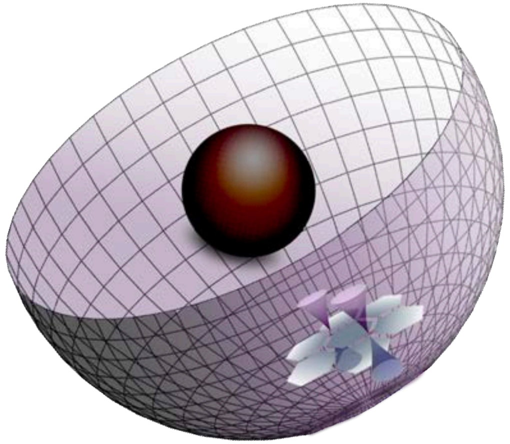
Spacetime picture of strange metal: SM vs QG

The origin of SYK and ER=EPR in QFT



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(2503.19431) Y. Wang+SJS

Introduction

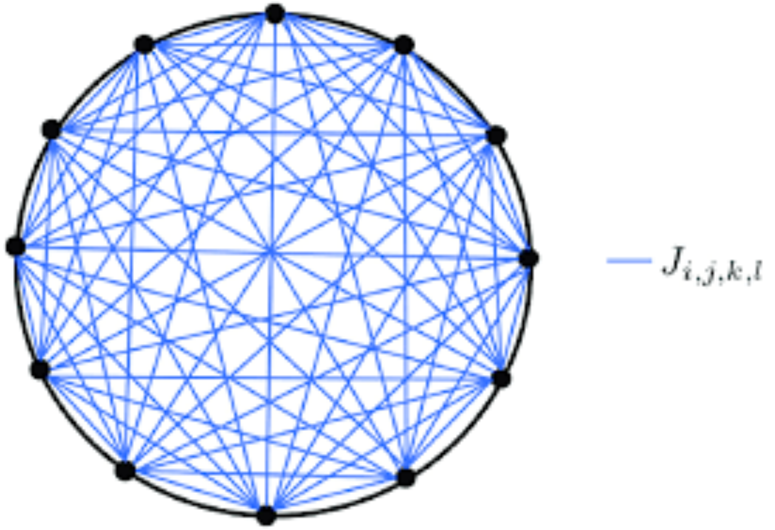
1. SYK
2. SM/Non-Fermi Liquid

SYK model: one of driving power of string theory of last 2 decades, hence a Juke box

$$H = \frac{1}{4!} \sum_{i,j,k,l=1}^{N} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l$$

$$\mathscr{L} = \chi_i \partial_t \chi_i - H$$

$$\overline{J_{ijkl}} = 0, \qquad \overline{J^2_{ijkl}} = \frac{3!\, J^2}{N^3}, \qquad \{\chi_i, \chi_j\} = \delta_{ij}$$



0-dim all to all

SYK model<-> JT: Holographic dual,

originally model for Strage metal , it recall BH, WH, Hawking, Coleman

Strange Metal

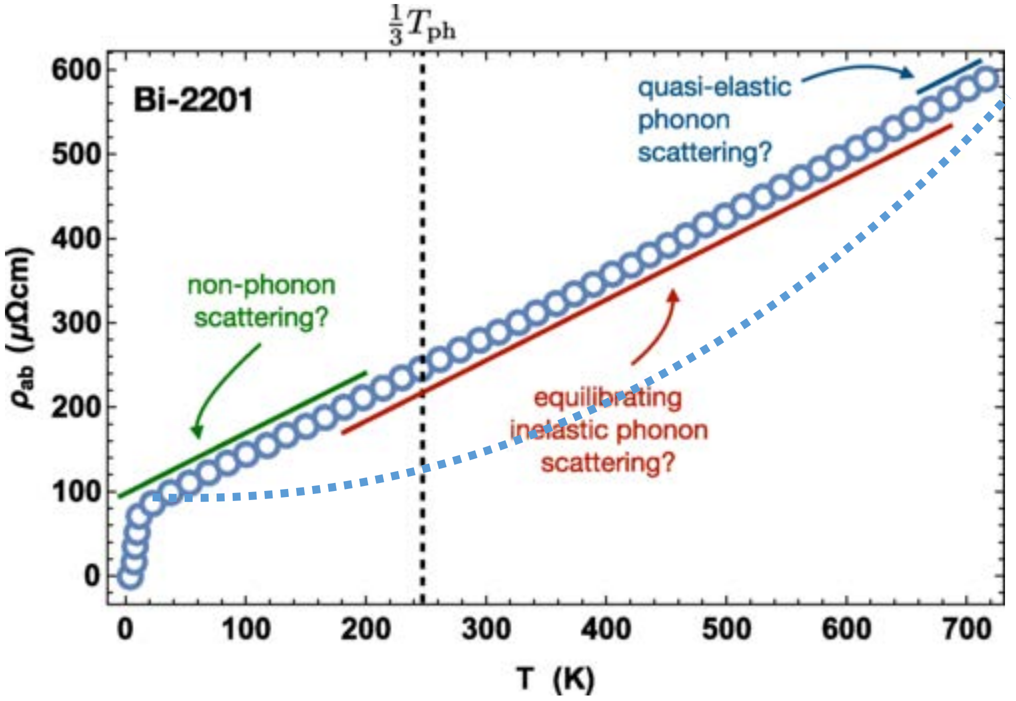
Main interest: expected to be the essence of High Tc SC by being its Normal state.

$$\rho \sim T$$

Not T^2, hence non Fermi liquid

References

- P.W Anderson, Phys. Rev. Lett. 67 (1991) 2092.
- P.W. Anderson, Physics Today 66 (2013) 9.



- Phenomena:
  i) momentum is dissipated Strangely Fast : $1/\tau \sim T$,
  ii) T_c is high.

- Exp: Universal behavior of strongly correlated metal.

- Theory: has been Difficult in writing a QFT model for 3 decades.

Brief History of SYK model

- SY model : $H = \Sigma_{i<j} J_{ij} S_i \cdot S_j,$ random Heisenberg model.
  Situation of the time: hot issue: High Tc SC; P. Anderson: Normal state of RVB: = massless spin liquid
  Motive: possibility of non-fermi liquid (=strange metal=metal without quasi-particle)?

- Main result: gapless spectrum. $\chi(\omega, T) \sim T^{1/2} f(\omega/T)$, $\Gamma \sim T$ -> QC behavior,
  non-Fermi: (Similar to Holography)
- Why disorder?
  i) frustration ~ disorder. But ….
  ii) Make the model solvable in large N.

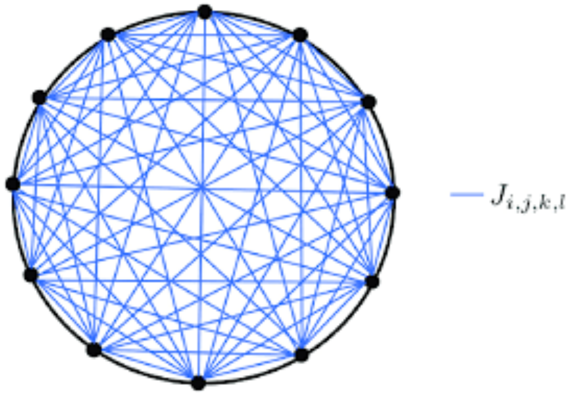
- Not very popular before Kitaev:
  i) Random coupling is too artificial, why connected to ….entanglement.
  ii) Spin no fermion
  iii) 0-d no transport.

- Kitaev(2015): I) Fermonize by $S_i \sim \sigma_{ijk} f_j f_k$, ii) OTOC 4point function ~ $e^{\lambda t}$ ~ Chaos
  SYK model ~ BH : holographic realization (still no transport)
- Dual gravity ~ JT gravity, wormhole. Randomness~Qm gravity

SYK : all to all int.

$$L = \frac{1}{2}\sum_{i=1}^{N} \chi_i \partial_\tau \chi_i - \frac{1}{4!}\sum_{i,j,k,l=1}^{N} J_{ijkl}\chi_i\chi_j\chi_k\chi_l,$$

$$\langle J_{ijkl} J_{i'j'k'l'} \rangle = J^2 \delta_{ii'}\delta_{jj'}\delta_{kk'}\delta_{ll'}$$



Hint: if I,j,k =discretized spacetime,
then all to all means 0-dim.
SM and H-Tc is reasonable!

- No Calulability for transport.
- But conceptually it explain everything: SM as well as high Tc SC.
- SYK : all to all int is postulated. If this is essence, how to derive it?

i.e, How non-local all-to-all interaction can be generated out of local field theory.

ANS: Disorder=> non-locality+ WH

A QFT model for Linear Resistivity

2023.Aug. Patel and Sachdev et.al, wrote 2+1 model based on SYK model.

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RESEARCH ARTICLE | PHYSICS

Universal theory of strange metals from spatially random interactions

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$$\boxed{\rho = \rho_0 + AT}$$

Yukawa-SYK in 2+1 dim.

$$\mathscr{L} = \mathscr{L}_\psi^0 + \mathscr{L}_\phi^0 + \sum_{ijk} g_{ijk}(x)\phi_i\psi_j\psi_k(t,x) + \sum_{ij} v_{ij}(x)\psi_i\psi_j$$

$$\begin{cases} \langle g_{ijl}(\mathbf{r})\rangle = 0 \\ \langle g^*_{ijl}(\mathbf{r})g_{i'j'l'}(\mathbf{r}')\rangle = g^2\delta_{ii'jj'll'}\delta(\mathbf{r}-\mathbf{r}') \end{cases}$$

Gaussian weight

$$\boxed{\rho = \rho_0 + AT}$$

Resistivity linear in T at Low tem.

References

- A.A. Patel, H. Guo, I. Esterlis and S. Sachdev, Science 381 (2023) abq6011
- I. Esterlis, H. Guo, A.A. Patel and S. Sachdev, Phys. Rev. B 103 (2021) 235129
- H. Guo, A.A. Patel, I. Esterlis and S. Sachdev, Phys. Rev. B 106 (2022) 115151

previous works:

Yili Wang, XianHui Ge, YK Han + SJS

1. Yili Wang, Xian-Hui Ge, and S. Sin, arXiv: 2406.11170
2. Yili Wang, Young-Kwon Han, Xian-Hui Ge, and S. Sin, arXiv: 2501.07792
3. Most general strange metal model. (2507.09442)
4. ER=EPR and strange metal from quantum entanglement. (2503.19431)

1 How unique is the spatially random disorder mechanism?

Apart from Yukawa-SYK, & Vector-SYK also works $\phi\psi\psi$, $a_\mu\psi\partial_\mu\psi$:

Conjecture: any minimal tensor model will work. $B_{\mu\nu\ldots}\psi\partial_{\mu\nu\ldots}\psi$ (if cubic, it works).

2 Does not work in 3+1 => CuO_2 plane is important.

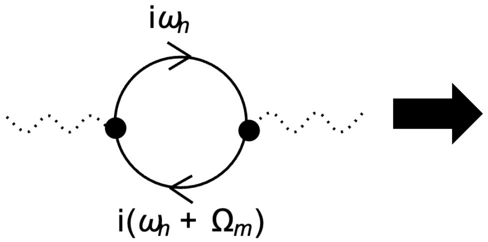
3. No T^2 inverse Hall angle behavior

4. Non-cubic interactions are irrelevant.

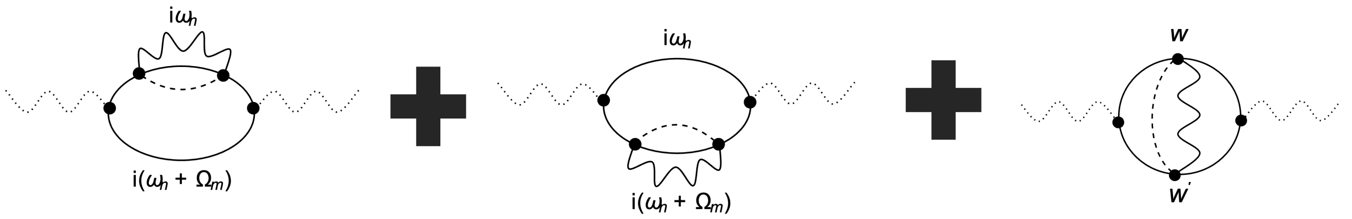
Why universal?

$\langle a_\mu a_\nu\rangle \sim \langle\phi\phi\rangle, \partial_\mu \sim k_{F,\mu}$ do not scale in the presence of FS

Q:Physical mechanism of linear T? Very hard to read off from calculation!



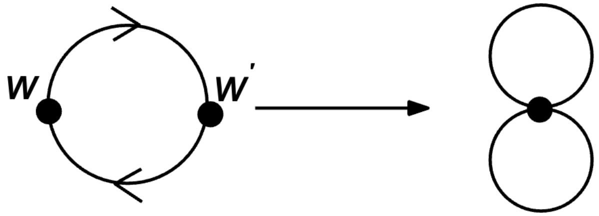
residual resistivity T^0



$$\left\{ \begin{array}{l} = 0 \cdot T,\ for\ g,\ or\ g_{ijk} \\ = T,\ for\ g_{ijk}(x) \end{array} \right.$$

$$\left\{ \begin{array}{l} \sim T^2,\ for\ g,\ or\ g_{ijk} \\ = 0,\ for\ g_{ijk}(x) \end{array} \right.$$

In the presence of local disorder



Summary of various model(Yili Wang+SJS, arXiv: 2406.11170)

scattering type	**resistivity**	
$v_{ij}(\boldsymbol{r})\psi_i^\dagger\psi_j$	$\sim \rho_0$	
$v_{ij}\psi_i^\dagger\psi_j$	no contribution	
$u\psi^\dagger\psi^\dagger\psi\psi$+impurities	$\sim \rho_0 + AT^2$	Textbook
$g\phi\psi^\dagger\psi$	$\cancel{T^{4/3}}$ absence	Kim,Wen [cond-mat/9405083], H. Guo, A.A. Patel, I. Esterlis and S. Sachdev,, Phys. Rev. B 106 (2022) 1151
$g_{ijk}\phi_i\psi_j^\dagger\psi_k$	$\cancel{T^{4/3}}$ absence	I. Esterlis, H. Guo, A.A. Patel and S. Sachdev, Phys. Rev. B 103 (2021) 235129 [2103.08615].
$g_{ijk}\phi_i\psi_j^\dagger\psi_k + v_{ij}(\boldsymbol{r})\psi_i^\dagger\psi_j$	$\sim \rho_0 + \cancel{AT} + BT^2$	*
$g_{ijk}(\boldsymbol{r})\phi_i\psi_j^\dagger\psi_k$	$\sim T$	
$g_{ijk}(\boldsymbol{r})\phi_i\psi_j^\dagger\psi_k + v_{ij}(\boldsymbol{r})\psi_i^\dagger\psi_j$	$\sim \rho_0 + AT$	**

Knowing the answer is one thing, understanding why is another.

Observation I: All to all int. out of local Action

$$\mathscr{L} = \mathscr{L}^0_\psi + \mathscr{L}^0_\phi + \sum_{ijk} g_{ijk}(x)\phi_i\psi_j\psi_k(t,x) + \sum_{ij} v_{ij}(x)\psi_i\psi_j$$

This is local theory for each fixed g. But the first term in g is zero, because

$$\Gamma = \Gamma^0_K + \Gamma_1 + \Gamma_2 + \ldots,$$

$$\Gamma_{1,int} = \langle \int dx g(x)\mathcal{O}_{int}(x)\rangle_g = 0.$$ from <g>=0

The leading order is n=2. $$\Gamma_2 = \int dx dt_1 dy dt_2 \langle g(x)g(y)\rangle_g \langle \mathcal{O}_{int}(x,t_1)\mathcal{O}_{int}(y,t_2)\rangle$$

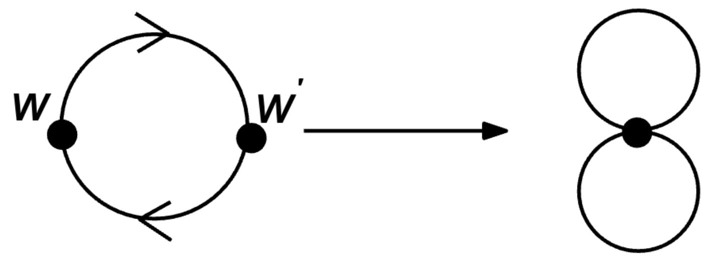
It contains wormholes=> Spatially all to all.

$$Why?\ \ \Gamma_2 = \iint dx dy \gamma_{xy}\mathscr{G}(x-y), with\ \mathscr{G}(x-y) = \prod_i G_i(x-y),\ \gamma_{xy} = \langle g(x)g(y)\rangle$$

=> Naturality of all to all

Observation II: Disorder brings us wormhole effect : a short cut for interaction in spacetime

In the presence of local disorder



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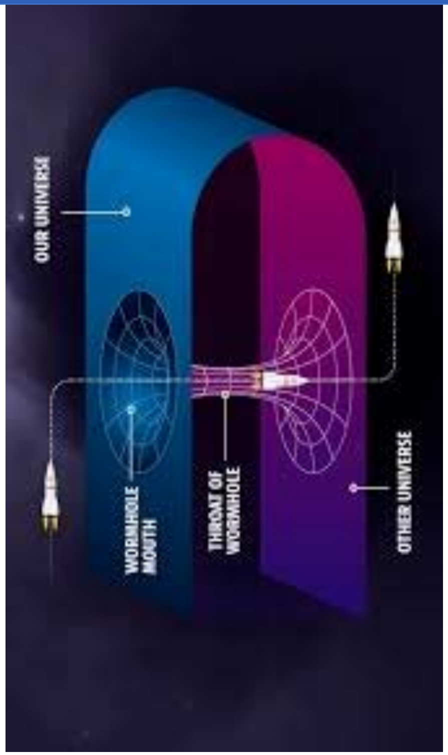
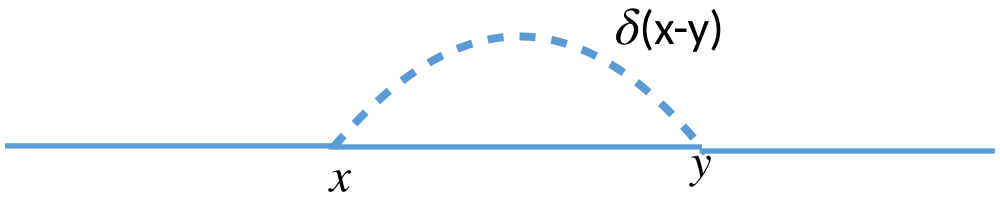


$$\langle g^*_{ijl}(\mathbf{r}) g_{i'j'l'}(\mathbf{r}')\rangle = g^2 \delta_{ii'jj'll'}\delta(\mathbf{r}-\mathbf{r}')$$

- gaussian disorder
  => effect of wormhole



- Gravitational Wormhole

ER=EPR in terms of QFT?

More on QFT wormhole (I)

Coleman's wormhole:

The result of wormhole gas (like instanton gas)

$$S_{int} = \iint dx_1 dx_2 \mathcal{O}(x_1) C(x_1, x_2) \mathcal{O}(x_2) \ \rightarrow \ localize\ by\ Hubbard\ Stratonovich$$

Our case is a special case where $C(x_1, x_2) = \delta(r_1 - r_2),\ x = (r, t)$

He reverse engineered by taking special case C=2α = constant.

$S_{int} \rightarrow -\alpha^2 - 2\alpha \int dx \mathcal{O}(x),$ Our coupling g is Coleman's α.

This process can be done for any (local, random) coupling.

=> wormholes introduces the disorder!

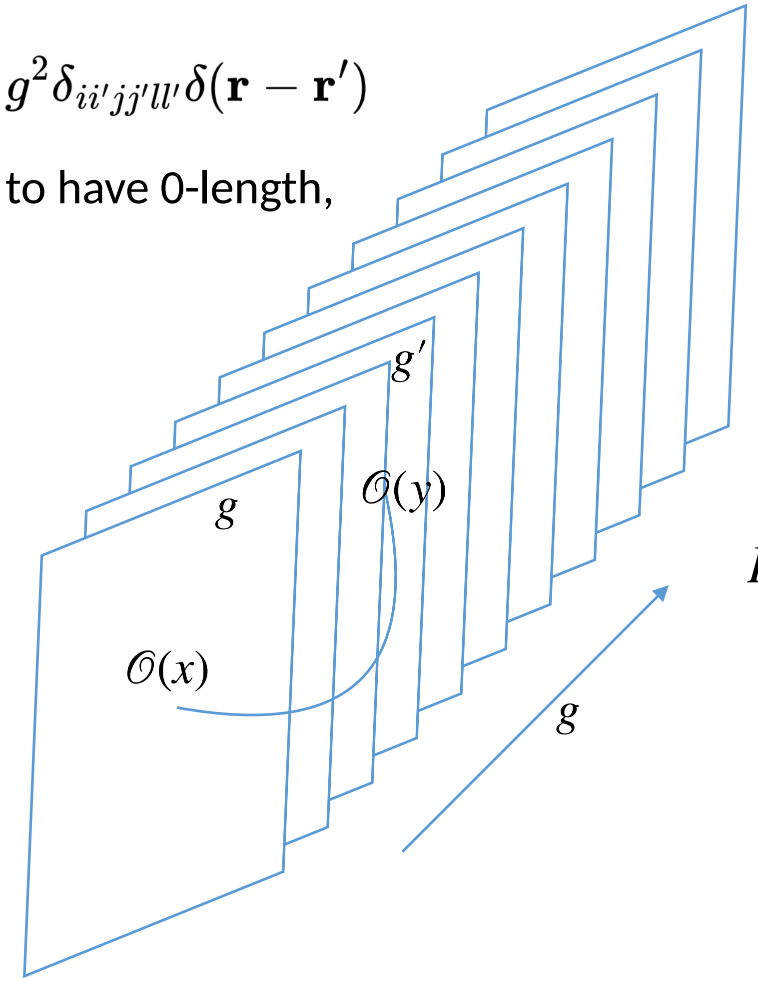
=> for each fixed g, there is a baby universe in QG or sample in CM. See figure ….

Wormholes: Disorder vs MFT: g, g(r) ; g(r,t)

$$\int D[g]P(g)e^{-g\int \mathcal{O}(x)dx} = e^{W}$$

$$\langle g^*_{ijl}(\mathbf{r})g_{i'j'l'}(\mathbf{r}')\rangle = g^2\delta_{ii'jj'll'}\delta(\mathbf{r}-\mathbf{r}')$$

makes the wormhole to have 0-length,



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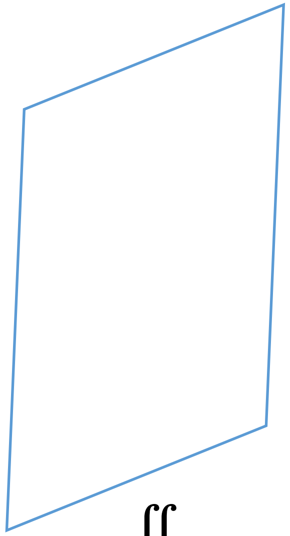


$$If\ g = const.\qquad W = \iint_{xy}\mathcal{O}(x)\mathcal{O}(y)dxdy$$ Fuzzy WH

If g(r,t) => $\int \mathcal{O}(r,t)\mathcal{O}(r,t)drdt$ MFT

If g(r) => $\iint dtdt'\int \mathcal{O}(r,t)\mathcal{O}(r,t')dr$ WH

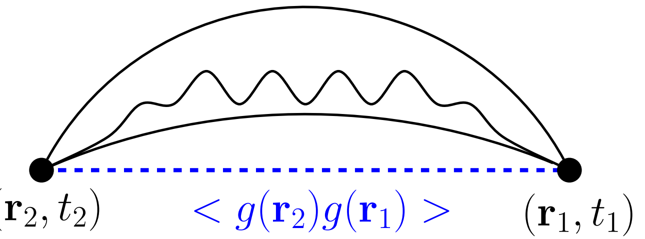
More on QFT wormhole (II)

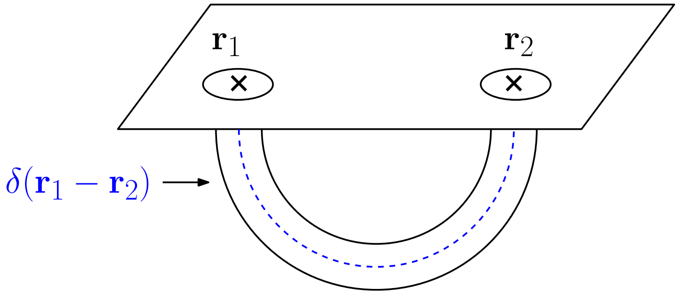
$$H_{ijk} = \phi_i \psi_j \psi_k (t, x)$$

$$-F_q = \langle \log Z \rangle_{dis} = \sum_{n=0}^{\infty} \int_{\mathbf{x}_1 \ldots \mathbf{x}_n} \Big\langle \langle \prod_{a=1}^{n} g_{ijk}(\mathbf{x}_a) H_{ijk}(\mathbf{x}_a) \rangle_{\Phi,c} \Big\rangle_{dis},$$

Later First Connected

n=2 : <g(x)g(y)> $\langle \phi\psi\psi(x)\phi\psi\psi(y) \rangle$





Effect of Wormhole

$$\langle g^*_{ijl}(\mathbf{r}) g_{i'j'l'}(\mathbf{r}') \rangle = g^2 \delta_{ii'jj'll'} \delta(\mathbf{r} - \mathbf{r}')$$

$$\Gamma_2 = \iint dxdy \gamma_{xy} \mathcal{G}(x-y), \text{with } \mathcal{G}(x-y) = \prod_i G_i(x-y), \text{ and } \gamma_{xy} = \langle g(x)g(y) \rangle$$

More on QFT wormhole (III)

$$\Gamma_2 = \iint dxdy\gamma_{xy}\mathcal{G}(x-y), with\ \mathcal{G}(x-y) = \prod_i G_i(x-y), \text{ and } \gamma_{xy} = \langle g(x)g(y)\rangle$$

1) $\gamma_{xy} \neq \delta(x-y);\ \Gamma_2 = Vol \cdot \int dz\tilde{\mathcal{G}}(z)$=finite, the averaged value of $\tilde{\mathcal{G}}(z) = \gamma_{xy}\mathcal{G}(x-y)$,

z=(x+y)/2, r=|x-y|; Assume V is large but finite. $\mathcal{G}(z)$ has causal suppressions like $\mathcal{G}(z) \sim e^{-mr}/r^{\alpha}$.

2) But if $\gamma_{xy} = \delta(x-y)$; $\Gamma_2 = Vol \cdot \mathcal{G}(0)$ > $Vol \cdot \int dz\tilde{\mathcal{G}}(z)$: <= detailed evaluation. (See table).

$$\mathcal{G}^{\#}(0) = \iint dtdt'\mathcal{G}(t-t';0) \text{ is independent of r=x-y.}$$

$$\Gamma_2 = Vol \cdot \int dz\frac{\mathcal{G}^{\#}(0)}{Vol} = \int dxdy\frac{\mathcal{G}^{\#}((0)}{Vol}:$$ propagation amplitude $\mathcal{G}(x-y) = \frac{\mathcal{G}^{\#}(0)}{Vol}$=r-indep, r=|x-y|.

=> the enhancement due to the contact interaction between the two points x and y.

-> Def. Of QFT WormHole

With Wormhole vs without WH = g vs $g_{ijk}(\mathbf{r})$

scattering type	resistivity	
$v_{ij}(\boldsymbol{r})\psi_i^\dagger\psi_j$	$\sim \rho_0$	
$v_{ij}\psi_i^\dagger\psi_j$	no contribution	
$u\psi^\dagger\psi^\dagger\psi\psi$+impurities	$\sim \rho_0 + AT^2$	Textbook
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$g_{ijk}\phi_i\psi_j^\dagger\psi_k$	$\cancel{T^{4/3}}$ absence	I. Esterlis, H. Guo, A.A. Patel and S. Sachdev, Phys. Rev. B 103 (2021) 235129 [2103.08615].
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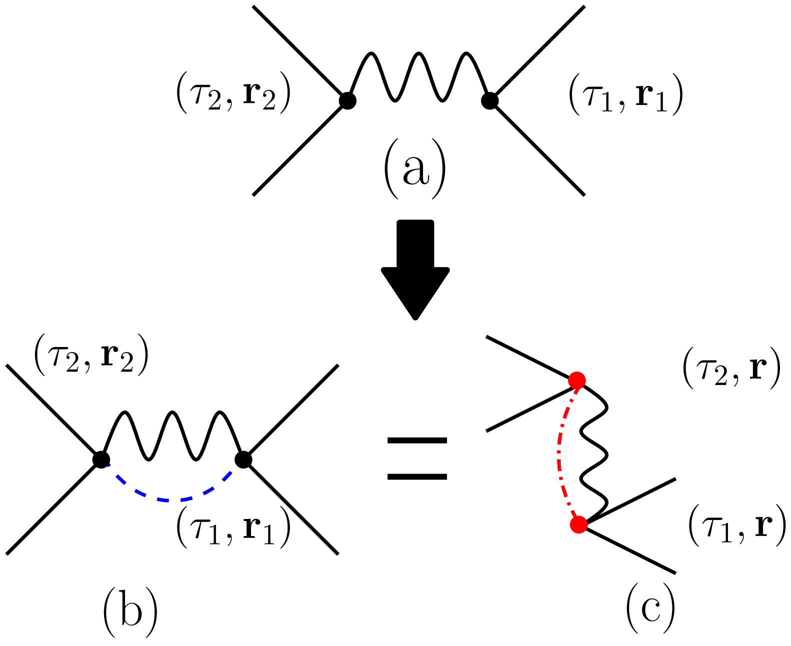
$g_{ijk}(\boldsymbol{r})\phi_i\psi_j^\dagger\psi_k$	$\sim T$	
$g_{ijk}(\boldsymbol{r})\phi_i\psi_j^\dagger\psi_k + v_{ij}(\boldsymbol{r})\psi_i^\dagger\psi_j$	$\sim \rho_0 + AT$	* * *

Summary: QFT-WormHole: distance independence of int. amp.

Without WH

With WH



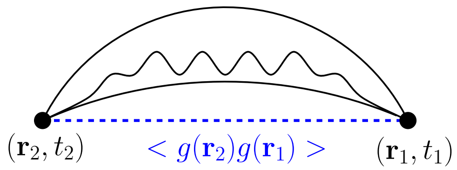
$$\int dxdy\delta(x-y)\prod G(x-y)$$

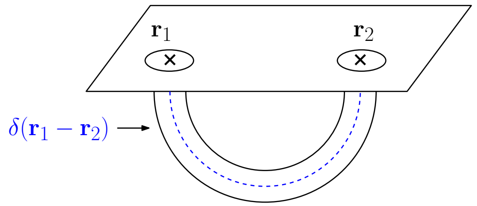
$$\int dr[\frac{1}{r}e^{-mr}]$$

Causal suppression!

Finite

In many body:
m -> 1/L,   L ~MFP





Universality of Wormhole and its relevance to Strange metal

The wormhole exists in any model with disorder, not just minimal Yukawa-SYK model.
The T-scaling depends on the details of WH.
Then, why wormhole is the mechanism of strange metal?

ANS: need to allow all possible interaction term $g_2(x)\psi\psi, \quad g_3(x)\psi\phi\psi, \quad g_5(x)(\psi\phi\psi)^2, \cdots.$
unless forbidden by a symmetry.

The first two terms give ρ_0 and aT respectively.

All others are irrelevant in low temperature.

Mean field th. vs Disorder theory in annealed pic.

Mean field : $S_{int} = \int d^3x[\frac{1}{2}(\nabla g(x))^2 + g(x)\mathcal{O}(x)]$,

Now integrate out V first: Then we get; $g(x) = \nabla^{-2}\mathcal{O}$.

$$S_{eff} = \frac{1}{2}\int_x\int_y \mathcal{O}(x)\nabla^{-2}(x,y)\mathcal{O}(y) = \frac{1}{2}\int d^3x' \frac{\mathcal{O}(x')\mathcal{O}(x)}{|x-x'|} .$$

On the other hand, ensemble average with gaussian $e^{-\frac{1}{2}V^2}$, $\quad S_{int} = \int gO$,

Then, $\int[-\frac{1}{2}g^2 + gO]dx = \int dx[-\frac{1}{2}(g(x)-O)^2 + \frac{1}{2}OO]$

$$\rightarrow S_{eff} = \frac{1}{2}\int O(x)O(x)dx = \frac{1}{2}\int dxdy\delta(x-y)O(x)O(y)$$

That is, the propagator for gaussian weight as kinetic term is delta function.

<g>=0, <g(x)g(y)>= $\delta(x-y)$ There is a Wormhole structure in the field theory.

Hubbard-Stratonovich vs Disorder

Hubbard-Stratonovich : $g(x, t)$

$$S_{eff} = \int dx \int dt \mathcal{O}^2(x, t),$$ a local field theory.

Disorder theory : $g(x)$

$$S_{eff} = \int dx \int dt_1 dt_2 \mathcal{O}(x, t_1)\mathcal{O}(x, t_2),$$ non-local in time.

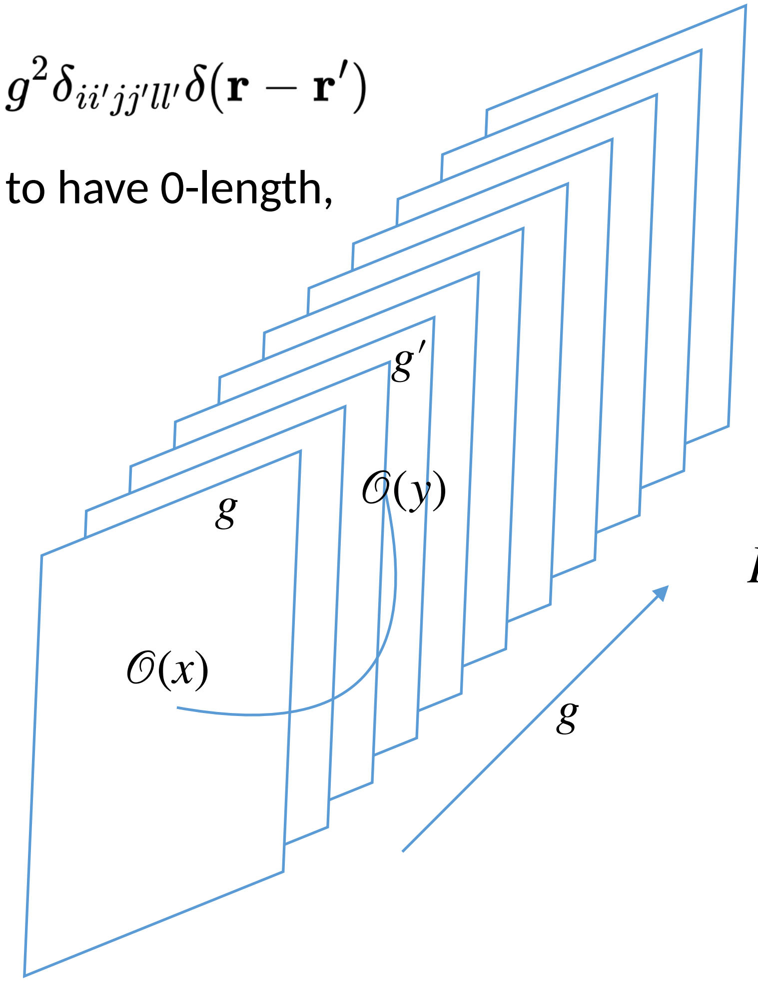
This difference makes different scaling behavior of resistivity.

Wormholes: Disorder vs MFT: g, g(r) ; g(r,t)

$$\langle g^*_{ijl}(\mathbf{r}) g_{i'j'l'}(\mathbf{r}') \rangle = g^2 \delta_{ii'jj'll'} \delta(\mathbf{r}-\mathbf{r}')$$

makes the wormhole to have 0-length,

$$\int D[g] P(g) e^{-g\int \mathcal{O}(x)dx} = e^W$$



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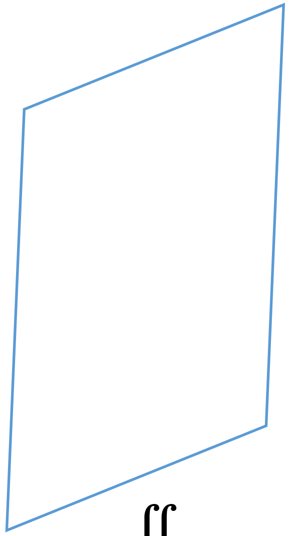


$If\ g = const.$ $\quad W = \iint_{xy} \mathcal{O}(x)\mathcal{O}(y)dxdy$ Fuzzy WH

If g(r,t) => $\int \mathcal{O}(r,t)\mathcal{O}(r,t)drdt$ MFT

If g(r) => $\iint dtdt' \int \mathcal{O}(r,t)\mathcal{O}(r,t')dr$ WH

Observation III: Quenched vs Annealed = ER vs EPR
If Q=A => ER=EPR

$$W_q = \int D[g]P[g]\left[\ln\left(\int D[\Psi]e^{-S_{\mathrm{tot}}[g,\psi,\phi]}\right)\right] ,= \langle \log Z\rangle_{dis}$$

=> Wormhole effect

$$W_a = \ln\left(\int D[\Psi]\int D[g]P[g]e^{-S_{\mathrm{tot}}[g,\psi,\phi]}\right) \quad = \log\langle Z\rangle_{dis}$$

=> Entanglement

Claim: In large N , Quenched=Annealed

=> analogue of ER=EPR

Annealed => EPR

$$H_{int}(x) = g(x)^2 + g(x)\int dt\mathcal{O}(x,t) = \left[g(x) + \int dt\mathcal{O}(x,t)\right]^2 - \iint dt_1 dt_2 \mathcal{O}(x,t_1)\mathcal{O}(x,t_2)$$

We can Integrate out disorder!

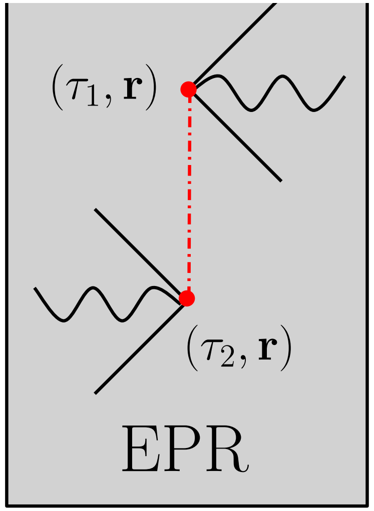
$$S_{\rm int} = -\frac{g^2}{2N^2}\int d\tau_1 d\tau_2 d^2\boldsymbol{r} \sum_{a,b,c=1}^{N} \psi_a^\dagger\psi_b(\boldsymbol{r},\tau_1)\phi_c(\boldsymbol{r},\tau_1) \times\psi_b^\dagger\psi_a(\boldsymbol{r},\tau_2)\phi_c(\boldsymbol{r},\tau_2). \quad (8)$$

vertex gives Entanglement!

*Only one p conservation for two vertex

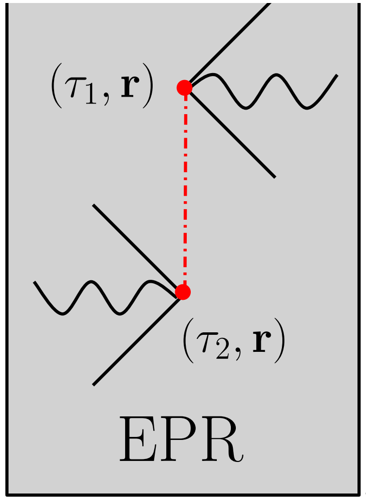
$$|\Psi\rangle = \int d\boldsymbol{p}\sqrt{P(\boldsymbol{p})}\,|\boldsymbol{p},-\boldsymbol{p}\rangle, \text{ with } \int d\boldsymbol{p}P(\boldsymbol{p}) = 1.$$

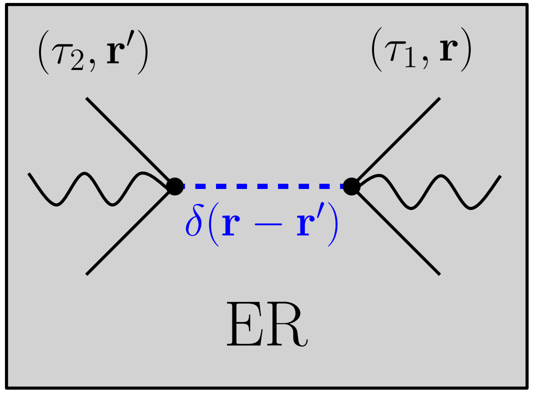
In annealed pic,3 particles
disappear at r,t with p, and
reappear with the same p
at the same r but at different time.



=> we will show
If we interpret LHS (Annealed)
in quenched pic, it is ER

ER (Wormhole) vs EPR





3 particle disappear at r,t with p, and reappear at the same r and same p but at different time.

3 particle disappear at r,t and reappear at r',t' through wormhole. It does not matter what is r- r'.

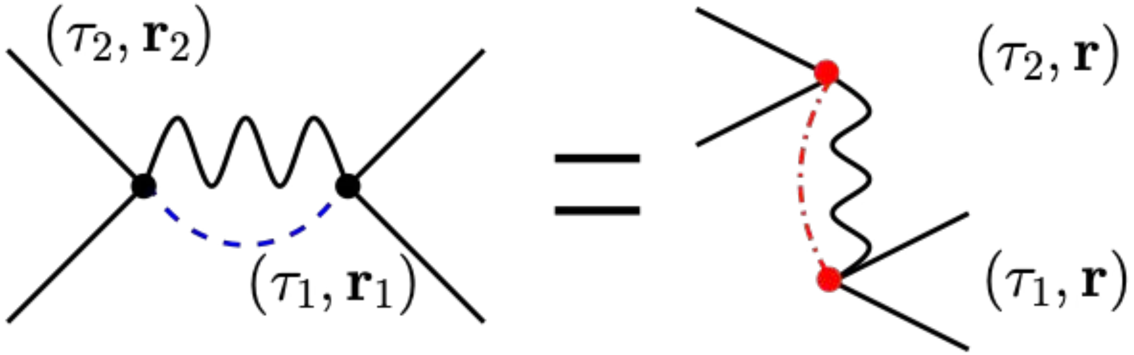
- 1. Quenched = Anealed in large N limit: -> 1-1 in diagram.
  <=> ER=EPR

It can imply the equivalence of the entanglement and disorder, justifying SY.

- 2. the origin of the Planckian Dissipation.
  (Because this is the mechanism of long range p delivery.)

ER=EPR

For any diagram containing the FT wormhole in quenched picture,
there is a corresponding diagram in effective theory of the Annealed picture

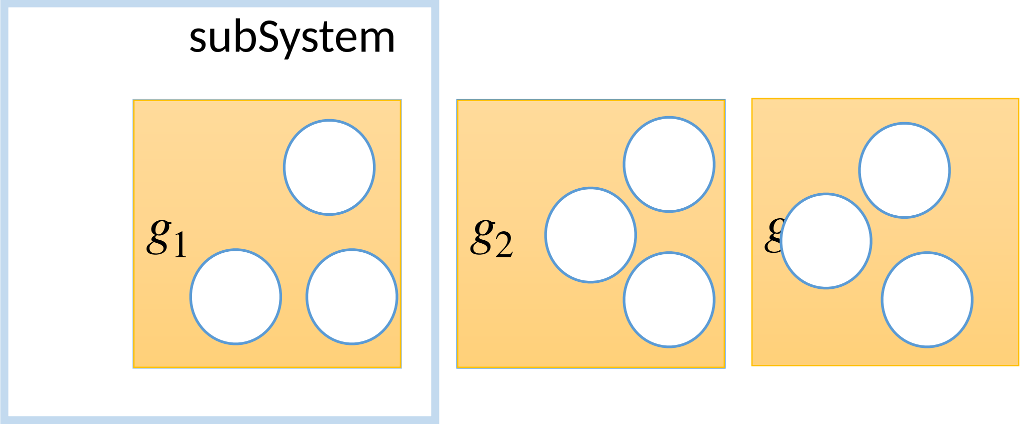


Also, Any process that gives entanglement can be interpreted as wormhole.

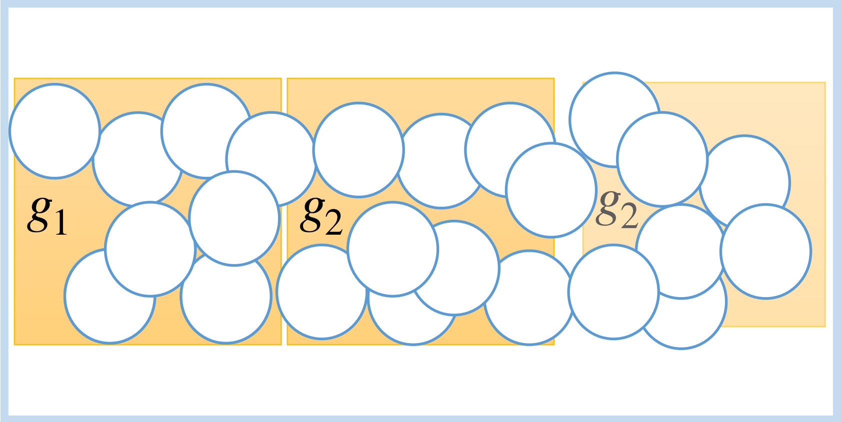
Impurity Density and spatially random disorder

$d =$ interparticle distance, $\lambda =$ correlation length = Size of wave function of single impurity

Case 1. $d \gg \lambda$



Case 2. $d \ll \lambda$



System: $\mathcal{O} = \phi\psi\psi$:

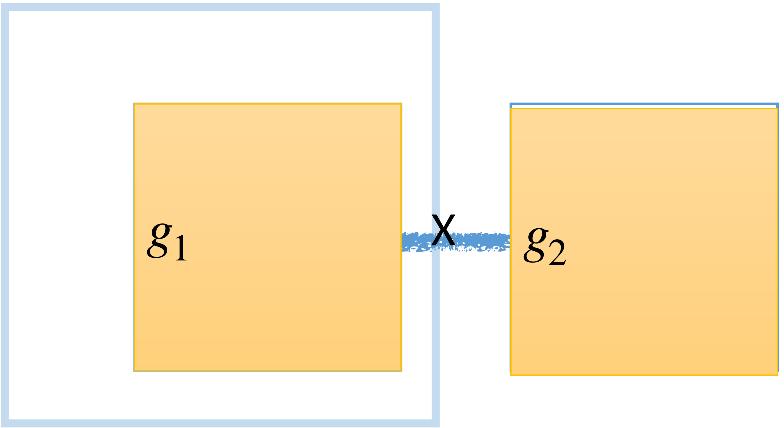
can divide the sample into
independent subsystems,
=> System= ensemble of subsystem.
Constant g
Open system: information leaks

no division possible
Not an ensemble.
g(x)
Closed system: No information loss
Very similar to Kondo-condensation

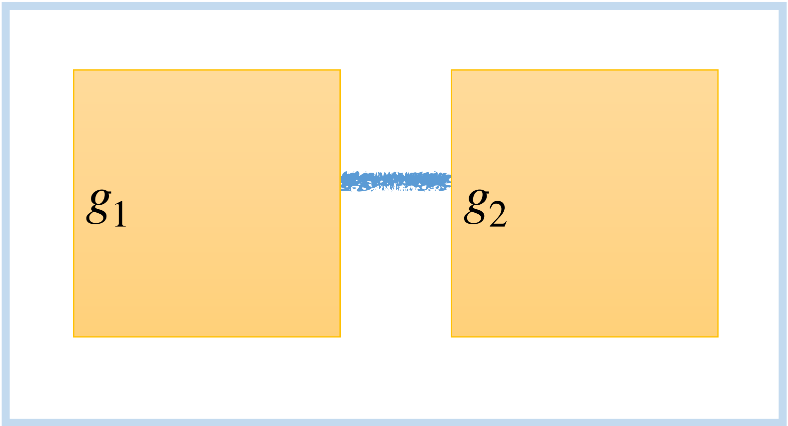
wormhole and information loss: g vs g(x) ?

$g \to$ Fuzzy wormhole

$g(x) \to$ wormhole



System



System include both

Open system: information leaks

Closed system: No information loss

sample = Each box ,

Mixed: sum of independent many ,

Conclusion:

1. The origin of the spatial all-to-all of SYK !

2. Qm gravity vs Disorder:
   Quenched = Annealed <=> ER=EPR

3. Weak interaction +disorder => long range interaction by WH.
   Not just mechanism of Strange metal but also of High Tc

Thank you

Part II

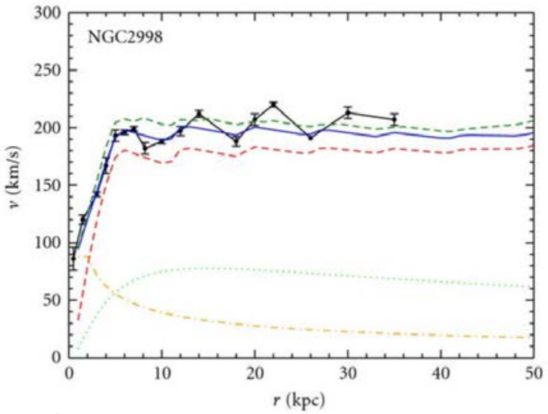
Distribution of dark matter :

From Liouville theorem to fuzzy dark matter.

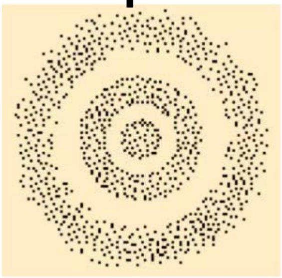
arxiv.org/hep-ph/9205208 :

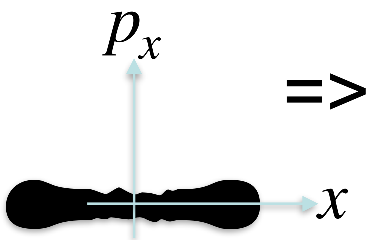
Galactic halo as bose liquid

Sikivie conjecture: Ripples in rotation curve



x space





=>

x, p space

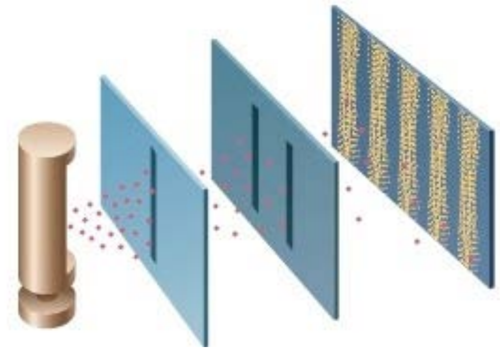
Created by Viktor Ostrovsky
from Noun Project

Calculating the distribution is non-trivial.
Analytically formidable.

Liouville theorem : A spiral by the gravitational pulling

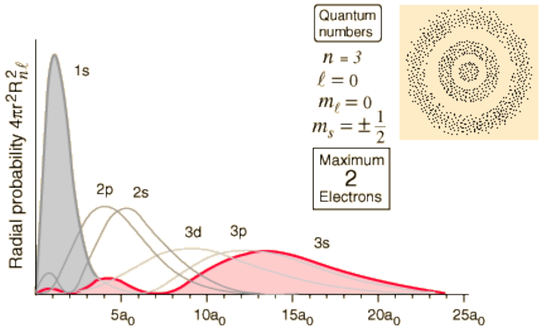
Give up to classically refine the result!

Quantum mechanics

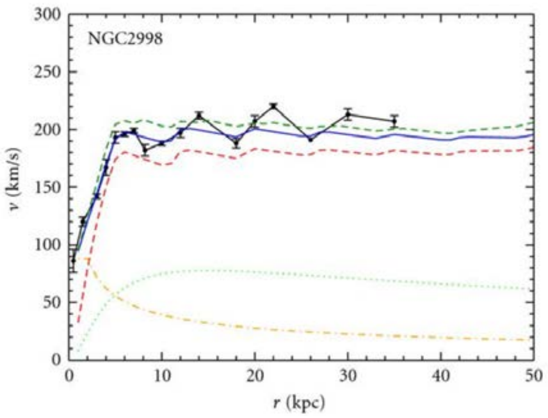


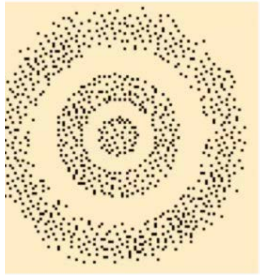
$$\rho(x) = |\psi(x)|^2,$$

양자역학=한개 입자를 여러번 측정한 결과를 한 입자의 파동함수로 해석
한 것



Vs





Quantum physics can make it easier

Manybody distribution from one particle's wave function?
Yes, if DM is in BEC.

$$\boxed{\rho = M_0 |\psi|^2,} \qquad \nabla^2 V = 4\pi\rho, \qquad i\hbar\partial_t \psi = \left[\frac{p^2}{2m} + V\right]\psi$$

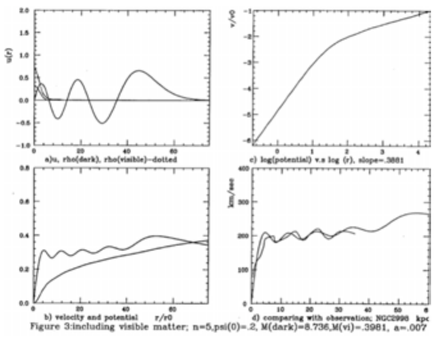
$$=> \quad i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\Delta^2\psi + GmM_0 \int_0^r dr' \frac{1}{r'^2} \int_0^r dr'' 4\pi r''^2 |\psi|^2 \cdot \psi(r)$$

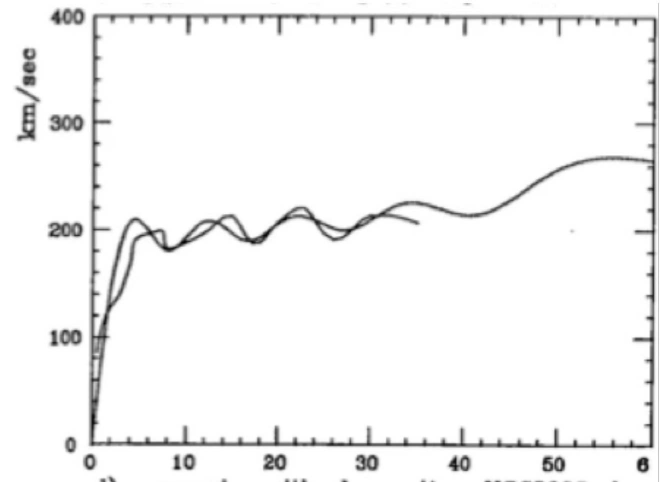
ψ_r

My idea turns out to be a special case of mean field theory, one particle theory. (non-int.)

Results

- Similar to the hydrogen atom wave function.
- $\psi \rightarrow \rho = M_0 |\psi|^2$ => ripple!





d) comparing with observation; NGC2998

ii(0)=.2, M(dark)=8.736,M(vi)=.3981, a=.007

ripple wavelength and the rotation velocity,

$$\boxed{m = 3.3 \cdot 10^{-23}} eV, \text{ and } M_0 = 0.69 \cdot 10^{12} M_\odot.$$

$$\rho_c = 1/4\pi |\hat{\psi}_0|^2 (M_0/r_0^3) = 1.08 \cdot 10^{-22} g/cm^3.$$

To simple to be wrong! What is it? But how they are created?

Thank you, again!