Department of Physics, Shanghai University

Stability of Neutron-Dark Matter Mixed Stars and Hybrid Stars

Kilar Zhang
kilar@shu.edu.cn

Dec 27, 2025

Outline

- We live in the multi-messenger era (GWs, EM, neutrinos)
- Compact stars - extreme cosmic labs
- A fundamental mystery: What is DM?

Can we **probe Dark Matter (DM)** inside compact stars?

Figure: LIGO

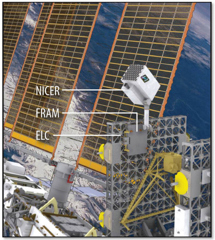


Figure: NICER

Figure: Hubble Image

How do **multi-fluid systems (e.g., DM+NM)** maintain equilibrium under extreme gravity?

Its **stability**?

Two Scenarios:

- Hybrid Stars:
  Strong non-gravitational interactions -> Phase separation -> Layered.
- Mixed Stars:
  Gravitational-only interactions -> Mixed core

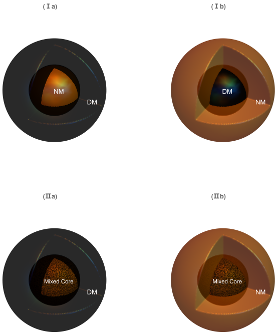


Figure: Star Model Setup

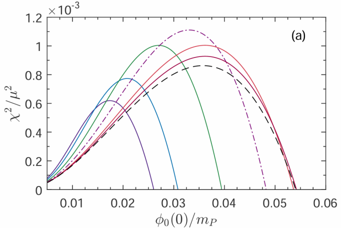
Dynamic Criterion (Standard)

Solve pulsation equations

criterion:
Sign of squared fundamental frequency ω_0^2

Rigorous **but** computationally expensive

[Chandrasekhar 1964, Kain 2020]

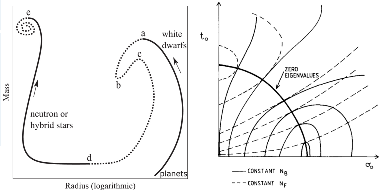


[Kain 2021]

Static Criterion (Efficient)

Analyze equilibrium solutions only

Key insight:
At marginal stability, conserved quantities can reach their extreme values simultaneously



[Alford 2017, Henriques et al. 1990]

Fast **but** the rigor has not been fully proven

[Henriques et al. 1990]

Are they equivalent?

Step 1 $\omega_n^2 = 0 \iff \nabla M \parallel \nabla N_1 \parallel \nabla N_2$

- zero-frequency mode
- infinitely slow transition between adjacent configurations
- modes preserve M and N_I

(with flow $J^\mu_{N_I} = n_I u^\mu, J^\mu_{M_I} = T_I^{\mu 0}$ and charge $Q_{N_I} = N_I, Q_{M_I} = M_I$)

- M and N_I both attain their extremal along certain direction

Theorem 1 Parallel Condition

For a two-fluid equilibrium configuration, the three gradients, ∇M, ∇N_1, and ∇N_2, are mutually parallel if and only if there exists at least a zero mode respect to radial purturbation ($\exists\, n \in \mathbb{N}$ s.t. $\omega_n^2 = 0$).

Step 2 *Reduce the condition to: two of the three gradients ∇M, ∇N_1 and ∇N_2 being parallel*

Theorem 2 Variational Principle

A two-fluid isentropic mixed star configuration satisfies the equilibrium equations if and only if the total mass M takes its extremal with respect to all time-independent variations that conserve the particle numbers of both fluids.

- See [Zhou et.al 2025] for detailed proof process

$\Rightarrow$ The critical curve ($\nabla M \parallel \nabla N_I$) exactly identifies the dynamical stability boundary ($\omega_0^2 = 0$)

Impact: Validates the static method for rigorous stability mapping
Efficiency: Static method (~1 hour) vs. Dynamic method (~1 week)

Numerical Validation:
Eigenfrequency vs. Critical Curve

Test Case: Bosonic DM ($B_4 = 0.1$) + Holographic NM.

Static Method

- *Compute normalized cross product:* $\mathcal{C}_{M,N_I} = \hat{u}_{\nabla M} \times \hat{u}_{\nabla N_I} \equiv \frac{\nabla M}{|\nabla M|} \times \frac{\nabla N_I}{|\nabla N_I|}$

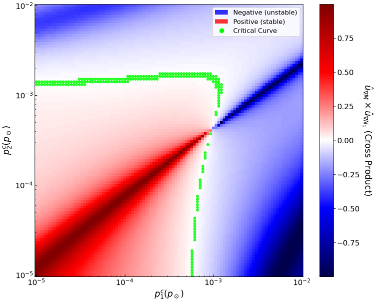


Figure: Static method result

Dynamic Method

- Distinguish the sign of squared fundamental frequency ω_0^2

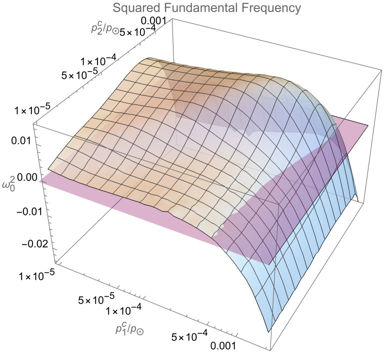


Figure: Dynamic method result

Static Method

- *Dividing Line between Pink and Green:* $|\mathcal{C}| \to 0$ = *critical curve*

Dynamic Method

- *Black Contour:* $\omega_0^2 \to 0$ = *stability marginal*

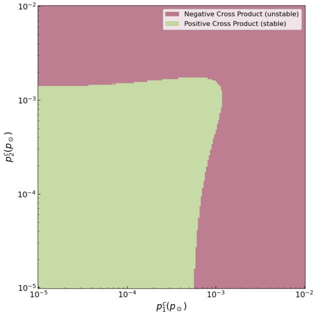


Figure: Static method result

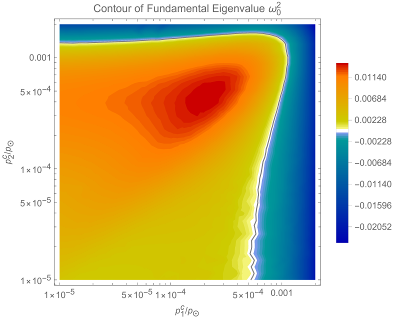


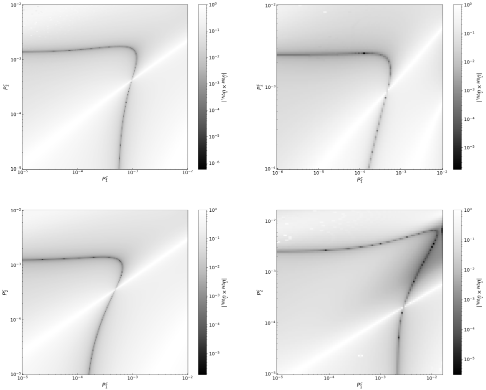
Figure: Dynamic method result

Result: Perfect overlap confirms equivalence!

Stability Boundaries
For Diverse EoS Combinations

EoS Combinations Tested:

1. Holographic NM + Bosonic DM
2. SLy4 (NM) + Bosonic DM
3. Holographic NM + Fermionic DM
4. SLy4 (NM) + Fermionic DM



Key Findings:

- The shape of the stability boundary is highly sensitive to the EoS combination and its parameters.
- Regions exist where both central pressures exceed their single-fluid critical values, yet the coupled system remains stable. Gravitational coupling enables new stable phases.

Does the critical curve only mark the vanishing of the fundamental mode?

Extended Parameter Scan:

Reveals **multiple nested critical curves**

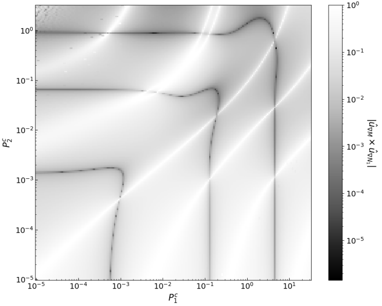


Figure: Multiple critical curves for higher-order modes

Interpretation:

- Innermost curve: $\omega_0^2 = 0$ (fundamental mode becomes unstable)
- Outer curves: $\omega_1^2 = 0$, $\omega_2^2 = 0$, ... (higher radial modes become unstable)
- Moving outward across each curve **gains one unstable mode**

white lines?

Single-Fluid Case

- BTM criterion: M-R curve extrema $\rightarrow$ stability change.

Multi-Fluid Mixed Stars

- Last stable point $\neq$ maximum mass point
  $\dashrightarrow$ **BTM breaks down.**

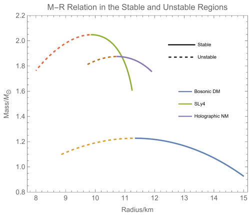


Figure: M-R relation for single fluid

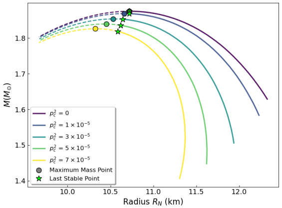


Figure: M-R_N curve: fixed p_1^c, varying p_2^c

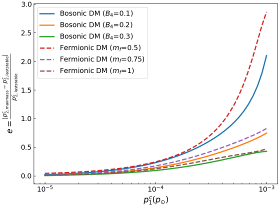


Figure: Relative separation: $p_{\text{last-stable}}$ vs. $p_{\text{max-mass}}$ as function of DM properties.

Resolving Degeneracies
The 3D Topological View

Problem: degeneracy in M-R diagram.

Multiple $(p^c_{\mathrm{DM}}, p^c_{\mathrm{NM}})$ can map to the same M - R region.

Solution: Lift to 3D Space

The stable region in p^c_1, p^c_2 space maps to a **2D surface** in M, R_t, p^c_I space.

Key Result & Benefit

The stable group unfolds into a **non-overlapping 2D surface**.
Clearly separates distinct solution branches.

Can identify potential **"Twin Stars"** (same M, R, different interior).

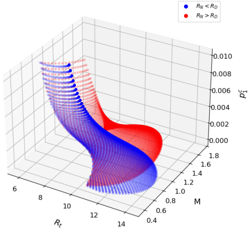


Figure: 3D surface: M-R_t-p^c_1 (DM)

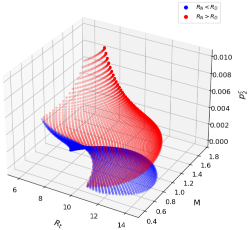


Figure: 3D surface: M-R_t-p^c_2 (NM)

Red: $R_N > R_D$, Blue: $R_N < R_D$

Mapping the stable region in p_1^c-p_2^c space onto the M-R plane yields **Stable Group**:

$$\mathcal{I}_{\text{stable}} = \{(M, R) \mid \exists \text{ stable } (p^c_{\text{DM}}, p^c_{\text{NM}})\}$$

Gravitational Wave (GW) observations

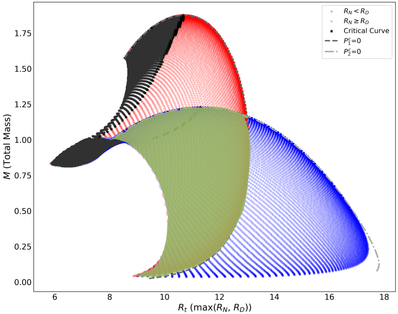


Figure: Stable group in M-R_t plane

Electromagnetic (EM) observations

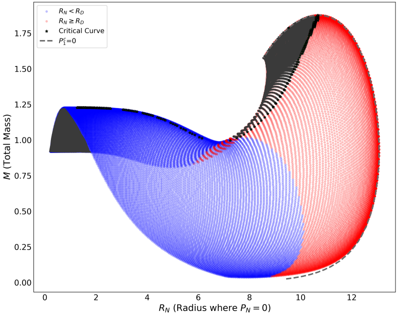


Figure: Stable group in M-R_N plane

Red points: $R_N > R_D$ Blue points: $R_N < R_D$

Twin Stars: Overlapping red and blue regions

Parameter combinations	$M_{\max}$ ($M_\odot$)	R_N (km)	R_D (km)	M/R_N	M/R_D	p_2^c ($p_\odot$)
B=0.0012, ratio=0.1	100.98	27.200	870.02	3.71	0.116	1.0481×10^{-6}
B=0.0022, ratio=0.1	55.340	15.254	495.75	3.63	0.112	3.5565×10^{-6}
B=0.0012, ratio=0.9	100.97	44.668	878.50	2.26	0.115	1.0000×10^{-7}
B=0.0022, ratio=0.9	55.434	29.046	494.51	1.91	0.112	3.3932×10^{-7}

Note: *ratio* $\equiv p^c_{\rm DM}/p^c_{\rm NM}$, the central pressure ratio of DM and NM.

Prediction prospects

Varying DM EoS and mixture ratio allows the **stable group** to span a wide M-R_N space:

Normal NS
(e.g., NICER targets)

Mass-Gap Objects
(3–$5M_\odot$, 50–$100M_\odot$)

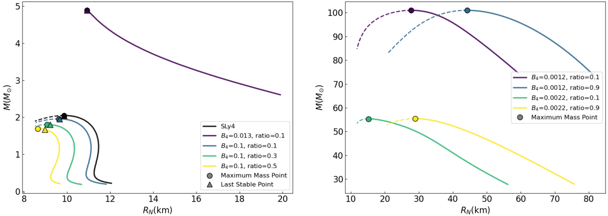


Figure: Prediction prospects for mixed star models displayed through the M-R plane.

The results also show the situations of **Massive Dark Objects** with the observable radius of conventional compact stars accompanied by a large dark halo.

Core Contributions

- **Equivalence Proven**: Rigorously established the equivalence between the dynamic ($\omega_0^2 = 0$) and static ($\nabla M \parallel \nabla N_I$) stability criteria.
- **Stability Mapped**: Conducted comprehensive stability analysis for diverse EoS combinations (SLy4, Holographic NM, Fermionic/Bosonic DM).
- **Astrophysical Framework**: Projected stable configurations to reveal **"Stable Groups"** on the *M*-*R* plane, identifying potential **"Twin Stars"**.
- **Predictive Power**: The model can explain regular NS, mass-gap events ($3 - 5M_\odot$, $50 - 100M_\odot$), and predict massive dark objects.

Future Outlook

- **Theory**: Extend framework to more sophisticated DM (e.g., self-interacting fermionic DM, axion-like particles) and include rotation/magnetic fields.
- **Synergy**: Our methodology provides a reference/tool for precision multi-messenger astronomy to jointly constrain the NS EoS and DM properties.

Thank You!

Questions & Discussion

kilar@shu.edu.cn

Shanghai University

Department of Physics, Shanghai University