Shanghai–APCTP–Sogang–GIST workshop on Gravity, Astroparticle, and Cosmology (SASGAC 2025)

Dec 26–29, 2025 Sogang University

Neural Network-Based Methods for Inverse Problems in Holography

Hyun-Sik Jeong

APCTP

Sogang University, 29 December 2025

asia pacific center for theoretical physics

The talk is mainly a **review**, but here are some of my contributions

Holographic CMT, QCD

arXiv:2401.00939 [pdf, other] hep-th cond-mat.dis-nn gr-qc hep-ph doi 10.1007/JHEP03(2024)141

Deep learning bulk spacetime from boundary optical conductivity

Authors: Byoungjoon Ahn, Hyun-Sik Jeong, Keun-Young Kim, Kwan Yun

arXiv:2502.10245 [pdf, other] hep-th cond-mat.dis-nn cond-mat.str-el gr-qc doi 10.1103/nh4n-ylnz **(PRD)**

Deep learning-based holography for T-linear resistivity

Authors: Byoungjoon Ahn, Hyun-Sik Jeong, Chang-Woo Ji, Keun-Young Kim, Kwan Yun

arXiv:2511.22522 [pdf, ps, other] hep-th cs.LG

AdS/Deep-Learning made easy II: neural network-based approaches to holography and inverse problems

Authors: Hyun-Sik Jeong, Hanse Kim, Keun-Young Kim, Gaya Yun, Hyeonwoo Yu, Kwan Yun

Holographic QI

arXiv:2406.07395 [pdf, other] hep-th cond-mat.dis-nn gr-qc doi 10.1007/JHEP01(2025)025

Holographic reconstruction of black hole spacetime: machine learning and entanglement entropy

Authors: Byoungjoon Ahn, Hyun-Sik Jeong, Keun-Young Kim, Kwan Yun

2020-2022

KAVLI ITS
Kavli Institute for Theoretical Sciences

2015-2020

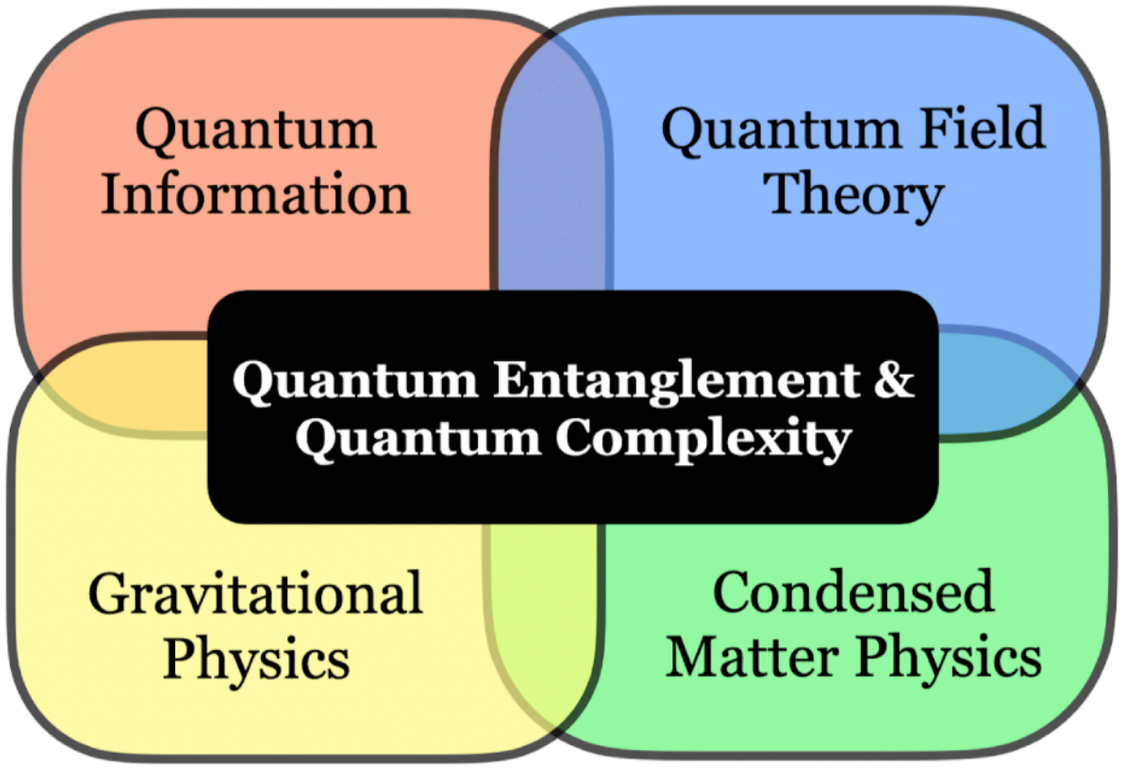
GIST
광주과학기술원
Gwangju Institute of Science and Technology

Advisor:
Keun-Young Kim

2022-2025

ift
Instituto de Física Teórica
UAM-CSIC

Quantum Information Black Holes & Holography



“Quantum Gravity & Quantum Many-Body”

PostDoc

Ashutosh Tripathi

PostDoc

Yongjun Ahn

PostDoc

???

Sep 2025 - Present

apctp

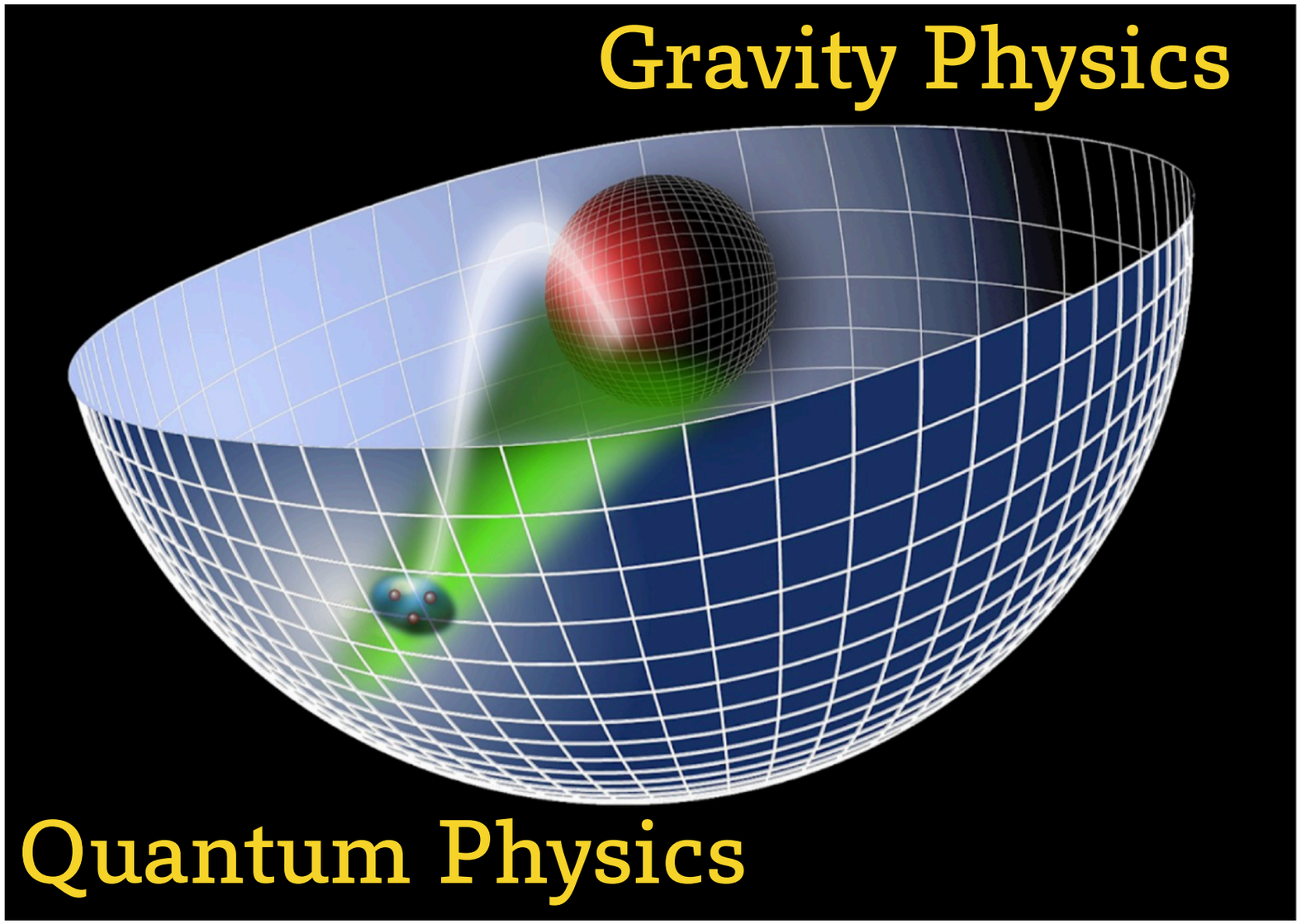
JRG Team

Today's Talk

(Inverse Problems in Holography)

Holography

Holographic Duality



Strongly Coupled

Perturbation Theory (X)

Ex: AdS/CFT Correspondence

- 1997 – Juan Maldacena proposed the AdS/CFT correspondence

iNSPIRE HEP High-Energy Physics

Top Cited Articles of All Time

As of 2025

The Large N limit of superconformal field theories and supergravity #1 AdS/CFT

Juan Martin Maldacena (Harvard U.) (Nov, 1997)

Published in: *Int.J.Theor.Phys.* 38 (1999) 1113-1133 (reprint), *Adv.Theor.Math.Phys.* 2 (1998) 231-252 • e-Print: hep-th/9711200 [hep-th]

pdf DOI cite claim reference search 20,774 citations

A Model of Leptons #2

Steven Weinberg (MIT, LNS) (Nov, 1967)

Published in: *Phys.Rev.Lett.* 19 (1967) 1264-1266

links DOI cite claim reference search 14,900 citations

Anti de Sitter space and holography #3 AdS/CFT

Edward Witten (Princeton, Inst. Advanced Study) (Feb, 1998)

Published in: *Adv.Theor.Math.Phys.* 2 (1998) 253-291 • e-Print: hep-th/9802150 [hep-th]

pdf links DOI cite claim reference search 13,246 citations

Gauge theory correlators from noncritical string theory #4

S.S. Gubser (Princeton U.), Igor R. Klebanov (Princeton U.), Alexander M. Polyakov (Princeton U.) (Feb, 1998)

Published in: *Phys.Lett.B* 428 (1998) 105-114 • e-Print: hep-th/9802109 [hep-th]

pdf DOI cite claim reference search 10,909 citations

CP Conservation in the Presence of Instantons #5

R.D. Peccei (Stanford U., ITP), Helen R. Quinn (Stanford U., ITP) (Mar, 1977)

Published in: *Phys.Rev.Lett.* 38 (1977) 1440-1443

links DOI cite claim reference search 8,540 citations

Juan Maldacena

Born	10 September 1968 (age 56) Buenos Aires, Argentina
Education	Balseiro Institute (BS) Princeton University (PhD)
Known for	AdS/CFT correspondence ER = EPR ABJM superconformal field theory
Awards	Alfred P. Sloan Foundation Fellowship Sackler Prize MacArthur Fellowship Xanthopoulos Pius XI Medal Dirac Medal

No Proof Yet

Evidence of AdS/CFT

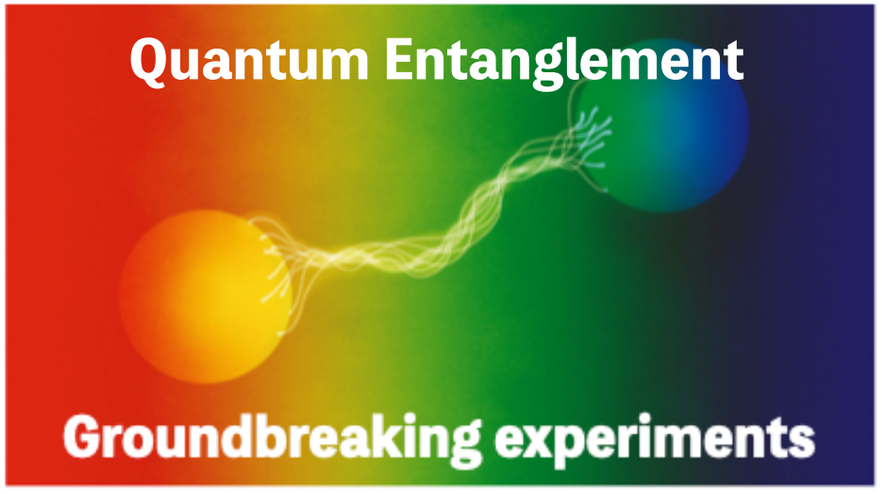
String Theory, SUSY

- SYM theory in (3+1) dimensions $\longleftrightarrow$ IIB supergravity on $\mathrm{AdS}_5 \times \mathrm{S}^5$
- ABJM theory in (2+1) dimensions $\longleftrightarrow$ 11d supergravity on $\mathrm{AdS}_4 \times \mathrm{S}^7/\mathrm{Z}_k$
- SCFT theory in (1+1) dimensions $\longleftrightarrow$ IIB supergravity on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{M}^4$

Entanglement Entropy

- Explicit example of AdS/CFT
- Measure of entanglement
- Quantum nature of gravity

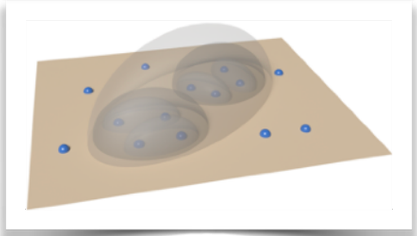
Nobel Prize in Physics 2022



© Johan Jarnestad/The Royal Swedish Academy of Sciences

Ryu-Takayanagi Formula

AdS/QI



Holographic Derivation of Entanglement Entropy from the anti–de Sitter Space/Conformal Field Theory Correspondence

Shinsei Ryu and Tadashi Takayanagi
Phys. Rev. Lett. **96**, 181602 – Published 9 May 2006

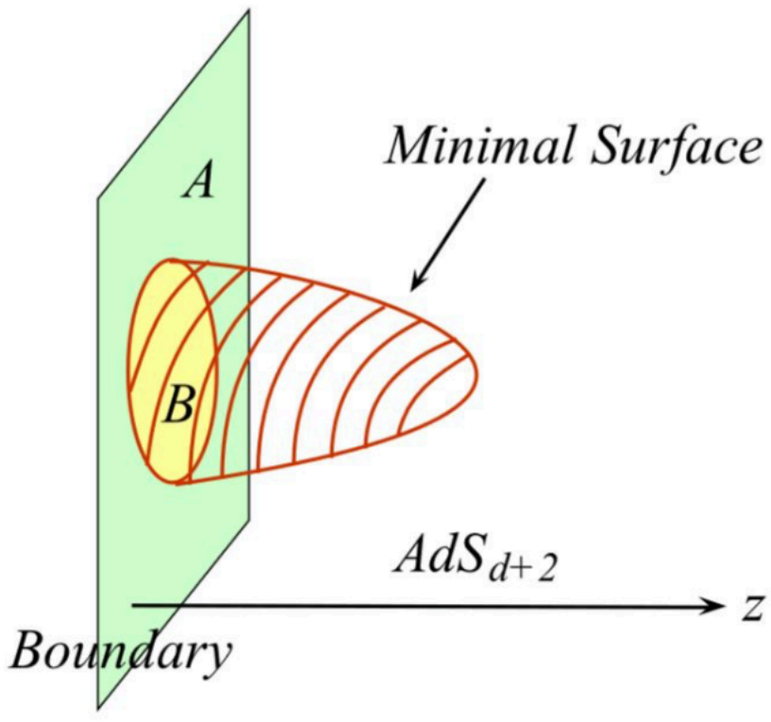
Physical Review Letters

Renyi entropy

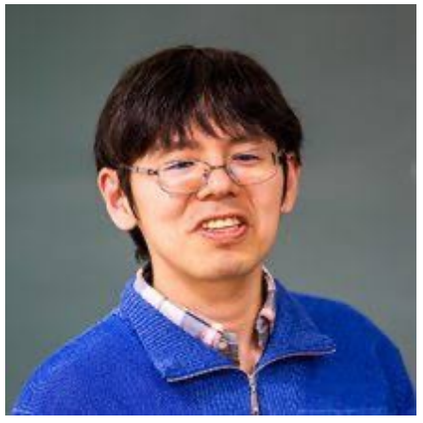
Reflected entropy

Krylov complexity

…



Shinsei Ryu



Tadashi Takayanagi

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N}$$

Quantum Gravity

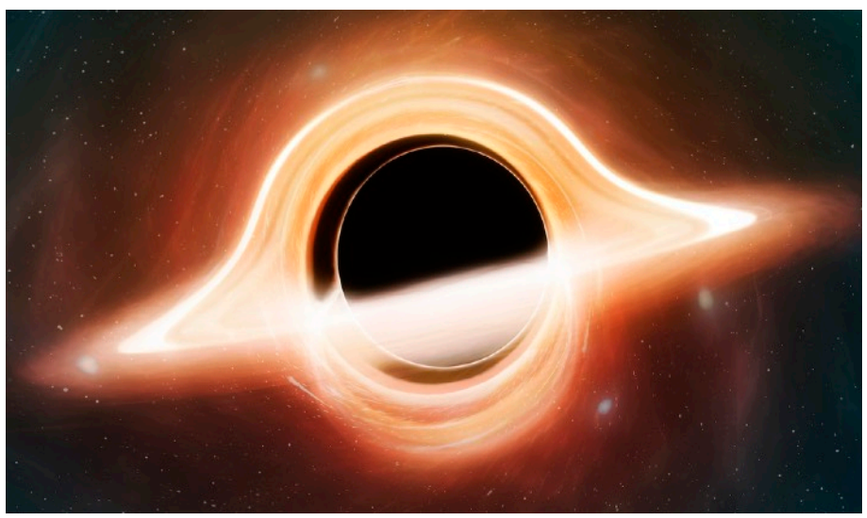
(Quantum) entanglement entropy
= Area of extremal surface

"New perspective of the nature of gravity"

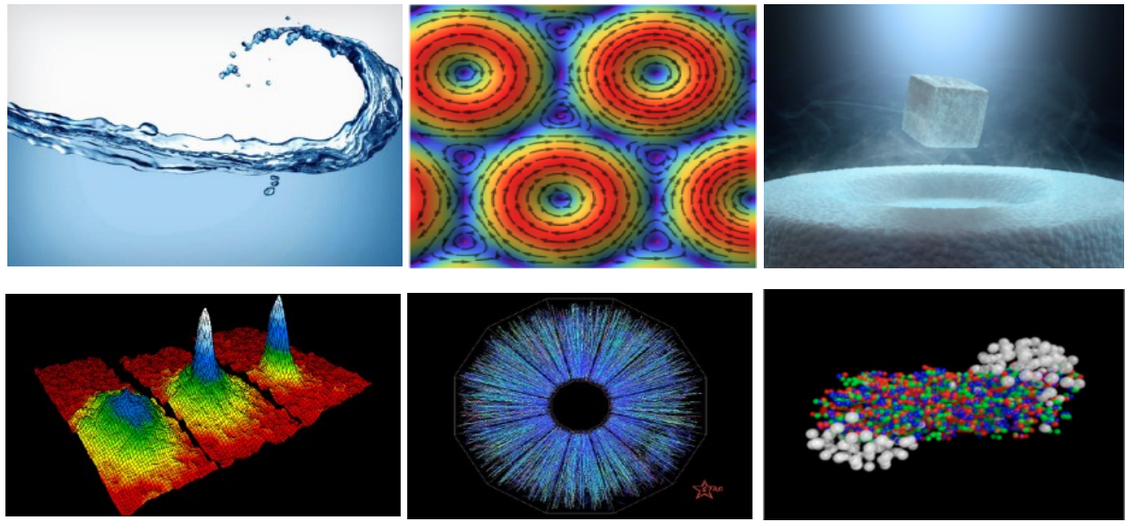
Applied Holography

Gravity Theory → **Quantum Physics**

Describing **gravitational** systems
(e.g., Black Holes)



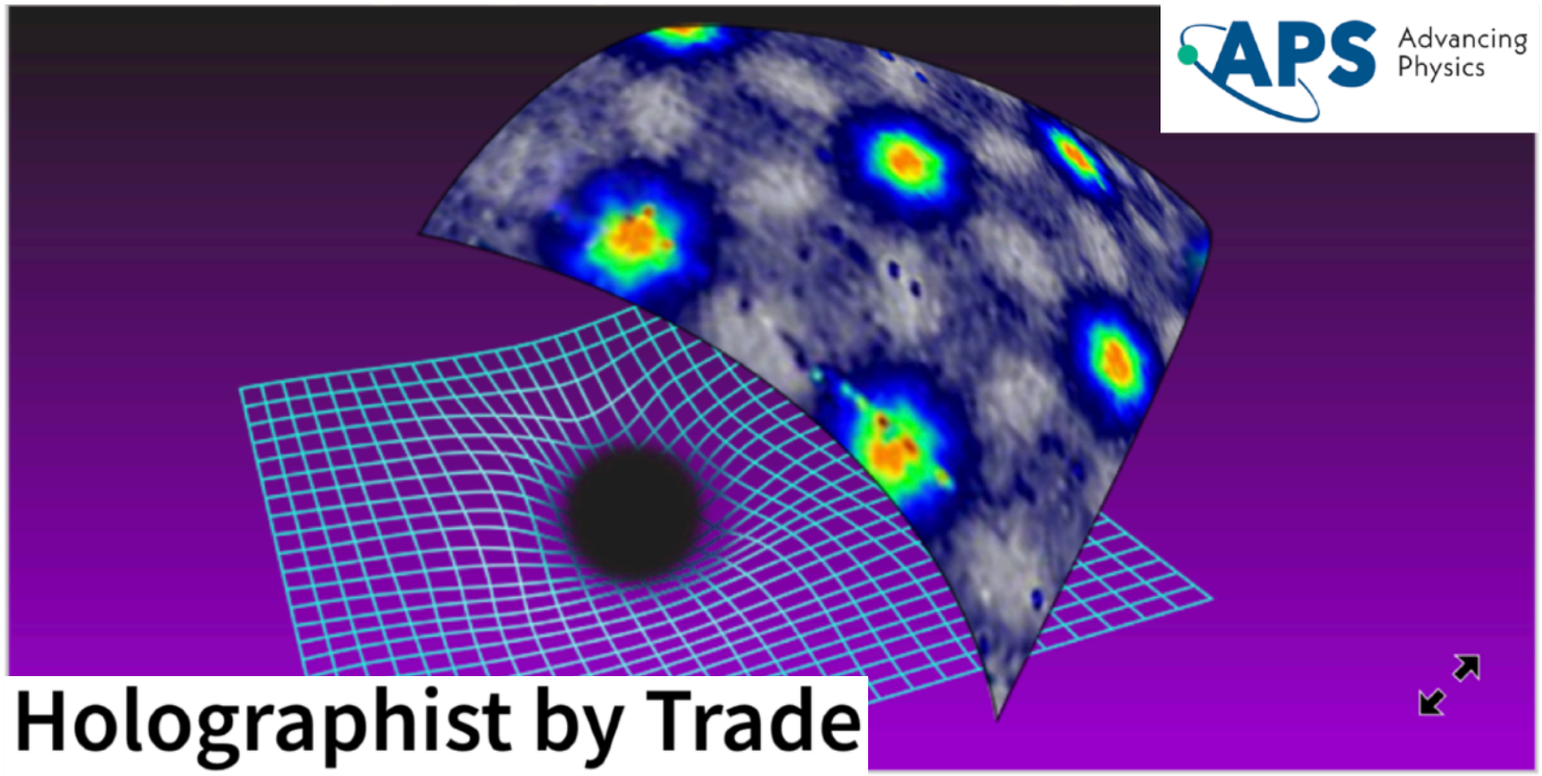
Describing **strongly interacting**
quantum systems



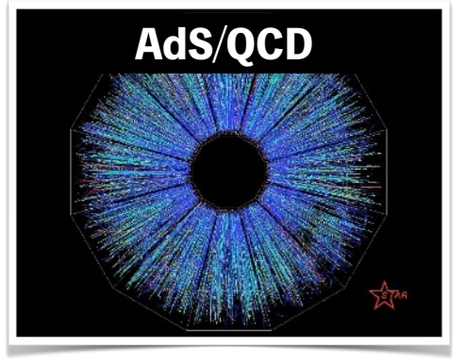
Perturbation Theory (X)

- Study **quantum gravity** in AdS (fundamental physics)
- Study **strongly interacting physics** (practical physics)

Holographic Methods for Many-Body Physics



Holographists use a string theory correspondence that relates black holes and condensed-matter systems, such as metals and superconductors.



Strongly Interacting Quark-Gluon Plasma

High-Tc Superconductors

- Holography has generated a new insights and perspective on many-body physics problem
- Holography has prompted efforts from field theory, hydrodynamics and even experiments

e.g, anomalous transport phenomena

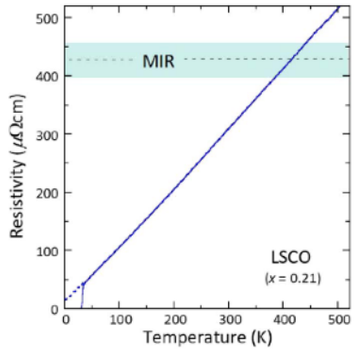
Science

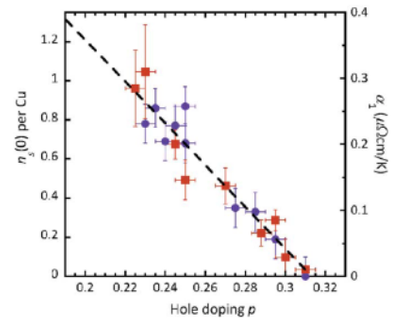
REVIEW | PHYSICS

Stranger than metals REVIEW

PHILIP W. PHILLIPS, NIGEL E. HUSSEY, AND PETER ABBAMONTE Authors Info & Affiliations

SCIENCE • 8 Jul 2022 • Vol 377, Issue 6602 • DOI: 10.1126/science.abh4273





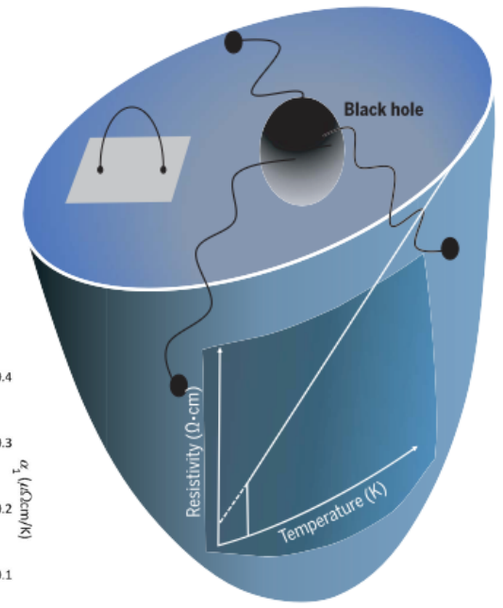


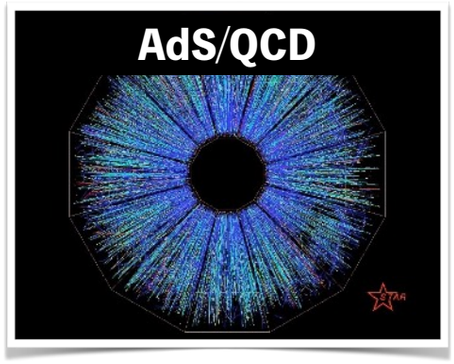
Table 2. Snapshot of current theoretical modeling of the strange metal regime. Indicated are consistency with *T*-linear resistivity, $\omega^{-2/3}$ scaling of the mid-IR optical conductivity, quadrature-scaling magnetoresistance, extended quantum criticality, and what predictions are made in terms of experimental observables. Scenarios: MFL, marginal Fermi liquid; EFL, ersatz Fermi liquid; SYK, Sachdev-Ye-Kitaev; AdS/CFT, anti–de Sitter space/conformal field theory conjecture; AD/EMD, anomalous dimensions/Einstein-Maxwell-dilaton; HM, Hubbard model; QMC, quantum Monte Carlo; ED, exact diagonalization; CA, cold atoms; DMFT/EDMFT, dynamical mean-field theory/embedded dynamical mean-field theory; A-B, Aharonov-Bohm effect; ECFL, extremely correlated Fermi liquid; QCP, quantum critical point. *T-linear resistivity is an input. **A slope change occurs through the MIR. ***Quadrature scaling obtained only for a bivalued random resistor model (*121*) with equal weights (*27*). ****Although this scaling was thought to arise in pure AdS with an inhomogeneous charge density (*123*), later studies (*124*, *125*) found otherwise.

T-linear Resistivity

Holography

	$\rho \propto T$ as $T \to 0$	$\rho \propto T$ as $T \to \infty$	$\sigma \propto \omega^{-2/3}$	Quadrature MR	Extended criticality	Experimental prediction
			Phenomenological			
MFL	√ (*67*)	× (*67*)	×	×	×	Loop currents (*107*)
EFL	— *	—	—	×	×	Loop currents (*108*)
			Numerical			
ECFL	×	(*109*)	—	—	×	×
HM (QMC/ED/CA)	— (*110*)	√ (*110*–*114*)	×	—	—	—
DMFT/EDMFT	√ (*115*)	√ (*116*, *117*)	×	—	√ (*117*)	—
QCP	(*118*)	—	—	—	×	—
			Gravity-based			
SYK	√ (*119*, *120*)	√** (*120*)	×	√*** (*121*)	—	×
AdS/CFT	√ (*122*)	√ (*122*)	√**** (*90*, *126*)	×	×	×
AD/EMD	√ (*127*–*129*)	√ (*90*, *126*, *127*, *129*, *130*)	√ (*90*, *126*, *130*)	×	√ (*126*)	Fractional A-B (*129*)

EX 1: Quark-Gluon Plasma



PHYSICAL REVIEW LETTERS

Highlights Recent Accepted Collections Authors Referees Search Press About Editorial Team

Viscosity in Strongly Interacting Quantum Field Theories from Black Hole Physics

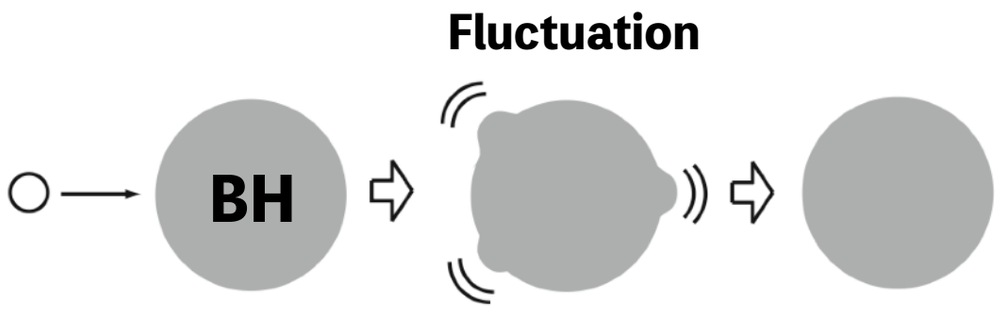
P. K. Kovtun, D. T. Son, and A. O. Starinets
Phys. Rev. Lett. **94**, 111601 – Published 22 March 2005

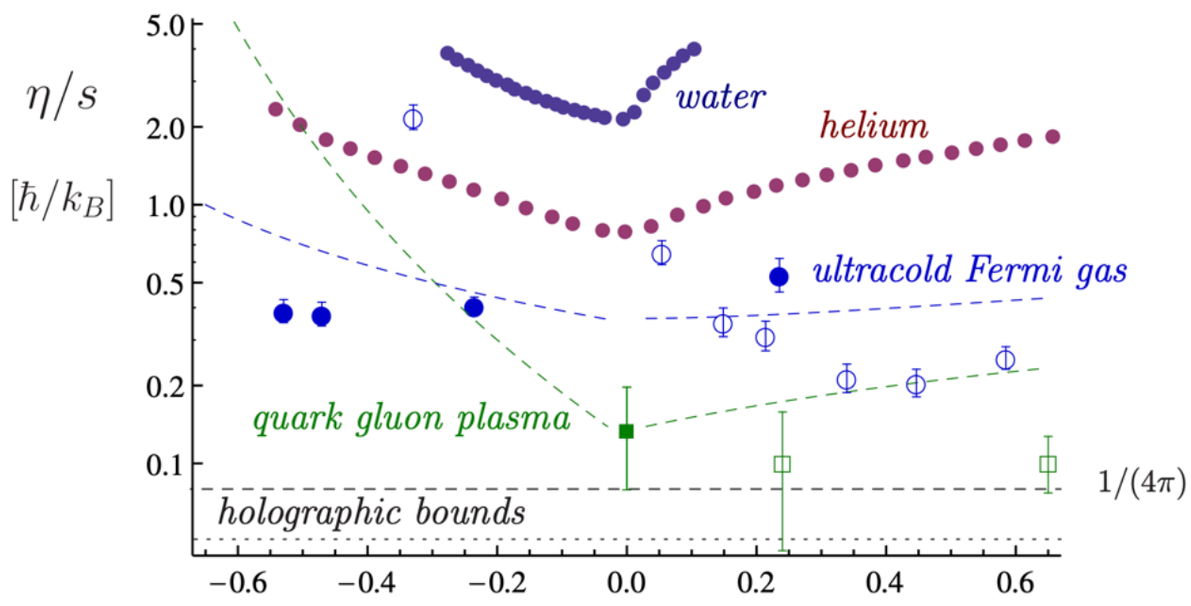
- Viscosity of Quark-Gluon Plasma (Insights into the early universe / heavy-ion collisions)

Perturbative Theory: $\eta/s = \dfrac{A}{\lambda^2 \log(B/\sqrt{\lambda})} \gg 1$ (small t' Hooft coupling λ)

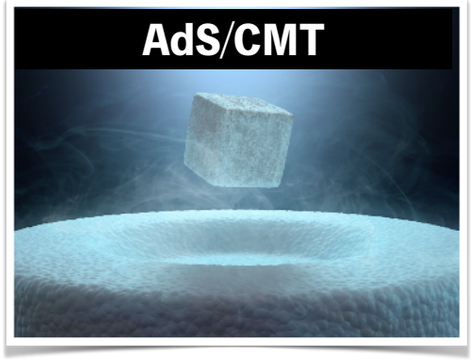
AdS/QCD Theory: $\eta/s = 1/(4\pi)$

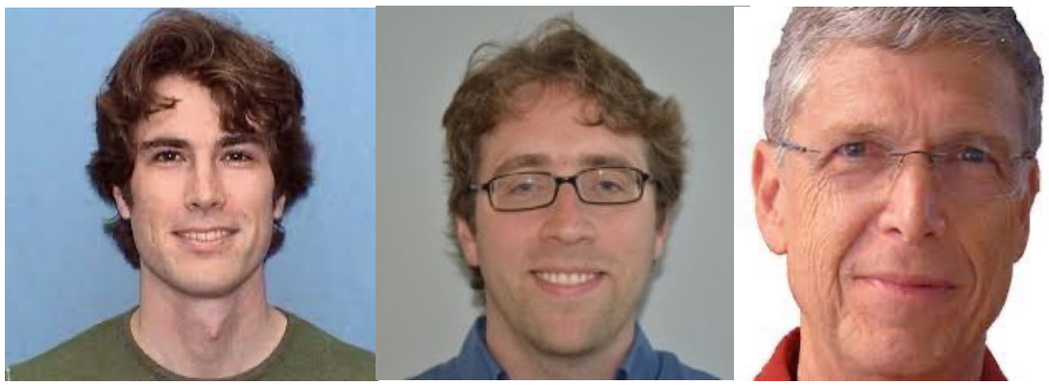
Experiment: $\eta/s \approx 0.19$





EX 2: Superconductors





PHYSICAL REVIEW LETTERS

Highlights Recent Accepted Collections Authors Referees Search Press About Editorial Team

Building a Holographic Superconductor

Sean A. Hartnoll, Christopher P. Herzog, and Gary T. Horowitz
Phys. Rev. Lett. **101**, 031601 – Published 14 July 2008

- New perspective of high-Tc superconductivity

- Spontaneous condensation, infinite conductivities, ⋯

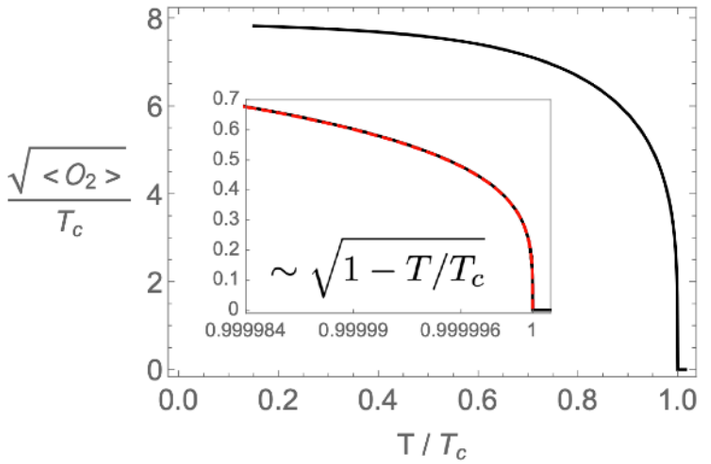
$$\sigma(\omega) = \sigma_0 + \left(\frac{i}{\omega} + \delta(\omega)\right)\frac{\rho_s}{\mu}$$

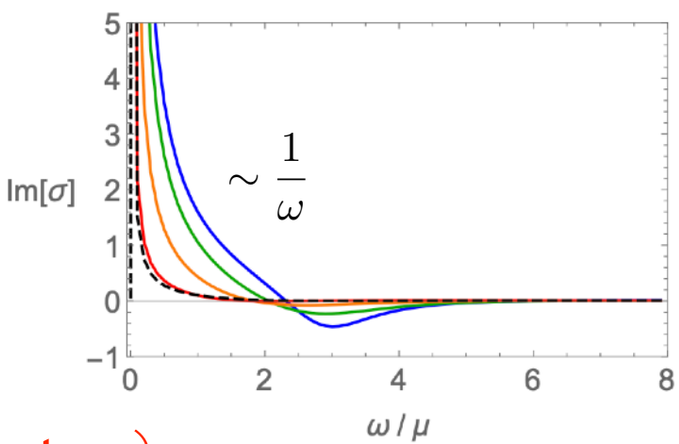
- Energy gap

BCS Theory: $\omega_g/T_c \approx 3.5$ (Weakly-coupled BCS Theory)

AdS/CMT Theory: $\omega_g/T_c \approx 8$ (Holographic Superconductors)

Experiment: $\omega_c/T_c \approx 7.9 \pm 0.5$ (High-Tc Cuprate Superconductors)





Nature **447**, 569–572 (2007)

AdS/Deep Learning

Forward Holography

（Standard Method）

Forward Problem

GRAVITY → **QUANTUM**

System/Model

（Unknown）

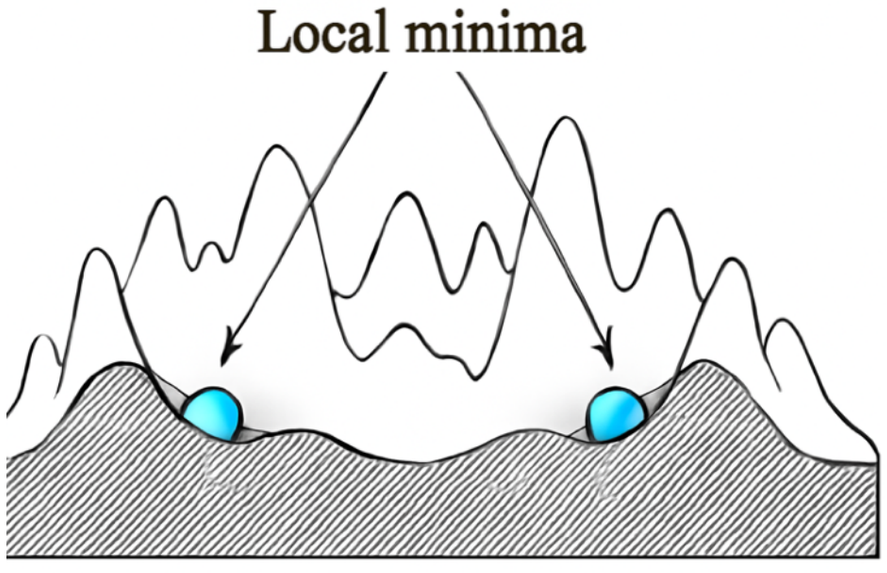
To model realistic systems

Observables

（Exp. accessible）

Model Space

Data Space



Data（fitted by some model）

Inverse Holography

GRAVITY

System/Model

（Unknown）

Forward Problem

New Predictions

To model realistic systems

Data-driven methods

QUANTUM

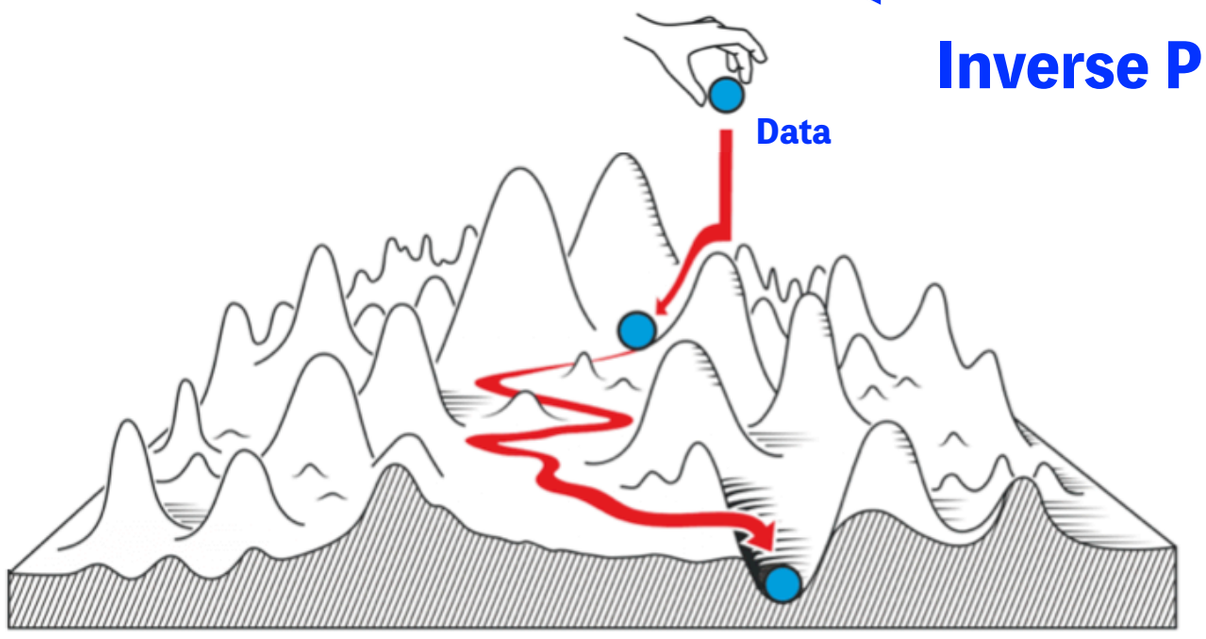
Observables

（Exp. accessible）

Model Space

Inverse Problem

Data Space

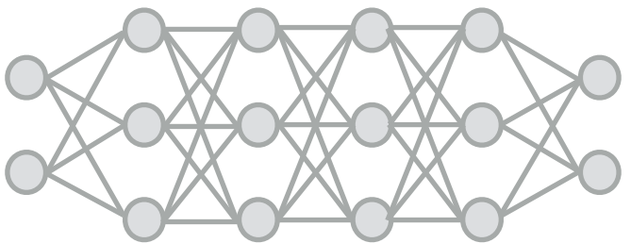


Data（fitted by some model）

Models（constrained by data）

Scientific Machine Learning（SciML）

Machine Learning



Scientific Knowledge

Scientific Machine Learning

Data-driven methods

More powerful, robust, interpretable ML modeling

Terminology

Scientific Machine Learning

Sub-field ↓ vs. purely data-driven approach

Physics-Informed Machine Learning

(+ Physics laws & equations into the learning process)

PDE / ODE ↙ ↘ ODE

Physics-Informed Neural Networks

PINNs

Neural Ordinary Differential Equations

Neural ODEs

Useful References

nature reviews physics

Review Article | Published: 24 May 2021

Physics-informed machine learning

George Em Karniadakis ✉, Ioannis G. Kevrekidis, Lu Lu, Paris Perdikaris, Sifan Wang & Liu Yang

Nature Reviews Physics **3**, 422–440 (2021) | Cite this article

PIML

REVIEWS OF MODERN PHYSICS

Machine learning and the physical sciences*

Giuseppe Carleo, Ignacio Cirac, Kyle Cranmer, Laurent Daudet, Maria Schuld, Naftali Tishby, Leslie Vogt-Maranto, and Lenka Zdeborová

Rev. Mod. Phys. **91**, 045002 – Published 6 December 2019

Physics

Science

Solving the quantum many-body problem with artificial neural networks

GIUSEPPE CARLEO AND MATTHIAS TROYER Authors Info & Affiliations

SCIENCE · 10 Feb 2017 · Vol 355, Issue 6325 · pp. 602-606 · DOI: 10.1126/science.aag2302

Many-Body Physics

TOPICAL REVIEW

Machine learning for condensed matter physics

Edwin Bedolla[1], Luis Carlos Padierna[2,1] and Ramón Castañeda-Priego[1]

Published 5 November 2020 • © 2020 IOP Publishing Ltd

Journal of Physics: Condensed Matter, Volume 33, Number 5

Physics Reports

Data science applications to string theory

Fabian Ruehle ✉

Volume 839, 21 January 2020

String Theory

Broad Applicability of SciML

Particle physics

Many-body physics

Cosmology

Quantum computing

String theory

...

My Interests..

New Prediction

GRAVITY

System/Model

（Unknown）

QUANTUM

Observables

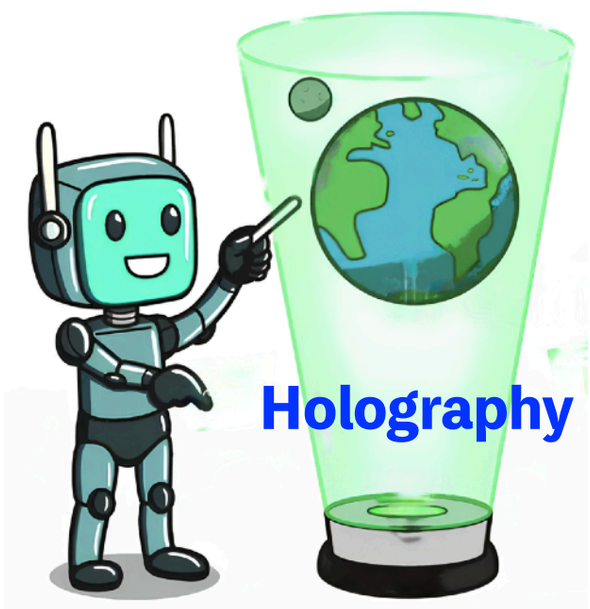
（Exp. accessible）

Strongly Coupled Physics

Quantum Gravity

Model Space

Data Space



Modeling Realistic Systems

Bulk Reconstruction

PINNs

Neural ODEs

AdS/DL Correspondence

Physical Review D

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Deep learning and the AdS / CFT correspondence

Koji Hashimoto[1], Sotaro Sugishita[1], Akinori Tanaka[2,3,4], and Akio Tomiya[5]

Phys. Rev. D **98**, 046019 – **Published 27 August, 2018**

DOI: https://doi.org/10.1103/PhysRevD.98.046019

Abstract

We present a deep neural network representation of the AdS / CFT correspondence, and demonstrate the emergence of the bulk metric function via the learning process for given data sets of response in boundary quantum field theories. The emergent radial direction of the bulk is identified with the depth of the layers, and the network itself is interpreted as a bulk geometry. Our network provides a data-driven holographic modeling of strongly coupled systems. With a scalar ϕ^4 theory with unknown mass and coupling, in unknown curved spacetime with a black hole horizon, we demonstrate that our deep learning (DL) framework can determine the systems that fit given response data. First, we show that, from boundary data generated by the anti–de Sitter (AdS) Schwarzschild spacetime, our network can reproduce the metric. Second, we demonstrate that our network with experimental data as an input can determine the bulk metric, the mass and the quadratic coupling of the holographic model. As an example we use the experimental data of the magnetic response of the strongly correlated material $Sm_{0.6}Sr_{0.4}MnO_3$. This AdS / DL correspondence not only enables gravitational modeling of strongly correlated systems, but also sheds light on a hidden mechanism of the emerging space in both AdS and DL.

Observables

AdS/QCD

High Energy Physics – Theory arXiv:1809.10536

[Submitted on 27 Sep 2018]

Deep Learning and Holographic QCD

Koji Hashimoto, Sotaro Sugishita, Akinori Tanaka, Akio Tomiya

Phys. Rev. D **98**, 106014 – Published 14 November 2018

Chiral condensate

High Energy Physics – Theory arXiv:2108.08091

[Submitted on 18 Aug 2021 (v1), last revised 31 Aug 2021 (this version, v2)]

Deriving dilaton potential in improved holographic QCD from meson spectrum

Koji Hashimoto, Keisuke Ohashi, Takayuki Sumimoto

Meson spectrum

Phys. Rev. D **105**, 106008 – Published 13 May 2022

High Energy Physics – Phenomenology arXiv:2401.06417

[Submitted on 12 Jan 2024 (v1), last revised 30 Mar 2024 (this version, v2)]

Machine learning holographic black hole from lattice QCD equation of state

Xun Chen, Mei Huang

Lattice QCD

Phys. Rev. D **109**, L051902 – Published 25 March 2024

High Energy Physics – Theory arXiv:2511.22522

[Submitted on 27 Nov 2025]

AdS/Deep-Learning made easy II: neural network-based approaches to holography and inverse problems

Hyun-Sik Jeong, Hanse Kim, Keun-Young Kim, Gaya Yun, Hyeonwoo Yu, Kwan Yun

QCD Equations of State

MANY MORE …

AdS/CMT

High Energy Physics – Theory arXiv:2004.12112

[Submitted on 25 Apr 2020 (v1), last revised 3 Nov 2020 (this version, v3)]

Deep learning black hole metrics from shear viscosity

Yu-Kun Yan, Shao-Feng Wu, Xian-Hui Ge, Yu Tian

Phys. Rev. D **102**, 101902(R) – Published 12 November 2020

Shear viscosity

High Energy Physics – Theory arXiv:2410.06523

[Submitted on 9 Oct 2024]

Phase Diagram from Nonlinear Interaction between Superconducting Order and Density: Toward Data-Based Holographic Superconductor

Sejin Kim, Kyung Kiu Kim, Yunseok Seo

Superconductivity

J. High Energ. Phys. **2025**, 77 (2025)

High Energy Physics – Theory arXiv:2401.00939

[Submitted on 1 Jan 2024]

Deep learning bulk spacetime from boundary optical conductivity

Byoungjoon Ahn, Hyun-Sik Jeong, Keun-Young Kim, Kwan Yun

J. High Energ. Phys. **2024**, 141 (2024) **Optical conductivity**

High Energy Physics – Theory arXiv:2502.10245

[Submitted on 14 Feb 2025]

Deep learning-based holography for T-linear resistivity

Byoungjoon Ahn, Hyun-Sik Jeong, Chang-Woo Ji, Keun-Young Kim, Kwan Yun

Phys. Rev. D **112**, 126008 – **Published 12 December, 2025**

T-linear resistivity

High Energy Physics – Theory arXiv:2411.16052

[Submitted on 25 Nov 2024]

Machine-learning emergent spacetime from linear response in future tabletop quantum gravity experiments

Koji Hashimoto, Koshiro Matsuo, Masaki Murata, Gakuto Ogiwara, Daichi Takeda

Machine Learning: Science and Technology, Volume 6, Number 1

AdS/QI

High Energy Physics – Theory arXiv:2110.01115

[Submitted on 3 Oct 2021]

Machine Learning Statistical Gravity from Multi-Region Entanglement Entropy

Jonathan Lam, Yi-Zhuang You

Phys. Rev. Research **3**, 043199 – Published 20 December 2021

High Energy Physics – Theory arXiv:2205.04445

[Submitted on 9 May 2022 (v1), last revised 11 Sep 2022 (this version, v3)]

Dual Geometry of Entanglement Entropy via Deep Learning

Chanyong Park, Chi-Ok Hwang, Kyungchan Cho, Se-Jin Kim

Phys. Rev. D **106**, 106017 – Published 28 November 2022

High Energy Physics – Theory arXiv:2406.07395

[Submitted on 11 Jun 2024 (v1), last revised 8 Sep 2024 (this version, v2)]

Holographic reconstruction of black hole spacetime: machine learning and entanglement entropy

Byoungjoon Ahn, Hyun-Sik Jeong, Keun-Young Kim, Kwan Yun

J. High Energ. Phys. **2025**, 25 (2025) **Entanglement entropy**

High Energy Physics – Theory arXiv:2509.25311

[Submitted on 29 Sep 2025]

Aspects of holographic entanglement using physics-informed-neural-networks

Anirudh Deb, Yaman Sanghavi

Wedge Cross Section

High Energy Physics – Theory arXiv:2509.10866

[Submitted on 13 Sep 2025 (v1), last revised 16 Sep 2025 (this version, v2)]

Physics-informed neural network solves minimal surfaces in curved spacetime

Koji Hashimoto, Koichi Kyo, Masaki Murata, Gakuto Ogiwara, Norihiro Tanahashi

Observables

AdS/QCD

High Energy Physics – Theory arXiv:1809.10536

[Submitted on 27 Sep 2018]

Deep Learning and Holographic QCD

Koji Hashimoto, Sotaro Sugishita, Akinori Tanaka, Akio Tomiya

Phys. Rev. D **98**, 106014 – Published 14 November 2018

Chiral condensate

High Energy Physics – Theory arXiv:2108.08091

[Submitted on 18 Aug 2021 (v1), last revised 31 Aug 2021 (this version, v2)]

Deriving dilaton potential in improved holographic QCD from meson spectrum

Koji Hashimoto, Keisuke Ohashi, Takayuki Sumimoto

Meson spectrum

Phys. Rev. D **105**, 106008 – Published 13 May 2022

High Energy Physics – Phenomenology arXiv:2401.06417

[Submitted on 12 Jan 2024 (v1), last revised 30 Mar 2024 (this version, v2)]

Machine learning holographic black hole from lattice QCD equation of state

Xun Chen, Mei Huang

Lattice QCD

Phys. Rev. D **109**, L051902 – Published 25 March 2024

High Energy Physics – Theory arXiv:2511.22522

[Submitted on 27 Nov 2025]

AdS/Deep-Learning made easy II: neural network-based approaches to holography and inverse problems

Hyun-Sik Jeong, Hanse Kim, Keun-Young Kim, Gaya Yun, Hyeonwoo Yu, Kwan Yun

QCD Equations of State

AdS/CMT

High Energy Physics – Theory arXiv:2004.12112

[Submitted on 25 Apr 2020 (v1), last revised 3 Nov 2020 (this version, v3)]

Deep learning black hole metrics from shear viscosity

Yu-Kun Yan, Shao-Feng Wu, Xian-Hui Ge, Yu Tian

Phys. Rev. D **102**, 101902(R) – Published 12 November 2020

Shear viscosity

High Energy Physics – Theory arXiv:2410.06523

[Submitted on 9 Oct 2024]

Phase Diagram from Nonlinear Interaction between Superconducting Order and Density: Toward Data-Based Holographic Superconductor

Sejin Kim, Kyung Kiu Kim, Yunseok Seo

Superconductivity

J. High Energ. Phys. **2025**, 77 (2025)

High Energy Physics – Theory arXiv:2401.00939

[Submitted on 1 Jan 2024]

Deep learning bulk spacetime from boundary optical conductivity

Byoungjoon Ahn, Hyun-Sik Jeong, Keun-Young Kim, Kwan Yun

J. High Energ. Phys. **2024**, 141 (2024) **Optical conductivity**

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MANY MORE ⋯

My Personal Goals（Almost Dream, lol）

Methodologies

Consistency

Can ML be a consistent methods?
(complement with traditional methods)

New Solutions

Can it find new solutions
not transparent in traditional methods?

Data

Robustness

How to construct models
that are both robust and accurate for data?

Predictions

What new insights and predictions
could ML provide?

• OUTLINE

1 | Methodology

: Physics-Informed Neural Networks (PINNs)

2 | Handful Examples

: AdS/QCD, CMT, QI

3 | Conclusion

Methodology

: Physics-Informed Neural Networks (PINNs)

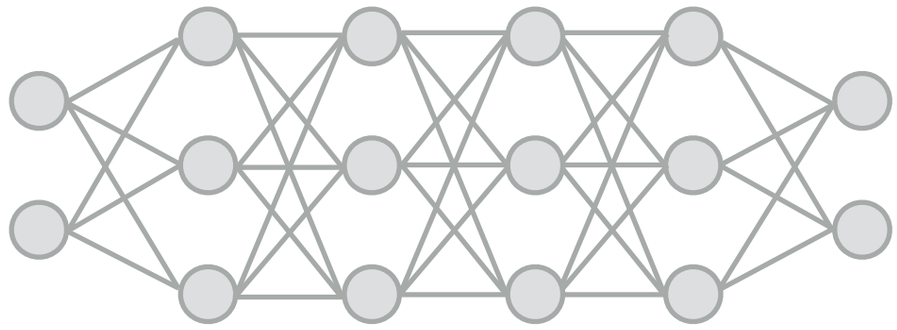
Neural Networks

Nobel Prize in Physics 2024

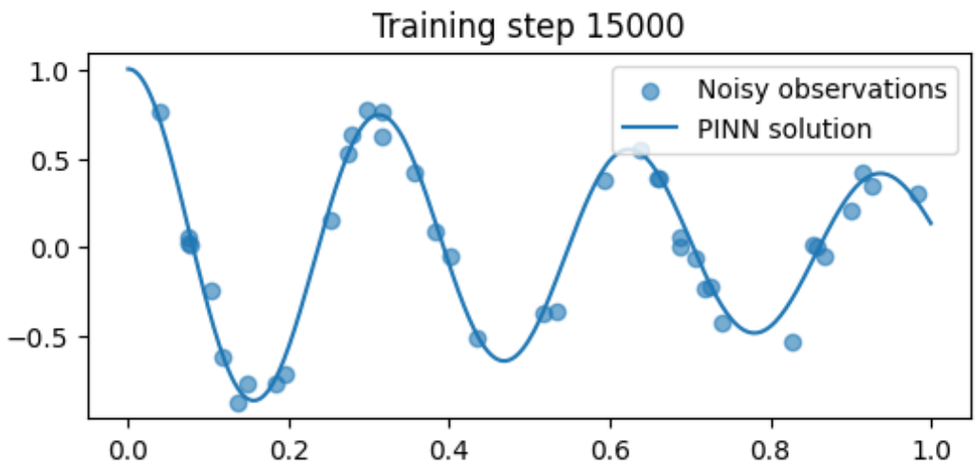
"For foundational discoveries and inventions that enable machine learning with artificial neural networks"

Neural Networks

Training parameters

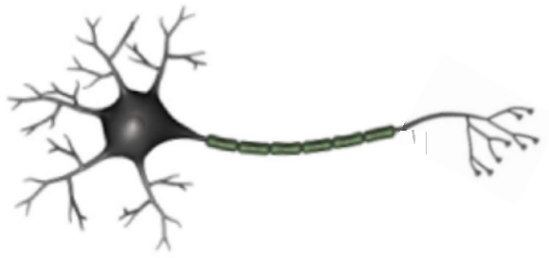
x  $y = \mathrm{NN}(x,\ \theta\ )$

Neural networks are simply **flexible functions** fit to data



Given data, tune the **parameter** θ so that the network approximates the true function

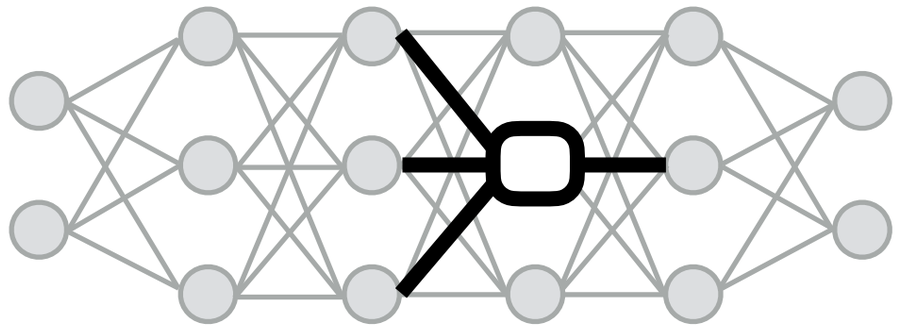
$$\mathrm{NN}(x,\ \theta\ ) \approx y_{\mathrm{true}}(x)$$



Biological Neuron

Perceptron

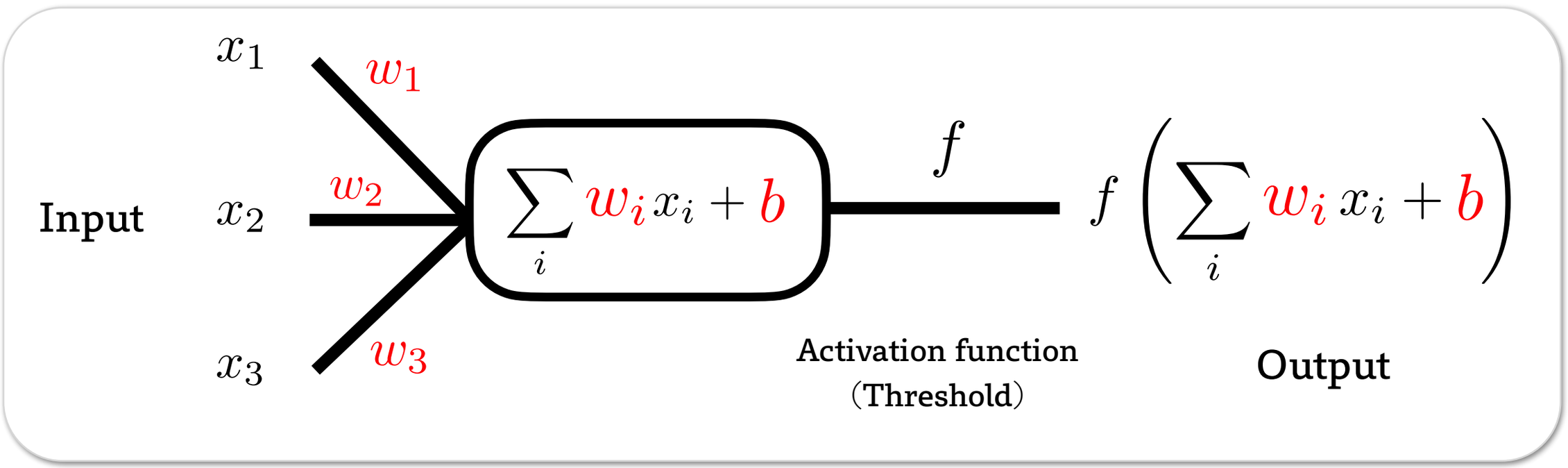
Training parameters



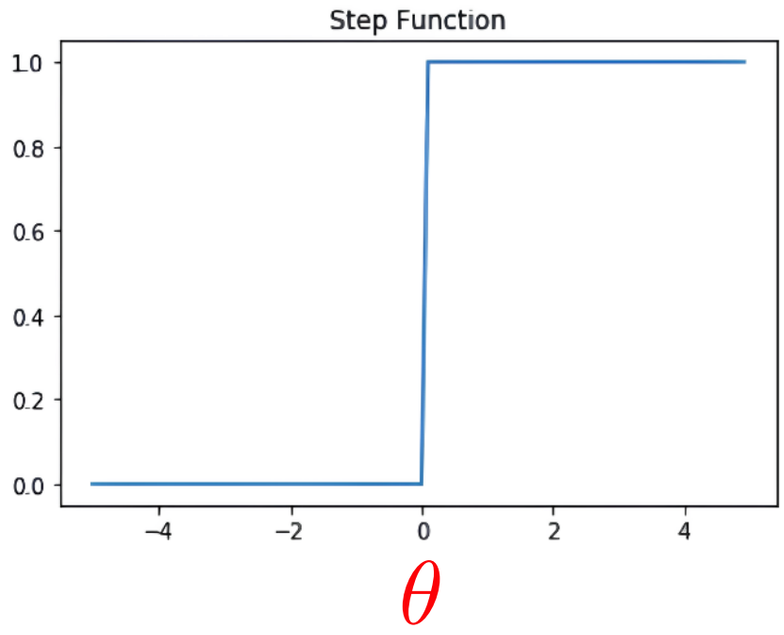
x

$$y = \mathrm{NN}(x,\ \theta\ )$$

Perceptron is the fundamental unit of a neural network



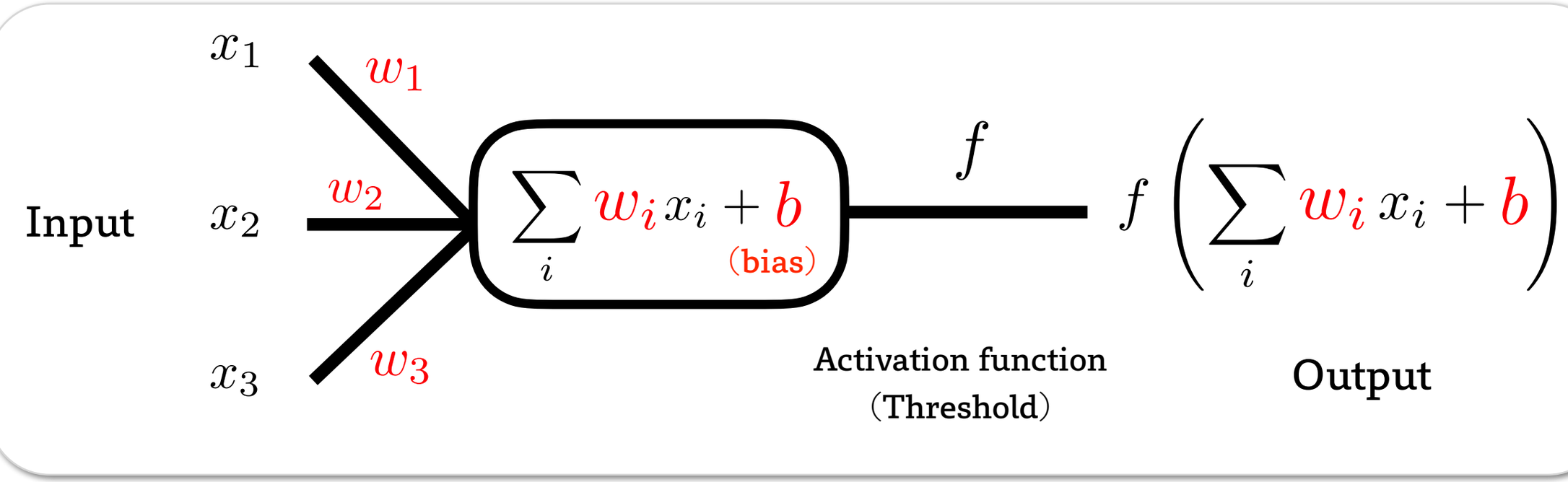
Activation Function



IF, $\sum_i w_i x_i \geq \theta$ (Threshold) $\longrightarrow$ $y = 1$ (Activated!)

IF, $\sum_i w_i x_i < \theta$ $\longrightarrow$ $y = 0$ (Deactivated!)

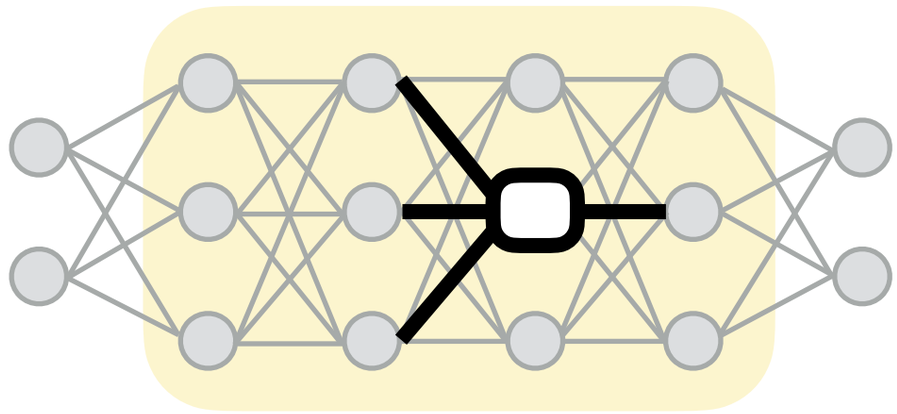
Activation function adds non-linearity by changing the output of a neuron, helping the model capture more complex patterns



Deep Neural Networks

Training parameters

$$x \quad \text{Hidden Layers} \quad y = \mathrm{NN}(x,\ \theta\ )$$



$$Wx + b$$

Deep neural networks are composition functions of NNs

$$\mathrm{NN}(x,\ \theta\ ) \approx W_M \cdot f\left[\cdots f\left\{W_2 \cdot f\left(W_1 \underbrace{x} + b_1\right) + b_2\right\}\cdots\right] + b_M$$

$\theta = \{W_i, b_i\}$ are learning parameters

# of layers

Universal Approximation Theorem

Original contribution Neural Networks Volume 2, Issue 5, 1989, Pages 359-366

Multilayer feedforward networks are universal approximators

Kurt Hornik, Maxwell Stinchcombe, Halbert White [1]

ELSEVIER

This paper rigorously establishes that standard multilayer feedforward networks with as few as one hidden layer using arbitrary squashing functions are capable of approximating any Borel measurable function from one finite dimensional space to another to any desired degree of accuracy, provided sufficiently many hidden units are available. In this sense, multilayer feedforward networks are a class of universal approximators.

DNNs, $\mathrm{NN}(x, \theta)$, can approximate **any continuous functions** $y(x)$

Existence Theorem

Definition 2.1

For any $r \in N \equiv \{1, 2, \ldots\}$, A^r is the set of all affine functions from R^r to R, that is, the set of all functions of the form $A(x) = w \cdot x + b$ where w and x are vectors in R^r, "$\cdot$" denotes the usual dot product of vectors, and $b \in R$ is a scalar. □

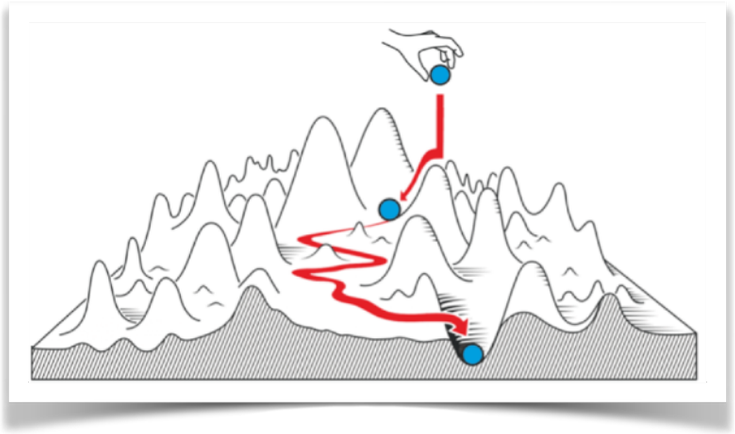
•
•
•

For every function g in M^r there is a compact subset K of R^r and an $f \in \Sigma^r(\Psi)$ such that for any $\varepsilon > 0$ we have $\mu(K) < 1 - \varepsilon$ and for every $x \in K$ we have $|f(x) - g(x)| < \varepsilon$, regardless of Ψ, r, or μ. □

$$| y(x) - \mathrm{NN}(x, \theta) | < \epsilon$$

How to find such solutions? or how to train DNNs?

Loss Function



We tune the parameters **minimizing** some loss function $L(\theta)$

Example:
Mean Squared Error
(L2 Loss)

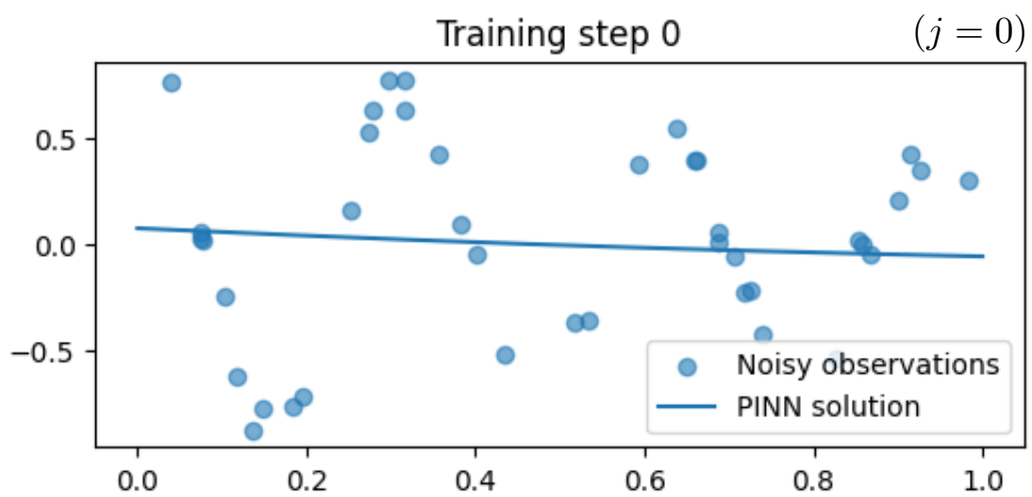
$$L(\theta) = \frac{1}{N}\sum_{i}^{N}[\,\mathrm{NN}(x_i, \theta) - y(x_i)]^2$$

Training DNNs — Data

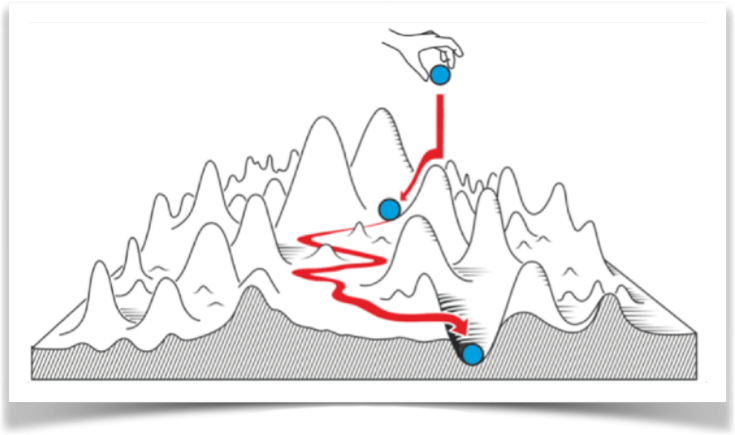
Typically, by using **gradient decent**, parameters are optimized iteratively

$$\theta^{j+1} = \theta^{j} - l_r \nabla_{\theta} L(\theta^{j})$$

the iteration (epoch) — learning rate (step size)



Loss Function



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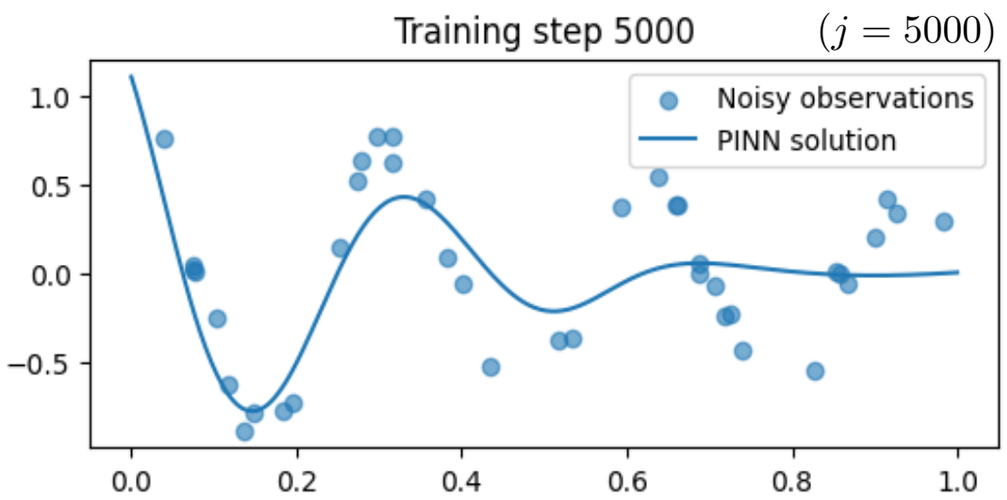
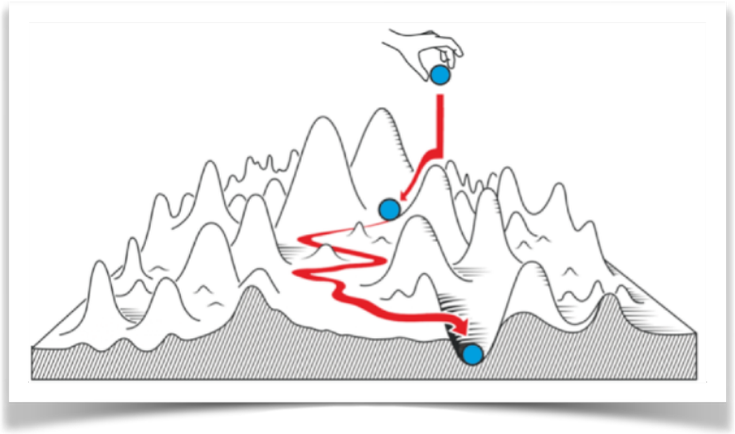


Deep Neural Networks

Automatic Differentiation

Adam Optimizer **L-BFGS**



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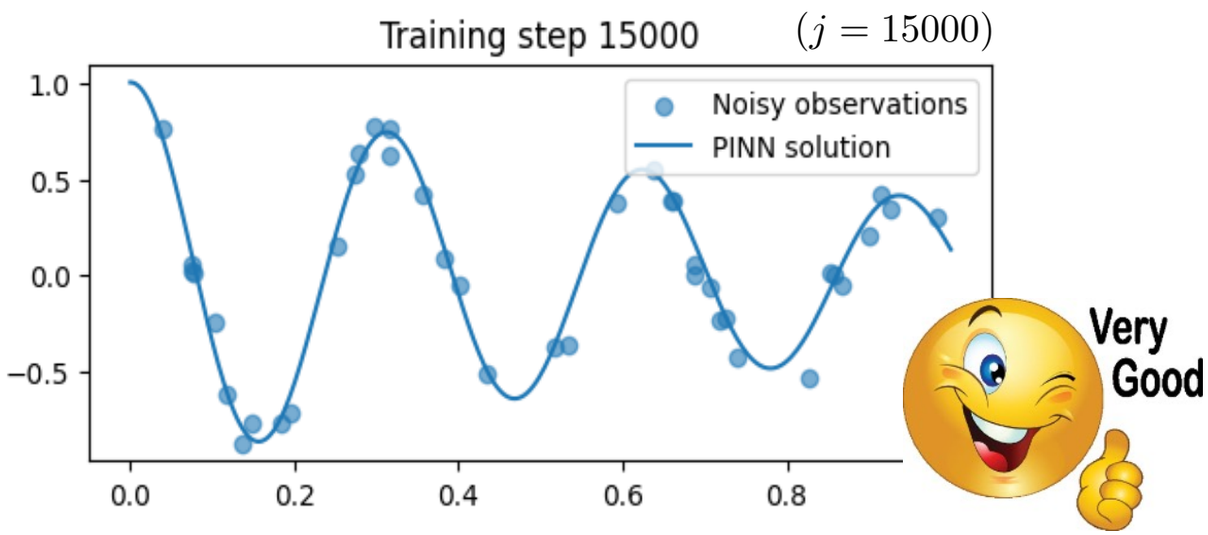


Physics-Informed Neural Networks (PINNs)

Physics-Informed Neural Networks

Journal of Computational Physics Volume 378, 1 February 2019, Pages 686-707

Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations

M. Raissi [a], P. Perdikaris [b], G.E. Karniadakis [a]

ELSEVIER

CITED BY	YEAR
18453	2019

PINNs incorporate **physical laws** (**PDE, ODEs**), directly into the training process.

$$L(\theta) = L(\theta)_{\text{Data}} + L(\theta)_{\text{Equations}}$$

Standard NN
(solely rely on data)

Data Loss

Physical Loss

To ensure the model
fits the available data points

To encode the governing
physical equations

1) to construct **robust models** (even with limited or noisy data) — preventing overfitting problem

2) a **new (forward problem) solver** to model system dynamics

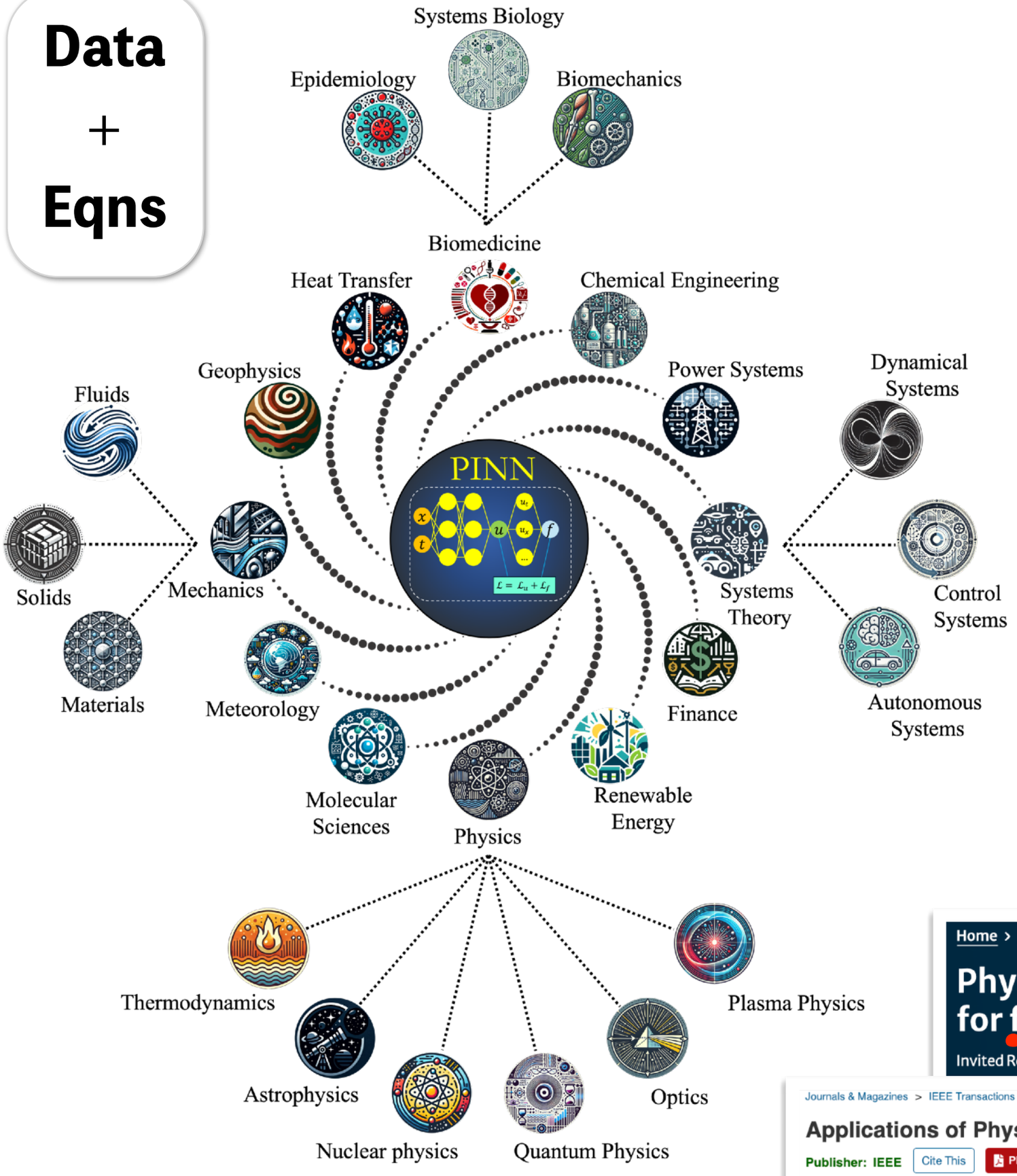
3) an **inverse problem solver** to learn the underlying physical parameters of the system

Broad Applicability of PINNs

Data

+

Eqns



Home > Journal of Scientific Computing > Article

Scientific Machine Learning Through Physics–Informed Neural Networks: Where we are and What's Next

Open access | Published: 26 July 2022

Volume 92, article number 88, (2022) Cite this article

Computer Science > Machine Learning arXiv:2410.13228

From PINNs to PIKANs: Recent Advances in Physics-Informed Machine Learning

Juan Diego Toscano[a,1], Vivek Oommen[b,1], Alan John Varghese[b,1], Zongren Zou[a], Nazanin Ahmadi Daryakenari[b,c], Chenxi Wu[b], George Em Karniadakis[a,b,2]

[a]Division of Applied Mathematics, Brown University, Providence, 02912, RI, USA
[b]School of Engineering, Brown University, Providence, 02912, RI, USA
[c]Center for Biomedical Engineering, Brown University, Providence, 02912, RI, USA

Computer Science > Machine Learning arXiv:2510.05433

Physics-Informed Machine Learning in Biomedical Science and Engineering

Home > Acta Mechanica Sinica > Article

Physics-informed neural networks (PINNs) for fluid mechanics: a review

Invited Review | Published: 23 January 2022

Journals & Magazines > IEEE Transactions on Power Sy... > Volume: 38 Issue: 1

Applications of Physics-Informed Neural Networks in Power Systems - A Review

Publisher: IEEE Cite This PDF

Bin Huang ; Jianhui Wang All Authors

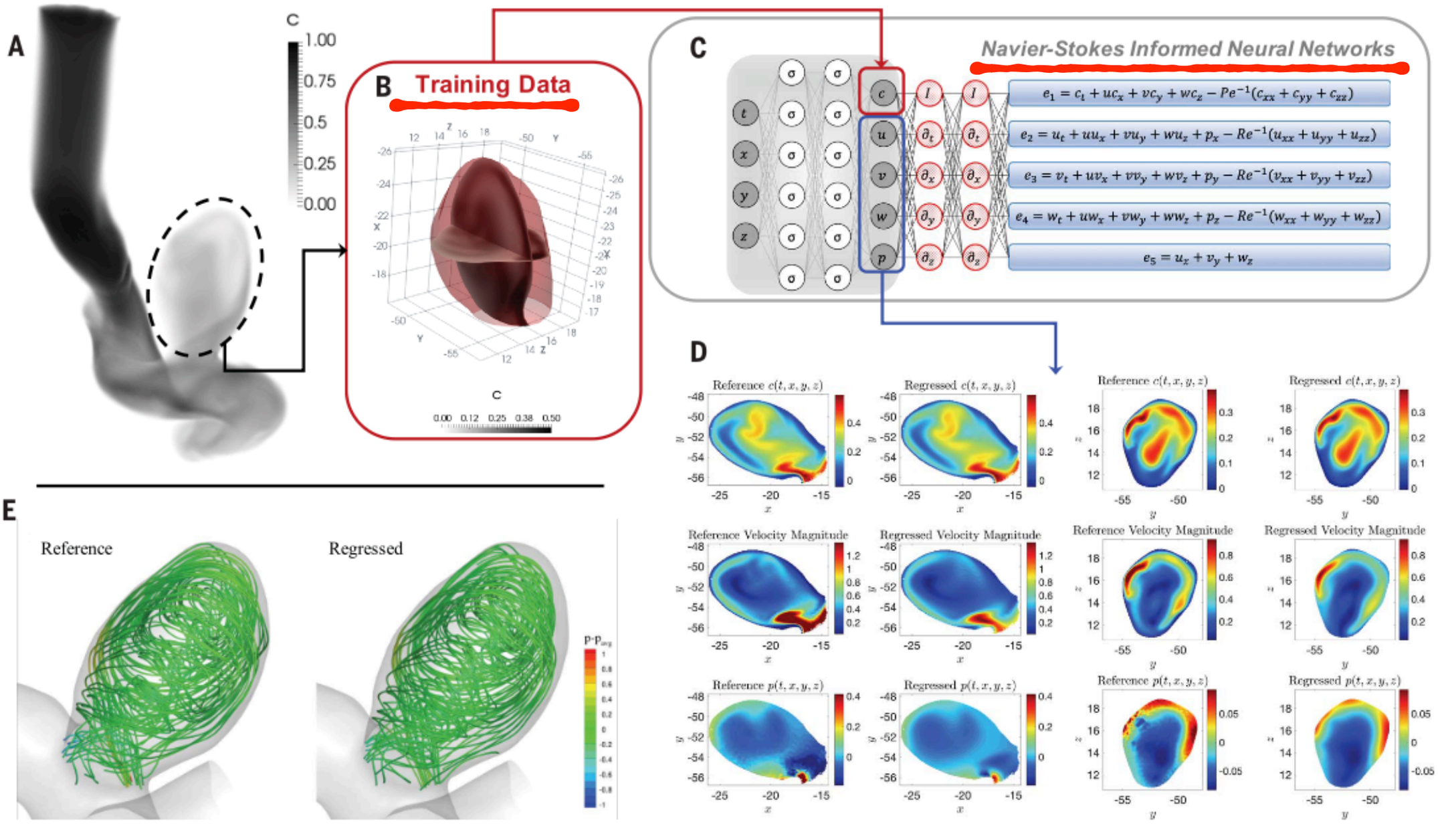
Science

Hidden fluid mechanics: Learning velocity and pressure fields from flow visualizations

MAZIAR RAISSI, ALIREZA YAZDANI, AND GEORGE EM KARNIADAKIS Authors Info & Affiliations

SCIENCE · 30 Jan 2020 · Vol 367, Issue 6481 · pp. 1026-1030 · DOI: 10.1126/science.aaw4741

Data +
Navier-Stokes Eqns

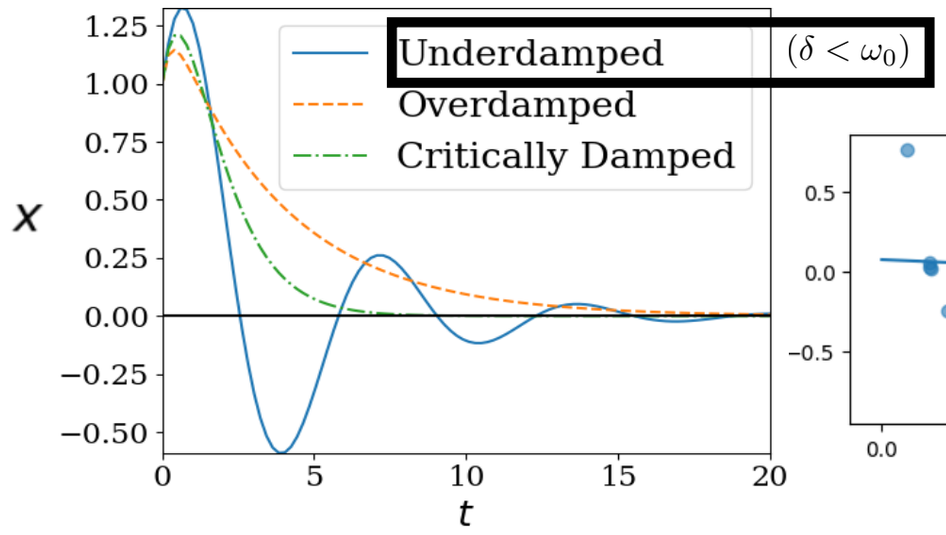


Example

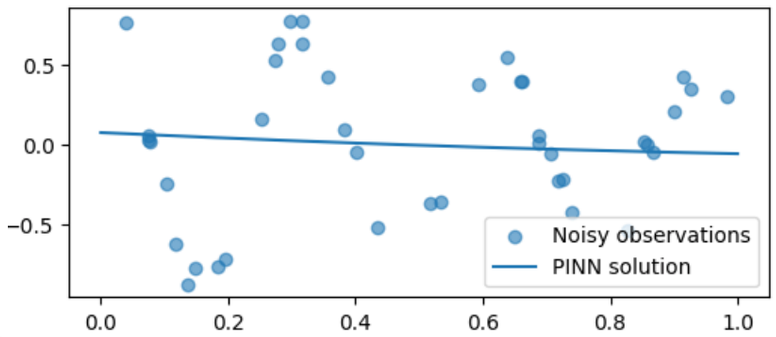
Damped Harmonic Oscillator

$$x''(t) + 2\,\delta\,x'(t) + \omega_0^2\,x(t) = 0$$

Damping coefficient — Spring constant



Data Preparation

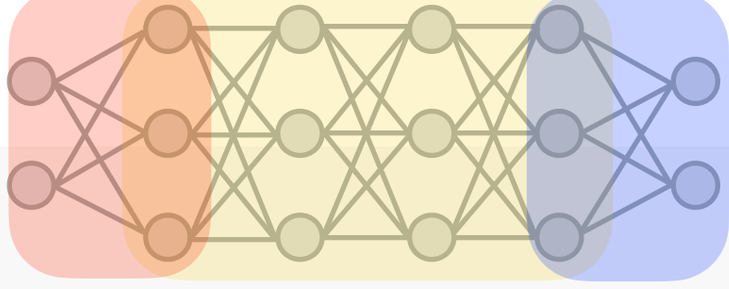


$(\delta,\, \omega_0) = (1,\, 20)$

PyTorch

```
import torch
import torch.nn as nn
```

nn: Neural Networks



DNNs

```
def __init__(self, N_INPUT, N_OUTPUT, N_HIDDEN, N_LAYERS):
    super().__init__()
    activation = nn.Tanh
    self.fcs = nn.Sequential(*[nn.Linear(N_INPUT, N_HIDDEN),activation()])
    self.fch = nn.Sequential(*[nn.Sequential(*[nn.Linear(N_HIDDEN, N_HIDDEN),activation()]) for _ in range(N_LAYERS-1)])
    self.fce = nn.Linear(N_HIDDEN, N_OUTPUT)
```

δ , ω_0

```
d = torch.nn.Parameter(torch.zeros(1, requires_grad=True))
w0 = torch.nn.Parameter(torch.empty(1).uniform_(0, 50), requires_grad=True)
```

Training parameters

```
torch.optim.Adam(list(pinn.parameters())+[d]+[w0],lr=1e-3)
```

Optimizer

```
dxdt = torch.autograd.grad(x, t_physics, torch.ones_like(x), create_graph=True)[0]
```

Auto-grad

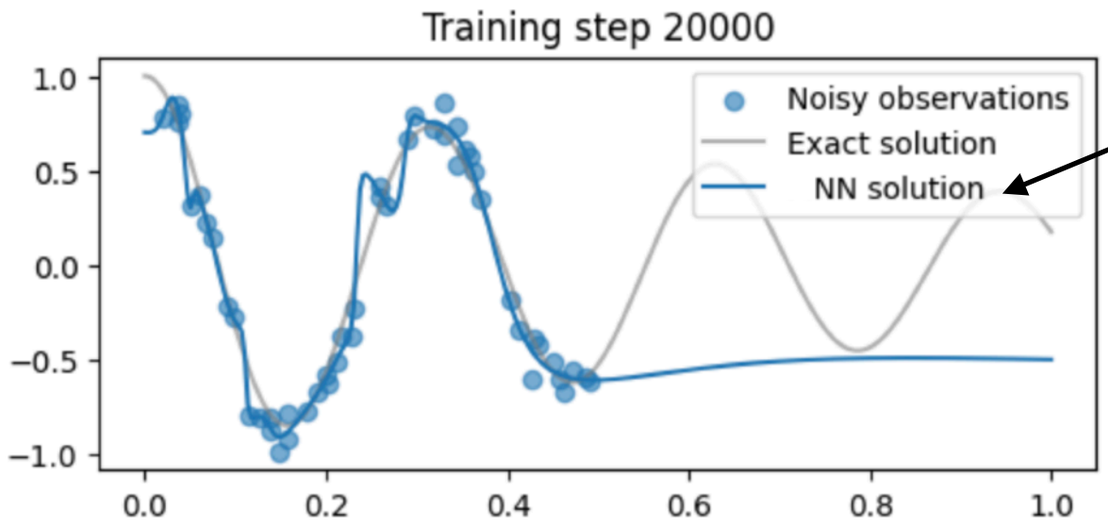
Loss

$$\mathcal{L}(\theta, \delta, \omega_0) = \frac{1}{N}\sum_{i}^{N}\left[x''_{\mathrm{PINN}}(t_i;\theta) + 2\,\delta\,x'_{\mathrm{PINN}}(t_i;\theta) + \omega_0^2\,x_{\mathrm{PINN}}(t_i;\theta)\right]^2 + \frac{\lambda}{M}\sum_{j}^{M}\left(x_{\mathrm{PINN}}(t_j;\theta) - x_{\mathrm{obs}}(t_j)\right)^2$$

Physical Loss — Data Loss

1) to construct **robust models** (even with limited or noisy data)

preventing overfitting problem

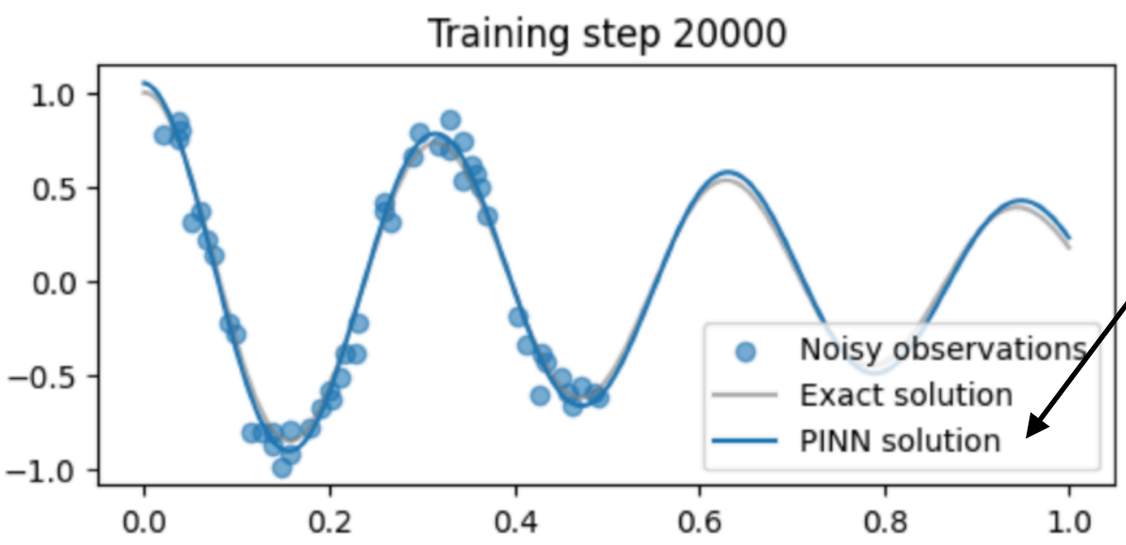


Standard Neural Network solution

(training only with **data**)

"It's suffering from the overfitting problem even in simple example"

"It's quite poor outside the data domain"



Physics-Informed Neural Network solution

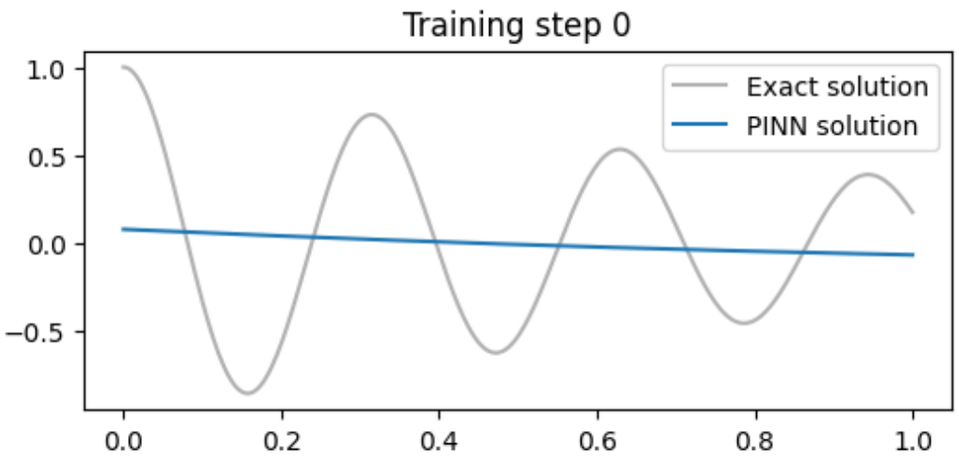
(training with **data + equations**)

"PINN allows accurate predictions even with limited training data"

"Working well even outside the data domain"

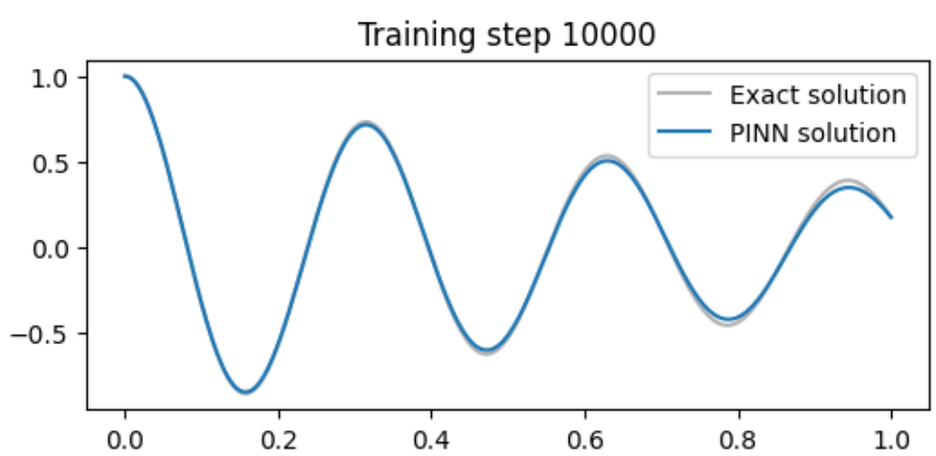
2) a new (forward problem) solver to model system dynamics

$$\mathcal{L}(\theta) = \underbrace{\left[x_{\mathrm{PINN}}(t=0;\theta) - 1\right]^2 + \lambda_1 \left[x'_{\mathrm{PINN}}(t=0;\theta) - 0\right]^2}_{\text{Boundary Conditions}} + \frac{\lambda_2}{N} \sum_i^N \underbrace{\left[x''_{\mathrm{PINN}}(t_i;\theta) \; + \; 2\,\delta\, x'_{\mathrm{PINN}}(t_i;\theta) \; + \; \omega_0^2\, x_{\mathrm{PINN}}(t_i;\theta)\right]^2}_{\text{Equation of Motion}}$$



Training

Minimizing Loss Function

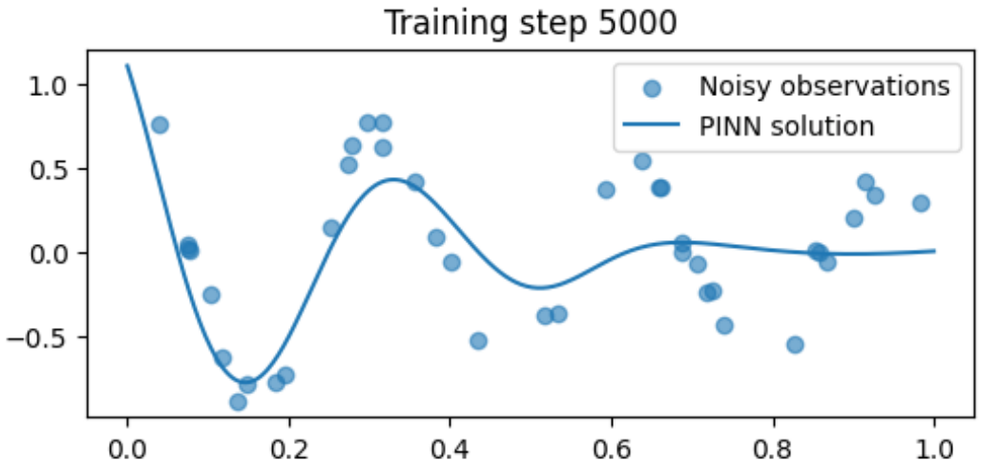


$$x_{\mathrm{Exact}}(t) = e^{-\delta t}\left(c_1 e^{i\omega t} + c_2 e^{-i\omega t}\right)$$

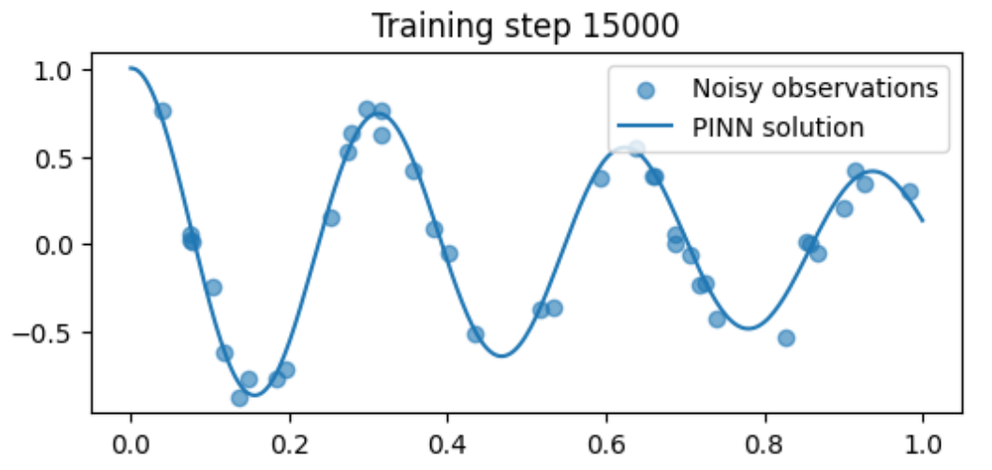
$$(\delta,\, \omega_0) = (1,\, 20) \qquad \omega := \sqrt{\omega_0^2 - \delta^2}$$

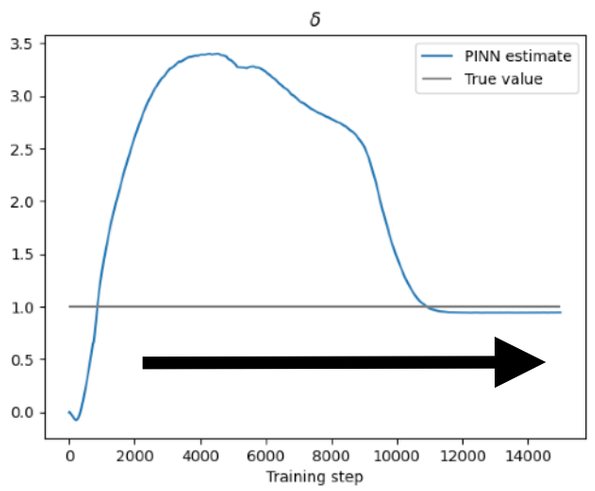
3) an **inverse problem solver** to learn the underlying physical parameters of the system

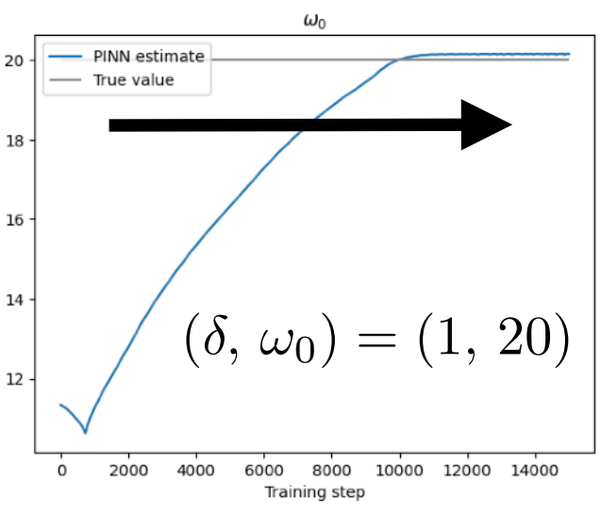
$$\mathcal{L}(\theta, \delta, \omega_0) = \underbrace{\frac{\lambda}{M}\sum_{j}^{M}\left(x_{\mathrm{PINN}}(t_j;\theta) - x_{\mathrm{obs}}(t_j)\right)^2}_{\text{Data or B.C.s}} + \underbrace{\frac{1}{N}\sum_{i}^{N}\left[x''_{\mathrm{PINN}}(t_i;\theta) + 2\,\delta\, x'_{\mathrm{PINN}}(t_i;\theta) + \omega_0^2\, x_{\mathrm{PINN}}(t_i;\theta)\right]^2}_{\text{Equation of Motion}}$$



Training



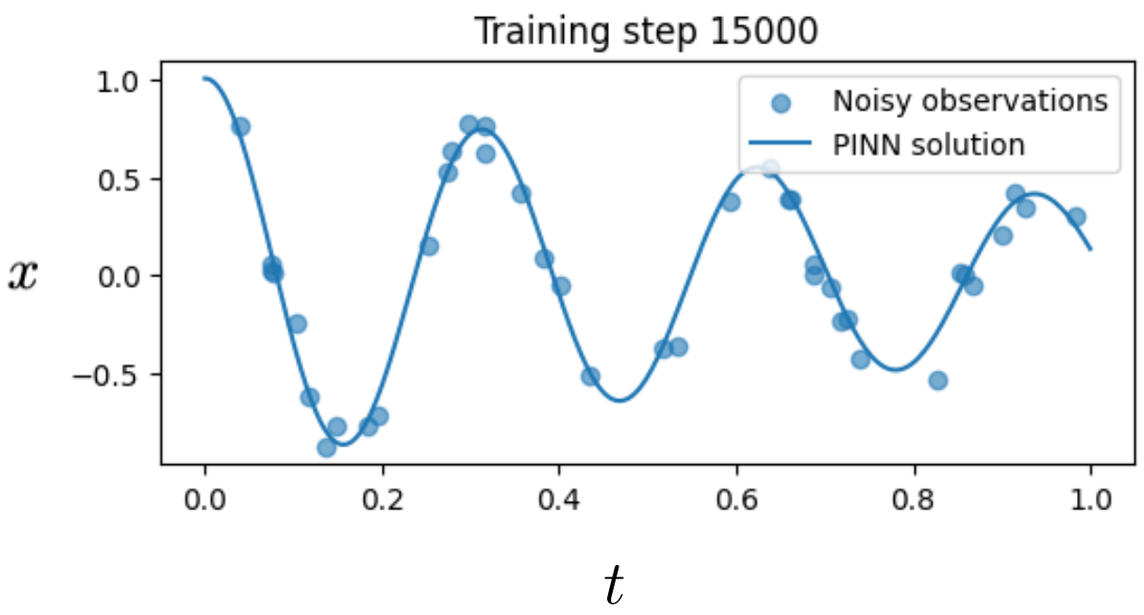


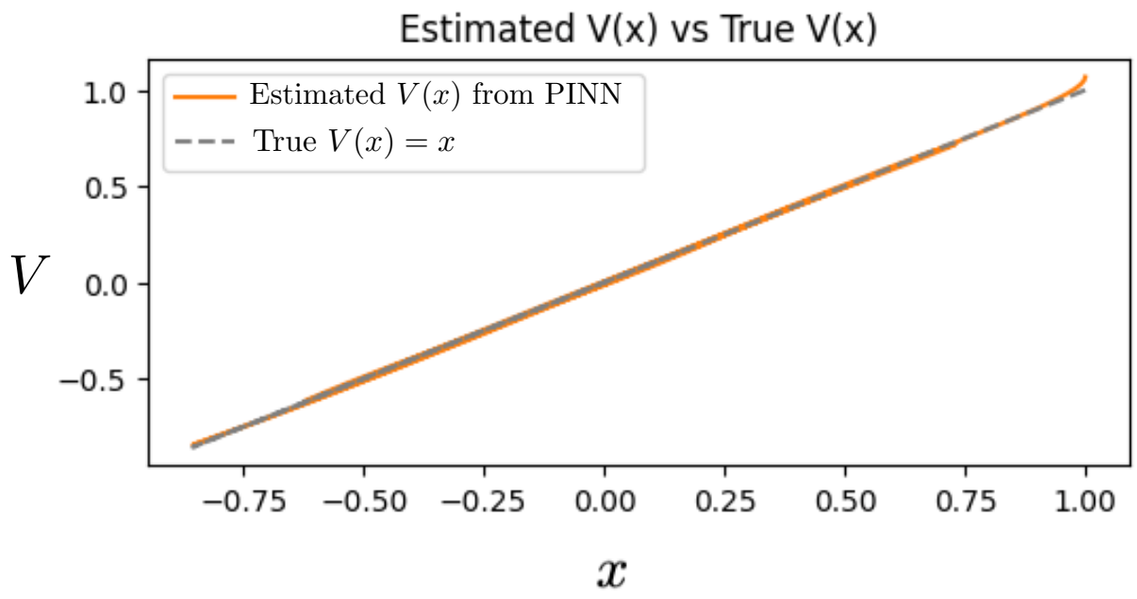


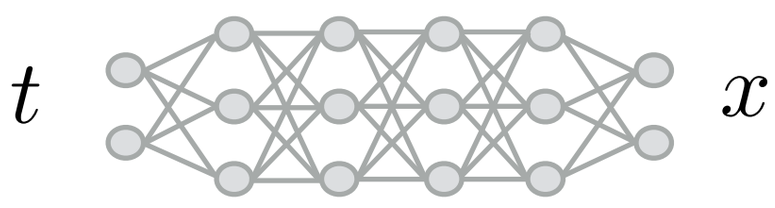
3) an **inverse problem solver** to learn the underlying physical parameters of the system

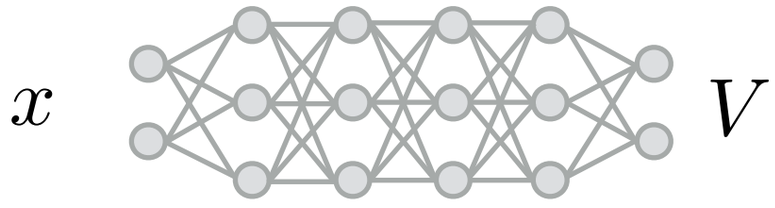
$$\mathcal{L}(\theta, \delta, \omega_0) = \underbrace{\frac{\lambda}{M}\sum_{j}^{M}\left(x_{\mathrm{PINN}}(t_j;\theta) - x_{\mathrm{obs}}(t_j)\right)^2}_{\text{Data or B.C.s}} + \underbrace{\frac{1}{N}\sum_{i}^{N}\left[x''_{\mathrm{PINN}}(t_i;\theta) + 2\,\delta\, x'_{\mathrm{PINN}}(t_i;\theta) + \omega_0^2\, V(x)\right]^2}_{\text{Equation of Motion}}$$

Finding potentials !!









Handful Examples

AdS/QCD, CMT, QI

AdS/QCD, CMT

arXiv:2401.00939 [pdf, other] hep-th cond-mat.dis-nn gr-qc hep-ph doi 10.1007/JHEP03(2024)141

Deep learning bulk spacetime from boundary optical conductivity

Authors: Byoungjoon Ahn, Hyun-Sik Jeong, Keun-Young Kim, Kwan Yun

arXiv:2502.10245 [pdf, other] hep-th cond-mat.dis-nn cond-mat.str-el gr-qc doi 10.1103/nh4n-ylnz **(PRD)**

Deep learning-based holography for T-linear resistivity

Authors: Byoungjoon Ahn, Hyun-Sik Jeong, Chang-Woo Ji, Keun-Young Kim, Kwan Yun

arXiv:2511.22522 [pdf, ps, other] hep-th cs.LG

AdS/Deep-Learning made easy II: neural network-based approaches to holography and inverse problems

Authors: Hyun-Sik Jeong, Hanse Kim, Keun-Young Kim, Gaya Yun, Hyeonwoo Yu, Kwan Yun

Setup

Action

$$S = \int \mathrm{d}^4x\sqrt{-g}\,(R + \mathcal{L}_m)\,, \quad \mathcal{L}_m = -\frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{1}{2}\sum_{i=1}^{2}(\partial\chi_i)^2$$

Bulk

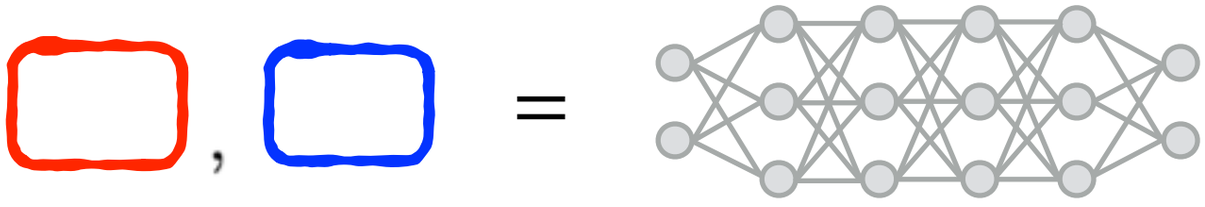
$$\mathrm{d}s^2 = \frac{1}{r^2}\left(-f(r)\mathrm{d}t^2 + \frac{1}{f(r)}\mathrm{d}r^2 + h(r)\sum_{i=1}^{2}\mathrm{d}x_i^2\right)$$

Fields

$$\phi = \phi(r)\,, \qquad A = A_t(r)\mathrm{d}t\,, \qquad \chi_i = \beta x_i\,,$$

"to break translational sym."

DNNs



Analogy

$$x(t) \to g_{\mu\nu}(r)\,, A_t(r)\,, \phi(r)$$

$$V(x) \to V(\phi)\,, Z(\phi)$$

$$\{\delta\,, \omega_0\} \to \beta$$

Damped Harmonic Oscillator

$$x''(t) + 2\,\delta\,x'(t) + \omega_0^2 V(x) = 0$$

Setup

$$\mathcal{L}_m = -\frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{1}{2}\sum_{i=1}^{2}(\partial\chi_i)^2$$

Einstein's equations,

Maxwell' equations,

Dilaton equations

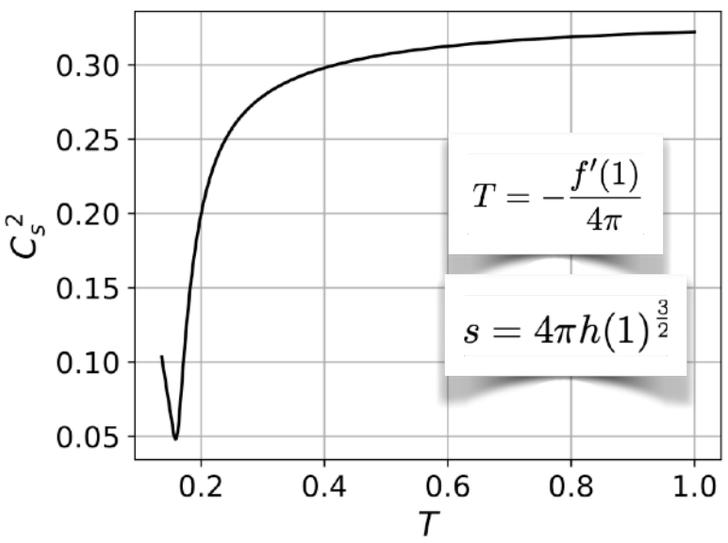
Loss Function

$$L = L_{\text{Data}} + L_{\text{EOM}}$$

QCD Equation of State (Speed of Sound)	Optical Conductivity	T-linear Resistivity

"Black hole thermodynamics"

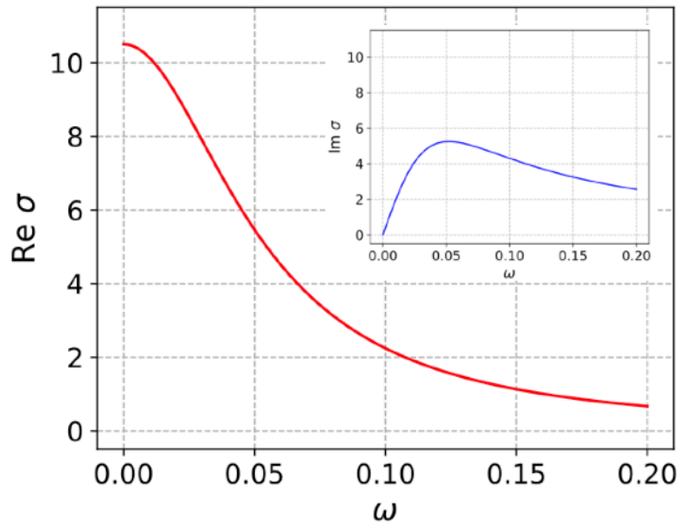
$$C_S^2 = \frac{d\,\log T}{d\,\log s}$$



"we give horizon constraints"

"Two-point functions"

$$\sigma(\omega) = \frac{1}{i\omega}G^R_{j^x j^x}(\omega) = \frac{1}{i\omega}\frac{a_x^{(R)}}{a_x^{(S)}}$$

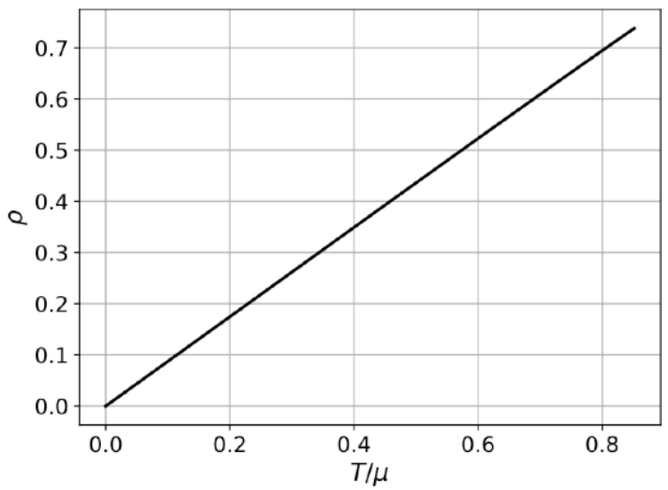


$$a_x = a_x^{(S)} + a_x^{(R)} z + \cdots$$

$$\rho = \frac{1}{\sigma_{DC}}$$

$$\sigma_{DC} = Z_H + \frac{q^2}{\beta^2\, h(1)}$$

$$q = Z_H\, h(1)\, A_t'(1)$$



"we give horizon constraints"

Results

Mostly, we haven't found anything new yet, but **the consistency** with standard holography results, and **some interesting observations**.

I. QCD Equation of State (**Speed of Sound**)

$$\int d^5x\sqrt{-g}\left(R-\frac{1}{2}(\partial\phi)^2-V(\phi)\right)$$

"By human learning (trial-error)"

$$V(\Phi) = -12\cosh(\gamma\,\Phi) + b\,\Phi^2\,,$$

$$\text{with} \quad \gamma = 0.606\,,\ b = 2.057$$

PRL **101,** 131601 (2008) PHYSICAL REVIEW LETTERS week ending 26 SEPTEMBER 2008

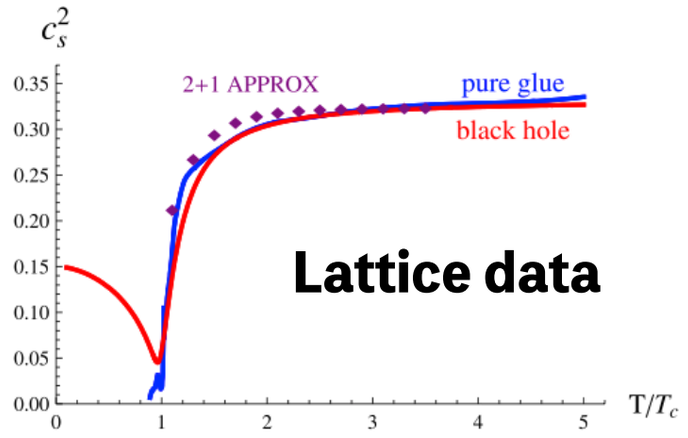
Thermodynamics and Bulk Viscosity of Approximate Black Hole Duals to Finite Temperature Quantum Chromodynamics

Steven S. Gubser,* Abhinav Nellore,† Silviu S. Pufu,‡ and Fábio D. Rocha§
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544, USA
(Received 30 June 2008; published 23 September 2008)

PHYSICAL REVIEW D **78,** 086007 (2008)

Mimicking the QCD equation of state with a dual black hole

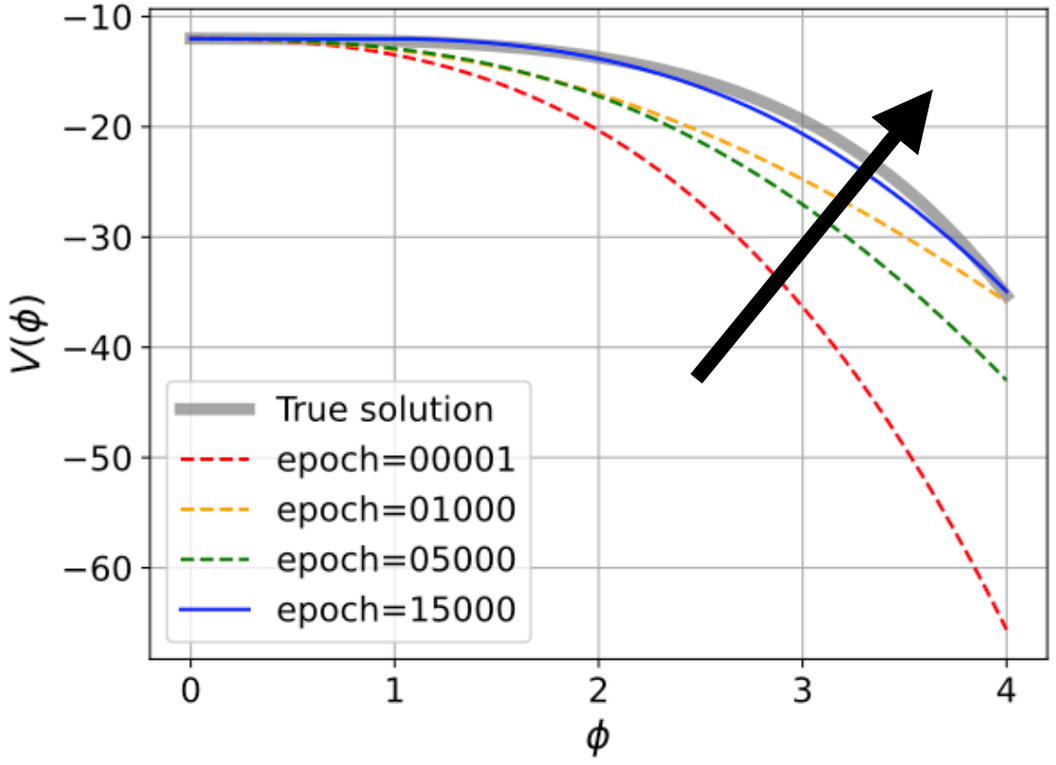
Steven S. Gubser and Abhinav Nellore
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544, USA
(Received 25 June 2008; published 29 October 2008)



Lattice data

Results

Mostly, we haven't found anything new yet, but **the consistency** with standard holography results, and **some interesting observations**.



$$\int d^5x\sqrt{-g}\left(R-\frac{1}{2}(\partial\phi)^2-V(\phi)\right)$$

"By human learning (trial-error)"

$$V(\Phi) = -12\cosh(\gamma\,\Phi) + b\,\Phi^2\,,$$

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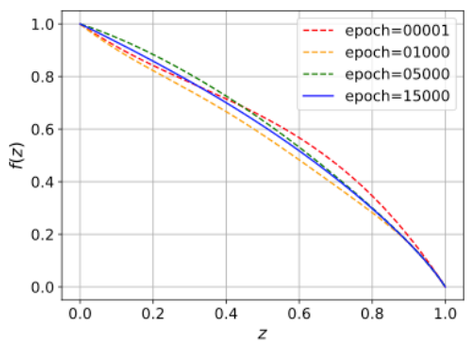
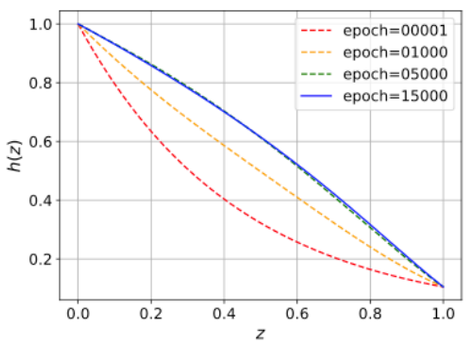
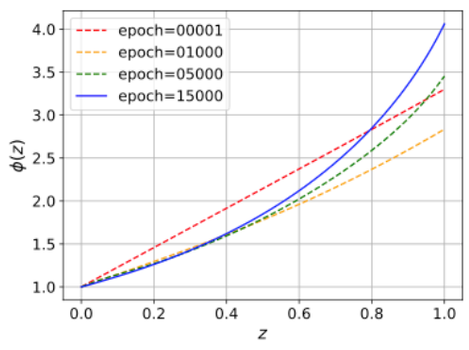
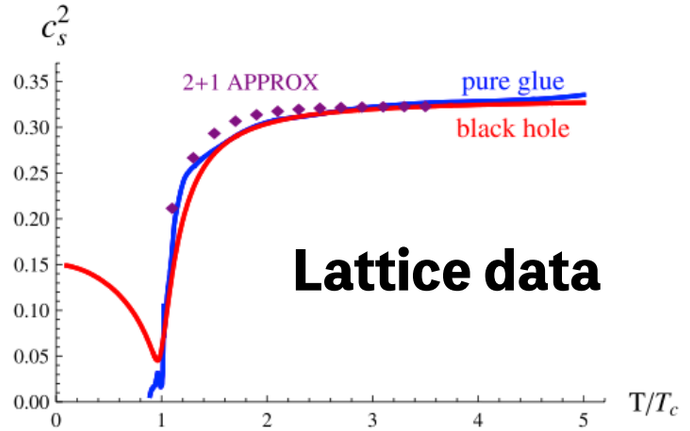








Results

$$\mathcal{L}_m = -\frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{1}{2}\sum_{i=1}^{2}(\partial\chi_i)^2$$

Mostly, we haven't found anything new yet, but **the consistency** with standard holography results, and **some interesting observations**.

II. Optical Conductivity

$$\int \mathrm{d}^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{2}\sum_{I=1}^{2}(\partial X_I)^2\right)$$

Home › Journal of High Energy Physics › Article

A simple holographic model of momentum relaxation

Open access | Published: 22 May 2014

Volume 2014, article number 101, (2014) Cite this article

Tomás Andrade & Benjamin Withers

Home › Science China Physics, Mechanics & Astronomy › Article

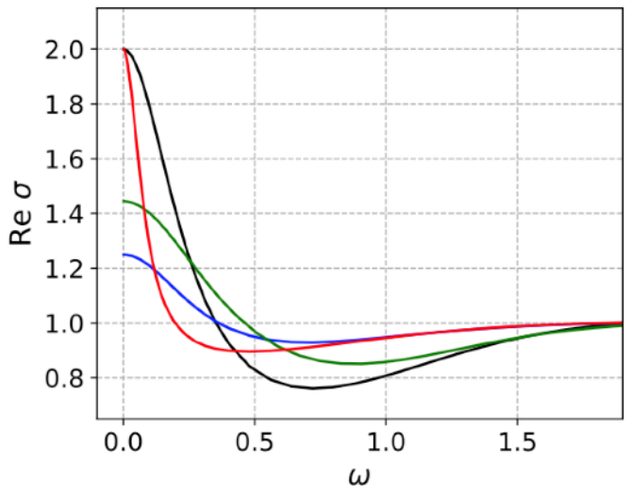
Holographic axion model: A simple gravitational tool for quantum matter

Invited Review | Published: 01 June 2021

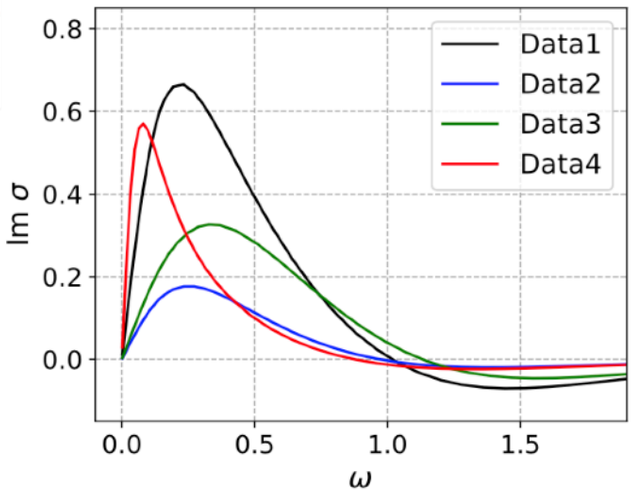
Volume 64, article number 270001, (2021) Cite this article

Matteo Baggioli, Keun-Young Kim, Li Li & Wei-Jia Li

"Linear-axion model": the effective toy model



$$\chi_i = \beta x_i$$



Holographic Optical Conductivity

Results

$$\mathcal{L}_m = -\frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{1}{2}\sum_{i=1}^{2}(\partial\chi_i)^2$$

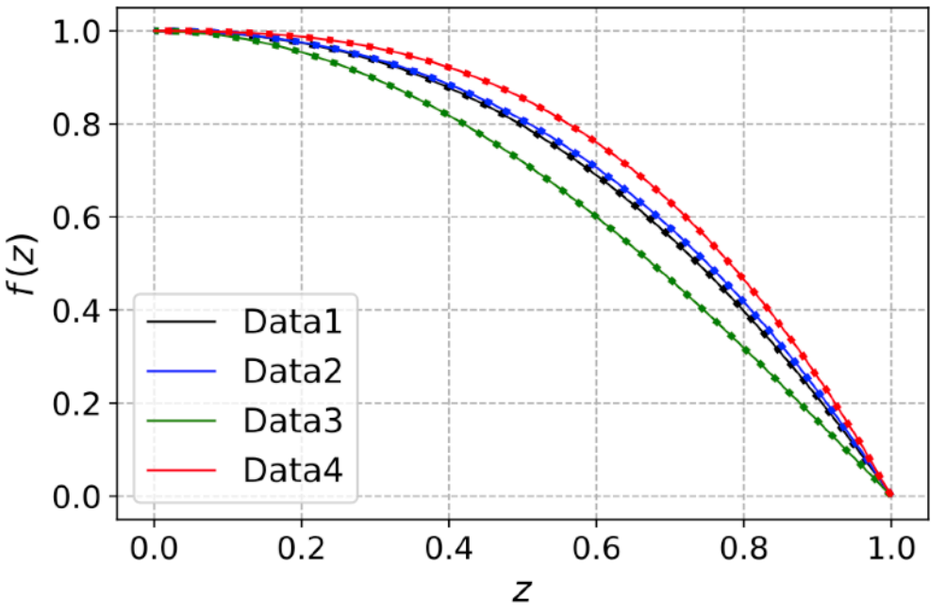
6 1

Mostly, we haven't found anything new yet, but **the consistency** with standard holography results, and **some interesting observations**.

II. Optical Conductivity

$$\int \mathrm{d}^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{2}\sum_{I=1}^{2}(\partial X_I)^2\right)$$

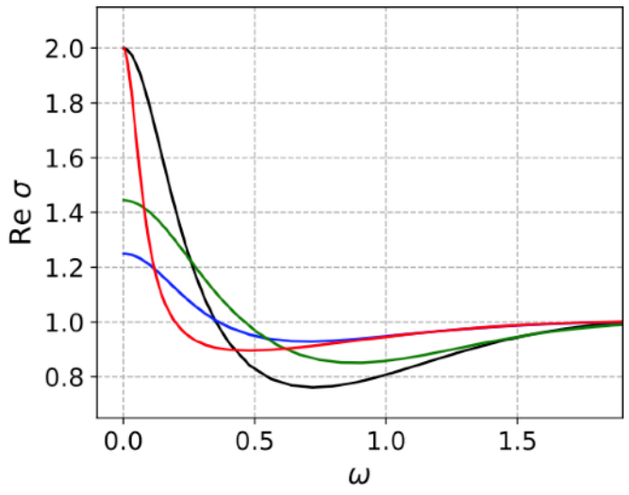
"Emblackening Factor"



Dots = True Solutions

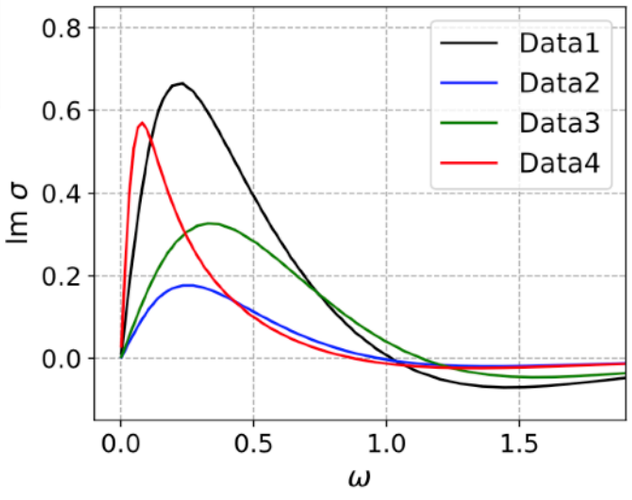
Lines = PINNs

"Linear-axion model": the effective toy model



$$\chi_i = \beta x_i$$

Holographic Optical Conductivity



Results

$$\mathcal{L}_m = -\frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{1}{2}\sum_{i=1}^{2}(\partial\chi_i)^2$$

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II. Optical Conductivity

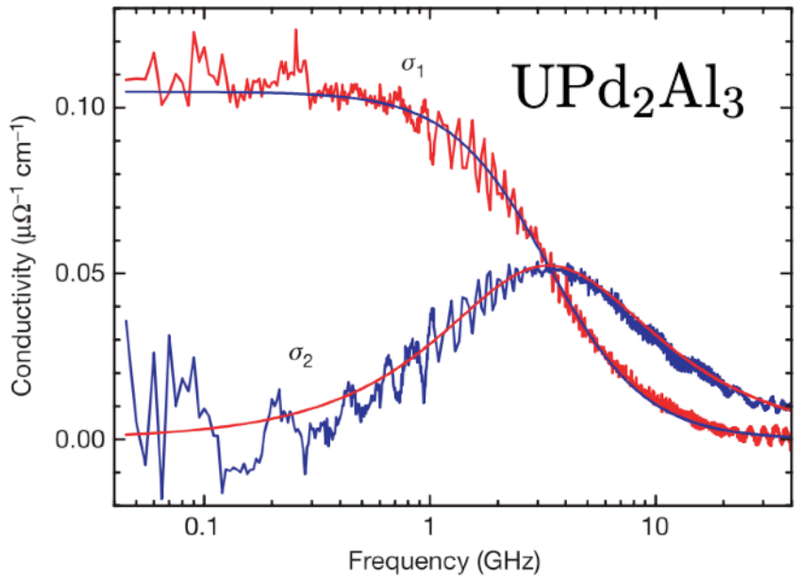
nature

Letter | Published: 22 December 2005

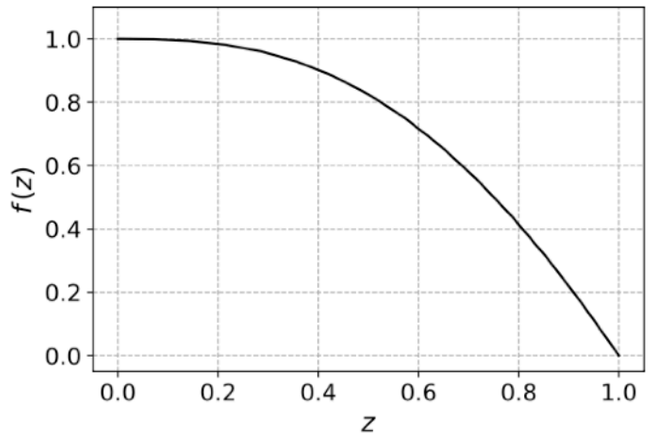
Extremely slow Drude relaxation of correlated electrons

Marc Scheffler ✉, Martin Dressel, Martin Jourdan & Hermann Adrian

Nature **438**, 1135–1137 (2005) | Cite this article



"Heavy Fermion Metals"



Linear-axion model with

$$\beta = 0.4825$$

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}$$

δ `d = torch.nn.Parameter(torch.zeros(1, requires_grad=True))`

ω_0 `w0 = torch.nn.Parameter(torch.empty(1).uniform_(0, 50), requires_grad=True)`

Results

$$\mathcal{L}_m = -\frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{1}{2}\sum_{i=1}^{2}(\partial\chi_i)^2$$

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III. T-linear Resistivity

PHYSICAL REVIEW D **81,** 046001 (2010)

Peculiar properties of a charged dilatonic black hole in AdS_5

Steven S. Gubser and Fabio D. Rocha
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544, USA
(Received 30 November 2009; published 1 February 2010)

PHYSICAL REVIEW B **89**, 245116 (2014)

Holographic duality and the resistivity of strange metals

Richard A. Davison,[1,*] Koenraad Schalm,[1,2,†] and Jan Zaanen[1,‡]
[1]*Instituut-Lorentz for Theoretical Physics, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands*
[2]*Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
(Received 12 November 2013; revised manuscript received 17 May 2014; published 12 June 2014)

$$V(\phi) = 6\cosh\frac{\phi}{\sqrt{3}}, \quad Z(\phi) = e^{\frac{\phi}{\sqrt{3}}},$$

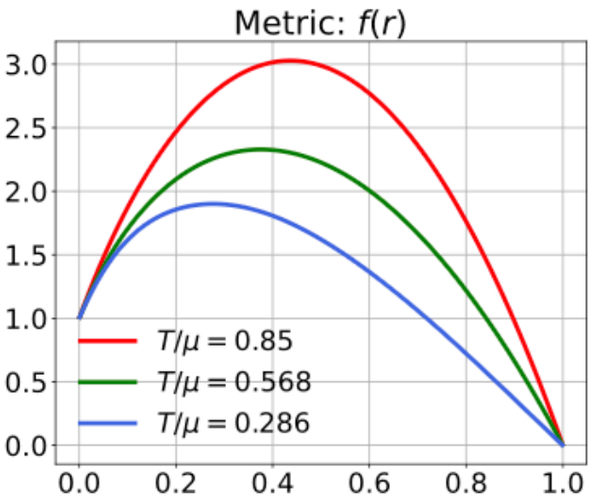
"Gubser-Rocha model"

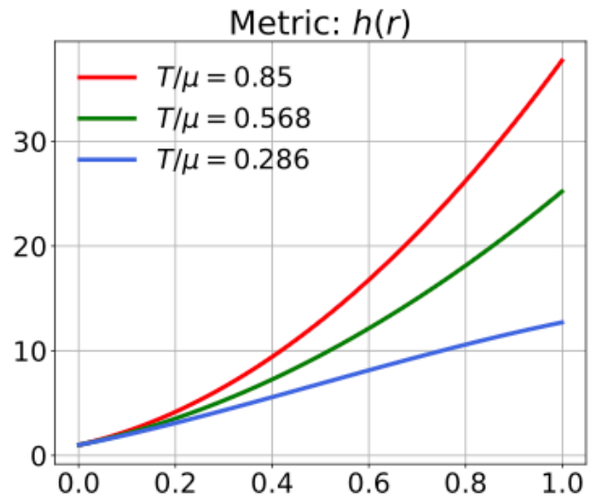
1) Top-down string theory models

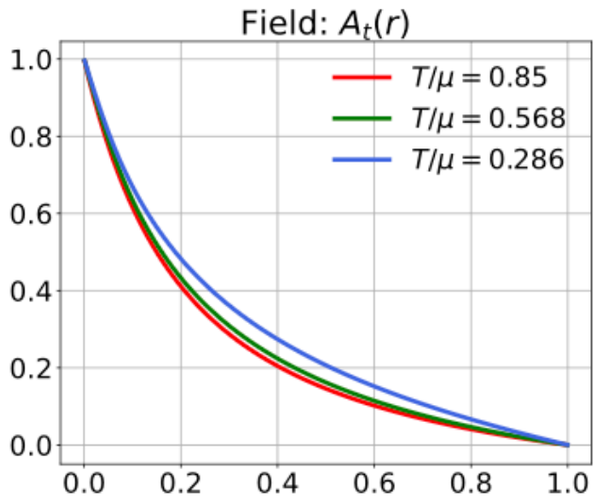
2) It allows the analytic background solutions

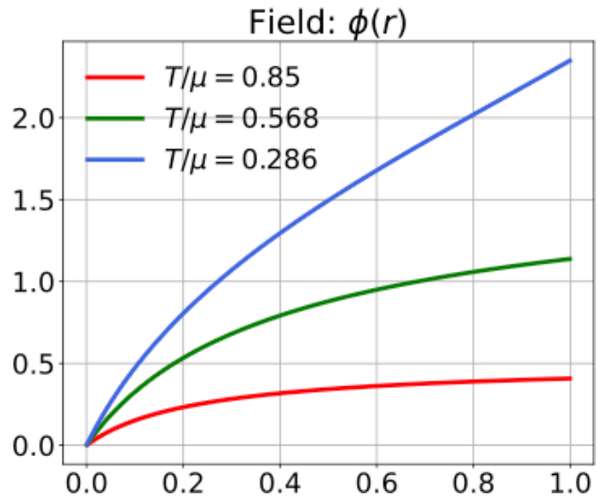
3) It shows T-linear resistivity and linear specific heat

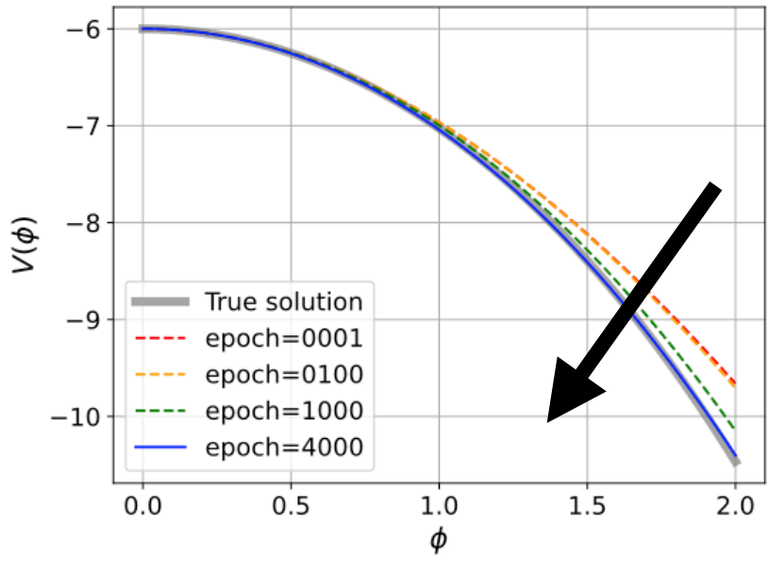
(as in cuprate strange metals)


Metric: $f(r)$
$T/\mu = 0.85$
$T/\mu = 0.568$
$T/\mu = 0.286$

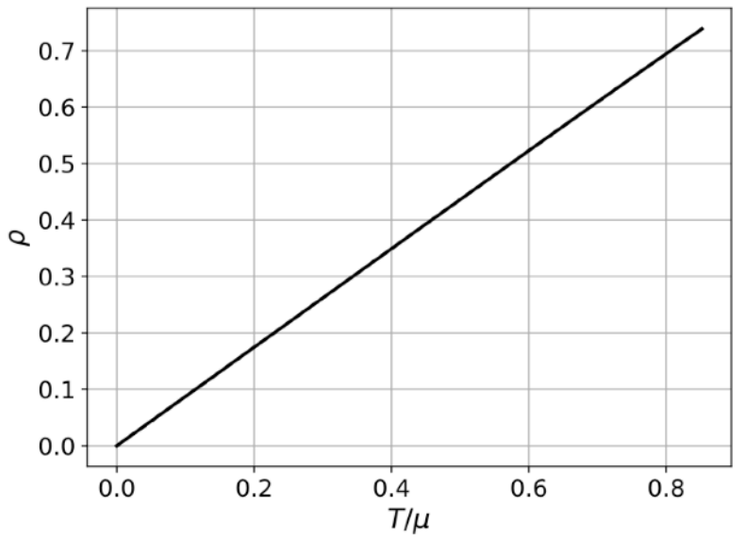

Metric: $h(r)$
$T/\mu = 0.85$
$T/\mu = 0.568$
$T/\mu = 0.286$

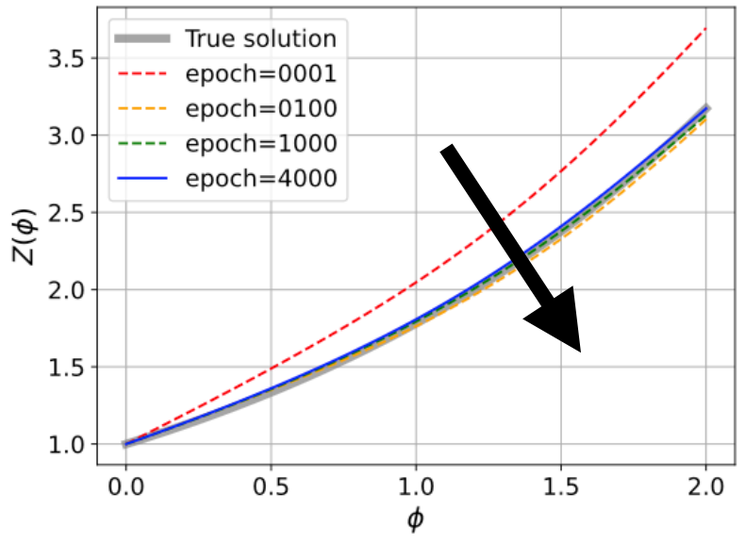

Field: $A_t(r)$
$T/\mu = 0.85$
$T/\mu = 0.568$
$T/\mu = 0.286$

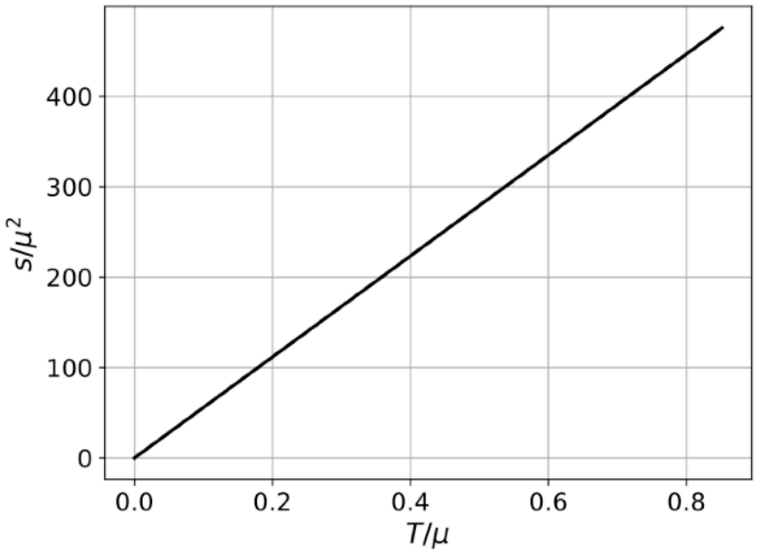

Field: $\phi(r)$
$T/\mu = 0.85$
$T/\mu = 0.568$
$T/\mu = 0.286$


$V(\phi)$
ϕ
True solution
epoch=0001
epoch=0100
epoch=1000
epoch=4000

Training


ρ
T/μ

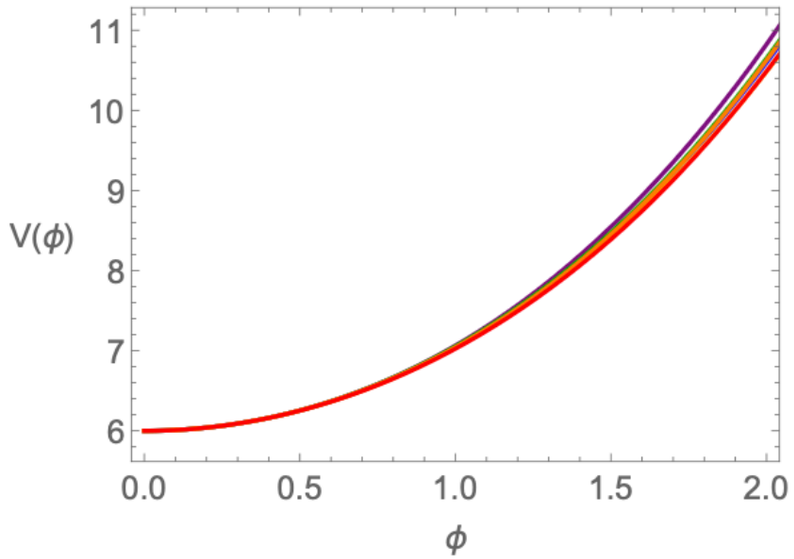

$Z(\phi)$
ϕ
True solution
epoch=0001
epoch=0100
epoch=1000
epoch=4000


s/μ^2
T/μ

Results

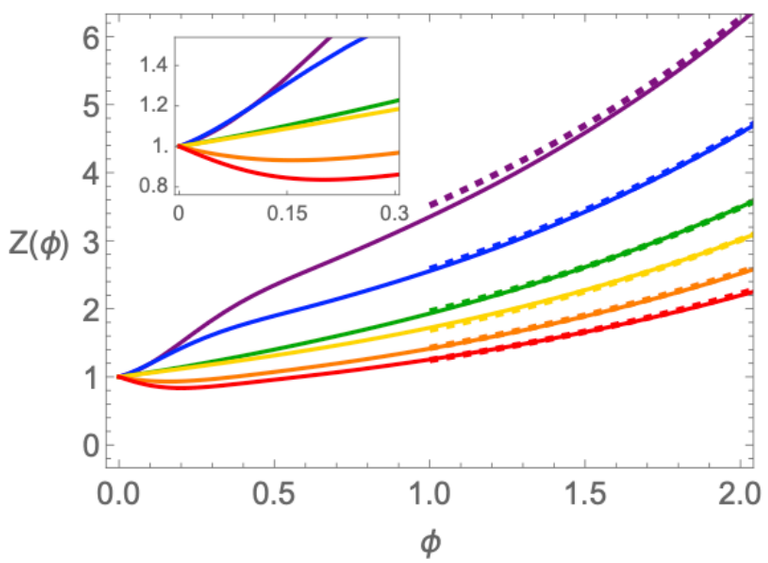
$$\mathcal{L}_m = -\frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{1}{2}\sum_{i=1}^{2}(\partial\chi_i)^2$$

Mostly, we haven't found anything new yet, but **the consistency** with standard holography results, and **some interesting observations**.



$$V(\phi) \approx e^{\frac{1}{\sqrt{3}}\phi}$$

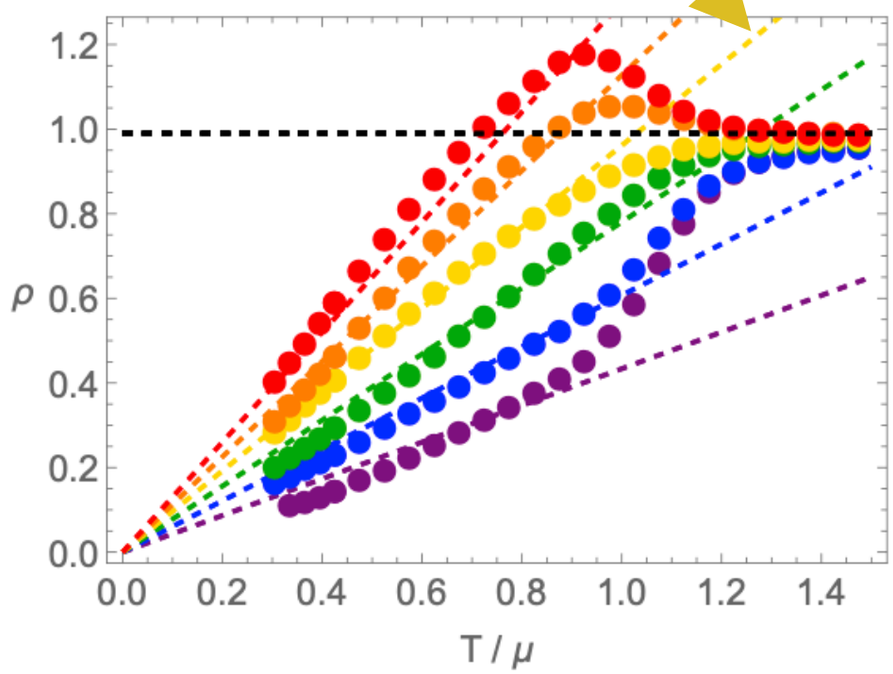
Training



$$Z(\phi) \approx \alpha\, e^{\frac{1}{\sqrt{3}}\phi}$$

$$\alpha = \alpha(\mathcal{A})$$

Gubser-Rocha model



$$\rho = \mathcal{A}\, T$$

Results

$$\mathcal{L}_m = -\frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{1}{2}\sum_{i=1}^{2}(\partial\chi_i)^2$$

Mostly, we haven't found anything new yet, but **the consistency** with standard holography results, and **some interesting observations**.

---

yses. Specifically, one can check that within the EMDA theories (3.1) [24–27, 36–43], where the axion-dilaton coupling $\frac{Y(\phi)}{2}\sum_{i=1}^{2}(\partial\chi_i)^2$ is simplified to $Y(\phi) = 1$, the corresponding IR geometry takes the form of the called η geometry:

$$\mathrm{d}s^2 = \xi^{-\eta}\left(\frac{-\mathrm{d}t^2 + \mathcal{L}^2\mathrm{d}\xi^2}{\xi^2} + \sum_{i=1}^{2}\mathrm{d}x_i^2\right), \qquad \mathcal{L}^2 := \frac{2(1+\eta)^2}{2-\beta^2}. \tag{4.1}$$

Here, the scaling parameters satisfy

$$z \to \infty, \qquad \theta \to -\infty, \qquad \theta/z \to -\eta. \qquad V_{\mathrm{IR}}(\phi) \approx Z_{\mathrm{IR}}(\phi) \approx e^{\frac{\phi}{\sqrt{3}}}$$

In this geometry, the resistivity and entropy scale at low T as

$$\rho \approx T^{\eta}, \qquad s \approx T^{\eta}.$$

Consistent with the IR Geometry Analysis

AdS/QI

arXiv:2406.07395 [pdf, other] hep-th cond-mat.dis-nn gr-qc doi 10.1007/JHEP01(2025)025

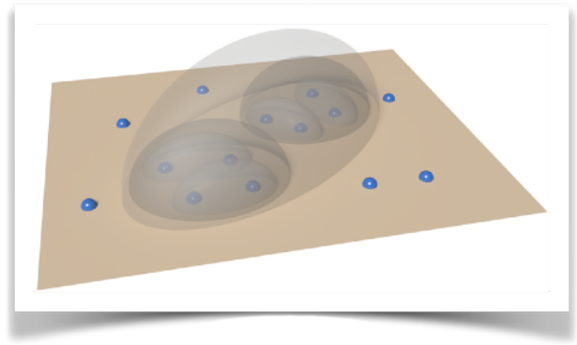
Holographic reconstruction of black hole spacetime: machine learning and entanglement entropy

Authors: Byoungjoon Ahn, Hyun-Sik Jeong, Keun-Young Kim, Kwan Yun

Setup

$$S = \frac{\text{Area}(\gamma)}{4G_N}$$

Bulk Reconstruction



Action — **We do not need the action**

Bulk Fields

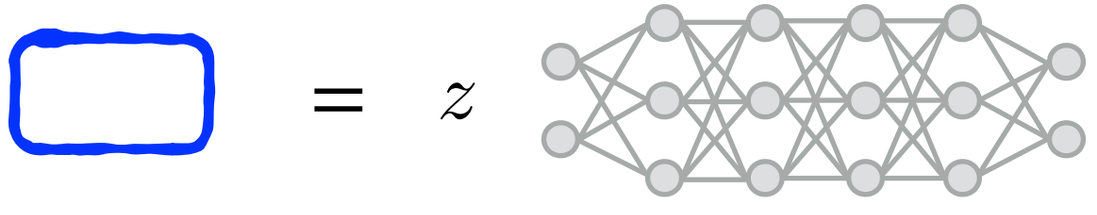
$$\mathrm{d}s^2 = \frac{L^2}{z^2}\left[-f(z)\,\mathrm{d}t^2 + \frac{\mathrm{d}z^2}{f(z)} + h(z)\left(\mathrm{d}x^2 + \mathrm{d}y^2\right)\right]$$

"General homogenous (isotropic) metric"

"Coordinate transformation"

$$\mathrm{d}s^2 = -D(r)\,\mathrm{d}t^2 + B(r)\,\mathrm{d}r^2 + C(r)\,\mathrm{d}\vec{x}_i^2$$

DNNs

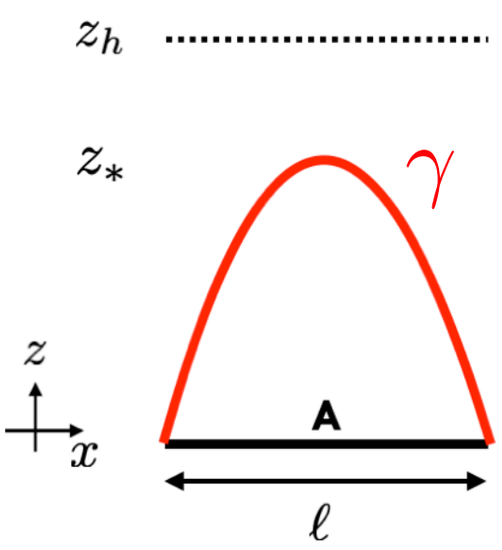


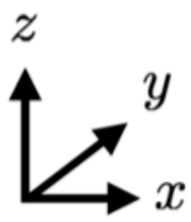
Holographic Entanglement Entropy

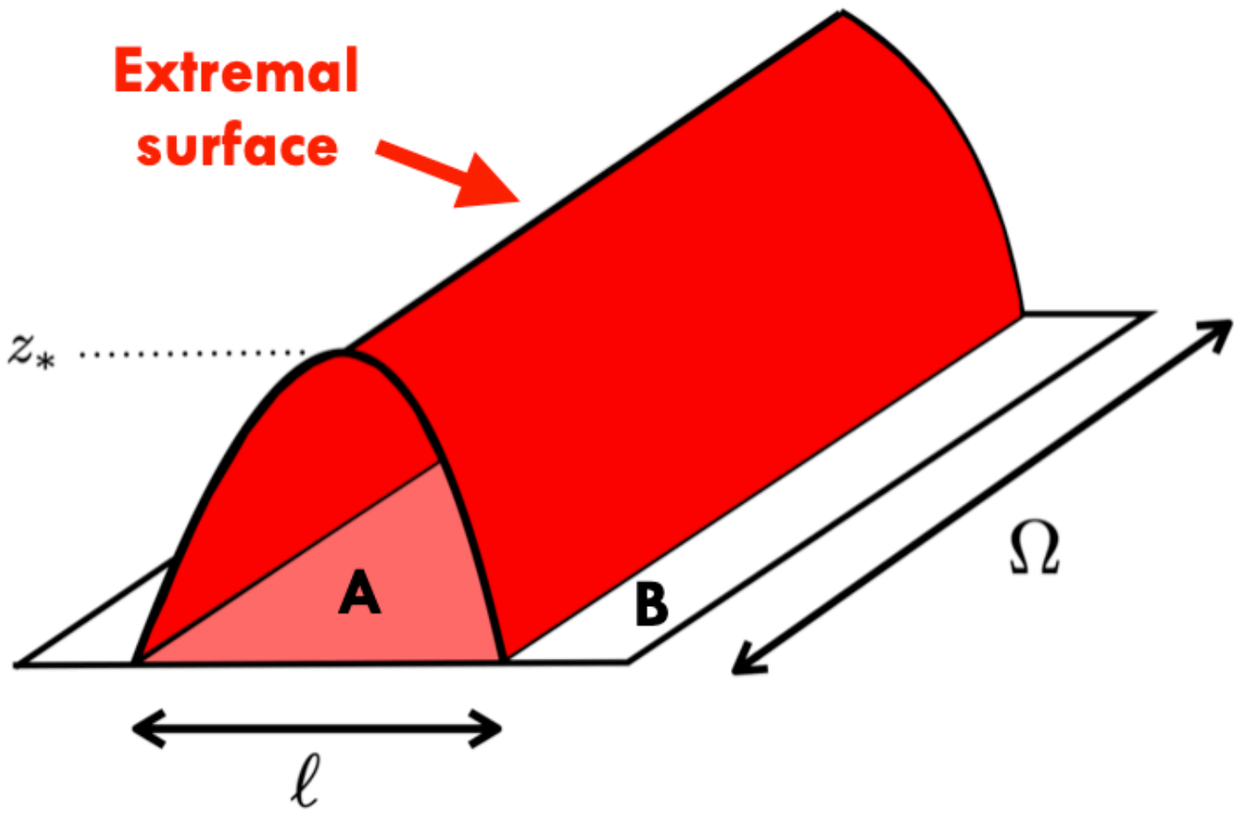
Entanglement entropy in the boundary theory can be computed from **the extremal surface** in the higher dimensional gravity theory

$$S = \frac{\mathrm{Area}(\gamma)}{4G_N}$$

$$= S(\ell)$$







Holographic Entanglement Entropy

Entanglement entropy in the boundary theory can be computed from **the extremal surface** in the higher dimensional gravity theory

$$S = \frac{\mathrm{Area}(\gamma)}{4G_N} = S(\ell)$$

$$\mathrm{Area}(\gamma) = \int \mathrm{d}y\mathrm{d}x\sqrt{\det\gamma_{\mu\nu}}$$

"Induced metric at time slice"

"Finding the ext. area by Euler-Langrangian method"

$$\mathrm{d}s^2 = \frac{L^2}{z^2}\left[-f(z)\,\mathrm{d}t^2 + \frac{\mathrm{d}z^2}{f(z)} + h(z)\,\left(\mathrm{d}x^2 + \mathrm{d}y^2\right)\right]$$

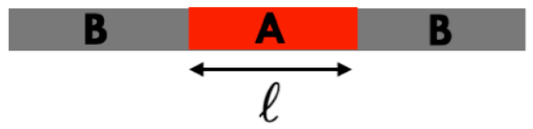
$$S = \frac{L^2\Omega}{2G_N}\int_\epsilon^{z_*} \frac{1}{z^2}\sqrt{\frac{h(z)}{1-\frac{z^4}{z_*^4}\frac{h(z_*)^2}{h(z)^2}}}\frac{1}{\sqrt{f(z)}}\mathrm{d}z\,,$$

$$\ell = 2\int_0^{z_*} \frac{1}{\sqrt{\frac{h(\alpha)^2}{h(z_*)^2}\frac{z_*^4}{\alpha^4}-1}}\frac{1}{\sqrt{h(\alpha)f(\alpha)}}\mathrm{d}\alpha\,,$$

One more thing…

Example: 1+1 CFT $S_A = \frac{c}{3}\log\frac{\ell}{\epsilon} + finite$

(c: central charge of the CFT; ϵ: UV cut off)

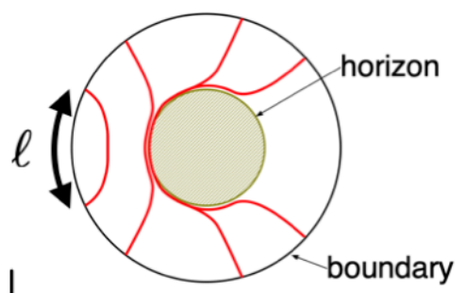


$$S = \frac{L^2\Omega}{2G_N}\int_\epsilon^{z_*} \frac{1}{z^2}\sqrt{\frac{h(z)}{1-\frac{z^4}{z_*^4}\frac{h(z_*)^2}{h(z)^2}}}\frac{1}{\sqrt{f(z)}}\,\mathrm{d}z$$

$$= \frac{L^2\Omega}{2G_N}\left[\int_\epsilon^{z_*}\frac{1}{z^2}\mathrm{d}z + \int_0^{z_*}\frac{1}{z^2}\left(\sqrt{\frac{h(z)}{1-\frac{z^4}{z_*^4}\frac{h(z_*)^2}{h(z)^2}}}\frac{1}{\sqrt{f(z)}}-1\right)\mathrm{d}z\right]$$

$$= \frac{L^2\Omega}{2G_N}\left[\frac{1}{\epsilon}+\left\{\int_0^{z_*}\frac{1}{z^2}\left(\sqrt{\frac{h(z)}{1-\frac{z^4}{z_*^4}\frac{h(z_*)^2}{h(z)^2}}}\frac{1}{\sqrt{f(z)}}-1\right)\mathrm{d}z-\frac{1}{z_*}\right\}\right]$$

$$:= \frac{L^2\Omega}{2G_N}\left[\frac{1}{\epsilon}+S_{\mathrm{Finite}}\right],$$



Loss Function

$$L = \frac{1}{N_\ell}\sum_\ell \left|S^{(m)}_{\mathrm{Finite}}(\ell) - S_{\mathrm{Finite}}(\ell)\right| + \left|s^{(m)} - s\right|$$

Technical Reason
(Must satisfy by construction)

Bulk Reconstruction

(Without Machine Learning)

$$S = \frac{L^2\Omega}{2G_N}\int_\epsilon^{z_*} \frac{1}{z^2}\sqrt{\frac{h(z)}{1-\frac{z^4}{z_*^4}\frac{h(z_*)^2}{h(z)^2}}}\frac{1}{\sqrt{f(z)}}\,\mathrm{d}z$$

Handbook of integral eqns

Output Input

$$F(z_*) = \int_\epsilon^{z_*} \frac{1}{\sqrt{G(z_*)-G(z)}}\,Y(z)\,\mathrm{d}z\,, \qquad G'(z_*)>0$$

$$\text{(Solution)}: \quad Y(z_*) = \frac{1}{\pi}\frac{\mathrm{d}}{\mathrm{d}z_*}\int_\epsilon^{z_*} \frac{G'(z)}{\sqrt{G(z_*)-G(z)}}\,F(z)\,\mathrm{d}z\,.$$

For the simple metric

$$h(z)=1$$

$$\mathrm{d}s^2 = \frac{L^2}{z^2}\left[-f(z)\,\mathrm{d}t^2 + \frac{\mathrm{d}z^2}{f(z)} + h(z)\left(\mathrm{d}x^2+\mathrm{d}y^2\right)\right]$$

$$\frac{2G_N}{L^2\Omega}\frac{S(z_*)}{z_*^2} = \int_\epsilon^{z_*} \frac{1}{\sqrt{z_*^4-z^4}}\frac{\sqrt{1/f(z)}}{z^2}\,\mathrm{d}z$$

$$G(z) := z^4$$

$$\frac{\mathrm{d}S(\ell)}{\mathrm{d}\ell} = \frac{L^2\Omega}{4G_N}\frac{1}{z_*^2}$$

$$\sqrt{\frac{1}{f(z)}} = \frac{8G_N}{\pi L^2\Omega}\,z^2\,\frac{\mathrm{d}}{\mathrm{d}z}\int_\epsilon^{z} \frac{z_*}{\sqrt{z^4-z_*^4}}\,S(z_*)\,\mathrm{d}z_*$$

We **do not need ML** to reconstruct the bulk from EE **when metric is simple**

Bulk Reconstruction

(Example) $h(z) = 1$

$$f(z) = 1 - \frac{\beta^2}{2} z^2 - \left(1 - \frac{\beta^2}{2} + \frac{\mu^2}{4}\right) z^3 + \frac{\mu^2}{4} z^4$$

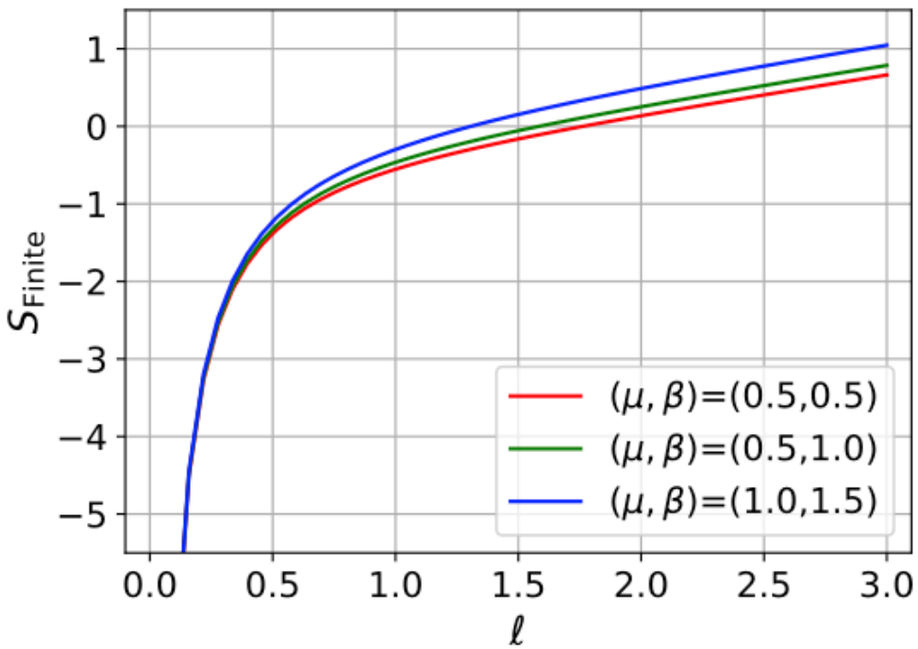
$$\int \mathrm{d}^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{2}\sum_{I=1}^{2}(\partial X_I)^2\right)$$

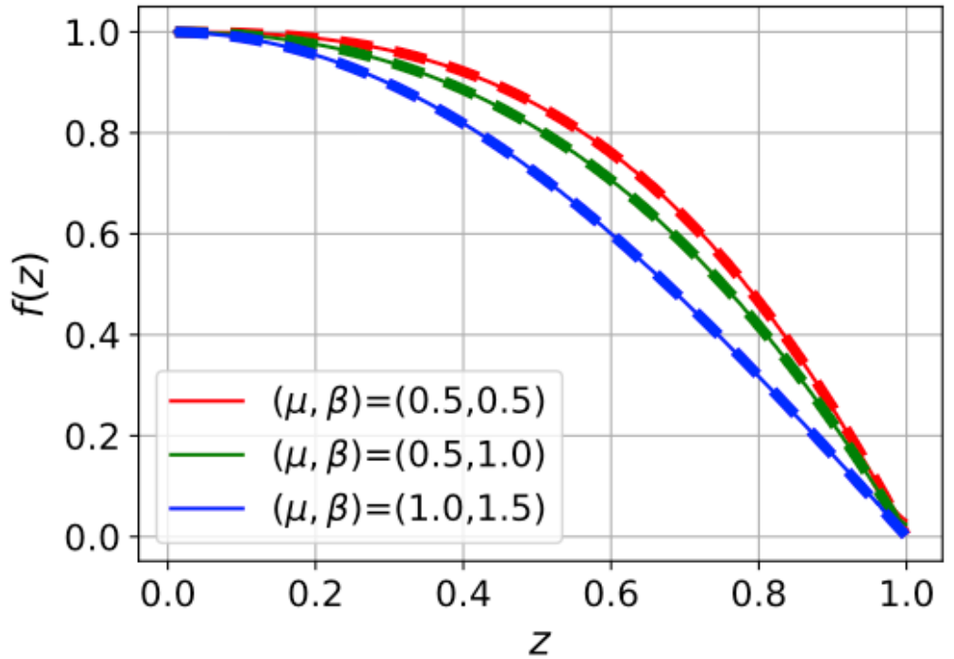
"Linear-axion model": the effective toy model

$$\sqrt{\frac{1}{f(z)}} = \frac{8G_N}{\pi L^2 \Omega} z^2 \frac{\mathrm{d}}{\mathrm{d}z} \int_\epsilon^z \frac{z_*}{\sqrt{z^4 - z_*^4}} S(z_*)\, \mathrm{d}z_*$$

Given holographic entanglement entropy

Bulk reconstruction without machine learning





Bulk Reconstruction

(Limitation)

$$S = \frac{L^2\Omega}{2G_N}\int_\epsilon^{z_*} \frac{1}{z^2}\sqrt{\frac{h(z)}{1-\frac{z^4}{z_*^4}\frac{h(z_*)^2}{h(z)^2}}}\frac{1}{\sqrt{f(z)}}\,\mathrm{d}z$$

For the **generic** metric, **ML becomes very powerful** (necessary) for reconstruction

$h(z) \neq 1$ (This is very very common situation in holography)

$$\mathrm{d}s^2 = \frac{L^2}{z^2}\left[-f(z)\,\mathrm{d}t^2 + \frac{\mathrm{d}z^2}{f(z)} + h(z)\left(\mathrm{d}x^2+\mathrm{d}y^2\right)\right]$$

Same Information

$S(z_*)$

(See details in our paper)

Output

$$\sqrt{\frac{1}{f(z)\,h(z)^3}} = \frac{2}{\pi}\frac{1}{z^2}\frac{\mathrm{d}}{\mathrm{d}z}\int_0^z \frac{\ell(z_*)}{\sqrt{\frac{z^4}{h(z)^2}-\frac{z_*^4}{h(z_*)^2}}}\frac{2z_*^3h(z_*)-z_*^4h'(z_*)}{2h(z_*)^3}\,\mathrm{d}z_*$$

Input

Single information

But, two outputs

$$\frac{\mathrm{d}S}{\mathrm{d}\ell} = \frac{L^2\Omega}{4G_N}\frac{h(z_*)}{z_*^2}$$

"Unless h(z) is provided, this method cannot fully reconstruct the metric"

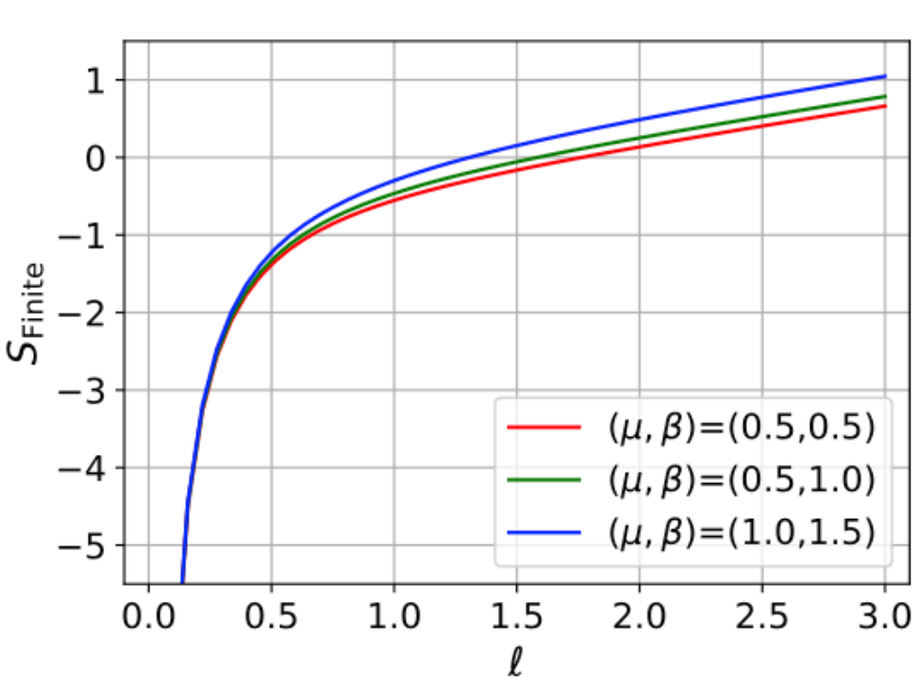
Bulk Reconstruction

(**Machine Learning** Example I)

$$\int \mathrm{d}^4x\sqrt{-g}\left(R+6-\frac{1}{4}F_{ab}F^{ab}-\frac{1}{2}\sum_{I=1}^{2}(\partial X_I)^2\right)$$

"Linear-axion model": the effective toy model

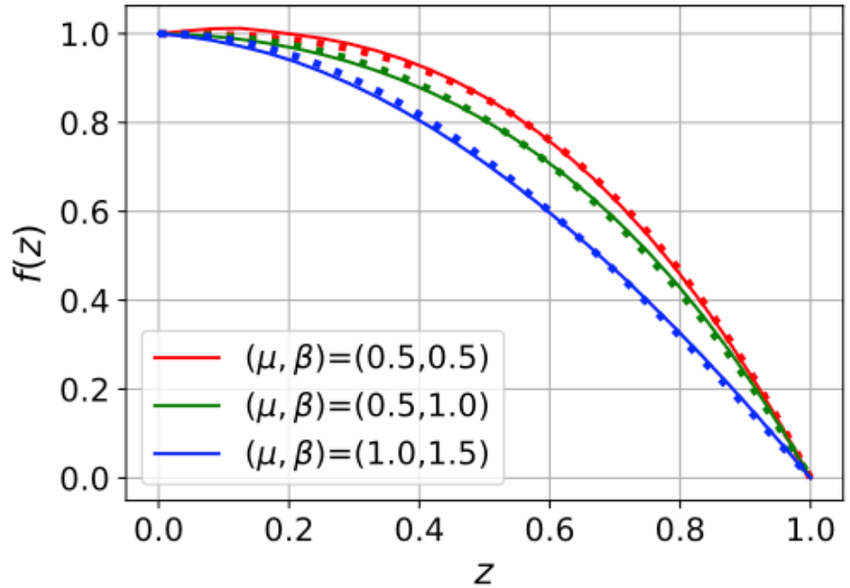
Given holographic entanglement entropy

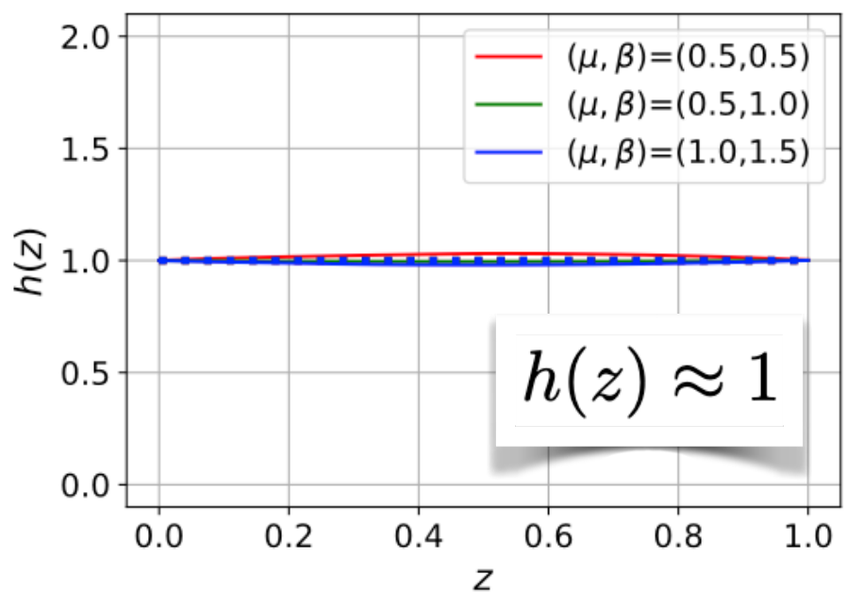


Dots = True Solutions

Lines = PINNs

Bulk reconstruction **with** machine learning





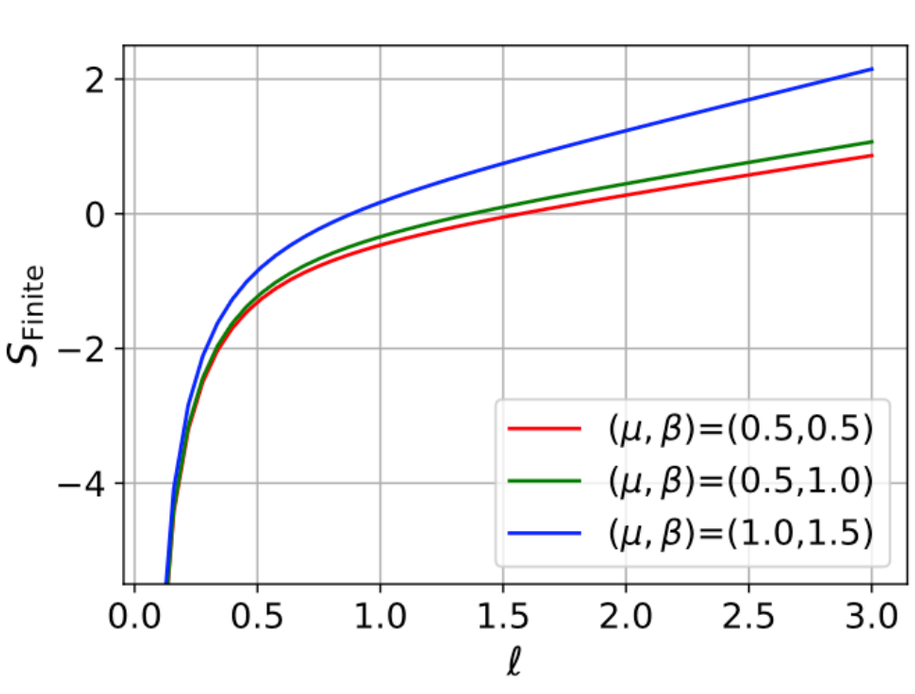
Bulk Reconstruction

(**Machine Learning** Example II)

$$V(\phi) = 6\cosh\frac{\phi}{\sqrt{3}}\,, \quad Z(\phi) = e^{\frac{\phi}{\sqrt{3}}}\,,$$

"Gubser-Rocha model"

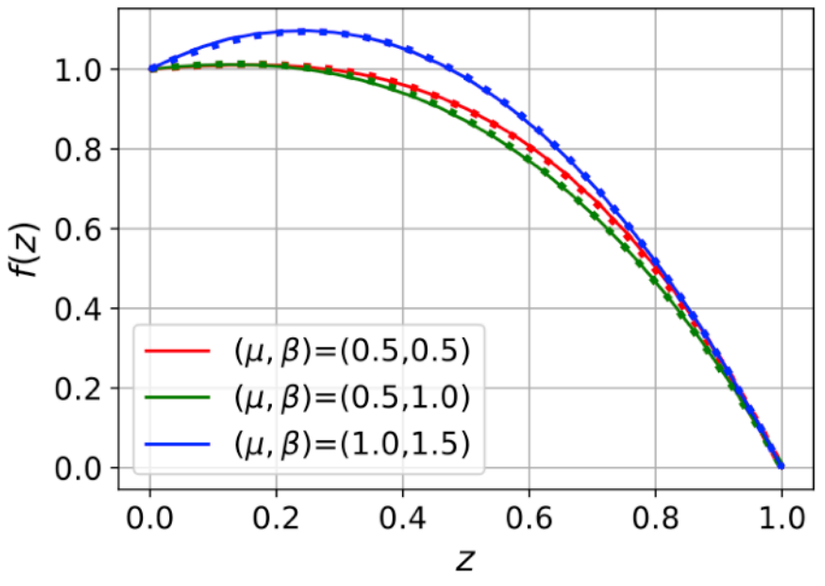
Given holographic entanglement entropy

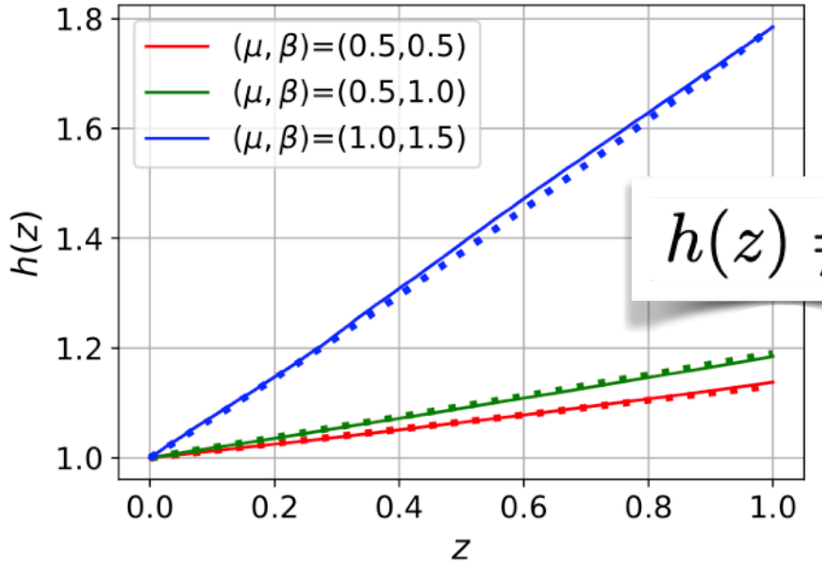


Dots = True Solutions

Lines = PINNs

Bulk reconstruction with machine learning



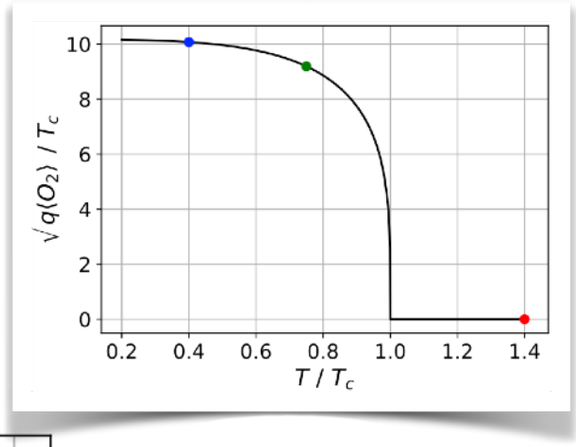


Bulk Reconstruction

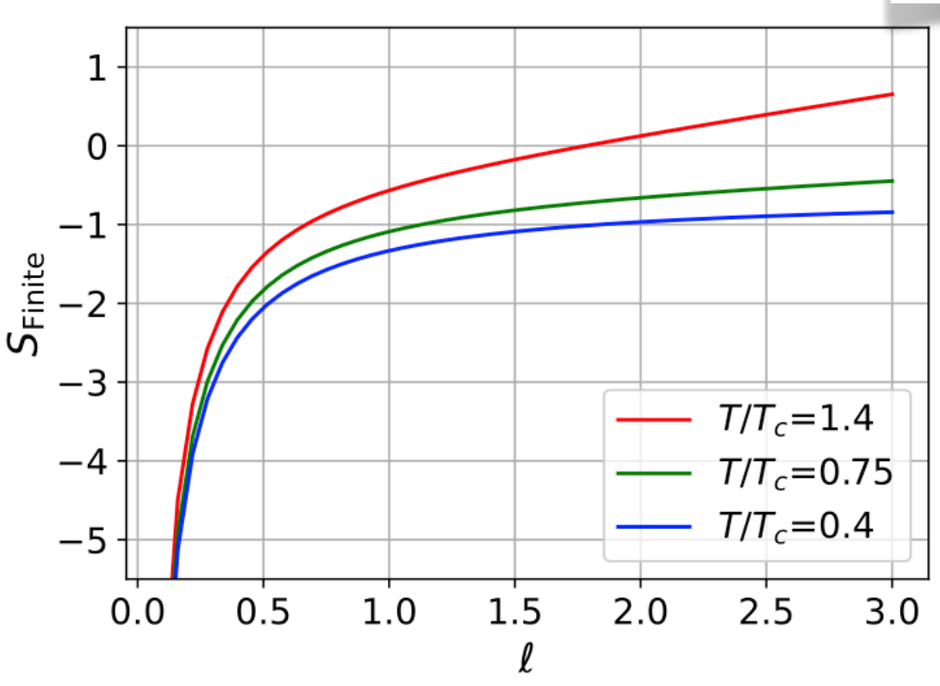
(**Machine Learning** Example III)

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F^2 - |D\Phi|^2 - m^2|\Phi|^2\right)$$

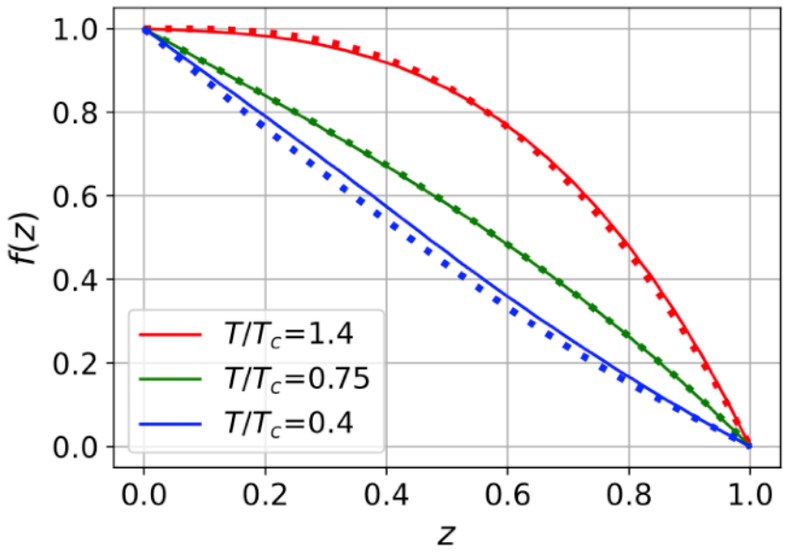
"Holographic Superconductor"

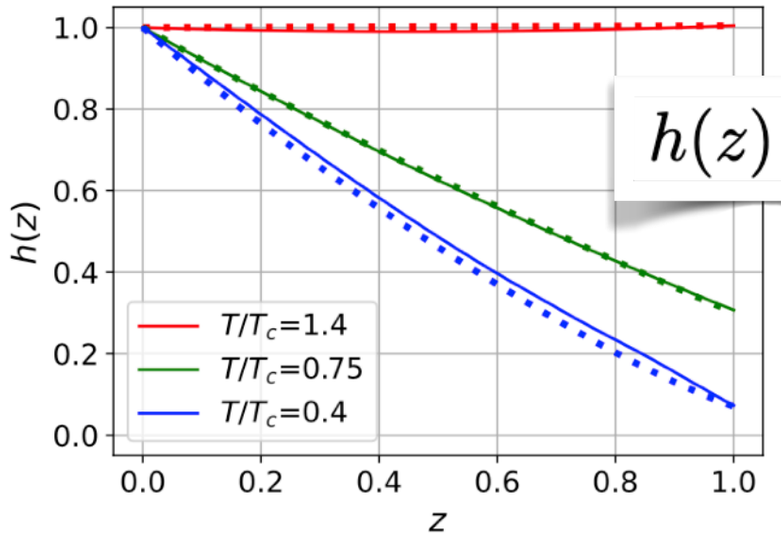


Given holographic entanglement entropy



Bulk reconstruction with machine learning





$h(z) \neq 1$

Dots = True Solutions

Lines = PINNs

Bulk Reconstruction

(**Machine Learning** Example IV)

$$ds^2 = \frac{L^2}{z^2}\left[-f(z)\,dt^2 + \frac{dz^2}{f(z)} + h(z)\,dx^2\right]$$

"AdS_3 metric" ⟷ (Dual) "(1+1) dimensional system"

$$S = \frac{L}{2G_N}\log\left[\frac{2}{\epsilon}\sinh\left(\frac{\ell}{2}\right)\right] \qquad S_{\mathrm{CFT}} = \frac{c}{3}\log\left[\frac{\beta}{\pi\epsilon}\sinh\left(\frac{\pi\ell}{\beta}\right)\right]$$

Given non-holographic entanglement entropy —??→ **Bulk reconstruction with machine learning**

PHYSICAL REVIEW LETTERS **127,** 040603 (2021)

Eigenstate Entanglement: Crossover from the Ground State to Volume Laws

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Department of Physics, Duke University, Durham, North Carolina 27708, USA

(Received 30 May 2019; revised 27 May 2021; accepted 14 June 2021; published 21 July 2021)

To confirm this numerically, we sample energy eigenstates $|E_n\rangle$ from small windows of width ΔE around energies E for fermionic tight-binding chains $\hat{H} = -\sum_i(\hat{c}_i^\dagger \hat{c}_{i+1} + \mathrm{H.c.}) = \sum_k \varepsilon_k \hat{n}_k$ at half filling [65]. These obey weak ETH and are

$$\hat{\rho}_n := \mathrm{Tr}_{\mathcal{B}}|E_n\rangle\langle E_n|$$

Similar to CFT_2 result
(so, similar to BTZ as well?)

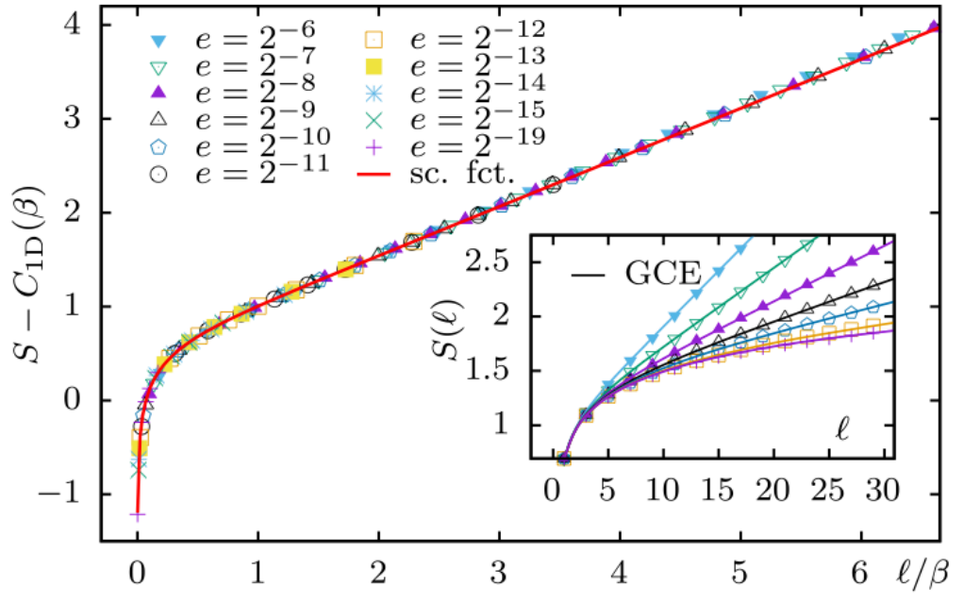
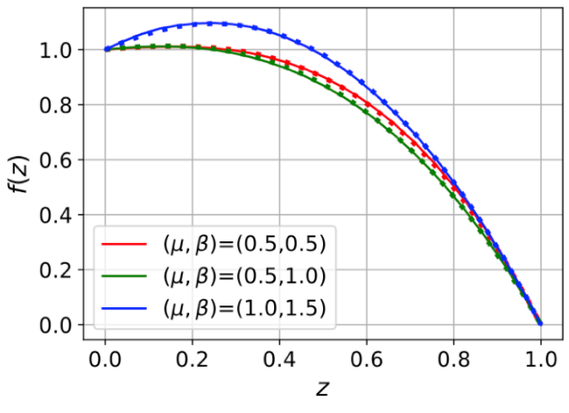
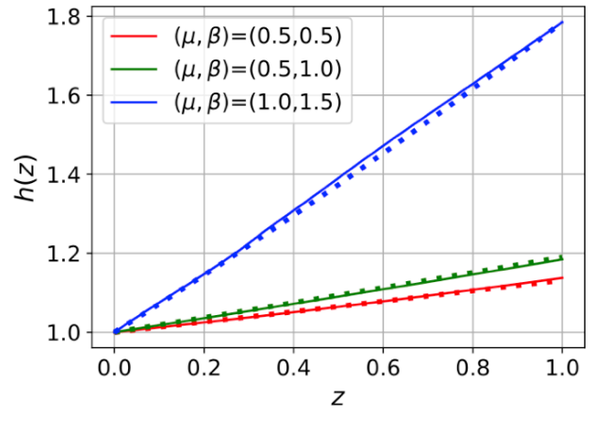


FIG. 1. Entanglement entropies of randomly sampled eigenstates for a fermionic tight-binding chain with half filling ($\mu = 0$, $v = 2$), system size $L \approx 4 \times 10^6$, excitation energy densities $e := (E - E_{\mathrm{gs}})/|E_{\mathrm{gs}}|$, and window size $\Delta E = 1$. The main panel confirms the data collapse to the crossover scaling function (7). The inset asserts the validity of the ETH by comparison to GCE subsystem entropies (lines).

Bulk Reconstruction

(**Machine Learning** Example IV)



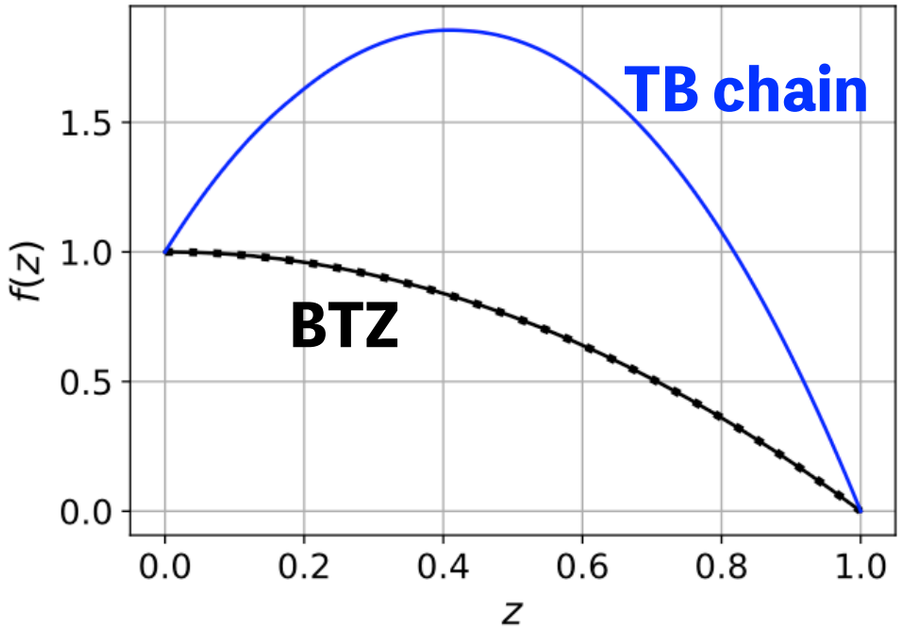


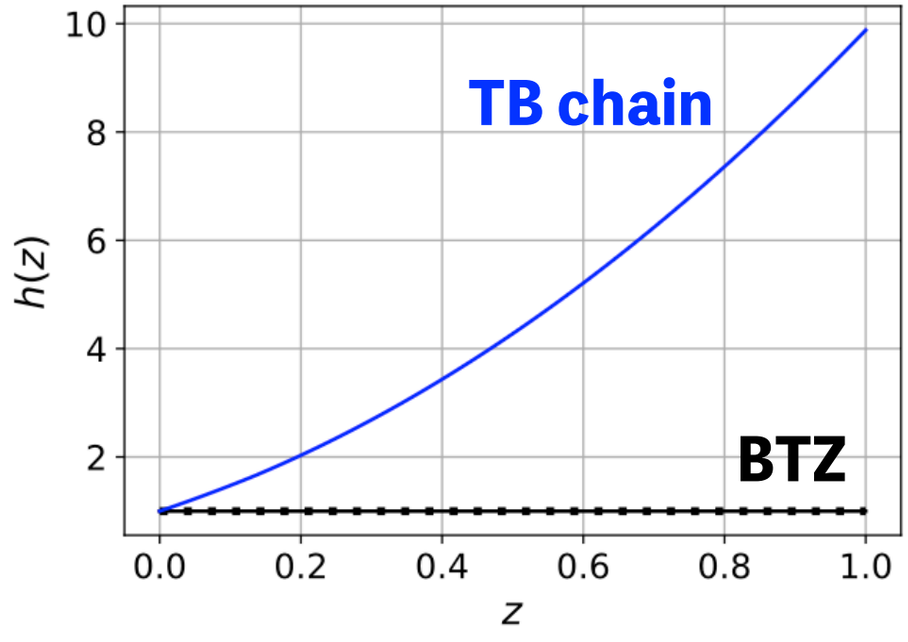
"Gubser-Rocha model"

Given non-holographic entanglement entropy

??

Bulk reconstruction with machine learning





The emergent bulk spacetime is not similar to BTZ geometries, instead..

Gubser-Rocha type.. the similarity is due to the metallic property??

Conclusion

Universal Approximation Theorem

PINNs, Neural ODEs

Neural networks are simply flexible functions fit to data

GRAVITY

System/Model

（Unknown）

New Prediction

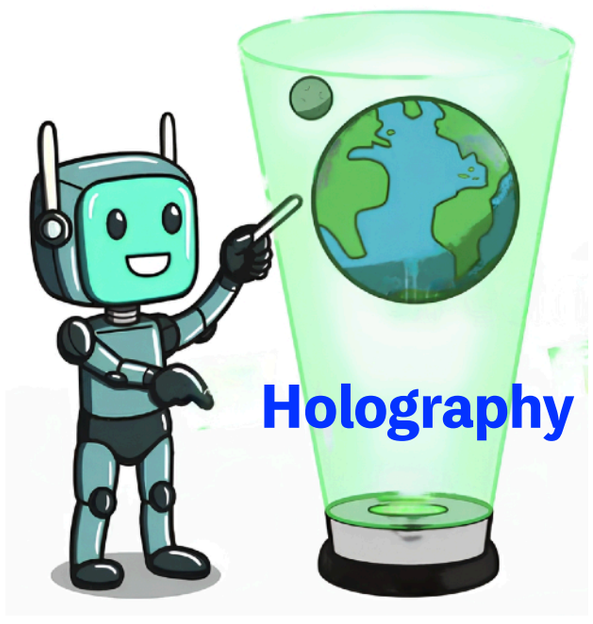
Strongly Coupled Physics

Quantum Gravity

QUANTUM

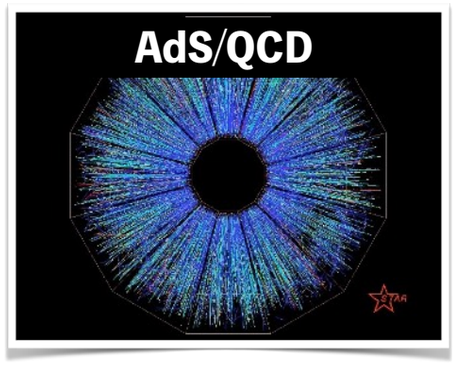
Observables

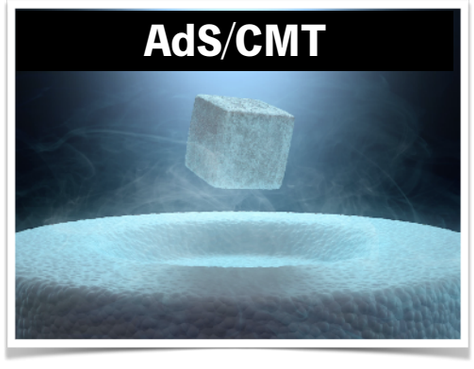
（Exp. accessible）

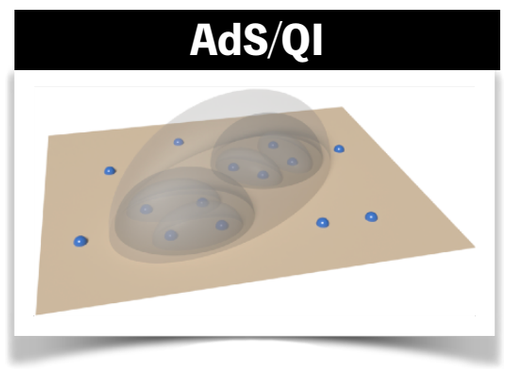


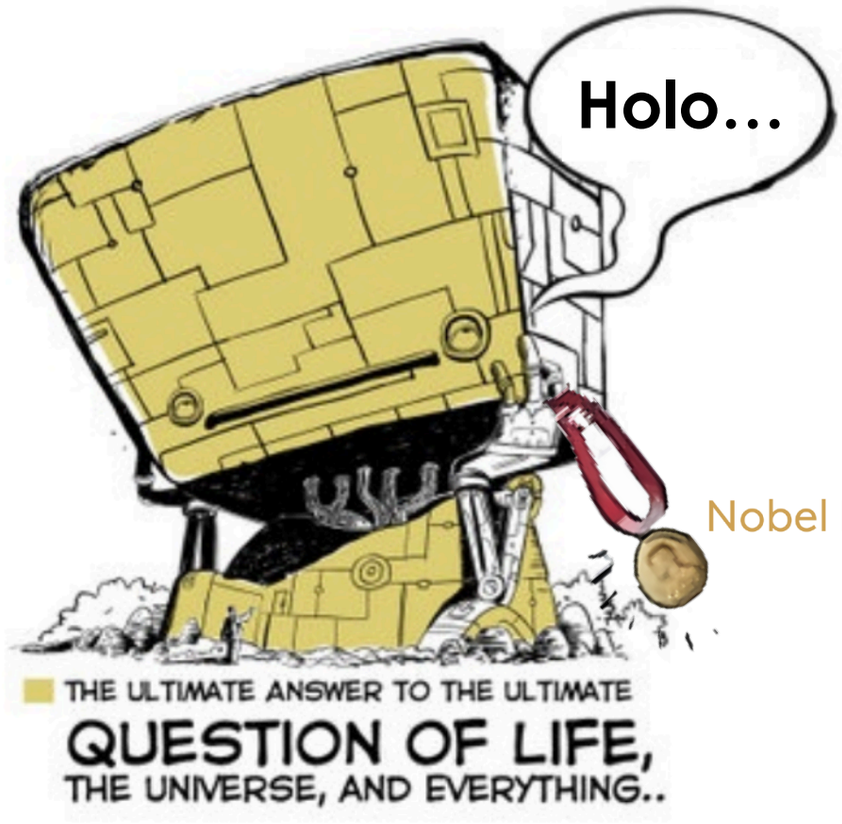
Modeling Realistic Systems

Bulk Reconstruction









Potential implications are far-reaching !!!

THANK YOU

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