text{ eV}$

why????

Considering tiny $m \geq 10^{-22} \text{ eV}$, this consistence
is non-trivial

ULDM and neutrino

3. How were such light particles created? We first notice that the required mass can be written as

Sin3
$$m \sim \frac{m_{\nu_\tau}^2}{M_{GUT}} \quad (23)$$

if we take a tau neutrino mass of about half an MeV. (The experimental upper limit is about a few tens of MeV). This is so far incidental. Recently, it was suggested by Hill *et al.* [4] that there is a mechanism to create a small-mass particle of $m \sim m_{\nu_e}^2 / M_{GUT} \sim 10^{-27}$ eV, and they suggested that if this particle is created by a phase transition after the decoupling then it can be used to solve the large scale structure formation. This idea has attracted a lot of attentions and we refer the reader to the literature on late-time phase-transition theories. (See Refs. 7-9 and references therein.) In this context, it is natural for

- $\tilde{m}$ might be related to neutrino mass (See-saw?)
- SIULDM as majorons?

ULDM as majoron?



real

$$\mathcal{L}_{\text{Yukawa}} = -y\phi\bar{\nu}^c\nu \quad \text{Majorana } \nu$$

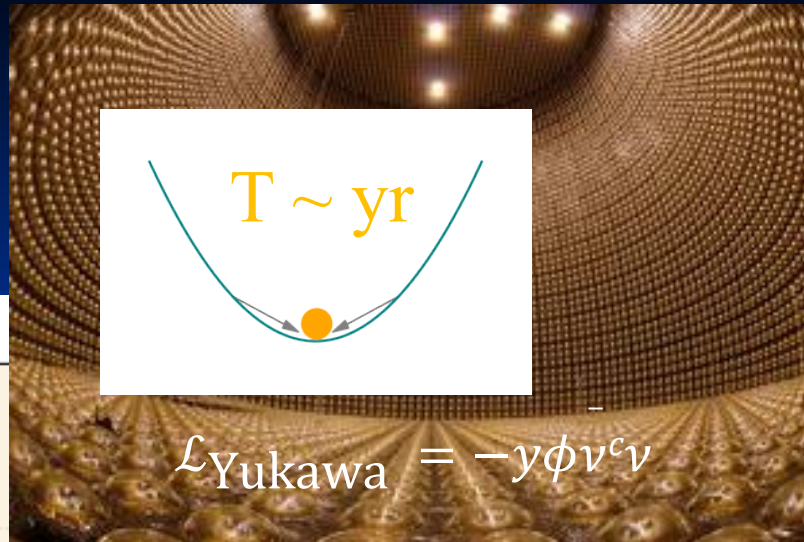
$$m_\nu = 0.1\text{eV} \left(\frac{y}{10^{-25}} \right) \left(\frac{v}{10^{15}\text{GeV}} \right)$$

$$y = \frac{m_\nu \sqrt{\lambda}}{m_\phi} = 10^{-25} \left(\frac{m_\nu}{0.1\text{eV}} \right) \left(\frac{m_\phi}{10^{-22}\text{eV}} \right)^{-1} \left(\frac{\lambda}{10^{-92}} \right)^{1/2}$$

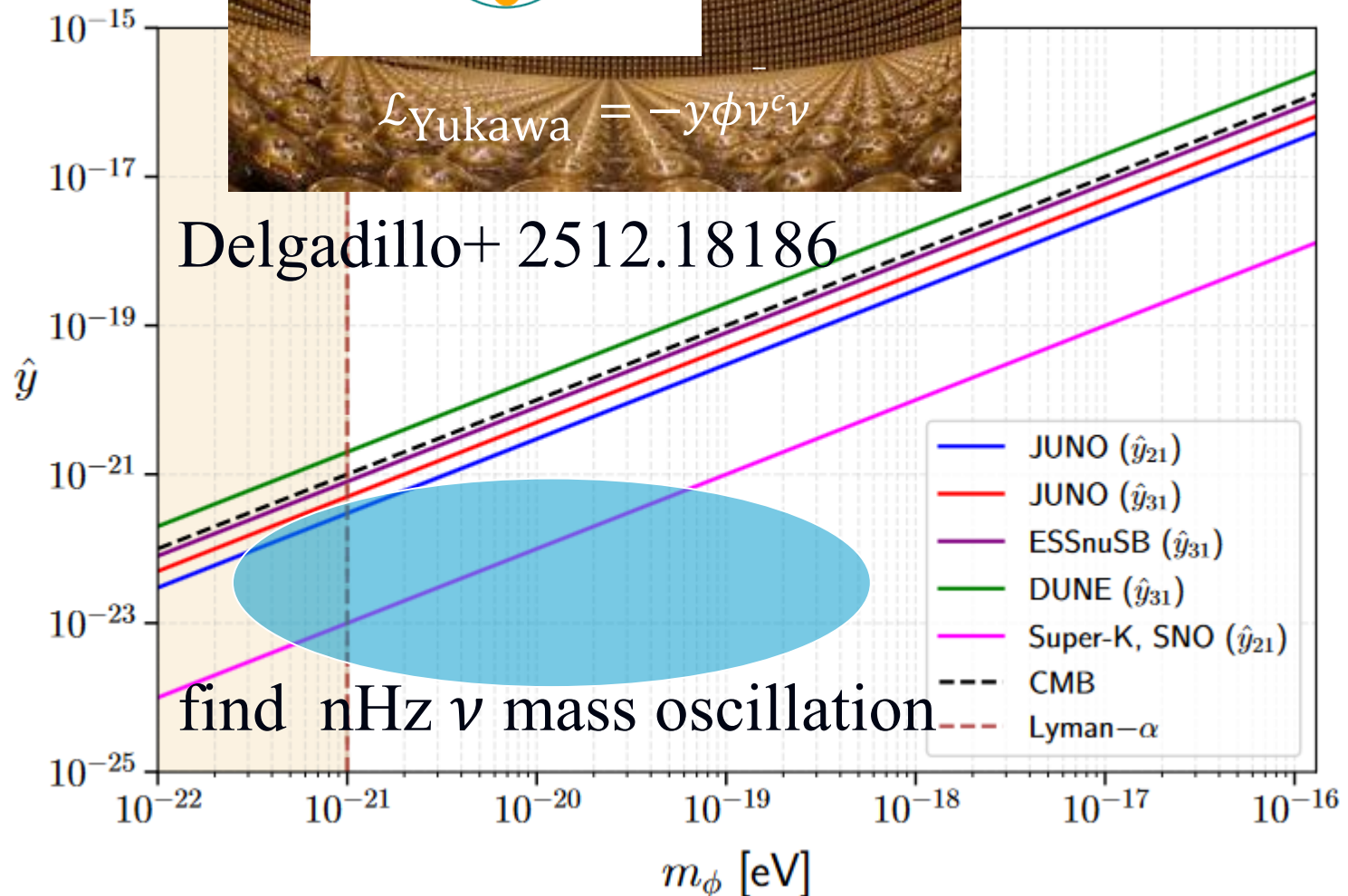
$$= 10^{-25} \left(\frac{m_\nu}{0.1\text{eV}} \right) \left(\frac{m_\phi}{10^{-22}\text{eV}} \right) \left(\frac{10\text{ eV}}{\tilde{m}} \right)^2$$

One-loop quantum correction from the Yukawa is $O(y^4) \simeq 10^{-8}\lambda \ll \lambda$

$$0.06\text{ eV} < \Sigma m_\nu < 0.071\text{ eV (DESI)}$$



Delgadillo+ 2512.18186



Hierarchy?

JLee 2410.02842

GUT

$$m^2(T) = m^2 + \lambda T^2/4$$
$$T_c \simeq m/\sqrt{\lambda} = \tilde{m}^2/m \sim 10^{15} \text{ GeV}$$

neutrino

$$\tilde{m} \equiv m/\lambda^{1/4} \sim 10 \text{ eV}$$

reverting seesaw?



EW

$$T_{EW} \sim (T_c \tilde{m})^{1/2} \sim 10^3 \text{ GeV}$$

induces EW scale

PNGB
ULDM
(Majoron?)

$$\text{ex) } m \sim 10^{-22} \text{ eV}, \lambda \sim 10^{-92}$$

galaxy observations

Conclusions

ULDM with $m \sim 10^{-22}$ eV or

self-interacting ULDM with $\frac{m}{\lambda^{1/4}} \sim 1 \text{ eV}$

seems to be a viable alternative to CDM

→ might solve many mysteries of cosmology and astrophysics.

→ can be detected soon

Cold and fuzzy dark matter

Wayne Hu (Princeton, Inst. Advanced Study), Rennan Barkana (Princeton, Inst. Advanced Study), Andrei Gruzinov (Princeton, Inst. Advanced Study)
Mar, 2000

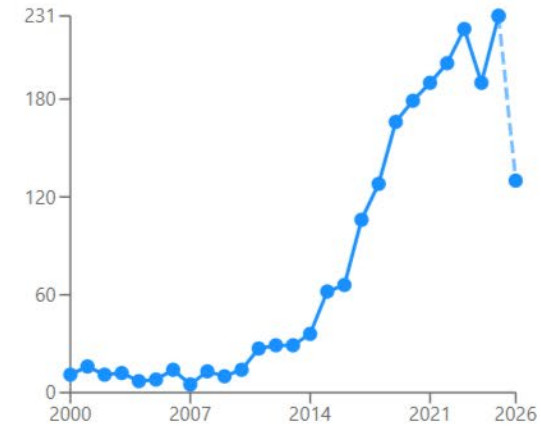
Suppress small scale

4 pages
Published in: *Phys.Rev.Lett.* 85 (2000) 1158-1161
e-Print: [astro-ph/0003365](#) [astro-ph]
DOI: [10.1103/PhysRevLett.85.1158](#)
View in: [ADS Abstract Service](#)

 pdf  cite  claim

 reference search  2,115 citations

Citations per year



Ultralight scalars as cosmological dark matter

Lam Hui (Columbia U.), Jeremiah P. Ostriker (Columbia U. and Princeton U. Observ.), Scott Tremaine (Princeton, Inst. Advanced Study), Edward Witten (Princeton, Inst. Advanced Study)
Oct 26, 2016

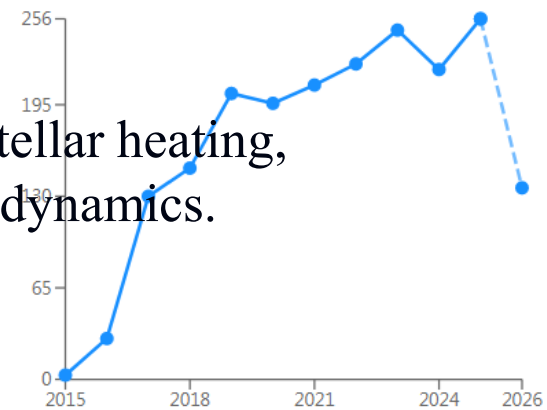
32 pages
Published in: *Phys.Rev.D* 95 (2017) 4, 043541
Published: Feb 28, 2017
e-Print: [1610.08297](#) [astro-ph.CO]
DOI: [10.1103/PhysRevD.95.043541](#)
View in: [ADS Abstract Service](#)

- FDM predictions to dwarf galaxies, stellar heating, subhalo survival, and globular-cluster dynamics.
- ULA miracle

 pdf  cite  claim

 reference search  2,004 citations

Citations per year



ULA miracle

$$I = \int d^4x \sqrt{g} \left[\frac{1}{2} F^2 g^{\mu\nu} \partial_\mu a \partial_\nu a - \mu^4 (1 - \cos a) \right]$$

$$m = \frac{\mu^2}{F}$$

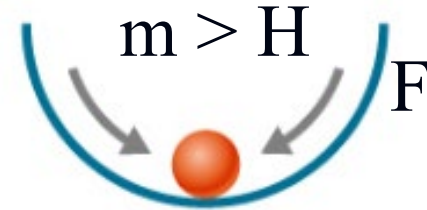
$$\ddot{a} + 3H\dot{a} + m^2 \sin a = 0$$

$$\text{oscillation starts at } H \sim \frac{T_{osc}^2}{M_P} = m$$

$$\text{MDE starts at } T_1 \sim 1 \text{ eV} \rightarrow \frac{\mu^4(DM)}{T_{osc}^4(rad)} \rightarrow \frac{\mu^4 T_{osc}}{T_{osc}^4 T_1} \sim 1$$

$$F = \frac{\mu^2}{m} \sim \frac{M_P^{3/4} T_1^{1/2}}{m^{1/4}} \sim 10^{17} \text{ GeV} \quad \text{typical field value}$$

$$\Omega_a \sim 0.1 \left(\frac{F}{10^{17} \text{ GeV}} \right)^2 \left(\frac{m}{10^{-22} \text{ eV}} \right)^{1/2} \quad \text{ULA miracle?}$$



Hui et al 2017

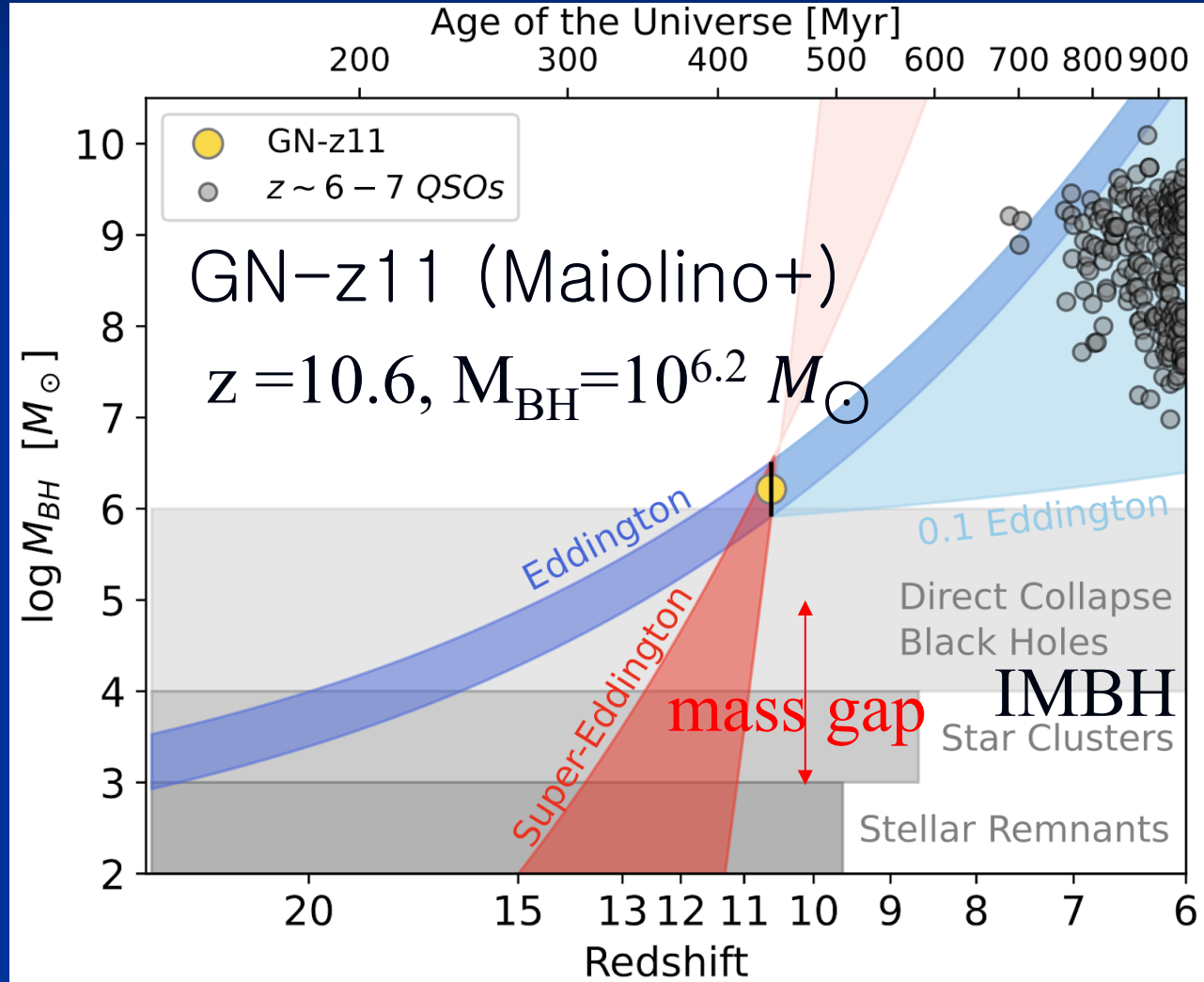
ULDM naturally explains DM density with GUT scale.
 This holds for generic ULDM with a quadratic pot.

Impossibly early SMBH?

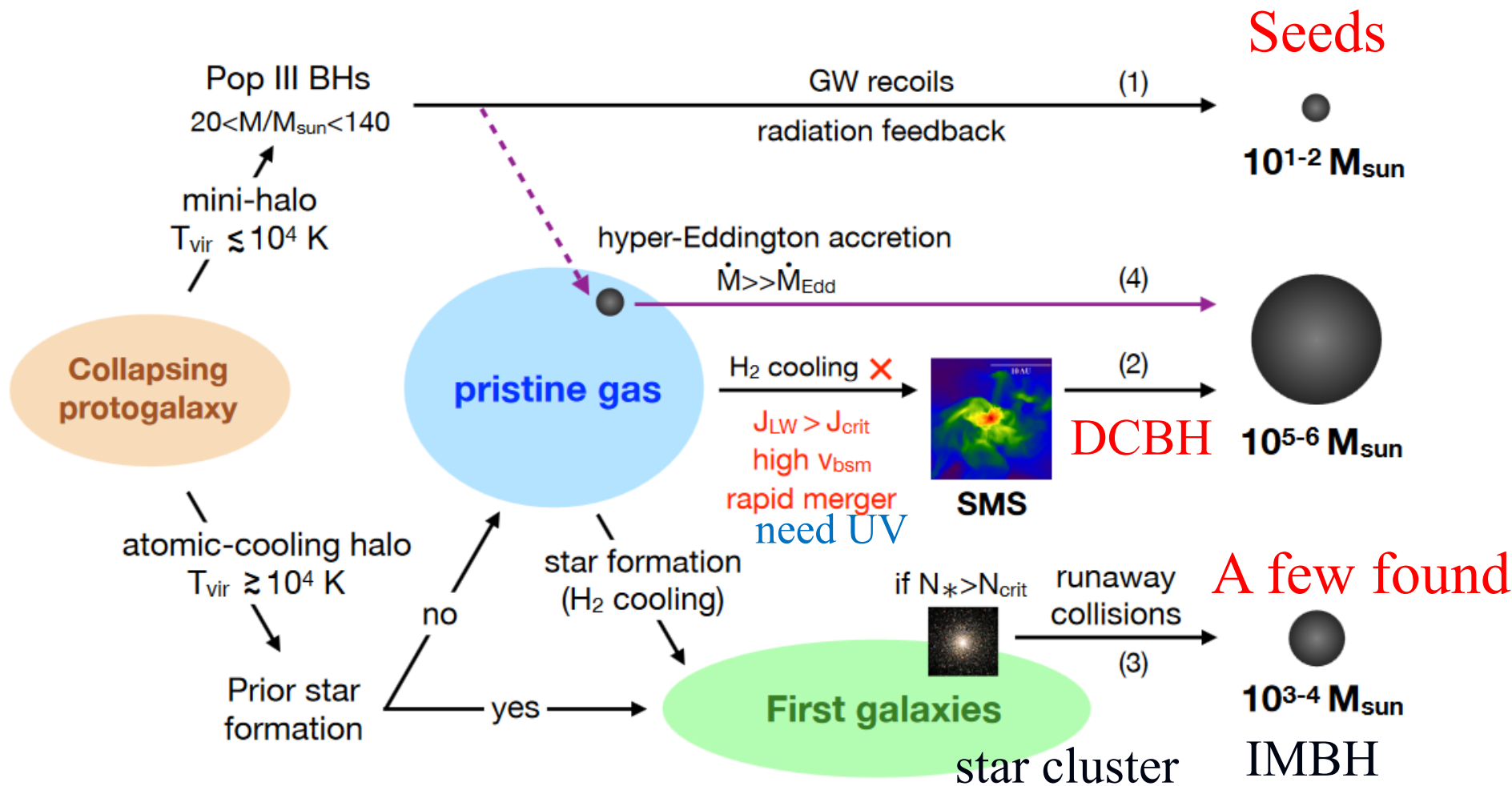
SMBHs (Quasar) with $M \sim 10^6 M_\odot$ were already mature at around $z=11$ (~ 0.45 Gyr)



Cosmic
chicken-egg
problem



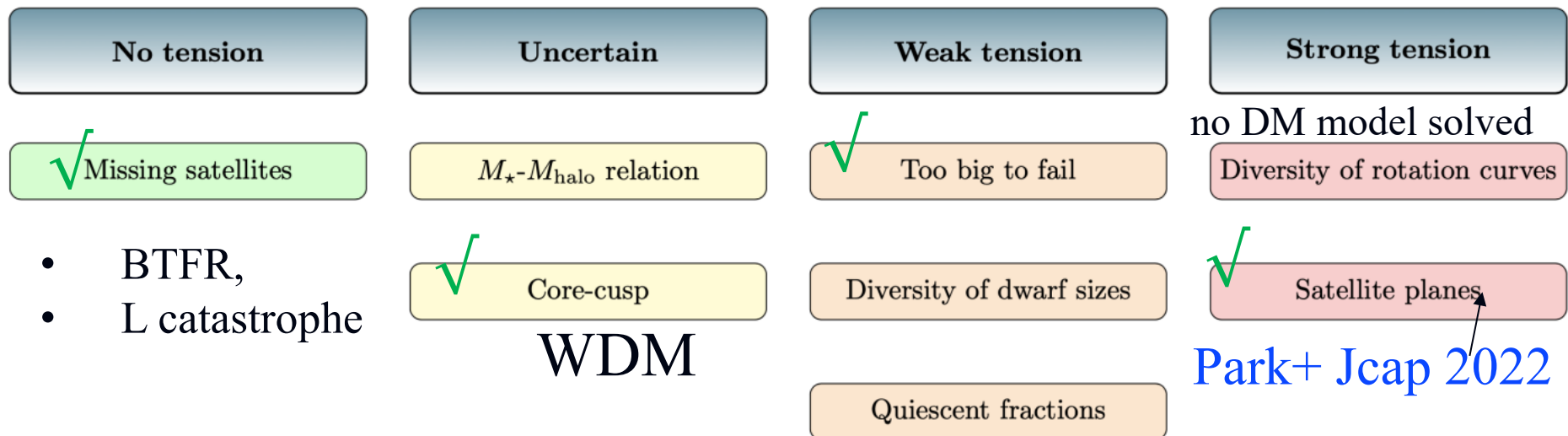
How SMBH seeds can form



Small scale issues with CDM (simulations)

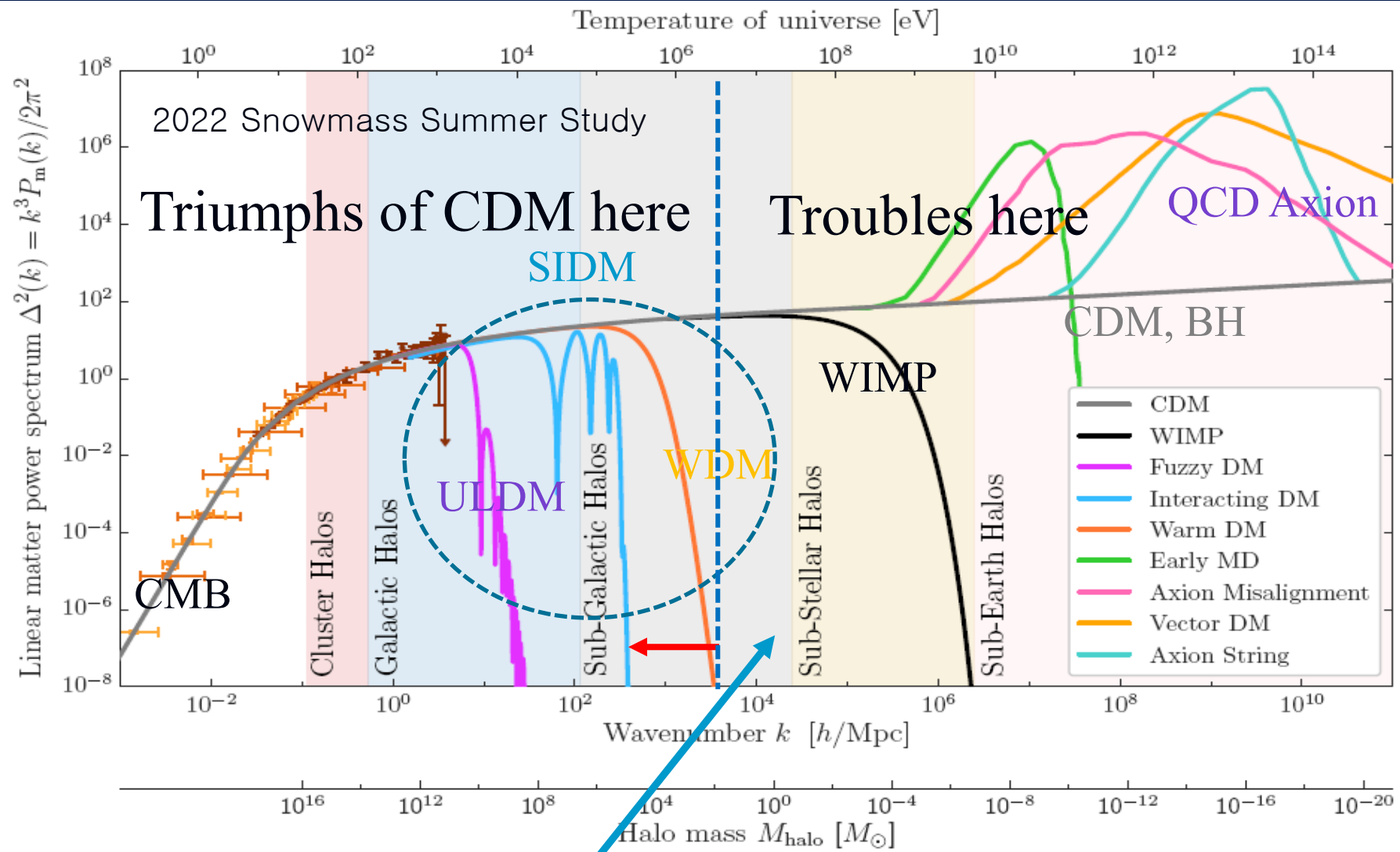
Sales+ 2206.05295

Λ CDM Tensions with Dwarf Galaxies



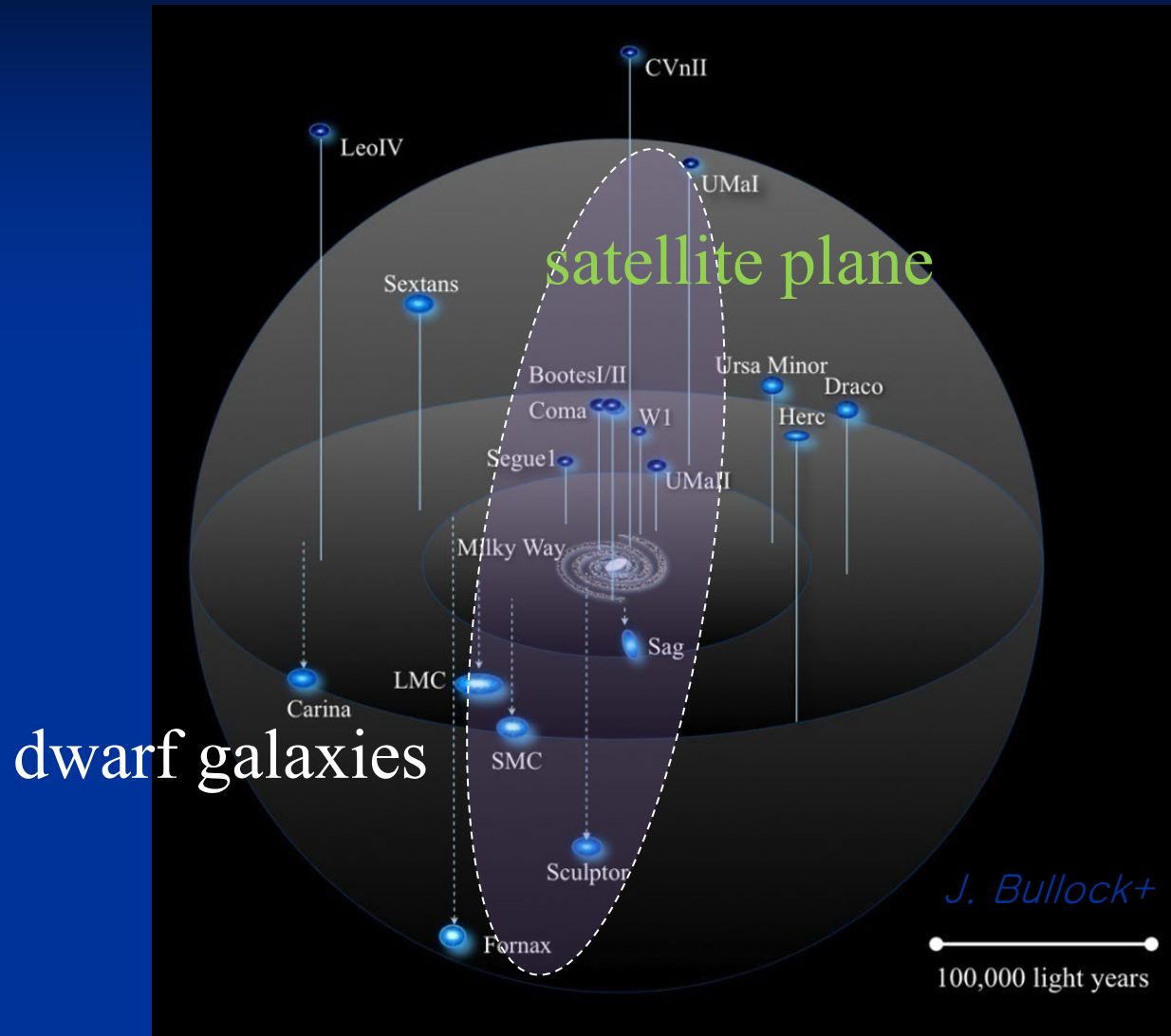
- Key problem is how to suppress small scale structures < dwarf galaxies.
 → we need a new CDM → ULDM with $m \sim 10^{-22}$ eV might solve many of these
- Still unsolved problems seem to be related to Baryon-DM relation
- Can baryon physics + more precise numerical simulation + more observations save CDM?

Galaxies are DM dominated and seem to have ~ kpc size scale



No DM star or planet found so far → DM has kpc length scale?

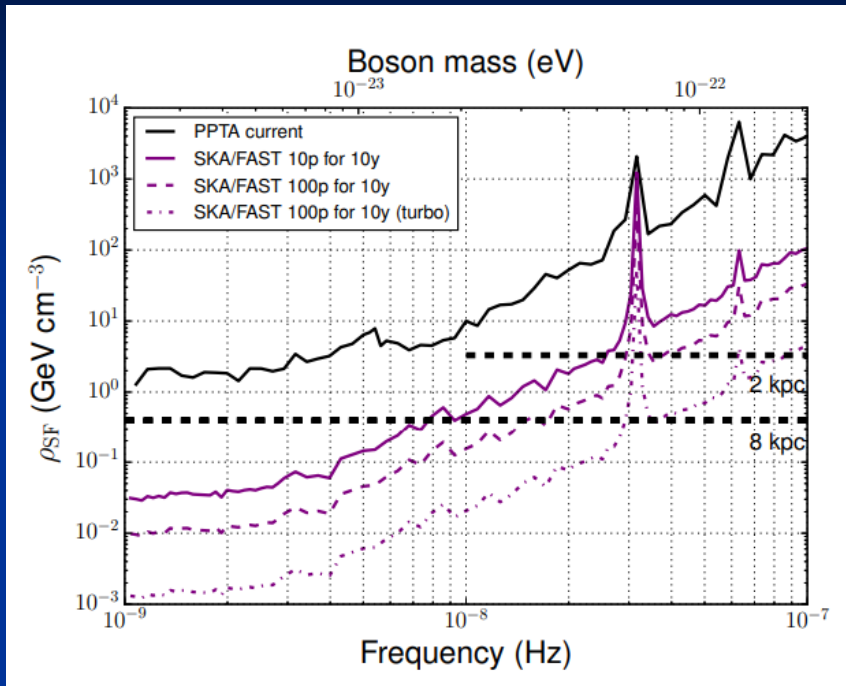
Galaxies observed



Minimum mass
 $\sim 10^6 M_\odot$

Any good DM model should explain observed galaxies

GW background detected by pulsar timing array



1810.03227

ULDM has
intrinsic osc time scale
period $\sim 1/m \sim \text{yrs}$

$$\omega = \frac{1}{2.5 \text{ months}} \frac{m}{10^{-22} \text{ eV}}$$

