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Geometric realization of the concept on the 'charge without charge'

Wonwoo Lee
(CQUeST, Sogang University)

based on

Charged traversable wormholes: charge without charge; Hyeong-Chan Kim, Sung-Won Kim, Bum-Hoon Lee, Wonwoo Lee, e-Print: 2405.10013 [gr-qc], Online version was published in JKPS (2025).

Charged wormholes in (anti-)de Sitter spacetime; Hyeong-Chan Kim, Wonwoo Lee, e-Print: 2505.09981 [gr-qc], It will be appeared in PLB.

The plan of this talk

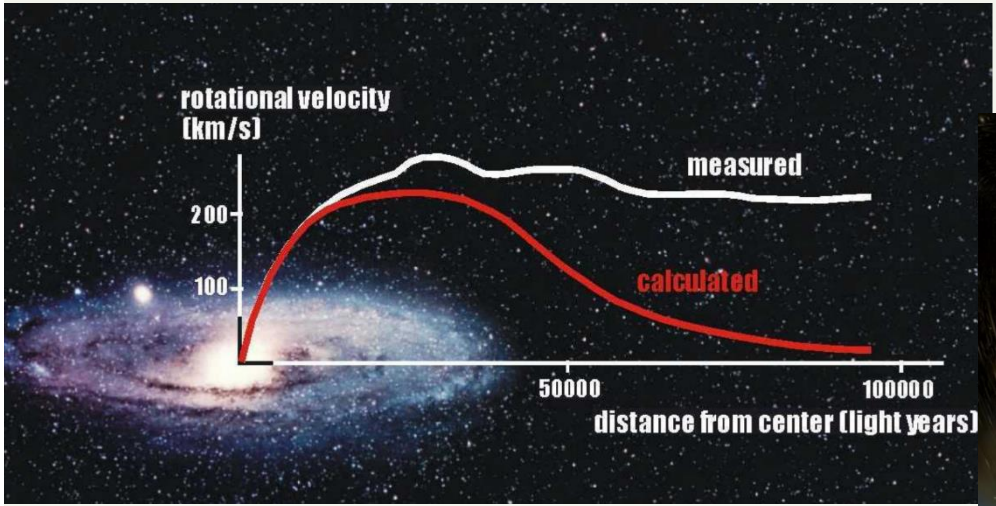
1. Motivations

2. Solutions to the source-free Maxwell equations in the Einstein-Maxwell system: with charge

3. Solutions to the source-free Maxwell equations in the Einstein-Maxwell system: charge without charge

4. Summary and discussions

1. Motivations



The absence of a clear explanation for dark matter and dark energy is igniting interest in modified theories of gravitation. In addition, the ongoing need for precise descriptions of astrophysical phenomena continues to propel research into finding and analyzing various solutions that incorporate matter fields beyond the vacuum solution.

Artist's conception of a wormhole. (Image credit: Shutterstock)

As one of the most fantastical solutions allowed by Einstein's theory, the wormhole solution, including time travel, has become an object of interest and awe for scientists and the public alike, as well as a great subject for study, fiction, and movies.

Charge without charge: electric field by a wormhole

ANNALS OF PHYSICS: 2, 525–603 (1957)

Classical Physics as Geometry

Gravitation, Electromagnetism, Unquantized Charge, and Mass as Properties of Curved Empty Space*

CHARLES W. MISNER† AND JOHN A. WHEELER‡

Lorentz Institute, University of Leiden, Leiden, Netherlands, and Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

If classical physics be regarded as comprising gravitation, source free electromagnetism, unquantized charge, and unquantized mass of concentrations of electromagnetic field energy (geons), then classical physics can be described in terms of curved empty space, and nothing more. No changes are made in existing theory. The electromagnetic field is given by the "Maxwell square root" of the contracted curvature tensor of Ricci and Einstein. Maxwell's equa-

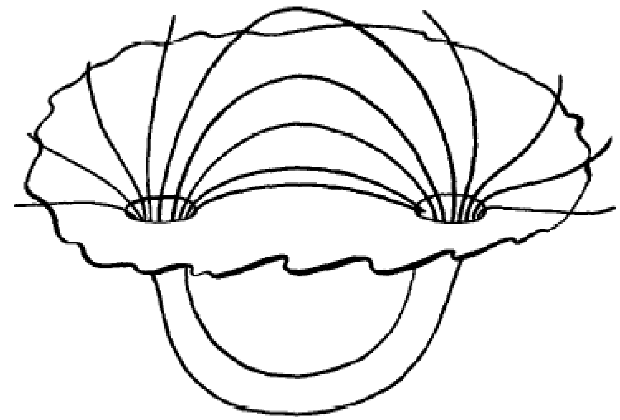
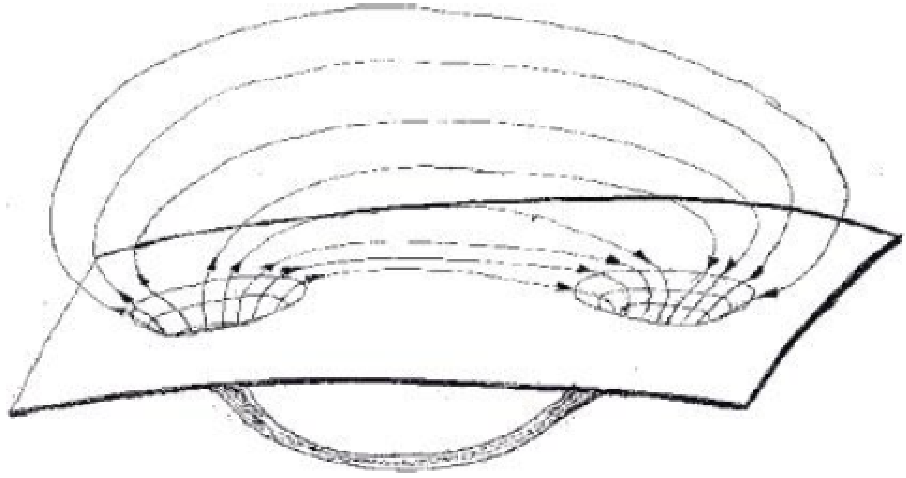


FIG. 3. Symbolic representation of the unquantized charge of classical theory. Fo

Misner and Wheeler highlighted the importance of this solution to the source-free Maxwell equations, noting that the electric field enters one side of the wormhole and exits the other. The full realization of this concept would require finding a charged wormhole solution.

GEONS, BLACK HOLES, AND QUANTUM FOAM

W. W. NORTON & COMPANY



The idea of "charge without charge": Electric field lines that seem to begin at one place and end at another may be connected, thanks to a wormhole in "multiply connected" space.

(Drawing by John Wheeler.)

2. Solutions to the source-free Maxwell equations: with charge

- We consider the action

$$I = \int_{\mathcal{M}} \sqrt{-g} d^4x \left[\frac{R}{16\pi G} - \frac{1}{16\pi} F_{\alpha\beta} F^{\alpha\beta} \right] + I_b$$

We obtain the Einstein field equations from the variation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

where

$$T_{\mu\nu} = \frac{1}{4\pi} \left(F_{\mu\alpha} F_{\nu}{}^{\alpha} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right)$$

and the source free Maxwell equations

$$\nabla_\mu F^{\mu\nu} = \frac{1}{\sqrt{-g}} [\partial_\mu (\sqrt{-g} F^{\mu\nu})] = 0$$

Bianchi identity

$$\nabla_\rho F_{\mu\nu} + \nabla_\mu F_{\nu\rho} + \nabla_\nu F_{\rho\mu} = \partial_\rho F_{\mu\nu} + \partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} = 0$$

- **In the Einstein-Maxwell system, to obtain the self-gravitating solution**

  (1) One takes the metric ansatz.

  (2) One solves the (source free) Maxwell equations.

  (3) The solution to the Maxwell equations is then substituted into the energy-momentum tensor in order to solve the Einstein equations.

- **When employing a solution generating method**

  (1) First, one uses a solution generating method to find the solution to the Einstein equations.

  (2) Solve the Maxwell equations in the geometry of the solution to the Einstein equations in order to obtain that solution.

• **We consider static spherically symmetric geometry (for a black hole)**

$$\begin{aligned} ds^2 &= -f(r)dt^2 + \frac{dr^2}{g(r)} + r^2 d\Omega_2^2 \\ &= -f(r)dt^2 + \frac{dr^2}{f(r)h(r)} + r^2 d\Omega_2^2 \,. \end{aligned}$$

Here, we will think about a RN black hole eventually, thus we take

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2$$

After we will show the components of the energy-momentum tensor. Then I will explain why this choice was made.

- **The source free Maxwell equations: the electric field**

$$\nabla_\mu F^{\mu\nu} = \frac{1}{\sqrt{-g}}[\partial_\mu(\sqrt{-g}F^{\mu\nu})] = 0$$

for μ=t, ν=r : $\partial_t(r^2 \sin\theta E) = 0$

for μ=r, ν=0 : $\partial_r(r^2 \sin\theta E) = 0$

The integration constant emerges. By comparing it to the classical electromagnetic case (Gauss's law), one could guess that the integration constant corresponds to the charge located at the origin.

$$\Rightarrow \quad E = \frac{Q}{r^2}, \qquad (\mathrm{Q = const.})$$

- **The source free Maxwell equations: the magnetic field**

$$\nabla_\rho F_{\mu\nu} + \nabla_\mu F_{\nu\rho} + \nabla_\nu F_{\rho\mu} = \partial_\rho F_{\mu\nu} + \partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} = 0$$

for ρ=t, μ=θ, ν=φ : $\partial_t(r^2 \sin\theta B) = 0$

for ρ=r, μ=θ, ν=φ : $\partial_r(r^2 \sin\theta B) = 0$

$$\Rightarrow \qquad B = \frac{P}{r^2}, \qquad (\mathrm{P} = \mathrm{const.})$$

$$\Rightarrow \quad T^\mu_{\ \nu} = \frac{E^2 + B^2}{8\pi}\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \frac{Q^2 + P^2}{8\pi r^4}\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

⇒ **traceless** $T^\mu_\mu = 0 \quad \Rightarrow \quad R = 0$

- **The energy-momentum tensor**

$$T^{\nu}_{\mu} = \mathrm{diag}(-\varepsilon, p_r, p_\theta, p_\psi), \;\; p_r(r) = -\varepsilon(r)\,, \quad p_\theta(r) = p_\psi(r) = \varepsilon(r).$$

$$\varepsilon(r) = \frac{Q^2}{8\pi r^4}\,, \qquad T^{\mu}_{\nu} = \begin{pmatrix} -\frac{Q^2}{8\pi r^4} & 0 & 0 & 0 \\ 0 & -\frac{Q^2}{8\pi r^4} & 0 & 0 \\ 0 & 0 & \frac{Q^2}{8\pi r^4} & 0 \\ 0 & 0 & 0 & \frac{Q^2}{8\pi r^4} \end{pmatrix}$$

The Maxwell charge is present in the energy-momentum tensor, which causes divergence at r = 0. However, the quantity that corresponds to the mass of a black hole is not present in the energy-momentum tensor.

Property of the metric function, (when f(r)=g(r))

$$p_r + \varepsilon = \frac{g(r)f'(r) - f(r)g'(r)}{rf(r)} = 0$$

The radial pressure is the same as the negative of the energy density. For the black hole written by $ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$ **.**

- **We check the components of the Einstein tensor**

$$\Rightarrow \quad G^r{}_r = -\frac{1}{r^2} + \frac{f(r)}{r^2} + \frac{f'(r)}{r} = -\frac{G(Q^2+P^2)}{r^4}$$

$$\Rightarrow \quad \frac{d}{dr}[rf(r)] = rf' + f = 1 - \frac{G(Q^2+P^2)}{r^2}$$

$$\Rightarrow \quad f(r) = 1 - \frac{2GM}{r} + \frac{G(Q^2+P^2)}{r^2}$$

$$T_{\mu\nu} \neq 0 \quad \Rightarrow \quad R_{\mu\nu} \neq 0\,, \quad R = 0$$

The Kretshmann scalar

$$R_{\mu\nu}R^{\mu\nu} = \frac{4G^2Q^4}{r^8}\,, \quad R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} = \frac{48}{r^6}\left[\left(GM - \frac{GQ^2}{r}\right)^2 + \frac{G^2Q^4}{6r^2}\right]$$

• One can employ the Newman-Janis algorithm to obtain the rotating black hole geometry.

Schwarzschild (1916) → Kerr (1963)

Newman and Janis, JMP 6, 915 (1965)

Reissner – Nordström (1916, 1918) → Kerr-Newman (1965)

Newman, Chinnapared, Exton, Prakash and Torrence, JMP 6, 918 (1965)

- **The Kerr-Newman black hole geometry**

$$ds^2 = -F(r,\theta)dt^2 - 2[1-F(r,\theta)]a\sin^2\theta dt d\phi + \frac{\Sigma}{\rho^2}\sin^2\theta d\phi^2 + \frac{\rho^2}{\triangle}dr^2 + \rho^2 d\theta^2\,,$$

$$= -\frac{\triangle}{\rho^2}(dt - a\sin^2\theta d\phi)^2 + \frac{\sin^2\theta}{\rho^2}[adt - (r^2+a^2)d\phi]^2 + \frac{\rho^2}{\triangle}dr^2 + \rho^2 d\theta^2\,,$$

where $F(r,\theta) = 1 - \frac{2Mr}{\rho^2} + \frac{Q^2}{\rho^2}$

$$\rho^2 = r^2 + a^2\cos^2\theta,\ \Sigma = (r^2+a^2)^2 - a^2\triangle\sin^2\theta,\ \text{and}\ \triangle = \rho^2 F(r,\theta) + a^2\sin^2\theta$$

- **Let us consider physical quantities in an orthonormal frame,** $(e_{\hat{t}}, e_{\hat{r}}, e_{\hat{\theta}}, e_{\hat{\phi}})$ **, introduced by Carter(1968), in which the stress-energy tensor for the anisotropic matter field is diagonal,**

$$e^{\mu}_{\hat{t}} = \frac{(r^2+a^2,0,0,a)}{\rho\sqrt{\triangle}}\,, \quad e^{\mu}_{\hat{r}} = \frac{\sqrt{\triangle}(0,1,0,0)}{\rho}\,,$$

$$e^{\mu}_{\hat{\theta}} = \frac{(0,0,1,0)}{\rho}\,, \quad e^{\mu}_{\hat{\phi}} = -\frac{(a\sin^2\theta,0,0,1)}{\rho\sin\theta}\,.$$

• The components of the energy-momentum tensor are expressed in terms of Gμν as

$$8\pi\varepsilon = e^{\mu}_{\hat{t}} e^{\nu}_{\hat{t}} G_{\mu\nu}, \quad 8\pi p_{\hat{r}} = e^{\mu}_{\hat{r}} e^{\nu}_{\hat{r}} G_{\mu\nu}, \; 8\pi p_{\hat{\theta}} = e^{\mu}_{\hat{\theta}} e^{\nu}_{\hat{\theta}} G_{\mu\nu}, \; 8\pi p_{\hat{\phi}} = e^{\mu}_{\hat{\phi}} e^{\nu}_{\hat{\phi}} G_{\mu\nu}.$$

It gives

$$\varepsilon = \frac{Q^2}{8\pi\rho^4}\,, \quad p_{\hat{r}} = (-\varepsilon) = -\frac{Q^2}{8\pi\rho^4}\,, \quad p_{\hat{\theta}} = (p_{\hat{\phi}}) = \frac{Q^2}{8\pi\rho^4}$$

The Maxwell tensor can be obtained as follows:

$$\begin{aligned} F_{tr} &= -F_{rt} = \frac{Q}{\rho^4}(a^2\cos^2\theta - r^2)\,, \quad F_{t\theta} = -F_{\theta t} = \frac{Q}{\rho^4}(a^2 r \sin 2\theta)\,, \\ F_{r\phi} &= -F_{\phi r} = \frac{Q}{\rho^4} a \sin^2\theta (a^2\cos^2\theta - r^2)\,, \quad F_{\theta\phi} = -F_{\phi\theta} = \frac{Q}{\rho^4} a r \sin 2\theta (r^2 + a^2)\,. \end{aligned}$$

and

$$\begin{aligned} F^{tr} &= -F^{rt} = \frac{Q}{\rho^6}(r^2 - a^2\cos^2\theta)(r^2 + a^2)\,, \quad F^{t\theta} = -F^{\theta t} = \frac{Q}{\rho^6}(-a^2 r \sin 2\theta)\,, \\ F^{r\phi} &= -F^{\phi r} = \frac{Q}{\rho^6} a (a^2\cos^2\theta - r^2)\,, \quad F^{\theta\phi} = -F^{\phi\theta} = \frac{Q}{\rho^6} 2ar\cot\theta\,. \end{aligned}$$

The Maxwell tensor satisfy the source-free Maxwell equations.

$$\nabla_\mu F^{\mu\nu} = \frac{1}{\sqrt{-g}}[\partial_\mu(\sqrt{-g}F^{\mu\nu})] = 0$$

- **When the Newman-Janis algorithm does not apply**

  Obtaining the rotating black hole solution becomes a challenging study when the Newman-Janis algorithm is not well defined in the geometry.

  (1) If the seed black hole has the metric with f(r) ≠ g(r).

  (2) If the seed black hole has a cosmological constant.

  (3) If the seed black hole is immersed in the magnetic field.

- **one can take the metric ansatz**

$$ds^2 = -\frac{\rho^2\triangle}{\Sigma}dt^2 + \frac{\Sigma}{\rho^2}\sin^2\theta(d\phi - \tfrac{[1-F(r,\theta)]\rho^2 a}{\Sigma}\,dt)^2 + \frac{\rho^2}{\triangle}dr^2 + \rho^2 d\theta^2\,,$$

$$ds^2 = -\frac{\triangle}{\rho^2}(dt - a\sin^2\theta d\phi)^2 + \frac{\sin^2\theta}{\rho^2}[adt - (r^2+a^2)d\phi]^2 + \frac{\rho^2}{\triangle}dr^2 + \rho^2 d\theta^2\,,$$

3. Solutions to the source-free Maxwell equations in the Einstein-Maxwell system: charge without charge

Charge without charge: magnetic field

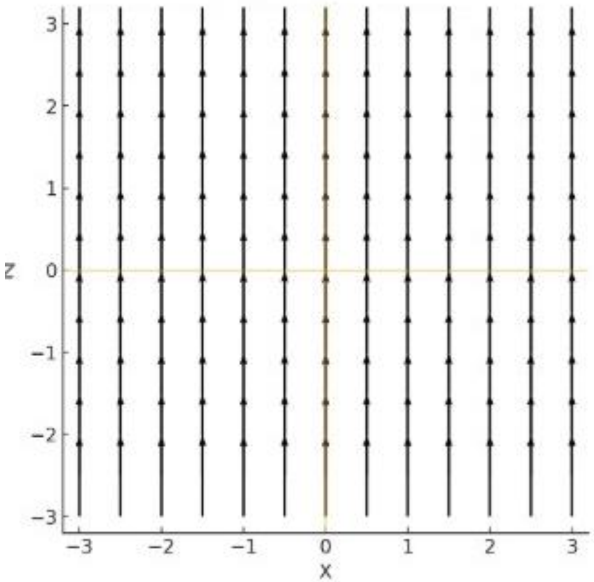
It is possible to imagine the Minkowski spacetime with a uniform magnetic field.

$$ds^2 = -dt^2 + d\rho^2 + \rho^2 d\phi^2 + dz^2$$

$$\mathbf{B} = B_0 \hat{\mathbf{z}}$$

$$\mathbf{A}(\rho, \phi, z) = \frac{1}{2} B_0 \rho \hat{\phi}$$

$$A_x = -\tfrac{1}{2} B_0 y, \qquad A_y = \tfrac{1}{2} B_0 x, \qquad A_z = 0,$$



It is probe limit! **There is no backreaction effect.**

Pure magnetic and electric geons; M. Melvin, Phys. Lett. 8, 65 (1964). **It is self-gravitating!**

The Einstein-Maxwell equations

(1) Einstein equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}, \quad T_{\mu\nu} = \frac{1}{4\pi}\left(F_{\mu\alpha}F_{\nu}^{\alpha} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta}F^{\alpha\beta}\right)$$

(2) Source-Free Maxwell equations

$$\nabla_{\mu} F^{\mu\nu} = 0$$

Melvin probably adopted the following metric ansatz and one-form consistent with cylindrical symmetry and stationary:

$$ds^2 = \Lambda(\rho)^2(-dt^2 + dz^2 + d\rho^2) + \frac{\rho^2}{\Lambda(\rho)^2} d\varphi^2, \quad A = A_{\varphi}(\rho)\, d\varphi,$$

To obtain the following, he would first have solved the source-free Maxwell equations.

$$B_z(\rho) = F_{\rho\phi} = \frac{B_o \rho}{\Lambda^2(\rho)} = \frac{B_o \rho}{(1 + \frac{1}{4}B_o^2 \rho^2)^2}\, , B^{\hat{z}} = \frac{B_o}{\Lambda^2(\rho)}$$

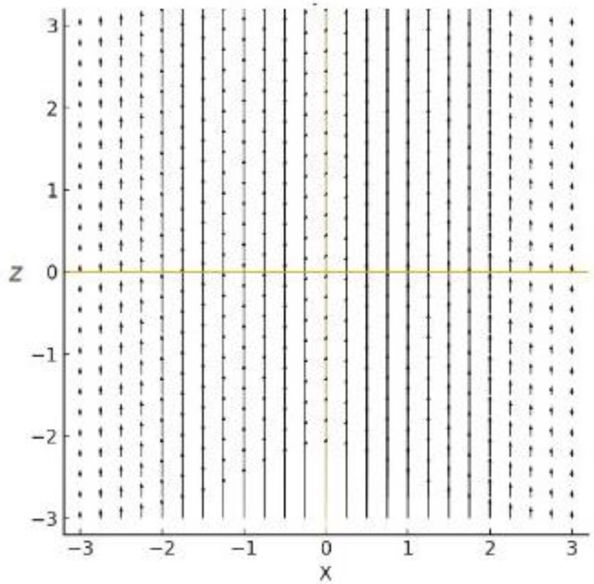
Ultimately, he would have obtained the solution by inserting this into the energy-momentum tensor and solving the Einstein equations.

$$ds^2 = \Lambda^2 \left[-dt^2 + dz^2 + d\rho^2\right] + \Lambda^{-2} \rho^2 d\phi^2$$

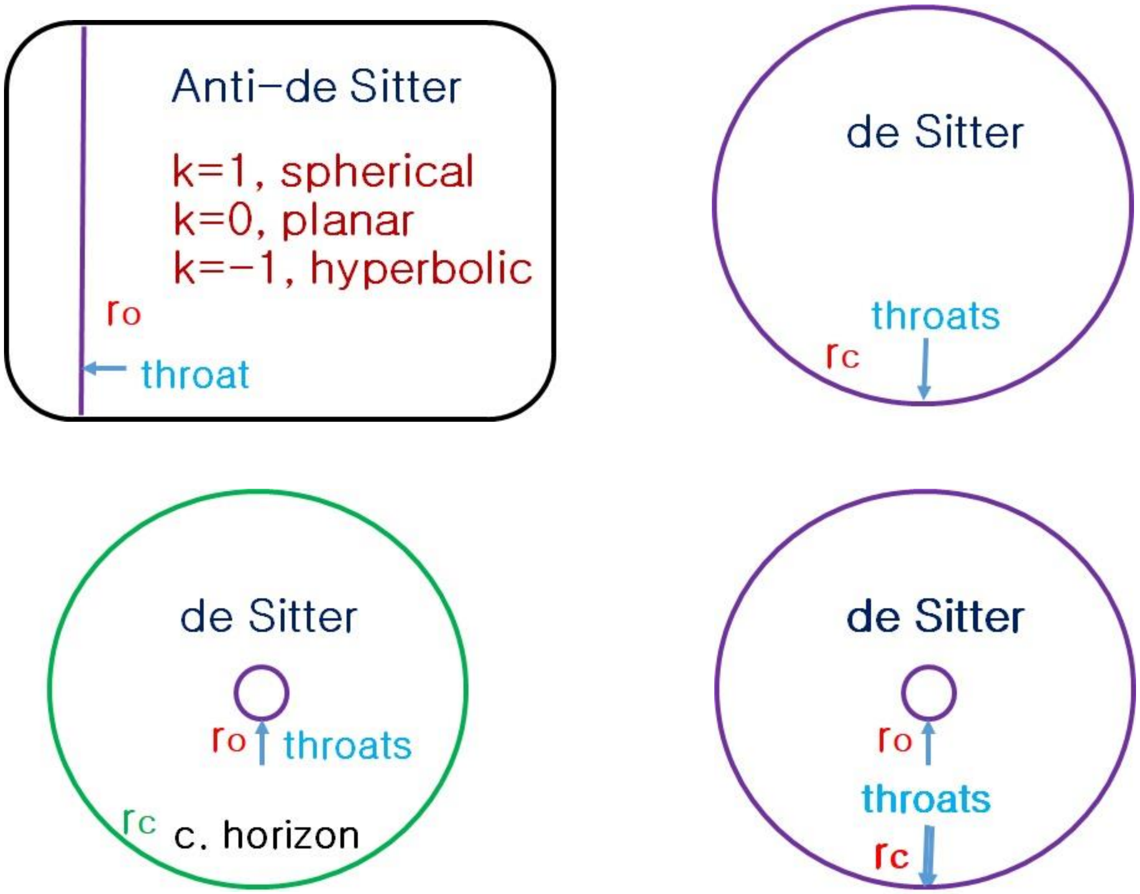
$$\text{where } \Lambda(\rho) = 1 + \tfrac{1}{4}B_o^2 \rho^2, \qquad A_\phi = \frac{B_o \rho^2}{2\Lambda(\rho)}$$

$$\varepsilon(\rho) = p_z = -\frac{B_o^2}{(1 + \frac{1}{4}B_o^2 \rho^2)^4}\, ,$$

$$p_r(\rho) = p_\theta = -\varepsilon(\rho) = \frac{B_o^2}{(1 + \frac{1}{4}B_o^2 \rho^2)^4}\, ,$$



Charge without charge: electric field
Charged wormholes with the cosmological constant?



Charged wormholes in (anti-)de Sitter spacetime; Hyeong-Chan Kim, Wonwoo Lee, e-Print: 2505.09981 [gr-qc], It will be appeared in PLB.

We consider the action

$$I = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi}(R-2\Lambda-F_{\mu\nu}F^{\mu\nu})+\mathcal{L}_{\rm am}\Big]+I_{\rm b},$$ (G = 1 for simplicity)

where $\mathcal{L}_{\rm am}$ describes effective anisotropic matter fields.

We obtain (1) the Einstein equation

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu} - \Lambda g_{\mu\nu} \ ,$$

(2) The source-free Maxwell equations are given by

$$\nabla_\nu F^{\mu\nu} = \frac{1}{\sqrt{-g}}[\partial_\nu(\sqrt{-g}F^{\mu\nu})] = 0$$

The additional (fluid) matter does not have an independent equation of motion.

The stress-energy tensor takes the form

$$T^{\mu\nu} - \frac{\Lambda}{8\pi} g^{\mu\nu} = T^{\mu\nu}_{\mathrm{M}} + T^{\mu\nu}_{\mathrm{am}} - \frac{\Lambda}{8\pi} g^{\mu\nu} \,,$$

The additional (fluid) matter does not have an independent equation of motion.

$$p_{r\mathrm{am}} = w_1 \varepsilon_{\mathrm{am}} \,, \quad p_{t\mathrm{am}} = w_2 \varepsilon_{\mathrm{am}} \,,$$

$$T^{\mu}_{\nu\mathrm{am}} = \mathrm{diag}\left(-\varepsilon_{\mathrm{am}}, w_1 \varepsilon_{\mathrm{am}}, w_2 \varepsilon_{\mathrm{am}}, w_2 \varepsilon_{\mathrm{am}}\right)$$

anisotropic matter : $p_{r\mathrm{am}} \neq p_{t\mathrm{am}}$

The static spherically symmetric wormhole geometry without the cosmological constant is given by

$$ds^2 = -f(r)dt^2 + \frac{1}{g(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\psi^2)\,,$$

$$e^{2\Phi(r)} = f(r), \quad \left(1 - \frac{b(r)}{r}\right) = g(r)$$

where Φ(r) and b(r) denote the redshift function and the wormhole shape function.

We consider static charged wormhole as the form

metric form for the charge without charge

$$ds^2 = -f(r)dt^2 + \frac{1}{g(r)}dr^2 + r^2 d\Sigma_k^2 \, ,$$

$$f(r) = \left(k + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2\right) \text{ and } g(r) = \left(k + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2 - \frac{b(r)}{r}\right),$$

In S.-W. Kim and H. Lee, PRD 63, 064014 (2001) [arXiv:gr-qc/0102077], H.-C. Kim, S.-W. Kim, B.-H. Lee and WL [arXiv: 2405.10013 [gr-qc]].

$$ds^2 = -\left(1 + \frac{Q^2}{r^2}\right)dt^2 + \left(1 + \frac{Q^2}{r^2} - \frac{b(r)}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\psi^2) \, ,$$

For the static electrically charged geometry,

$F^{r0} = -\sqrt{\frac{g(r)}{f(r)}}\frac{Q}{r^2}$ **satisfy the source-free Maxwell equations.**

We express the electric field with upper indices as $F^{0r} = E^r.$

This field should be defined in an orthonormal frame, we adopt covariant tetrad shown as

$$e^{\hat{t}}_{\mu} = (\sqrt{f(r)}, 0, 0, 0), \quad e^{\hat{r}}_{\mu} = (0, \frac{1}{\sqrt{g(r)}}, 0, 0),$$

$$e^{\hat{\theta}}_{\mu} = (0, 0, r, 0), \quad e^{\hat{\psi}}_{\mu} = (0, 0, 0, r\sin\theta)$$

The electric field can be obtained through $F^{\hat{a}\hat{b}} = e^{\hat{a}}_{\mu} e^{\hat{b}}_{\nu} F^{\mu\nu}$

The electric field is $E^{\hat{r}} = \frac{Q}{r^2}.$

We now consider Einstein equations. The nonvanishing components of the Einstein tensor are given by

$$
\begin{aligned}
G^t_t &= -8\pi\varepsilon = 8\pi(-\varepsilon_c - \varepsilon_\Lambda - \varepsilon_{\rm am}) \\
&= 8\pi(-\varepsilon_c) - \Lambda - \frac{b'(r)}{r^2}\,, \\
G^r_r &= 8\pi p_r = 8\pi(-\varepsilon_c - \varepsilon_\Lambda + w_1\varepsilon_{\rm am}) \\
&= 8\pi(-\varepsilon_c) - \Lambda + \frac{3(Q^2 - kr^2 + \Lambda r^4)b(r)}{3r^3(Q^2 + kr^2) - \Lambda r^7}\,, \\
G^\theta_\theta &= 8\pi p_t = 8\pi(\varepsilon_c - \varepsilon_\Lambda + w_2\varepsilon_{\rm am}) \\
&= 8\pi(\varepsilon_c) - \Lambda + \frac{3A(r)b(r) + B(r)b'(r)}{2r^3[3(Q^2 + kr^2) - \Lambda r^4]^2}\,,
\end{aligned}
$$

where the prime denotes the derivative with respect to r.

where $A(r) = kr^2(\Lambda r^4 - 9Q^2) + 3r^4 - 6Q^4 + 10Q^2\Lambda r^4$,
$B(r) = r^3(-3k + 2\Lambda r^2)[3(Q^2 + kr^2) - \Lambda r^4]$, $\varepsilon_c = \frac{Q^2}{8\pi r^4}$,

We obtain the solutions

$$b(r) = b_o \left(\frac{r b_o}{kr^2 + Q^2 - \frac{\Lambda}{3} r^4} \right)^{1/w_1}, \quad b_o = (kr_o^2 + Q^2)/r_o - \frac{\Lambda}{3} r_o^3,$$

$$\varepsilon_{\rm am} = -\frac{kr^2 - Q^2 - \Lambda r^4}{8\pi w_1 r^4} \left(\frac{b_o r}{kr^2 + Q^2 - \frac{\Lambda}{3} r^4} \right)^{(w_1+1)/w_1}$$

$$w_2(r) = -1 + \frac{(2Q^2 - kr^2) w_1}{kr^2 - Q^2 - \Lambda r^4} + \frac{3(2Q^2 + kr^2)(w_1 + 1)}{6(Q^2 + kr^2) - 2\Lambda r^4}$$

$$\varepsilon = \frac{Q^2}{8\pi r^4} + \frac{\Lambda}{8\pi} + \varepsilon_{\rm am} ,$$

$$p_r = -\frac{Q^2}{8\pi r^4} - \frac{\Lambda}{8\pi} + w_1 \varepsilon_{\rm am} ,$$

$$p_t = \frac{Q^2}{8\pi r^4} - \frac{\Lambda}{8\pi} + w_2 \varepsilon_{\rm am} .$$

Conditions to be a wormhole geometry

Now let us describe the conditions for the above solution to Einstein's equations to be a wormhole geometry.

Let us check out the flare-out condition and the energy condition.

※ To construct and maintain the structure of the traversable wormhole, there is the geometric flare-out condition that must be satisfied at the throat and the neighborhood of that, which is related to the energy condition of the matter supporting the wormhole structure.

We consider the flare-out condition of the wormhole through the embedding geometry at t = const. and θ = π/2 (sinh θ=1) :

$$ds^2_{\rm eq} = \left(1 + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2 - \frac{b(r)}{r}\right)^{-1} dr^2 + r^2 d\phi^2 = \left[1 + \left(\frac{dz}{dr}\right)^2\right] dr^2 + r^2 d\phi^2 \,,$$

The condition is given by

$$\frac{d^2 r}{dz^2} = \frac{r[r(b(r) - rb'(r)) - 2Q^2 - \frac{2\Lambda}{3}r^4]}{2[krb(r) - k(Q^2 - \frac{\Lambda}{3}r^4 + kr^2) + r^2]^2} > 0 \,,$$

At the throat

$$N(r_o) = (kr_o^2 - Q^2 - \Lambda r_o^4)\,(1 + 1/w_1) > 0$$

One could choose w1 > 0 or w1 < −1.

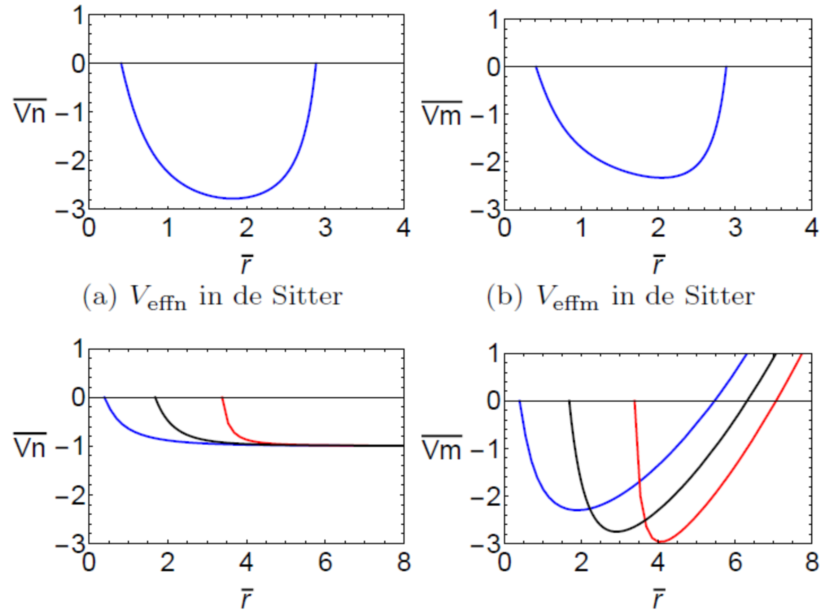
Radial geodesics

We now consider radial geodesics by both the light and massive particle. One can take m ≠ 0 for timelike geodesics and m = 0 for null geodesics.

The radial geodesics are given by

$$\left(\frac{dr}{d\lambda}\right)^2 + g(r)\left[-\frac{\left(E \mp \frac{eQ}{r}\right)^2}{f(r)} + \frac{L_z^2}{r^2}\right] = 0$$

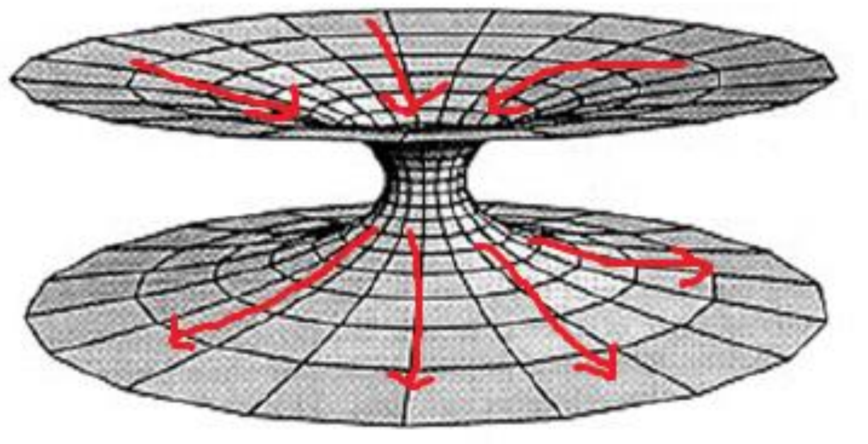
$$\left(\frac{dr}{d\tau}\right)^2 + \frac{g(r)}{m^2}\left[m^2 - \frac{\left(E \mp \frac{eQ}{r}\right)^2}{f(r)} + \frac{L_z^2}{r^2}\right] = 0\,,$$



(a) V_{effn} in de Sitter (b) V_{effm} in de Sitter

(c) V_{effn} in anti-de Sitter (d) V_{effm} in anti-de Sitter

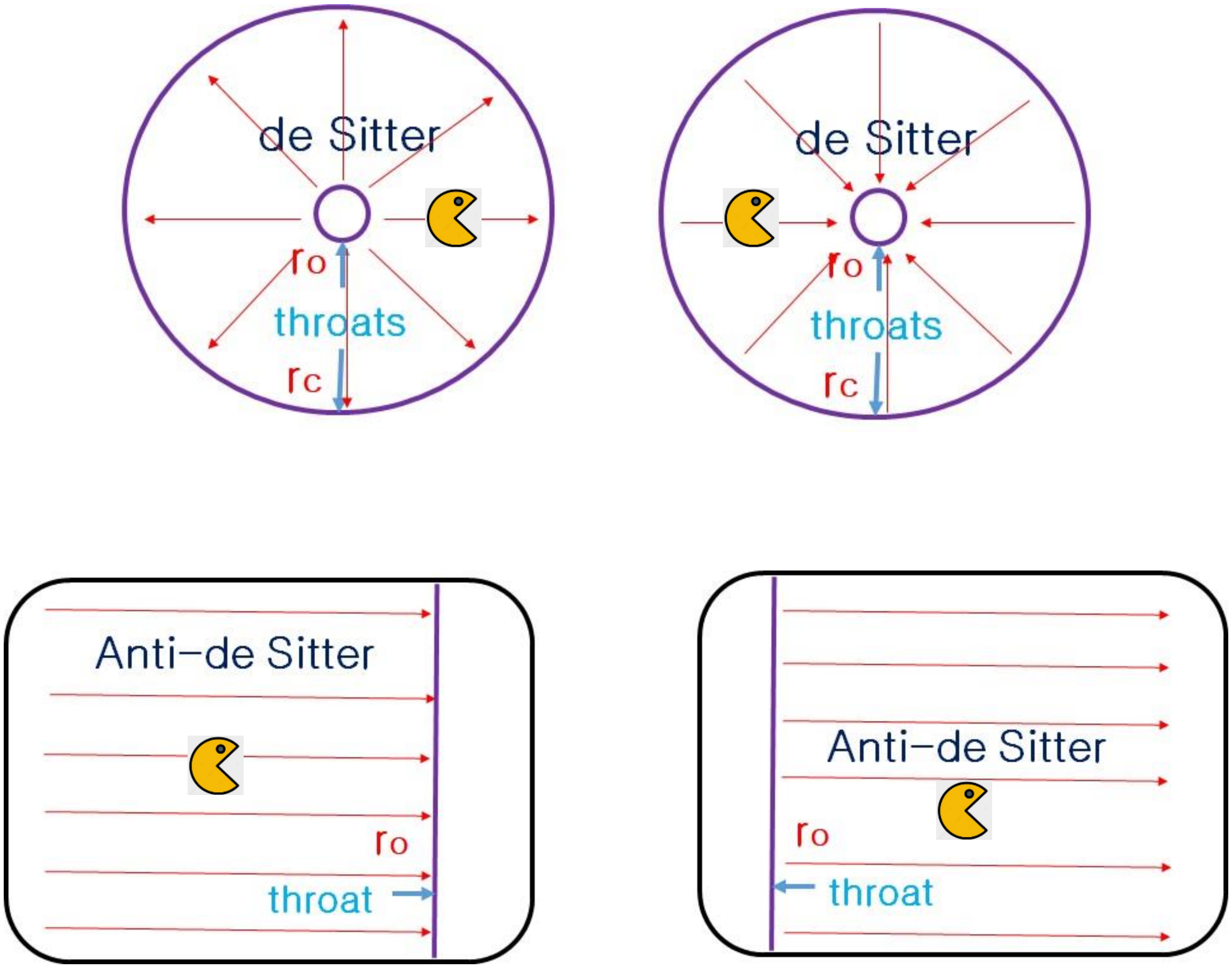
Geometric realization of the concept of 'charge without charge'



Embedded shapes of the wormhole.

This figure illustrates a conceptual embedded diagram of a wormhole featuring electric field lines. In this diagram, the red electric field lines converge toward the wormhole from one universe, traverse through it, and exit into another universe. At the throat of the wormhole, the Maxwell tensor, Ftr, goes to zero, which ensures continuity across this region. If one considers a Gaussian surface that surrounds the asymptotic regions of both universes, there is an equal flux of electric field lines entering and exiting, indicating that no net charge exists within the Gaussian surface.

Conceptual diagram of the wormhole in de Sitter and anti-de Sitter spacetimes



3. Summary and discussions

- We looked at Misner and Wheeler's ideas about the solution to the source-free Maxwell equations in the Einstein-Maxwell system.

- We examined well-known solutions by dividing them into cases with charge and the charge without charge.

- As one example of such a solution, we presented a wormhole solution with a cosmological constant.

Have fun with wormhole physics!

Thank you for your attention!