Type IIB Supergravity Action
Holography Beyond the $AdS_5 \times S_5$ Solution

Soumya Adhikari

Sogang University
Center of Quantum SpaceTime (CQUeST)

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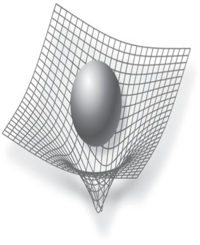
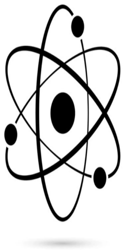
Outline of This Talk

1. Introduction
2. Motivation — Why This Problem Matters.
3. Type IIB Supergravity.
4. Pasti–Sorokin–Tonin (PST) Action.
5. Duality Symmetric Formalism.
6. On-Shell Action on $\mathrm{AdS}_5 \times S^5$ and Holography
7. Lunin-Maldacena Solution: S^5-deformation and non-trivial 2-forms.
8. Comment on $\mathrm{AdS}_4 \times M_6$ and $\mathrm{AdS}_6 \times M_4$
9. Discussion

Introduction

- The two main pillars of theoretical physics :





- Over the last century, the frontier of physics has been dedicated to unifying these domains: Quest for quantum gravity

- Superstring theory is one of the potential candidates in the pursuit of quantum gravity.
- The fundamental building blocks of this theory are strings.
- Their vibrational modes correspond to different particles.
- The graviton, the gravity mediator, and its supersymmetric partner, the gravitino, are examples of massless particles in the superstring theory spectrum.

- The spectrum also features a tower of massive Planck-scale particles.
- However, these massive particles are unlikely to be detected within the near future.
- The low-energy limit: An effective field theory: **Supergravity**
- In this talk, we will focus exclusively on Type IIB supergravity.

Motivation — Why This Problem Matters

Type IIB String Theory

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Type IIB Supergravity In 10D

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$\mathcal{N} = 4$ **Supergravity In 5D dual to** $\mathcal{N} = 4$ **SYM**

↓

Action$^{10D}_{AdS_5 \times S_5} = 0$; **Action**$^{5D}_{AdS_5} \neq 0 =$ **Holography result**.

↓

Improved Action[1]

Contains an additional 5-form dependent “topological” term.

↓

Now, the 10D On-Shell matches with 5D and become consistent with holography

[1] hep-th/2206.14522 , S.A. Kurlyand, A.A. Tseytlin

Motivation — Why This Problem Matters

- The original work did not consider a general treatment for the problem.
- We seek an improved action that resolves this mismatch for **more general AdS backgrounds**.
- It is crucial and indispensable that the holography must work for backgrounds beyond $AdS_5 \times S_5$.

Core Idea

Add a suitable **topological term** to the PST action to restore agreement between 10D and lower-dimensional supergravity action echoing the holographic descriptions.

Type IIB Supergravity

- **Type IIB theory bosonic field content:** $(g_{\mu\nu}, B_2, \phi, C_0, C_2, C_4)$

$$H_3 = dB_2\,,\, F_{n+1} = dC_n\,,\, \widetilde{F}_3 = F_3 - C_0 H_3\,,$$
$$\widetilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3\,.$$

The 'pseudo' Euclidean Type IIB action for the bosonic sector:

$$\begin{aligned} S_{\text{IIB}} = &-\frac{1}{2\kappa^2} \int \left(*\,R - \frac{1}{2} d\phi \wedge * d\phi - \frac{1}{2} e^{-\phi} H_3 \wedge * H_3 \right) \\ &+ \frac{1}{4\kappa^2} \int \left(e^{2\phi} F_1 \wedge * F_1 + e^{\phi} \widetilde{F}_3 \wedge * \widetilde{F}_3 + \frac{1}{2} \widetilde{F}_5 \wedge * \widetilde{F}_5 \right) \\ &+ \frac{i}{4\kappa^2} \int C_4 \wedge H_3 \wedge F_3\,. \end{aligned} \quad (1)$$

Type IIB Supergravity

- The **self-duality** of 5-form field strength $\widetilde{F}_5$: $\widetilde{F}_5 = -i * \widetilde{F}_5$
  1. Supersymmetry closure and Bosonic d.o.f.=Fermionic d.o.f..
  2. String origin.
- The self-duality **cannot** be derived from a manifestly **Lorentz-covariant** action.
- The self-duality has to be **imposed 'by hand'** in the field equations.

Pasti–Sorokin–Tonin (PST) Action

- In the PST formulation, the self-duality condition of the five-form field strength is obtained directly from an action principle.
- This is achieved by introducing an auxiliary scalar field $a(x)$.
- An additional gauge invariance is imposed.
- **Pasti–Sorokin–Tonin (PST)[2] Action:**

$$S_{\rm IIB}^{(\rm PST)} = -\frac{1}{2\kappa^2}\int\Big(*R - \frac{1}{2}d\phi\wedge *d\phi - \frac{1}{2}e^{-\phi}H_3\wedge *H_3\Big)$$

$$+\frac{1}{4\kappa^2}\int\Big(e^{2\phi}F_1\wedge *F_1 + e^{\phi}\widetilde{F}_3\wedge *\widetilde{F}_3 + \frac{1}{2}\widetilde{F}_5\wedge *\widetilde{F}_5$$

$$+\frac{1}{2}\mathfrak{i}_v\mathcal{F}_5\wedge *\mathfrak{i}_v\mathcal{F}_5\Big) + \frac{i}{4\kappa^2}\int C_4\wedge H_3\wedge F_3\,, \qquad (2)$$

Where, $\mathcal{F}_5 \equiv \widetilde{F}_5 + i*\widetilde{F}_5$, $v_\mu \equiv \frac{1}{\sqrt{-\partial^\mu a\partial_\mu a}}\partial_\mu a\,.$

[2] hep-th/9611100 , P. Pasti, D. Sorokin, M. Tonin

- Gauge invariance:
  $\delta_\ell a = 0 \,, \delta_\ell C_4 = a\ell_4 \,, (d\ell_4 = 0)$ Upto a boundary.
- Using the above gauge invariance, one can derive the self-duality from the equations of motions.

Duality Symmetric Formalism

1. This formulation has a and $Q_5 = dR_4$ auxiliary fields.
2. Obtains the self-duality of the combination $\widetilde{F}_5 + aQ_5$ without using a gauge invariance.
3. For background without boundary, gauge invariance plays the crucial role to give back the old set of fields without any hazards.
4. This formulation has the boundary term, $F_5 \wedge aQ_5$, from the beginning
5. No straightforward way to partition the physical self-dual field strength.
6. From the equations of motions, one cannot fix F and aQ separately.
7. F and aQ come in a similar fashion in the action except $F \wedge aQ$.
8. There is no prescription to partition the self-dual five-form field strength for cases beyond $AdS_5 \times X_5$.

On-Shell Action on $AdS_5 \times S^5$ and Holography

- **The $\mathrm{EAdS}_5 \times S^5$ vacuum background:**

$$ds^2_{10} = (ds^2_{\mathrm{EAdS}_5} + ds^2_{S^5})\,, \widetilde{F}_5 = F_5 = -4(i\epsilon_5 + *\epsilon_5)\,,$$
$$C_0, \phi = \mathrm{const}\,, B_2 = C_2 = 0\,, \tag{3}$$

- **Euclidean PST-IIB action** $\xrightarrow{\mathbf{EAdS}_5 \times S^5 \textbf{ Field configuration}} 0$
- **Contradiction with Low-Dimensional Supergravity:**

$$S_5 = -\frac{1}{2\kappa_5^2}\int d^5x\,\sqrt{g}\left(R_5 + 12L^{-2} + \dots\right) \tag{4}$$

- **Evaluating this action on the AdS_5 vacuum solution gives a non-zero result.**

$$S_5 = \frac{8}{2\kappa_5^2}\mathrm{Vol}(\mathrm{AdS}_5) \tag{5}$$

- **Gives correct holography result.**

AdS/CFT Correspondence

$$\text{Type IIB string theory on } AdS_5 \times S^5 \quad \leftrightarrow \quad \mathcal{N} = 4 \; SU(N) \text{ SYM on } \partial AdS_5$$

- The AdS/CFT Dictionary: $Z_{\text{grav}}[g^{(0)}] = Z_{\text{CFT}}[g^{(0)}]$,
  - Z_{grav} is the **gravitational partition function**,
  - Z_{CFT} is the **CFT partition function** on the boundary metric $g^{(0)}$.
- $Z_{\text{grav}}(\text{Classical (supergravity) limit}) \sim e^{-S_{\text{on-shell}}}$.
- The free-energy: $F_{\text{CFT}} = -\ln Z_{\text{CFT}} = S^{(\text{ren})}_{\text{on-shell}}$.
- The 10D on-shell action on $\text{AdS}_5 \times \text{S}_5$ is in contradiction with the AdS/CFT result.
- The 10d action, especially its on-shell value, should arise from a well-defined quantum string path integral, making the 10d approach essential for going beyond leading order in α' within AdS/CFT.

Topological Term As An Improvement To PST Action

- A natural way to resolve this problem is to assume that the 10d action is missing some "boundary term" that restores the equivalence of its on-shell value with that of the 5d action.
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    - The boundary term is also required for gauge invariance in the PST formulation on $M_5 \times X_5$ backgrounds.[3]
    - Construct a self-dual (electric + magnetic) IIB action that includes the boundary term from the start.[4]
    - Derive and justify the same boundary term using Sen's string field theory approach to IIB supergravity.[5]

[3] hep-th/2206.14522 , S.A. Kurlyand, A.A. Tseytlin

[4] hep-th/2207.00626 , K. Mkrtchyan, F. Valach

[5] hep-th/2211.02345 , S. Chakrabarti, D. Gupta, A. Manna

$M_5 \times X_5$ Background

$M_{10} = M_5 \times X_5$, where X_5 is compact without boundary and M_5 may be non-compact. The 4-form potential and field strength decompose as:

$$C_4 = C_{4M} \oplus C_{4X}, \qquad F_5 = F_{5M} \oplus F_{5X}, \qquad \delta_\xi C_4 = \delta_\xi C_{4M} \oplus \delta_\xi C_{4X}. \quad (6)$$

The variation of the Euclidean PST-IIB action:

$$\begin{aligned} \delta_\ell S^{(\mathrm{PST})}_{\mathrm{IIB}} &= \frac{i}{4\kappa^2} \int d(\widetilde{F}_5 \wedge \delta_\ell C_4) = -\frac{i}{4\kappa^2} \int F_5 \wedge d\delta_\ell C_4 \\ &= \frac{i}{4\kappa^2} \int_{\partial M^5} a\ell_4 \int_{X^5} F_{5X}. \end{aligned} \quad (7)$$

Gauge invariance is restored by adding the topological term

$$S_{\mathrm{top}} = -\frac{i}{4\kappa^2} \int F_{5M} \wedge F_{5X}\,. \quad (8)$$

whose variation cancels $\delta_\xi S_{\mathrm{PST}}$ and which reduces to a boundary integral, leaving the equations of motion unchanged.

We evaluate the on-shell action of the $\mathrm{EAdS}_5 \times S^5$ vacuum (3) using the improved action to confirm that it yields the value expected from holography. Substituting the field configuration (3) into the improved Euclidean PST-IIB action $S_{\mathrm{IIB}}^{(\mathrm{PST}+)}$, we only need to calculate the non-vani shing topological term:

$$\begin{aligned} S_{\mathrm{top}} &= -\frac{i}{4\kappa_{10D}^2}\int F_{5\mathrm{EAdS}_5} \wedge F_{5S^5} \\ &= \frac{4}{\kappa_{10D}^2}\int_{\mathrm{EAdS}_5} \epsilon_5 \int_{S^5} *\epsilon_5 = \frac{4}{\kappa_{10D}^2}\mathrm{vol}(\mathrm{EAdS}_5)\mathrm{vol}(S^5) \end{aligned} \qquad (9)$$

- This approach has two limitations.
  1. The improvement term is said to be background dependent even in $M^5 \times X^5$ types of solutions. We do not know what the improvements should be for solutions with non-trivial 2-forms.
  2. There is no straightforward generalizations to discuss beyond $M^5 \times X^5$.

Topological Term: $AdS_n \times X^{10-n}$ For $n \leq 5$

$$\delta S_{\text{IIB}}^{(\text{PST})} = \frac{i}{4\kappa_{10D}^2} \int \left[d\left(F_5 \wedge \delta C_4\right) + d\left(X_5 \wedge \delta C_4\right) \right] \tag{10}$$

We add topological terms to the PST action:

$$S_{\text{top}} = -\frac{i}{4\kappa_{10D}^2} \int \left(S_{\text{top}}^{(C_4)} + d\left(X_5 \wedge C_4\right) \right) \tag{11}$$

Assumption:

- The k-th de-Rahm Cohomology groups for the internal manifold: $H^5 \neq 0$ but others are zero.

We write: $F_5 = F_{5E} + F_{5NE}(X^{(10-n)})$ & $\delta F_5 =$ A globally exact form.
We **propose** the following topological term:

$$S_{\text{top}}^{(C_4)} = -\frac{i}{4\kappa_{10D}^2} \int F_{5E} \wedge F_{5NE} \,. \tag{12}$$

The variation of the above cancels the first term in (10).

Key Points:

- Inside F_{5E} we cannot have a global exact form that is a function of the internal manifold.
- For the simple case of $AdS_5 \times X^5$, F_{5E} and F_{5NE} have only contribution from AdS_5 and X^5, respectively.

However, we need the know:

- **How do we explicitly construct these two parts from the F_5?**
- **Does it give correct on-shell action?**

Lunin-Maldacena Solution: S^5-deformation and non-trivial 2-forms

The Lunin-Maldacena (LM) background [6]:

$$ds^2 = G^{-1/4}\left[ds^2_{EAdS_5} + \Sigma_i(d\mu_i^2 + G\mu_i^2 d\phi_i^2) + 9(\gamma^2+\sigma^2)G\mu_1^2\mu_2^2\mu_3^2 d\psi^2\right],$$

$$e^{-\phi} = G^{-1/2}H^{-1},\ \chi = \gamma\sigma g_{0,E}H^{-1},$$

$$B_2 = \ \gamma G w_2 - 12\sigma w_1 \wedge d\psi,\ C_2 = \ -\sigma G w_2 - 12\gamma w_1 \wedge d\psi,$$

$$\widetilde{F}_5 = -4(i\omega_{EAdS_5} + G dw_1 \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3),$$

where

$$G^{-1} = 1 + (\gamma^2+\sigma^2)g_{0,E},\ H = 1 + \sigma^2 g_{0,E},\ g_{0,E} = \mu_1^2\mu_2^2 + \mu_2^2\mu_3^2 + \mu_3^2\mu_1^2,$$

$$dw_1 = \mu_1\mu_2\mu_3 *_2 1,\ w_2 = \mu_1^2\mu_2^2 d\phi_1\wedge d\phi_2 + \mu_2^2\mu_3^2 d\phi_2 \wedge d\phi_3 + \mu_3^2\mu_1^2 d\phi_3 \wedge d\phi_1.$$

[6] hep-th/2206.14522 , O. Lunin, J. Maldacena

On-Shell Action For LM Solutions

- The PST action for Lunin-Maldacena is not zero but gives the following boundary term.

$$\frac{i}{4\kappa^2}\int F_5 \wedge X_5. \tag{13}$$

- Adding the proposed improvement term:

$$S_{IIB}^{\text{On-Shell}} = -\frac{i}{4\kappa^2}\int F_{5E} \wedge F_{5NE}\,. \tag{14}$$

- For the Lunin-Maldacena Solution:

$$F_5 = \widetilde{F}_5 + \tfrac{1}{2}C_2 \wedge H_3 - \tfrac{1}{2}B_2 \wedge F_3 = -4(i\omega_{AdS_5} + dw_1 \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3)$$

$$S_{IIB}^{\text{On-Shell}} = \frac{4}{\kappa_{10}^2}\text{Vol}_{EAdS_5}\int_{\tilde{S}_5}\mu_1\mu_2\mu_3 d^5\theta\,. \tag{15}$$

The deformation parameter does not play any role in the on-shell action for the Lunin-Maldacena solution.

Low-Dim Supergravity Action

We are interested in reducing the gravitational part of the type IIB action on the internal space $\tilde{S}^5$, so we write

$$\begin{aligned} S_{10} &= -\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{G_{10}}\,[R_{10}+\ldots] \\ S_5 &= -\frac{A}{2\kappa_{10}^2}\int d^5x\sqrt{g_5}\,[R_5+12+\ldots]\,. \end{aligned} \tag{16}$$

We see that the factor A in equation (16) is given by:

$$A = \int d^5\theta(\mu_1\mu_2\mu_3). \tag{17}$$

Going on-shell on $EAdS_5$:

$$S_5^{\text{On-Shell}} = \frac{4A}{2\kappa_{10}^2}\mathrm{Vol}(\mathrm{EAdS}_5) \tag{18}$$

Schematic Picture In $\mathrm{AdS}_4 \times S_1 \times S_1 \times M_4$

- We take a simple case of $\mathrm{AdS}_4 \times S_1 \times S_1 \times M_4$ with the following metric asatz: $ds_{10}^2 = \Delta\left(ds_4^2 + d\phi^2 + d\psi^2\right) + d\tilde{s}_4^2$
- For the above case, the determinant of the ten-dimensional vielbein: $e_{10} = \Delta^3 e_4 \tilde{e}_4$
- Low-dimensional on-shell action: $\propto \mathrm{Vol}_{\mathrm{AdS}_4} \int \Delta^2 \tilde{e}_4 d^6 y$.
- Now, we take a simplified ansatz for the self-dual field strength (we take the two-forms to be zero):
  $F_5 = \left(a(\Delta) e^4 \wedge e^8 + b\tilde{e}^4 \wedge e^9\right) \equiv F_E + F_M$, where b is a constant.
- The self-duality can be implemented by:
  $* \left(a(y) e^4 \wedge e^8\right) = \left(b\tilde{e}^4 \wedge e^9\right)$.
- The above gives: $a(y) = \Delta^2 b$.
- The topological term: $\propto F_E \wedge F_M \propto \mathrm{Vol}_{\mathrm{AdS}_4} \int \Delta^2 \tilde{e}_4 d^6 y$.

Comments on $\mathrm{AdS}_6 \times M_4$

- The proposal of (12) does not work for $n > 5$, as all closed five-forms will be exact forms globally.
- However, the global exact forms will be tensor product products of AdS_n global exact forms and non-exact forms from the internal manifolds.

We leave this as a future investigation.

Discussion

- We want to test our methods by explicitly working out the S-fold solution of the type IIB background where the 10d metric of the form $EAdS_4 \times S^2 \times S^2 \times \Sigma_2$, $EAdS_4 \times S^2 \times S^2 \times S_1 \times S_1$, and $EAdS_4 \times S^1 \times S^5$.
- Another non-trivial type IIB background of our interest is the $EAdS_6 \times S_2 \times \Sigma_2$ as the 10D solution of the type IIB background.
- How the string field theory motivated approach is related to the approaches discussed here.
- We have a parallel issue with the EAdS4 $\times$ S7 on-shell action of 11d supergravity, but do not have a good enough resolution unlike the IIB supergravity. ***Work In Progress***


감 사 합 니 다
Thank you.