The cosmological potential of gravitational wave memory with future detectors

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Phys.Rev.D 111(2025)8, Phys.Rev.D 112(2025)2

SASGAC 2025, CQeST, Sogang University

December 28, 2025

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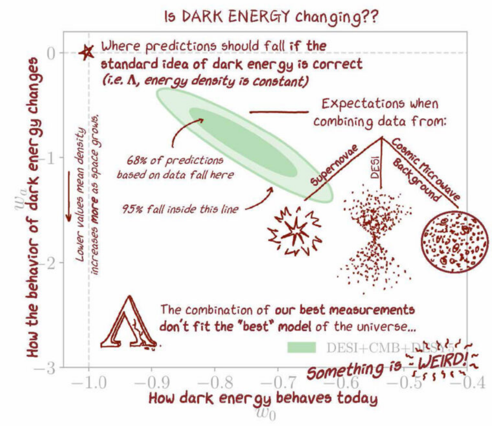
Background



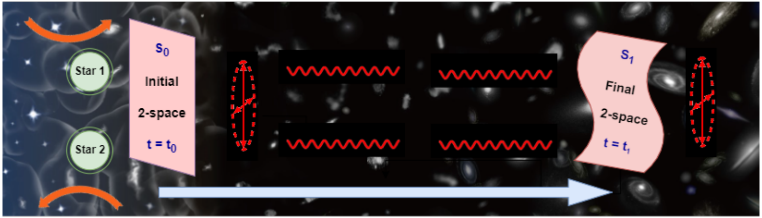
Credit: Claire Lamman/DESI collaboration

- The Λ cold dark matter(ΛCDM) model successfully explains accelerated expansion and flatness of the Universe, uniformity of CMBR etc.
- Recent DESI results reveal significant deviation from ΛCDM model in cosmic expansion.
- Correct cosmological model still remains an illusion!
- The suitable theory describing gravity in cosmological length scale is another mystery.

Can gravitational wave(GW) solve these riddles?

- Detection of GW by LIGO-VIRGO-KAGRA has opened a new window to probe various aspects of gravity and cosmology. [2009 Satya & Schutz]
- GW observations, originated from binary black hole(BBH) merger encodes luminosity distance(D_L) of the source absolutely [1986 Schutz]
- Redshift(z), extracted from the electromagnetic (EM) counter part, originated from binary neutron stars (BNS) determines ($D_L - z$) relation necessary to decode cosmic expansion.
- Does GW originated at higher redshifts, pick up an integrated effect of cosmological background to determine ($D_L - z$) relation? — akin to Integrated Sachs-Wolfe effect on CMB photons.

Our aim

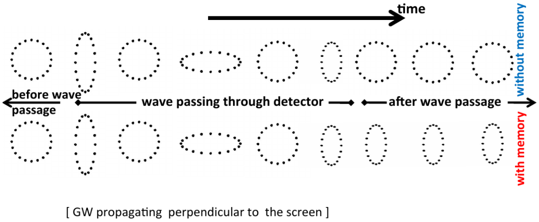


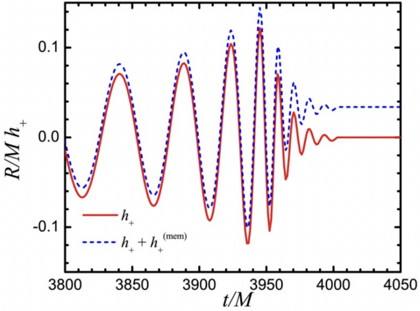
GWs propagating through cosmological distances, induce subsequent GWs, creating successive waves — *GW memory*.

1. Part I - The GW memory in cosmological background can distinguish between cosmological models— constrain cosmological parameters.
2. Part II - Parity violating gravity theory— amplitude difference between h_+ & $h_\times$ gets amplified in their memory in cosmic environment.

What is GW memory?

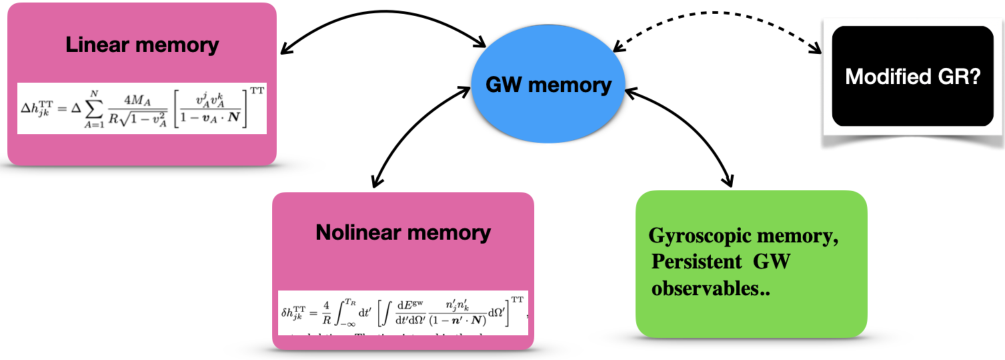
- Permanent distortion in the space-time(state of the detector), retains after passing of GW. (Favata, ICTS Talk)





- $\Delta h^{\rm mem}_{+,\times} = \lim_{t\to+\infty} h_{+,\times}(t) - \lim_{t\to-\infty} h_{+,\times}(t)$

Types of GW memory



Limitations in current approach

- GW memory, $\Delta h^{\rm mem}_{+,\times} \propto 1/r$ in asymptotically flat (AF) spacetime.
- It is well studied in AF spacetimes $\rightarrow$ Bondi Metzner Sachs (BMS) symmetries and charges.
- The analysis is not well established for asymptotically non-flat spacetimes.
- Complete understanding of GW memory requires formulation of GW memory master equation in generic spacetime.

Q. Can we build a formalism to obtain a master equation for GW for a larger class of spacetimes??

A. Using $1+1+2$ *covariant formalism* we develop an analytical technique to develop GW memory.

Understanding properties of gravity, Eulerian(static) to Lagrangian (comoving) observer might be insightful.

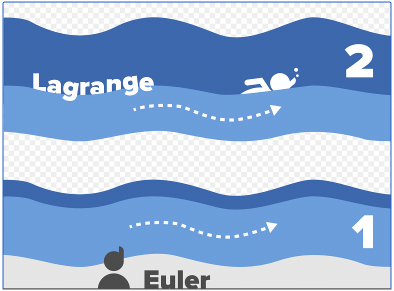
Fluid mechanics: detour

- In fluid mechanics, we can describe the same phenomena using two descriptions — Eulerian or Lagrangian.
- **Eulerian:** Variations are described at all fixed position:

$$\frac{\partial \vec{V}(\vec{x},\, t)}{\partial t} + \left[\vec{V}(\vec{x},\, t)\cdot\vec{\nabla}\right]\vec{V}(\vec{x},\, t) = -\frac{1}{\rho}\vec{\nabla}p(\vec{x},\, t)$$

- **Lagrangian:** Observer follows fluid particles and describes the variations around each fluid particle along its trajectory

$$\frac{\partial \vec{V}(\vec{x}_0,\, t)}{\partial t} = -\frac{1}{\rho}\vec{\nabla}p(\vec{x}_0,\, t)$$



Covariant $1+3$ formalism

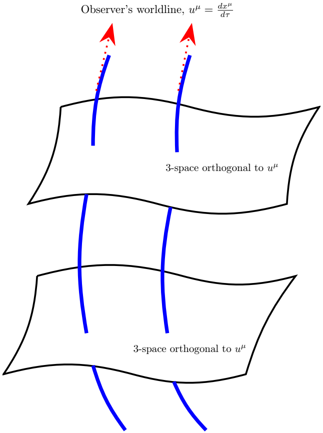


Figure: Visualisation of $1+3$ formalism.

- Timelike 4-velocity vector $u^a = \frac{dx^a}{d\tau}$, satisfying $u^a u_a = -1$.
- Define the following projection tensors:

$$U^a{}_b \equiv -u^a u_b, \text{ projects } \rightarrow u^a.$$

$$h_{ab} \equiv g_{ab} + u_a u_b; \quad h_{ab}\, u^b = 0$$

In 3-space:

$$\nabla_a \longrightarrow D_a + u_a u^b \nabla_b$$

$$\nabla_b u_a = \frac{\Theta}{3} h_{ab} + \sigma_{ab} + \omega_{ab} - \dot{u}_a u_b.$$

- Kinematic properties of space-time are determined by the covariant derivative of the observer's velocity $\rightarrow$ 1) Expansion + 2) Shear + 3) Vorticity.

$$\nabla_b u_a = \frac{\Theta}{3} h_{ab} + \sigma_{ab} + \omega_{ab} - \dot{u}_a u_b, \ \dot{u}^a = u^b \nabla_b u^a.$$

- Raychaudhuri's equation for Θ,

$$\dot{\Theta} = -\frac{\Theta^2}{3} + D_a \dot{u}^a + \dot{u}^a \dot{u}_a + \frac{1}{2}(a + 3p) - 2(\sigma^2 - \omega^2)$$

$\frac{1}{3}\Theta = \frac{\dot{S}}{S}$, where S is average length scale.

$$3\frac{\ddot{S}}{S} = D_a \dot{u}^a + \dot{u}^a \dot{u}_a + \frac{1}{2}(a + 3p) - 2(\sigma^2 - \omega^2)$$

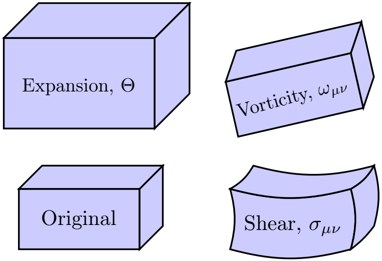


Figure: Kinematic quantities.

$1+1+2$ formalism

- Existence of a symmetric space-like direction, $1+3 \longrightarrow 1+1+2$ form.
- Considering a spacelike vector n^a, 3-space can be split into $1+2$ form:

$$h_{ab}(3) = n_a n_b\,(1) + N_{ab}\,(2); \quad N_{ab}\,n^b = 0$$

- **In** 2-space:

$$D_a \longrightarrow \delta_a + n_a\,(n^b D_b)$$

$$D_b n_a = \zeta_{ab} + \xi\epsilon_{ab} + \frac{1}{2}\phi N_{ab} + n_a a_b,\;\; a_b = \hat{n}_b = n^a D_a n_b$$

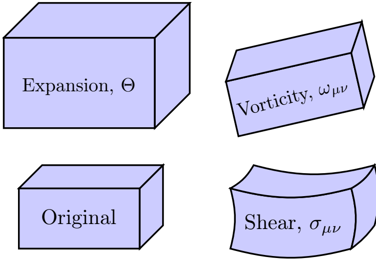


Figure: Kinematic quantities.

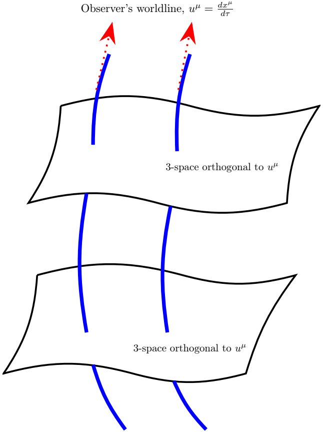
GR in $1+1+2$

- Acceleration of u^a: $\dot{u}^a = \mathcal{A}\, n^a + \mathcal{A}^a$
- Shear: $\sigma_{ab} = \Sigma \left(n_a n_b - \frac{1}{2} N_{ab}\right) + 2\Sigma_{(a} n_{b)} + \Sigma_{ab}$,
- Vorticity: $\omega^a = \Omega\, n^a + \Omega^a$
- Electric part of Weyl: $E_{ab} = \mathcal{E} \left(n_a n_b - \frac{1}{2} N_{ab}\right) + 2\mathcal{E}_{(a} n_{b)} + \mathcal{E}_{ab}$
- Magnetic part of Weyl: $B_{ab} = \mathcal{B} \left(n_a n_b - \frac{1}{2} N_{ab}\right) + 2\mathcal{B}_{(a} n_{b)} + \mathcal{B}_{ab}$
- The shear corresponding to the spacelike vector n^a , ζ_{ab}, the vorticity ξ and acceleration $\dot{n}^a = \mathcal{A}\, u^a + \alpha^a$.

LRS space-time

LRS or *locally rotationally symmetric* space-time:

- One chooses a spatial direction r or x.
- The Weyl tensor is of type D (or zero)
- All kinematic and observable quantities are rotationally symmetric about x.



The *kinematic* variables related to timelike congruence u^a and the spacelike direction n^a and *geometric* quantities:

- Type I:

$$\mathcal{D}_0 = \{\Theta, \Omega, \Sigma, \xi, \mathcal{A}, \mathcal{B}, \phi, \mathcal{E}\}$$

- Type II:

$$\mathcal{D}_0 = \{\Theta, \phi, \mathcal{A}, \Sigma, \mathcal{E}\}$$

Set up

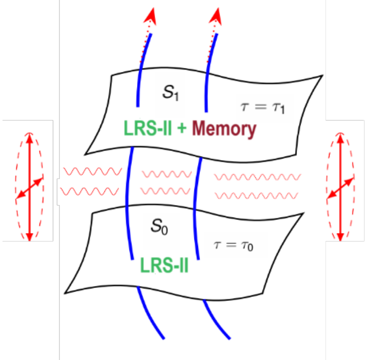


Figure: The presence of GW alters the background LRS type II spactime.

Gravitational wave: LRS-II $\rightarrow$ non-LRS-II

Let's consider a scenario: $g_{ab} = g_{ab}^{\rm LRS} + \epsilon g_{ab}^{\rm GW}$ (very small)

$$\mathcal{D}_0 \rightarrow \mathcal{D}_1 = \{\underbrace{\Theta, \mathcal{A}, \phi, \Sigma, \mathcal{E}}_{\text{LRS}}, \underbrace{\mathcal{B}, \Omega, \xi, \mathcal{A}_a, \Omega_a, \Sigma_a, \alpha_a, a_a, \mathcal{E}_a, \mathcal{B}_a, \Sigma_{ab}, \zeta_{ab}, \mathcal{E}_{ab}, \mathcal{B}_{ab}}_{\text{Non-LRS}}\}$$

As the ground-based GW detector observes GWs in the 2−d plane, our focus $g_{ab} \rightarrow N_{ab}$.

Evolution of N_{ab}: change wrt observer u^a encoded in Lie drag, $\mathcal{L}_u N_{ab}$.

$$\mathcal{X}_{ab} \equiv \mathcal{L}_u N_{ab} = u^c \nabla_c N_{ab} + N_{cb} \nabla_a u^c + N_{ca} \nabla_b u^c$$

In terms of perturbed quantities in $\mathcal{D}_1$:

$$\mathcal{X}_{ab} = 2 n_{(a} \left(-\alpha_{b)} + \Sigma_{b)} - \epsilon_{b)c}\, \Omega^c \right) + 2\, \Sigma_{ab} + N_{ab} \left(\frac{2\Theta}{3} - \Sigma \right)$$

Gravitational wave memory tensor

Demanding the projection tensor N_{ab} to be metric, the Greenberg vector: $\Sigma^a + \epsilon^{ab}\Omega_b - \alpha^a = 0$.

The traceless symmetric part of $\mathcal{X}_{ab}$ is:

$$\mathcal{X}_{\{ab\}} = 2\,\Sigma_{ab} \text{ (transverse \& trace-less, 2 DoF)}$$

We claim $\int dt\,\Sigma_{ab}$ contain information about the change in the spacetime or arm length of the detector:

$$\Delta h^{\mathrm{mem}} = \int_{t1}^{t2} dt\,\Sigma_{ab}$$

Next step: Relate Σ_{ab} with the passing GW through LRS spacetime.

Incoming GW

Incoming GW we represent by:

- Electric part of Weyl: $E_{ab} = \mathcal{E}\left(n_a n_b - \frac{1}{2}N_{ab}\right) + 2\mathcal{E}_{(a}n_{b)} + \mathcal{E}_{ab}$
- Magnetic part of Weyl: $B_{ab} = \mathcal{B}\left(n_a n_b - \frac{1}{2}N_{ab}\right) + 2\mathcal{B}_{(a}n_{b)} + \mathcal{B}_{ab}$

Considering 1st order perturbation to the LRS-II spacetime, evolution and propagation equations of GW components:

$$\mathcal{E}_a = \varepsilon_{ab}\mathcal{B}^b \ ; \ \mathcal{E}_{ab} = \varepsilon_{ad}\mathcal{B}^d{}_b \text{ (Total DoF 2)}$$

Constraints equations,

$$\hat{\mathcal{E}} = -\delta_a\mathcal{E}^a - 3\phi\mathcal{E}/2; \quad \hat{\mathcal{B}} = -\delta_a\mathcal{B}^a - 3\phi\mathcal{B}/2\,.$$

Part I: GW memory Master equation in LRS type II, consistent with GR.

Arrival to master equation

The evolution equation of $\mathcal{E}_{ab}$:

$$\partial_u \mathcal{E}_{\{ab\}} + \partial_u \left(\varepsilon_{c\{a} \mathcal{B}_{b\}}{}^c \right) = -\varepsilon_{c\{a} \delta^c \mathcal{B}_{b\}} - \frac{3\mathcal{E}}{2} \Sigma_{ab} - \frac{3\mathcal{B}}{2} \varepsilon_{c\{a} \zeta_{b\}}{}^c - \left(\Theta + \frac{3\Sigma}{2} \right) \mathcal{E}_{ab}$$
$$+ \left(\frac{\phi}{2} + 2\mathcal{A} \right) \varepsilon_{c\{a} \mathcal{B}_{b\}}{}^c - \tfrac{1}{2}(a+p)\, \Sigma_{ab}$$

Substituting magnetic & electric Weyl relations,

$$3\mathcal{E}\, \Sigma_{ab} + [2\Theta + 3\Sigma + \phi]\, \mathcal{E}_{ab} + [a+p]\, \Sigma_{ab} = -\mathcal{G}_{ab}; \text{ where, } \mathcal{G}_{ab} \equiv 2\, \epsilon_{c\{a} \delta^c \mathcal{B}_{b\}}$$

The Σ_{ab} evolution:

$$\dot{\Sigma}_{\{ab\}} = -\left(2\Theta/3 + \Sigma/2\right) \Sigma_{ab} - \mathcal{E}_{ab}.$$

Master equation combining the above two equations:

$$(2\Theta + 3\Sigma + \phi) \left[\dot{\Sigma}_{\{ab\}} + \left(\tfrac{2}{3}\Theta + \tfrac{1}{2}\Sigma \right) \Sigma_{ab} \right] - [3\mathcal{E} + a + p]\, \Sigma_{ab} = \mathcal{G}_{ab}$$

Consistency check in Minkowski

The master equation in Minkowski background (non-zero $\phi = 2/r$):

$$2\,\dot{\Sigma}_{ab} = r\,\mathcal{G}_{ab} \quad \rightarrow \quad \ddot{N}_{ab} = r\,\mathcal{G}_{ab}\,.$$

Assuming incoming GW profile for the $h_+ \sim h_\times$ [1978 Kovacs & Thorne]:

$$h_+ = \frac{4m_A m_B}{b\,r}\left[\frac{1}{4}\left(\frac{1}{\sqrt{l^2+1}} + \frac{1}{(l^2+1)^{3/2}}\right) + \frac{1}{2}\left(\frac{l}{\sqrt{l^2+1}} + \frac{l}{(l^2+1)^{3/2}} + 1\right) - \frac{1}{4\,(l^2+1)^{3/2}}\right]$$

where $l = \sqrt{1+u^2}$, b is the impact parameter, and r is the distance between the source and the detector.

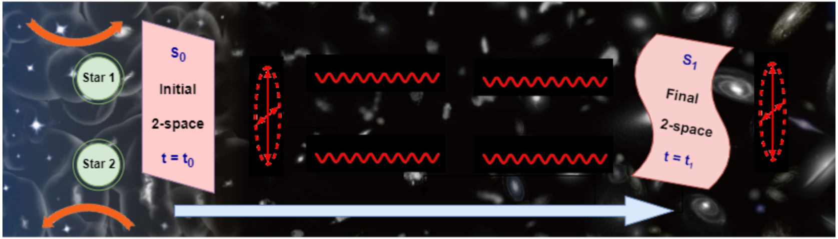
Evaluating $\mathcal{B}_a$ in terms of h_+ GW, [2011 Nichols. et al]:

$$\ddot{N}_{\vartheta\vartheta}(u) = r^2\,\cos\vartheta\,\cos(2\varphi)\,\ddot{h}_{\vartheta\vartheta}(u) \rightarrow \Delta N_{\vartheta\vartheta} \sim r^2 \Delta h_{\vartheta\vartheta} \tag{1}$$

h_+ falls as $\mathcal{O}(1/r)$, we find $\Delta N_{\vartheta\vartheta} \sim \mathcal{O}(r)$ & $\Delta N_{\hat{\vartheta}\hat{\vartheta}} \sim \mathcal{O}(1/r)$ [2017-Flanagan & Nichols]

Key application: Cosmological memory

GW memory in cosmological settings enables us to obtain the correct cosmological model describing the exact expansion rate.



FLRW spacetime

Consider conformally flat FLRW line-element $ds^2 = a^2(\eta)[-d\eta^2 + d\mathrm{x}^2]$.

We obtain, $\Theta = 3\mathcal{H}_0/a, \mathcal{H}_0 = a'/a \equiv d(\ln a)/d\eta$ and $\Sigma = -\Theta/3$, doest not $\sim \mathcal{O}(1/r)$.

The master equation in FLRW:

$$2\,\Theta\,\dot{\Sigma}_{ab} + \Theta^2\,\Sigma_{ab} - 2\,(\mu + p)\,\Sigma_{ab} = 2\,\mathcal{G}_{ab}\,,$$

$(.)\ \to \tilde{u} = \eta - r$ and μ, p refers to the energy density and pressure of the cosmological fluid.

$$\Sigma'_{\vartheta\vartheta} + \Gamma(\eta)\,\Sigma_{\vartheta\vartheta} = \frac{a(\eta)}{3\mathcal{H}_0}\,r\cos\vartheta\cos(2\varphi)\,h''_{\vartheta\vartheta}(\eta),$$

where, $(\prime) \sim \partial_\eta$. In terms of $N_{\vartheta\vartheta}$(memory) above, Eq. reads as:

$$\left[\frac{N_{\vartheta\vartheta}}{a^2}\right]'' + (\mathcal{H}_0 + \Gamma(\eta))\left[\frac{N_{\vartheta\vartheta}}{a^2}\right]' = \frac{2r}{3\mathcal{H}_0}\cos\vartheta\cos(2\varphi)\,h''_{\vartheta\vartheta}$$

Final results

Dimensionless Hubble parameter: $E(z) = \Omega_{\rm m}(1+z)^3 + (1-\Omega_{\rm m})f(z)$, & $f(z) = \exp\left[\int_0^z dz' \frac{1+w(z')}{(1+z')}\right]$.

$$N_+ = h_\oplus \frac{(1+z_0)^{1/2}}{E(z_0)^{1/3}} \int_0^{z_0} dz' \frac{(1+z')^{1/2}}{E(z')^{5/3}}; \quad h_\oplus = \mathscr{A}\, f(\vartheta,\varphi)(1+z)/D_L$$

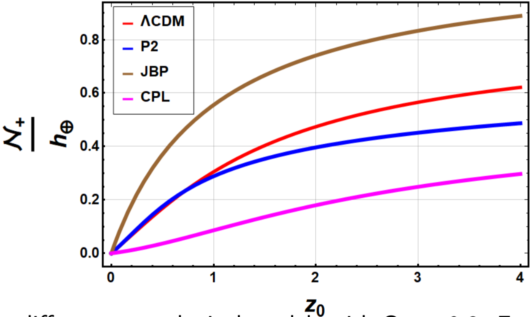
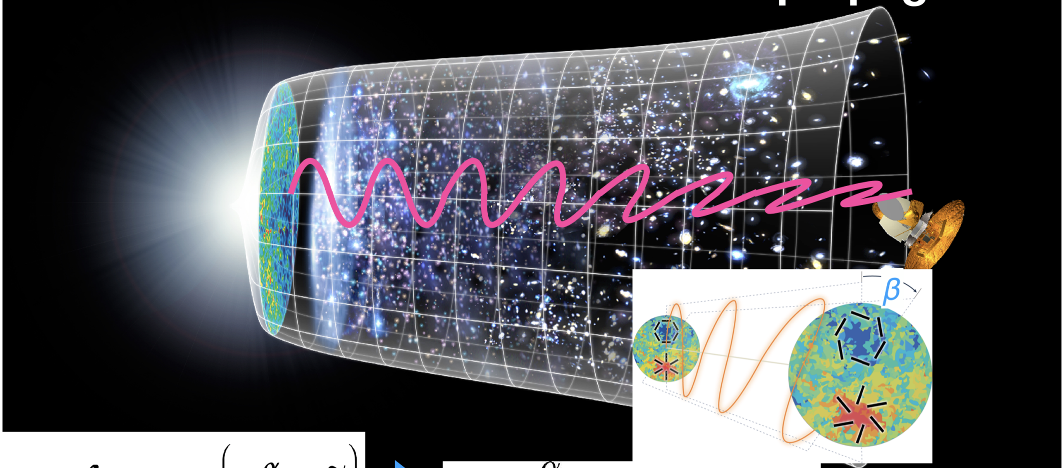


Figure: Integrated memory for different cosmological models with $\Omega_m = 0.3$. For ΛCDM, $N_+/h_\oplus \sim 180$ at $z = 4$ compared to $z = 0.01$. For CPL, JBP and P2 models, it is around 600, 80 and 145 respectively. [Chevallier et al. 2001, Linder 2003, Jassal et al. 2005, Sahni et al. 2003.]

Part II - What happens if we consider a parity violating gravity theory?

How does the EM wave of the CMB propagate?

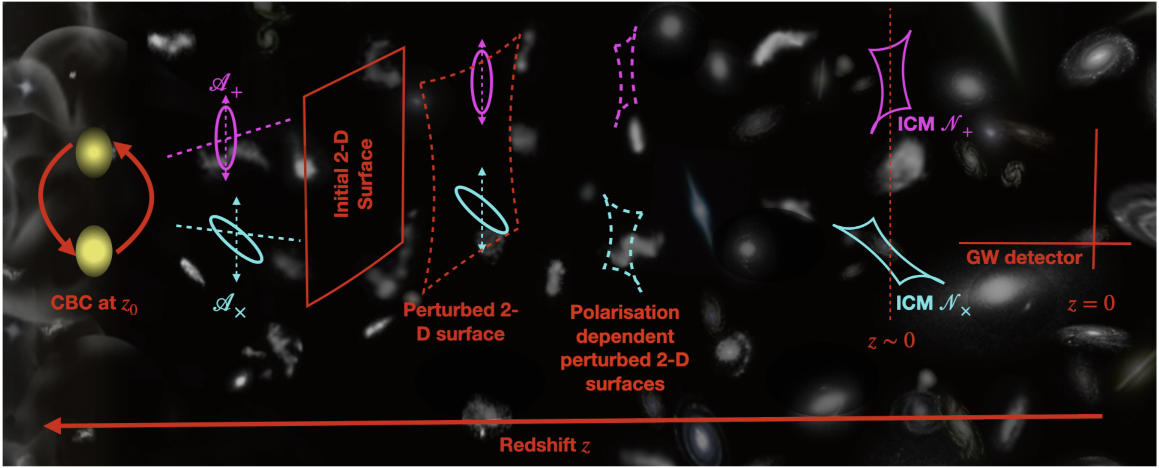


$$I_{\rm CS} = \int d^4x \sqrt{-g}\left(-\frac{\alpha}{4f}\chi F\widetilde{F}\right)$$

$$\beta = +\frac{\alpha}{2f}\left[\chi(\tau_{\rm obs}) - \chi(\tau_{\rm em})\right]$$

[Credit: WMAP Science Team & 2023 Eiichiro Komatsu XXV SIGRAV]

Does GW memory accumulates parity violation?



Parity violating gravity theory

We consider a 4−D dynamical Chern-Simons(dCS) gravity theory:

$$S = \int \mathrm{d}^4x \sqrt{-g} \left(\kappa \, R + \frac{\alpha \, \rho}{4} ({}^*RR) - \frac{1}{2} \, \nabla_a \rho \, \nabla^a \rho + \mathcal{L}_m \right).$$

where, $\kappa = 1/(16\pi G)$, α is the coupling constant, ${}^*RR$ represents the Pontryagin density, ρ is the scalar field, and $\mathcal{L}_m$ perfect fluid Lagrangian present in background.

$$G_{ab} = \frac{1}{2\kappa}(T_{ab} - 2\alpha \, C_{ab}) := \tilde{T}_{ab},$$
$$\Box\rho = -\frac{\alpha}{4} \, {}^*RR.$$

The Cotton tensor as per [2014 Witek. et al]:

$$C_{ab} = 2v^c \, \nabla^d \; {}^*W_{d(ab)c} + v^{dc} \; {}^*W_{d(ab)c}.$$

${}^*W_{d(ab)c}$ is the dual of the Weyl tensor, $v^c := \nabla^c \rho$ is the velocity of the scalar fluid, and $v^{dc} := \nabla^{(d} v^{c)}$.

GW Master equation in dCS theory

Performing the analysis in FLRW background, similar to GR with modified $\tilde{T}_{ab}$ leads to,

$$\frac{\dot{\Theta}}{2}\Sigma_{ab} + \frac{\Theta}{2}\dot{\Sigma}_{ab} - \frac{1}{2}(\mu + p)\Sigma_{ab} = \epsilon_{c\{a}\,\delta^c\mathcal{B}_{b\}} + \dot{\Pi}_{ab}$$

$\mathcal{B}_b$, containing the information of the primary GW, and $\Pi_{ab} \equiv -2\alpha\mathcal{C}_{ab}$ contribution from dCS gravity.

Defining $\Gamma_{ab} := \frac{\Theta}{2}\Sigma_{ab} - \Pi_{ab}$ and massaging the above equation:

$$\dot{\Gamma}_{ab} - \left(\frac{\mu + p}{\Theta}\right)\Gamma_{ab} = \epsilon_{c\{a}\,\delta^c\mathcal{B}_{b\}} + \left(\frac{\mu + p}{\Theta}\right)\Pi_{ab}\,.$$

Master equation for $+$ and $\times$ polarization

$$\frac{d\Gamma_+}{d\eta} + Q(\eta)\Gamma_+ = \frac{1}{r}[h''_+(\eta)F_1(\eta,\vartheta,\varphi) + h''_\times(\eta)F_2(\eta,\vartheta,\varphi)]$$
$$\frac{d\Gamma_\times}{d\eta} + Q(\eta)\Gamma_\times = \frac{1}{r}[h''_+(\eta)F_3(\eta,\vartheta,\varphi) - h''_\times(\eta)F_4(\eta,\vartheta,\varphi)].$$

where,

$$Q(\eta) = \left(\frac{2\mathcal{H}'}{3\mathcal{H}} - \frac{8\mathcal{H}}{3}\right), \qquad F_1 = \cos(2\varphi)f_1 + \sin(2\varphi)f_2, \qquad F_2 = \sin(2\varphi)f_1 - \cos(2\varphi)f_2,$$

$$\mathcal{H} = \frac{a'}{a}, \qquad F_3 = \cos(2\varphi)f_3 - \sin(2\varphi)f_4, \qquad F_4 = \sin(2\varphi)f_3 + \cos(2\varphi)f_4,$$

$$f_1 = 1/2(\cot\vartheta + \cos\vartheta/2), \qquad f_2 = \frac{4\alpha H_0}{3a^4}(\cos\vartheta - \cot\vartheta/2)\left(\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}}\right),$$

$$f_3 = \frac{1}{2}\left(1 - \frac{\sin^3\vartheta}{2}\right), \qquad f_4 = \frac{4\alpha H_0}{3a^4}\left(1 - \frac{\sin^3\vartheta}{2}\right)\left(\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}}\right).$$

Results

We introduce a parameter Δ, defined as:

$$\mathcal{A}_\times = \mathcal{A}_+(1+\Delta)\,.$$

$\mathcal{A}_{+,\times}$ are GW amplitude at source, and $\Delta \sim \alpha^2/L^4$. [2018 Bhattacharyya, Shankaranarayanan]

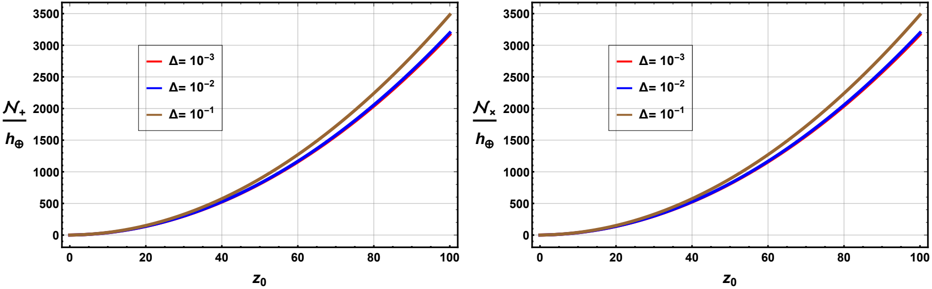


Figure: ICM for different values of Δ for $+$ (left) and $\times$ (right) polarization modes in dCS gravity for ΛCDM cosmology. We have set $\Omega_m = 0.3$. In the left (right) plot, the value of $\varphi = \pi/4$ $(\pi/2)$.

Results

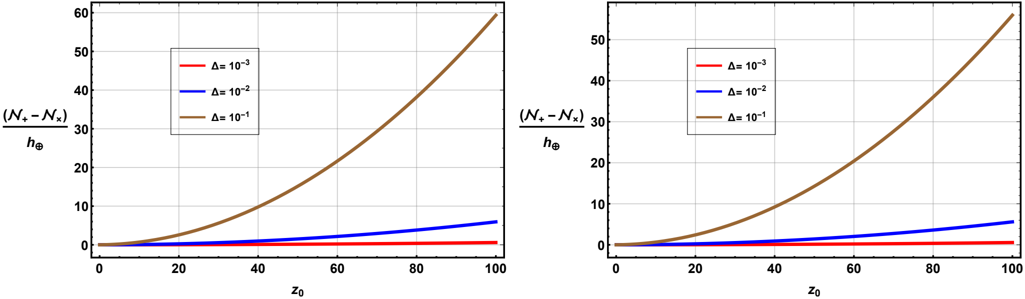


Figure: Difference between the ICM for the two polarizations in dCS gravity for ΛCDM cosmology. In the left (right) plot, the value of $\vartheta = \pi/4\,(\pi/2)$, and $\varphi = \pi/8$, $\Omega_m = 0.3$ in both plots.

Key take away

- In the covariant formalism, the GW is described by electric and magnetic parts of Weyl tensor, projected onto 2−D surface.
- The GW memory $\Delta N_{ab} = \int \; dt\Sigma_{ab}$.
- We obtain master equation for GW memory in LRS type II background.
- Integrated GW memory ratio in cosmological background $\mathcal{N}_+/h_\oplus$ is more for higher redshift — $N_+/h_\oplus \sim 180$ at $z \sim 4$ compared to $z \sim 0.01$.
- For parity violating dCS theory the correction in GW memory appears through cotton tensor C_{ab}, projected onto 2−D surface.
- It lead to polarization dependent master equation for memory corresponding h_+ & $h_\times$.
- The difference in memory ratio $(N_+ - N_\times)/h_\oplus$ differs for separate $\Delta \sim \alpha^2/L^4$.

Thank you