Effective Field Theory of coupled dark energy and dark matter

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Motivation and outline

⇒ ΛCDM is a successful model

- Cosmological constant problem: why the value of CC is very small.
- Coincidence problem: why DE dominates at very recent redshift?
- Observational Anomalies: Hubble tension, S_8 anomaly, and DESI observation.

⇒ Dynamical DE like, Quintessence, k-essence, proca field were introduced to address them.

⇒ Interacting dark energy and dark matter is introduced to address the coincidence problem.

⇒ Interacting dark energy and dark matter is shown to be successful to address H_0, and S_8 tension, as well as the DESI observations.

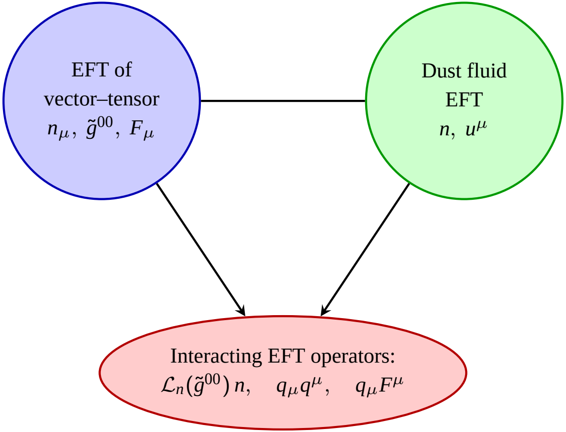
⇒ There are many models of interacting DE and DM, some of which predict a suppressed $G_{\rm eff}$, potentially reducing structure growth and addressing tension in large-scale observations.

Motivation and outline

⇒ Can we formulate a theory agnostic interacting description of Dark sector?

⇒ The phenomenology can be approached either via phenomenological parameterizations or by constructing a unified EFT framework, which captures all allowed operators consistent with symmetry principles.

⇒ Our EFT approach yields a well-defined, stable, and predictive theory, providing explicit stability conditions and controlled perturbation evolution, enabling meaningful inference from the data.



EFT of vector tensor: Symmetry breaking pattern

We assume existence of preferred direction by timelike vector, v^μ, which spontaneously break spacetime symmetry

$$v_\mu = \partial_\mu \tilde{t} + g_M A_\mu$$

$\tilde{t}$ is Stükelberg field associated with the $U(1)$ transformation of the vector field A_μ, and g_M is the gauge coupling. Defining $\tilde{t} = t$ the time coordinate, the preferred vector is

$$v_\mu = \tilde{\delta}^0_\mu \equiv \delta^0_\mu + g_M A_\mu \,.$$

The residual symmetries are

$$\begin{aligned} &\textbf{Combinet time and } U(1) &:&\quad t \to t - g_M \chi\,, \quad A_\mu \to A_\mu + \partial_\mu \chi\,, \\ &\textbf{Space(diff)} &:&\quad \vec{x} \to \vec{x}'(t, \vec{x}) \end{aligned}$$

preferred vector $\neq$ preferred slicing

EFT of vector tensor: Building blocks

Norm of the preferred vector

$$\tilde{g}^{00} = \tilde{\delta}^0_\mu \tilde{\delta}^0_\nu g^{\mu\nu}$$

We can define a time like vector

$$\tilde{n}_\mu \equiv \frac{\tilde{\delta}^0_\mu}{\sqrt{-\tilde{g}^{00}}} , \qquad \tilde{n}_\alpha \tilde{n}^\alpha = -1$$

Then a projection tensor can be defined

$$\tilde{h}_{\mu\nu} = g_{\mu\nu} + \tilde{n}_\mu \tilde{n}_\nu \, .$$

$\tilde{n}_\mu$ is not hypersurface orthogonal

The derivative of the preferred vector can have two parts

$$\nabla_\mu \tilde{\delta}^0_\nu = \frac{\tilde{n}_\nu \nabla_\mu \tilde{g}^{00}}{2\sqrt{-\tilde{g}^{00}}} - \sqrt{-\tilde{g}^{00}} \nabla_\mu \tilde{n}_\nu \, .$$

$\nabla_\mu \tilde{n}_\nu \Rightarrow$ decomposed in to

$$\begin{aligned} \tilde{K}_{\mu\nu} &= \tilde{h}_{(\mu|}{}^{\alpha} \nabla_\alpha \tilde{n}_{|\nu)} \, , \\ \tilde{\omega}_{\mu\nu} &= \tilde{h}_{[\mu|}{}^{\alpha} \nabla_\alpha \tilde{n}_{|\nu]} \, , \\ \tilde{a}_\mu &= \tilde{n}^\alpha \nabla_\alpha \tilde{n}_\mu \, . \end{aligned}$$

$\tilde{K}_{\mu\nu}, \tilde{\omega}_{\mu\nu}, \tilde{a}_\mu$, specify the geometric properties of the preferred vector.

EFT of vector tensor: Building blocks

For the gauge field A_μ, we have the field strength

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

Decomposed in to

$$\begin{aligned} \textbf{Electric} \quad &: \quad \tilde{F}_\mu \equiv \tilde{n}^\alpha F_{\mu\alpha} \, , \\ \textbf{Magnetic} \quad &: \quad \tilde{F}_{\mu\nu} \equiv \tilde{h}_\mu{}^\alpha \tilde{h}_\nu{}^\beta F_{\alpha\beta} \, . \end{aligned}$$

We have the follow relation

$$\begin{aligned} \tilde{\omega}_{\mu\nu} &= -\frac{g_M}{2\sqrt{-\tilde{g}^{00}}} \tilde{F}_{\mu\nu} \, , \\ \tilde{a}_\mu &= \frac{g_M}{\sqrt{-\tilde{g}^{00}}} \tilde{F}_\mu - \frac{\tilde{h}^\alpha{}_\mu \partial_\alpha \tilde{g}^{00}}{2\tilde{g}^{00}} \, . \end{aligned}$$

Hence $\tilde{\omega}_{\mu\nu}$, and a_μ are redundant.

We can have derivative operator

$$\tilde{D}_\mu T^{\nu\cdots}{}_{\rho\ldots} = \tilde{h}^\alpha{}_\mu \tilde{h}^\nu{}_\beta \ldots \tilde{h}^\gamma{}_\rho \nabla_\alpha T^{\beta\cdots}{}_{\gamma\ldots} \, .$$

Then the gauss equation can be written

$$\begin{aligned} \tilde{h}^\alpha{}_\mu \tilde{h}^\beta{}_\nu \tilde{h}^\gamma{}_\rho \tilde{h}^\delta{}_\sigma R_{\alpha\beta\gamma\delta} &= {}^{(3)}\tilde{R}_{\mu\nu\rho\sigma} + 2\tilde{B}_{[\rho|\mu}\tilde{B}_{|\sigma]\nu} \, , \\ \tilde{B}_{\mu\nu} &= \tilde{K}_{\mu\nu} + \tilde{\omega}_{\mu\nu} \, . \end{aligned}$$

Hence

$${}^{(3)}\tilde{R}_{\mu\nu} = {}^{(3)}\tilde{R}^\alpha{}_{\mu\alpha\nu} \, , \quad {}^{(3)}\tilde{R} = {}^{(3)}\tilde{R}^\mu{}_\mu = {}^{(3)}\tilde{R}^{\mu\nu}{}_{\mu\nu} \, ,$$

and

$$R = {}^{(3)}\tilde{R} + \tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu} - \tilde{K}^2 - 2\nabla_\mu(\tilde{a}^\mu - \tilde{K}\tilde{n}^\mu) \, .$$

EFT of vector tensor: Building blocks

The EFT building blocks are

$$\tilde{g}^{00}, \qquad {}^{(3)}\tilde{R}_{\mu\nu\rho\sigma}\,, \qquad \tilde{F}_{\mu\nu}\,, \qquad \tilde{F}_{\mu}\,, \qquad \tilde{K}_{\mu\nu}\,,$$
$$\mathfrak{L}_{\tilde{n}}\,, \qquad \tilde{D}_{\mu}\,, \qquad \text{and} \qquad g^{\mu\nu}\,(\text{or} \quad h^{\mu\nu})$$

The general action

The most general action in unitary gauge is

$$S = \int \mathrm{d}^4x\, \mathcal{L}_{\mathrm{DE}} + S_{\mathrm{m}}[\psi, g]$$

where

$$\mathcal{L}_{\mathrm{DE}} = \mathcal{L}_{\mathrm{DE}}\left(\tilde{g}^{00}\,, {}^{(3)}\tilde{R}_{\alpha\beta\gamma\delta}\,, \tilde{F}_{\mu\nu}\,, \tilde{F}_{\mu}\,, \tilde{K}_{\mu\nu}\,, \mathfrak{L}_{\tilde{n}}\,, \tilde{D}_{\mu}\right)$$

With no transverse degrees of freedom

Without transverse mode, gauge field is

$$A_\mu = \left[A_0, \partial_i A\right]$$

We can use the residual gauge freedom of the combined $U(1)$ and time diff to set $A = 0$.

Then

$$A_\mu = \left[A_0, \mathbf{0}\right].$$

In this case the temporal freedom is completely fixed except the time reparameterization $t \to t'(t)$.

The vector $\tilde{n}_\mu$ coincide with the vector orthogonal to the hypersurface, n_μ.

Then

$$\tilde{n}^\mu = n^\mu \,, \tilde{D}_\mu = D_\mu \,, \tilde{K}_{\mu\nu} = K_{\mu\nu} \,, {}^{(3)}\tilde{R}_{\alpha\beta\gamma\delta} = {}^{(3)}R_{\alpha\beta\gamma\delta} \,.$$

The preferred vector

$$v_\mu = \tilde{\delta}^0_\mu = \delta^0_\mu + g_M A_\mu = (1 + g_M A_0, \mathbf{0}).$$

The norm

$$\tilde{g}^{00} = \tilde{\delta}^0_\alpha \tilde{\delta}^0_\beta g^{\alpha\beta} = (1 + g_M A_0)^2 \, g^{00} \,.$$

Hence, the building blocks becomes

$$n_\mu \,, \quad \tilde{g}^{00} \,, \quad F_\mu \,, \quad K_{\mu\nu} \,, \quad {}^{(3)}R_{\mu\nu} \,.$$

Luminal EFT of Dark energy action

Using the above ansatz and considering the Luminal EFT of vector-tensor DE: $c_T = c$, where c speed of light

$$\begin{aligned}\mathcal{L}_{\rm DE} \quad &= \quad \frac{M_*^2}{2} f(t)\left[{}^{(3)}R + K_{\mu\nu}K^{\mu\nu} - K^2\right] - \hat{\Lambda}(t) - \hat{c}(t)\tilde{g}^{00} - d(t)K \\ &\quad + \frac{1}{2}\hat{M}_2^4(t)\left(\frac{\delta\tilde{g}^{00}}{-\tilde{g}^{00}_{\rm BG}}\right)^2 - \frac{1}{2}\bar{M}_1^3(t)\left(\frac{\delta\tilde{g}^{00}}{-\tilde{g}^{00}_{\rm BG}}\right)\delta K + \frac{1}{2}\gamma_1(t)F_\mu F^\mu\,,\end{aligned}$$

In this case we have the following consistency conditions

$$\begin{aligned}\dot{\hat{\Lambda}} + 3H\dot{d} + \dot{\hat{c}}\tilde{g}^{00}_{\rm BG} \quad &= 0\,, \\ 2\hat{M}_2^4\frac{\mathrm{d}}{\bar{N}\mathrm{d}t}\ln(-\tilde{g}^{00}_{\rm BG}) + 3\bar{M}_1^3\dot{H} + 2\dot{\hat{c}}\tilde{g}^{00}_{\rm BG} \quad &= 0\,, \\ \dot{d} + \frac{1}{2}\bar{M}_1^3\frac{\mathrm{d}}{\bar{N}\mathrm{d}t}\ln(-\tilde{g}^{00}_{\rm BG}) \quad &= 0\,, \\ \dot{f} \quad &= 0\,.\end{aligned}$$

Dark Matter action

Here we assume that that dust fluid is described by three scalars

$$\phi^i(t,\boldsymbol{x})\,, \qquad i = 1,2,3$$

these scalar fields can be understood as comoving coordinates $\phi^i = x^i$.

Fluid is described by the invariance under volume preserving diffs

$$\phi^i \to \phi'^i \quad \text{s.t.} \quad \det\frac{\partial\phi'^i}{\partial\phi^i} = 1\,.$$

the invariant building blocks are

$$n \equiv \sqrt{\det g^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j}$$

$$u^\mu \equiv -\frac{1}{6}\varepsilon_{ijk}\epsilon^{\mu\nu\rho\sigma}\partial_\nu\phi^i\partial_\rho\phi^j\partial_\sigma\phi^k\,.$$

In this formulation the current $\mathcal{J}^\mu \equiv nu^\mu$ conservation is off-shell

$$\nabla_\mu\mathcal{J}^\mu = 0$$

in the comoving gauge $\phi^i = x^i$

$$n = \sqrt{\det g^{ij}}\,, \qquad u^\mu = \frac{\delta^0_\mu}{\sqrt{-g_{00}}}$$

$$\boxed{\mathcal{S}_{\rm DM} = \int \mathrm{d}^4x\sqrt{-g}\,\mathcal{L}_{\rm DM}\,, \qquad \mathcal{L}_{\rm DM} = -\hat{m}_{\rm c}n\,, \qquad \hat{m}_{\rm c} \text{ is a constant.}}$$

Interacting action

Here are the basic ingredients from dark sectors

$$\underbrace{n_\mu\,, \quad \tilde{g}^{00}\,, \quad F_\mu\,,}_{\text{DE}} \qquad \underbrace{n\,, \quad u^\mu}_{\text{DM}}\,.$$

The possible interactions are

$$\mathcal{L}_n(\tilde{g}^{00})n\,, \qquad n_\mu u^\mu\,, \qquad F_\mu u^\mu\,.$$

The first one is energy transfer, the second two are momentum transfer.

Introducing $\mathcal{U} \equiv n_\mu u^\mu$, and $q^\mu \equiv u^\mu + n^\mu \mathcal{U}$.

Related by

$$q_\mu q^\mu = -1 + \mathcal{U}^2$$

In comoving gauge

$$q^\mu = \left(0, \frac{N^i}{\sqrt{N^2 - N_i N^i}}\right)\,.$$

We use q_μ instead of u_μ, hence interactions are

$$q_\mu q^\mu\,, \qquad q_\mu F^\mu\,.$$

Interacting action

Then the general interacting Lagrangian is given by

$$
\begin{aligned}
\mathcal{L}_{\rm int} &= \mathcal{L}_n(\tilde{g}^{00})n + \mathcal{L}_{q^2}(\tilde{g}^{00})q^\mu q_\mu + \mathcal{L}_{q\cdot F}(\tilde{g}^{00})F_\mu q^\mu + \cdots \\
&= -\Delta\Lambda(t) - \Delta c(t)\tilde{g}^{00} - \Delta m_{\rm c}(t)n \\
&\quad + \frac{1}{2}\Delta M_2^4(t)\left(\frac{\delta\tilde{g}^{00}}{-\tilde{g}^{00}_{\rm BG}}\right)^2 - m_1^4(t)\frac{\delta n}{\bar{n}}\left(\frac{\delta\tilde{g}^{00}}{-\tilde{g}^{00}_{\rm BG}}\right) - m_2^4(t)q^\mu q_\mu - \bar{m}_1^2(t)q^\mu F_\mu \\
&\quad + \cdots ,
\end{aligned}
$$

where

$$
\begin{aligned}
&\Delta\Lambda(t) = \bar{\mathcal{L}}_{n\tilde{g}^{00}}\,\bar{n}\,\tilde{g}^{00}_{\rm BG} , \qquad \Delta c(t) = -\bar{\mathcal{L}}_{n\tilde{g}^{00}}\,\bar{n} , \qquad \Delta m_{\rm c}(t) = -\bar{\mathcal{L}}_n , \\
&\Delta M_2^4(t) = \bar{\mathcal{L}}_{n\tilde{g}^{00}\tilde{g}^{00}}\,\bar{n}\,(\tilde{g}^{00}_{\rm BG})^2 , \qquad m_1^4(t) = \bar{\mathcal{L}}_{n\tilde{g}^{00}}\,\tilde{g}^{00}_{\rm BG}\,\bar{n} , \\
&m_2^4(t) = -\bar{\mathcal{L}}_{q^2} , \qquad \bar{m}_1^2(t) = -\bar{\mathcal{L}}_{q\cdot F} ,
\end{aligned}
$$

and

$$
\delta n = n - \bar{n}(t) , \qquad \mathcal{L}_{n\tilde{g}^{00}} = \frac{{\rm d}\mathcal{L}_n}{{\rm d}\tilde{g}^{00}} , \qquad \mathcal{L}_{n\tilde{g}^{00}\tilde{g}^{00}} = \frac{{\rm d}\mathcal{L}_{n\tilde{g}^{00}}}{{\rm d}\tilde{g}^{00}} .
$$

EFT action for coupled Dark Energy and Dark Matter

Action

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\,\left(\mathcal{L}_{\mathrm{D}}^{\mathrm{NL}} + \mathcal{L}_{\mathrm{D}}^{(2)}\right) + \mathcal{S}_{\mathrm{m}}\,,$$

$$\begin{aligned}
\mathcal{L}_{\mathrm{D}}^{\mathrm{NL}} &= \frac{M_*^2}{2}f(t)\left({}^{(3)}R + K_{\mu\nu}K^{\mu\nu} - K^2\right) - \Lambda(t) - \tilde{c}(t)\tilde{g}^{00} - d(t)K - m_{\mathrm{c}}(t)n\,,\\
\mathcal{L}_{\mathrm{D}}^{(2)} &= \frac{1}{2}M_2^4(t)\left(\frac{\delta\tilde{g}^{00}}{-\tilde{g}^{00}_{\mathrm{BG}}}\right)^2 - \frac{1}{2}\bar{M}_1^3(t)\left(\frac{\delta\tilde{g}^{00}}{-\tilde{g}^{00}_{\mathrm{BG}}}\right)\delta K + \frac{1}{2}\gamma_1(t)F_\mu F^\mu\\
&\quad - m_1^4(t)\frac{\delta n}{\bar{n}}\left(\frac{\delta\tilde{g}^{00}}{-\tilde{g}^{00}_{\mathrm{BG}}}\right) - m_2^4(t)q^\mu q_\mu - \bar{m}_1^2(t)q^\mu F_\mu\,.\\
&\dot{\rho}_{\mathrm{c}} + 3H\rho_{\mathrm{c}} = \frac{\mathrm{d}}{\mathrm{d}t}\ln(m_{\mathrm{c}})\bar{\rho}_{\mathrm{c}}\,, \qquad \rho_{\mathrm{c}} \equiv m_{\mathrm{c}}\bar{n}_{\mathrm{c}}\,.
\end{aligned}$$

Mapping

For the Horndeski case

$$\mathcal{L}_{\rm DM} + \mathcal{L}_{\rm int} = -f_1(t, X, Z)n + f_2(t, X, Z)\,.$$

where $X = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$, and $Z = -u^\mu\nabla_\mu\phi$

The mapping for interacting coefficient

$$\begin{aligned}
m_{\rm c} &= f_1\,,\\
m_1^4 &= -\bar{n}\left(f_{1,X}X_{\rm BG} + \frac{1}{2}\sqrt{2X_{\rm BG}}f_{1,Z}\right)\,,\\
m_2^4 &= \frac{1}{2}\sqrt{2X_{\rm BG}}\,(f_{1,Z}\,\bar{n} - f_{2,Z})\,,\\
g_M &= \gamma_1 = \bar{m}_1^2 = 0\,.
\end{aligned}$$

For GP theory

$$\mathcal{L}_{\rm DM} + \mathcal{L}_{\rm int} = -f_1(\tilde{X}, \tilde{Z}, \tilde{E})n + f_2(\tilde{X}, \tilde{Z}, \tilde{E})\,,$$

$\tilde{X} = -A_\mu A^\mu/2$, $\tilde{Z} = A_\mu u^\mu$, and $\tilde{E} = -A^\mu F_{\mu\nu}u^\nu$.

The mapping

$$\begin{aligned}
\gamma_1 &= 1\,,\\
m_{\rm c} &= f_1\,,\\
m_1^4 &= -\bar{n}\left(f_{1,\tilde{X}}\tilde{X}_{\rm BG} + \frac{1}{2}\sqrt{2\tilde{X}_{\rm BG}}\,f_{1,\tilde{Z}}\right)\,,\\
m_2^4 &= \frac{1}{2}\sqrt{2\tilde{X}_{\rm BG}}\,\left(f_{1,\tilde{Z}}\,\bar{n} - f_{2,\tilde{Z}}\right)\,,\\
\bar{m}_1^2 &= -\sqrt{2\tilde{X}_{\rm BG}}\,(f_{1,\tilde{E}}\,\bar{n} - f_{2,\tilde{E}})\,.
\end{aligned}$$

[A. De Felice, S. Nakamuara S. Tsujikawa- arXiv:2004.09384, MCP, K. Koyama-arXiv:2405.06565]

EFT action in α- Basis

From the definition

$$\frac{\delta\tilde{g}^{00}}{-\tilde{g}^{00}_{\rm BG}} = \frac{\delta g^{00}}{-g^{00}_{\rm BG}} - \frac{2g_M}{(1+g_M A_0)}\delta A_0\,, \qquad F_\mu F^\mu = (n^0\nabla_i A_0)(n^0\nabla^i A_0) = \frac{\delta^{ij}\nabla_i\delta A_0\nabla_j\delta A_0}{a^2\bar{N}^2}\,,$$

$$q^\mu q_\mu = \frac{N^i N_i}{N^2 - N_j N^j} \simeq \frac{N^i N_i}{\bar{N}^2}\,, \qquad q^\mu F_\mu = \frac{n^0 N^i\nabla_i\delta A_0}{\sqrt{N^2 - N_j N^j}} \simeq \frac{N^i\nabla_i\delta A_0}{\bar{N}^2}\,.$$

We can write the Lagrangian as

$$\mathcal{L}^{(2)}_{\rm D} = \mathcal{L}^{(2)}_{\rm ST} + \mathcal{L}^{(2)}_{\delta\hat{A}_0}\,,$$

where

$$\mathcal{L}^{(2)}_{\rm ST} = 2M_2^4\left(\frac{\delta N}{\bar{N}}\right)^2 - \bar{M}_1^3\frac{\delta N}{\bar{N}}\delta K - 2m_1^4\frac{\delta N}{\bar{N}}\frac{\delta n}{\bar{n}} - m_2^4\frac{N^i N_i}{\bar{N}^2}\,,$$

$$\mathcal{L}^{(2)}_{\delta\hat{A}_0} = \frac{1}{2}\left(\nabla_i\delta\hat{A}_0\nabla^i\delta\hat{A}_0 + g_{\rm eff}^2 M_2^4\delta\hat{A}_0^2\right) + \left[\frac{\bar{m}_1^2\nabla_i N^i}{\sqrt{\gamma_1}\bar{N}} + g_{\rm eff}\left(\frac{1}{2}\bar{M}_1^3\delta K - 2M_2^4\frac{\delta N}{\bar{N}} + m_1^4\frac{\delta n}{\bar{n}}\right)\right]\delta\hat{A}_0\,, \qquad g_{\rm eff} \equiv \frac{2g_M\bar{N}}{\sqrt{\gamma_1}(1+g_M\bar{A}_0)}\,, \quad \delta\hat{A}_0 \equiv \frac{\sqrt{\gamma_1}}{\bar{N}}\delta A_0\,.$$

EFT action in α- Basis

After integrating out $\delta\hat{A}_0$, we can write the EFT action in real space,

$$
\begin{aligned}
\mathcal{S}_{\mathrm{D}}^{(2)} &= \int \mathrm{d}^4x\sqrt{-g}\,\mathcal{L}_{\mathrm{D}}^{(2)} \\
&= \int \mathrm{d}^4x\,\bar{N}a^3\Bigg[2M_{2,\mathrm{eff}}^4\left(\frac{\delta N}{\bar{N}}\right)^2 - \bar{M}_{1,\mathrm{eff}}^3\frac{\delta N}{\bar{N}}\delta K - 2m_{1,\mathrm{eff}}^4\frac{\delta N}{\bar{N}}\frac{\delta n}{\bar{n}} - m_2^4\frac{N_iN^i}{\bar{N}^2} \\
&\quad -\bar{m}_{1,\mathrm{eff}}^4\frac{(\nabla_iN^i)^2}{\bar{N}^2} - \frac{\mu_1^4}{2}\left(\frac{\delta n}{\bar{n}}\right)^2 - \frac{\mu_2^2}{8}(\delta K)^2 - \frac{\mu_3^3}{2}\delta K\frac{\delta n}{\bar{n}} - \frac{\mu_4^4}{2}\delta K\frac{\nabla_iN^i}{\bar{N}} \\
&\quad +2\mu_5^5\frac{\delta N}{\bar{N}}\frac{\nabla_iN^i}{\bar{N}} - \mu_6^5\frac{\delta n}{\bar{n}}\frac{\nabla_iN^i}{\bar{N}}\Bigg],
\end{aligned}
$$

where

$$
M_{2,\mathrm{eff}}^4 \equiv (1-\mathcal{G})\,M_2^4\,, \qquad \bar{M}_{1,\mathrm{eff}}^3 \equiv (1-\mathcal{G})\,\bar{M}_1^3\,, \qquad m_{1,\mathrm{eff}}^4 \equiv (1-\mathcal{G})\,m_1^4\,,
$$

$$
\bar{m}_{1,\mathrm{eff}}^4 \equiv \frac{\mathcal{G}\bar{m}_1^4}{2\gamma_1 g_{\mathrm{eff}}^2M_2^4}\,, \qquad \mu_1^4 \equiv \mathcal{G}\,\frac{m_1^8}{M_2^4}\,, \qquad \mu_2^2 \equiv \mathcal{G}\,\frac{\bar{M}_1^6}{M_2^4}\,, \qquad \mu_3^3 \equiv \mathcal{G}\,\frac{\bar{M}_1^3m_1^4}{M_2^4}\,,
$$

$$
\mu_4^4 \equiv \frac{\mathcal{G}\bar{M}_1^3\bar{m}_1^2}{\sqrt{\gamma_1}\,g_{\mathrm{eff}}M_2^4}\,, \qquad \mu_5^5 \equiv \frac{\mathcal{G}\bar{m}_1^2}{\sqrt{\gamma_1}\,g_{\mathrm{eff}}}\,, \qquad \mu_6^5 \equiv \frac{\mathcal{G}m_1^4\bar{m}_1^2}{\sqrt{\gamma_1}\,g_{\mathrm{eff}}M_2^4}\,, \qquad \mathcal{G} \equiv \frac{g_{\mathrm{eff}}^2M_2^4}{-\nabla_i\nabla^i + g_{\mathrm{eff}}^2M_2^4}\,.
$$

EFT action in α- Basis

As in the case of the EFT of uncoupled vector-tensor DE theories, we have three unified descriptions of the EFT of coupled DE and DM:

1. $g_{\rm eff} \neq 0$, it corresponds to the EFT of coupled vector DE,
2. $g_{\rm eff} = 0$, together with the consistency conditions, we obtain the EFT of coupled scalar DE in shift-symmetric scalar-tensor theories,
3. $g_{\rm eff} = 0$, without the consistency conditions, we have the EFT of coupled scalar DE in non-shift-symmetric scalar-tensor theories.

EFT action in α- Basis

Action in α- Basis

$$
\begin{aligned}
\mathcal{S}^{(2)} \quad = \quad & \int \mathrm{d}^4x\, \bar{N}a^3 \frac{M^2}{2}\Bigg[\delta K_{\mu\nu}\delta K^{\mu\nu} - \delta K^2 + \delta_2\left(\frac{\sqrt{h}}{a^3}{}^{(3)}R\right) + \frac{\delta N}{\bar{N}}\delta^{(3)}R - 6\Omega_c H^2\delta_2\left(\frac{\sqrt{h}}{a^3}\frac{n}{\bar{n}}\right) \\
& + H^2\tilde{\alpha}_K\left(\frac{\delta N}{\bar{N}}\right)^2 + 4\tilde{\alpha}_B H\delta K\frac{\delta N}{\bar{N}} + (\tilde{\alpha}_{m_1} - 6\Omega_c)H^2\frac{\delta N}{\bar{N}}\frac{\delta n}{\bar{n}} + \alpha_{m_2}H^2\frac{N^iN_i}{\bar{N}^2} \\
& + \tilde{\alpha}_{\bar{m}_1}\frac{(\nabla_iN^i)^2}{\bar{N}^2} + \alpha_{\mu_1}H^2\left(\frac{\delta n}{\bar{n}}\right)^2 + \alpha_{\mu_2}\delta K^2 + \alpha_{\mu_3}H\delta K\frac{\delta n}{\bar{n}} \\
& - \alpha_{\mu_4}\delta K\frac{\nabla_iN^i}{\bar{N}} - \alpha_{\mu_5}H\frac{\delta N}{\bar{N}}\frac{\nabla_iN^i}{\bar{N}} - \alpha_{\mu_6}H\frac{\delta n}{\bar{n}}\frac{\nabla_iN^i}{\bar{N}}\Bigg] + \tilde{\mathcal{S}}_{\mathrm{m}}^{(2)}\,,
\end{aligned}
$$

$$
\tilde{\mathcal{S}}_{\mathrm{m}}^{(2)} \equiv \mathcal{S}_{\mathrm{m}}^{(2)} + \delta_2\int \mathrm{d}^4x\,\bar{N}\sqrt{h}\left(\frac{\delta N}{\bar{N}}\bar{\rho}_{\mathrm{m}} - \bar{p}_{\mathrm{m}}\right)\,,
$$

Ghost conditions

$$N = \bar{N}(t)\,, \qquad N_i = 0\,, \qquad h_{ij} = a^2(t)(\delta_{ij} + \tilde{h}_{ij})\,, \qquad \tilde{h}_{11} = -\tilde{h}_{22} = h_+(t,z)\,, \qquad \tilde{h}_{12} = \tilde{h}_{21} = h_\times(t,z)\,.$$

Tensor modes

The action reduces to

$$\mathcal{S}_{\mathrm{T}}^{(2)} = \sum_{\lambda=+,\times} \int \frac{\mathrm{d}t\mathrm{d}^3k}{(2\pi)^3} \bar{N} a^3 \frac{M^2}{4} \left(|\dot{h}_\lambda|^2 - \frac{k^2}{a^2} |h_\lambda|^2 \right)\,,$$

where we have changed the action into Fourier space, and k is the Fourier mode.

Expected as we have focused on theories with the luminal speed of gravitational waves.

$$\ddot{h}_\lambda + H\,(3 + \alpha_M)\,\dot{h}_\lambda + \frac{k^2}{a^2} h_\lambda = 0\,, \qquad \alpha_M \equiv \frac{2\dot{M}}{HM}\,.$$

Ghost conditions

$$N = \bar{N}(t)\,(1+\alpha)\ , \quad N_i = \bar{N}(t)\,(\partial_i \chi)\ , \quad h_{ij} = a^2(t)\left[(1+2\zeta)\delta_{ij} + 2\partial_i\partial_j E\right]\ .$$

Scalar modes

write the action in the form

$$\mathcal{S}^{(2)} = \int \frac{\mathrm{d}t\mathrm{d}^3k}{(2\pi)^3}\,\bar{N}a^3\left(\dot{\vec{\mathcal{X}}}{}^t_{\boldsymbol{k}}\boldsymbol{K}\dot{\vec{\mathcal{X}}}_{-\boldsymbol{k}} - \vec{\mathcal{X}}^t_{\boldsymbol{k}}\tilde{\boldsymbol{G}}\vec{\mathcal{X}}_{-\boldsymbol{k}} - \frac{k}{a}\vec{\mathcal{X}}^t_{\boldsymbol{k}}\boldsymbol{B}\dot{\vec{\mathcal{X}}}_{-\boldsymbol{k}}\right)\ ,$$

where

$$\vec{\mathcal{X}}^t_{\boldsymbol{k}} = \left[\frac{\delta_{\mathrm{c}}(\boldsymbol{k})}{k}, \zeta(\boldsymbol{k}), \frac{\delta_{\mathrm{m}}(\boldsymbol{k})}{k}\right]\ , \qquad \text{and} \quad \tilde{\boldsymbol{G}} = \frac{k^2}{a^2}\boldsymbol{G} + \boldsymbol{M}\,.$$

$\boldsymbol{K}$ and $\tilde{\boldsymbol{G}}$ are symmetric 3×3 matrices and $\boldsymbol{B}$ is anti-symmetric matrix.

Ghost conditions

High k limit

$$
\begin{aligned}
q_{\rm c} &\equiv \frac{K_{11}}{a^2} = \frac{M^2H^2}{2}\left(3\Omega_{\rm c} + \alpha_{m_2} - \alpha_{\bar{m}_1}^2\right) > 0\,, \\
q_{\rm s} &\equiv K_{22} = \frac{M^2(6\alpha_B^2 + \tilde{\alpha}_K)}{2(1+\alpha_B)^2} > 0\,,
\end{aligned}
$$

with the strong energy condition for the standard matter

$$\bar{\rho}_m(1+\omega_m) > 0\,.$$

Speed of propagation

High k limit

$$c_{\rm s}^2 = \hat{c}_{\rm s}^2 + \Delta c_{\rm s}^2\,,$$

where

$$\begin{aligned}
\hat{c}_{\rm s}^2 &= \frac{G_{22}}{K_{22}} = -\frac{1}{(6\alpha_B^2+\tilde{\alpha}_K)}\Bigg[3\Omega_{\rm c} + 3\Omega_{\rm m}(1+w_{\rm m}) + 2(1+\alpha_B)\left(\alpha_B - \alpha_M - \epsilon_H\right) + \frac{2\dot{\alpha}_B}{H} \\
&\qquad -4\alpha_B^2\alpha_g^2 + \alpha_{m_2} - \alpha_{\bar{m}_1}^2 - 4\alpha_g\alpha_B\alpha_{\bar{m}_1}\Bigg]\,, \\
\Delta c_{\rm s}^2 &= \frac{B_{12}^2}{K_{11}K_{22}} = \frac{(\alpha_{m_1} + 2\alpha_{m_2} - 2\alpha_{\bar{m}_1}^2 - 4\alpha_g\alpha_B\alpha_{\bar{m}_1})^2}{4(3\Omega_{\rm c} + \alpha_{m_2} - \alpha_{\bar{m}_1}^2)(6\alpha_B^2 + \tilde{\alpha}_K)}\,.
\end{aligned}$$

To avoid Laplacian instabilities

$$c_{\rm s}^2 > 0\,.$$

Effective gravitational coupling

Small-scale CDM evolution:

$$\ddot{\delta}_{\rm c} + \mathcal{C}\dot{\delta}_{\rm c} - 4\pi G_{\rm eff}\bar{\rho}_{\rm c}\delta_{\rm c} \simeq 0$$

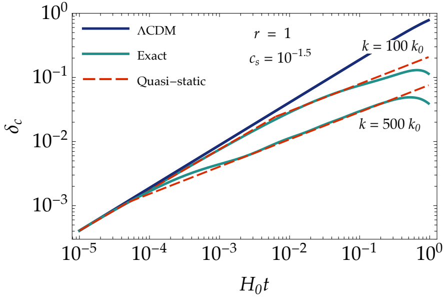
Damping coefficient:

$$\mathcal{C} = \frac{(5Hq_{\rm c} + \dot{q}_{\rm c})\nu_{\rm s}^2 + (2b_{12}\dot{b}_{12} + 5Hb_{12}^2)\nu_{\rm s} - b_{12}^2\dot{\nu}_{\rm s}}{q_{\rm c}\nu_{\rm s}^2 + b_{12}^2\nu_{\rm s}}$$

Effective gravitational coupling:

$$G_{\rm eff} = \frac{4\mu_{11}\nu_{\rm s}^2 + 4g_{12}^2\nu_{\rm s} - \left[3H^2(7-2\epsilon_H)b_{12}^2 + \dot{b}_{12}^2\right]\nu_{\rm s}}{16\pi\bar{\rho}_{\rm c}(q_{\rm c}\nu_{\rm s}^2 + b_{12}^2\nu_{\rm s})}$$

$$- \frac{2(\ddot{b}_{12} + 8H\dot{b}_{12} + 2\dot{g}_{12} + 4Hg_{12})b_{12}\nu_{\rm s} - 2(\dot{b}_{12} + 3Hb_{12} + 2g_{12})b_{12}\dot{\nu}_{\rm s}}{16\pi\bar{\rho}_{\rm c}(q_{\rm c}\nu_{\rm s}^2 + b_{12}^2\nu_{\rm s})}$$



Summary & Future direction

Take-home message

- We successfully constructed interacting EFT of Dark Energy and Dark Matter
- This Int-EFT can accommodate Vector-Tensor Dark Energy, Scalar-Tenor Dark Energy (both shift symmetric and non-shift symmetric)
- We derived the stability conditions that theory should posses.
- We explored the phenomenological impact of the possible interactions.
- With some interaction terms, we can find a possible suppression for the growth of CDM ovedensity.

Future directions

EFT-param — Study the effect of interactions with different EFT α-parametrization.

Constraints — Parameter estimation with cosmological data, CMB and LSS.

Non-linear — Extending the formulation to non-linear scales.

Thank you!