Gravitational Wave signals of FOPT in SO(10)

Based on

Gravitational Wave Signals in a Promising Realization of SO(10) Unification

(arXiv:2506.07182)

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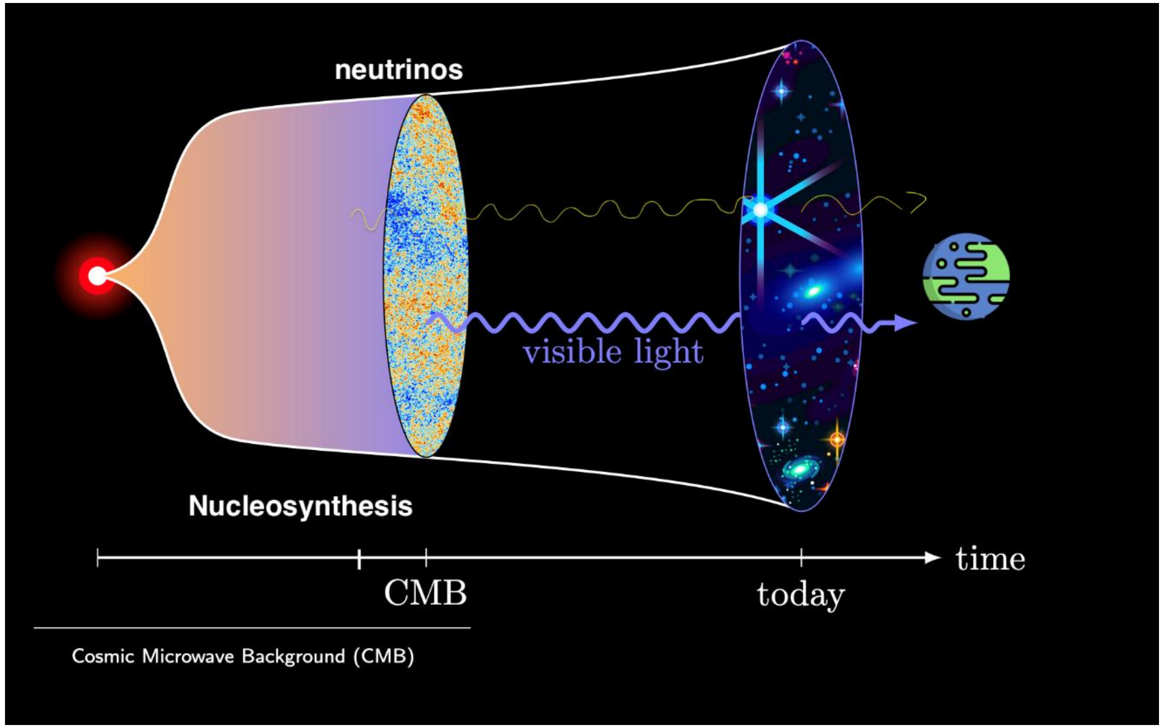
SASGAC 2025

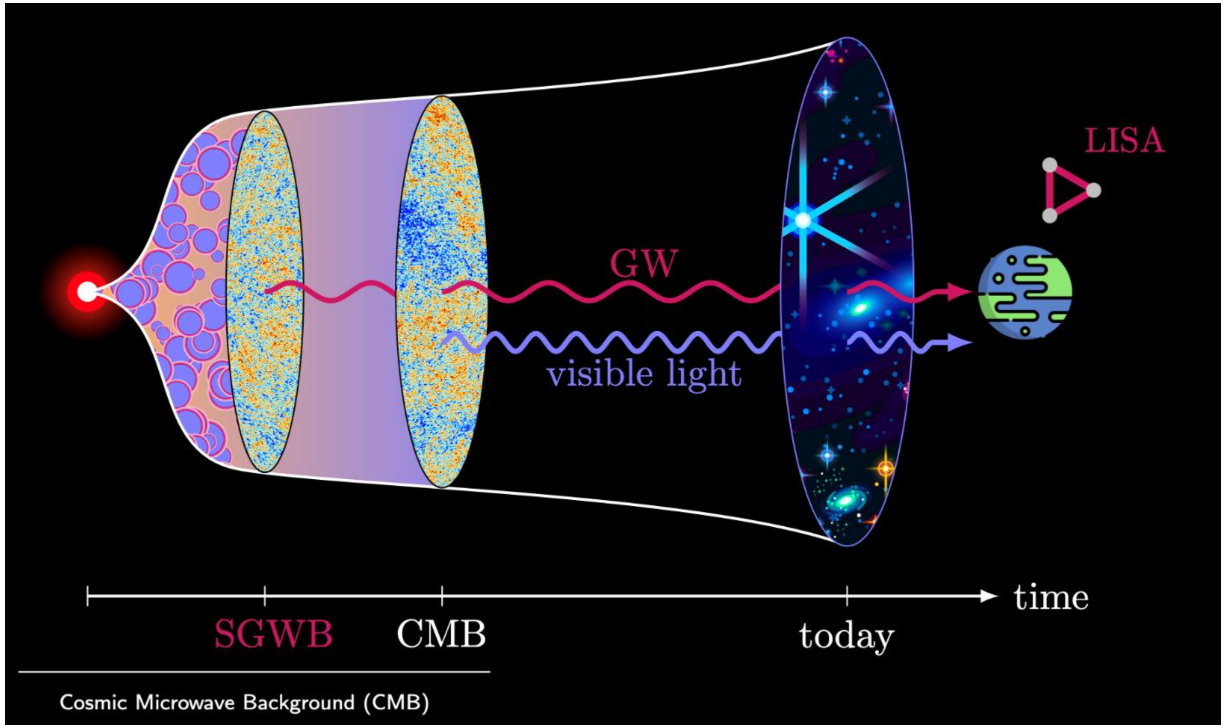
Center for Quantum SpaceTime

Introduction

- To explore beyond the Standard Model (BSM) in the early Universe

More than photons and neutrinos is needed





Introduction

- For times at Grand Unified Theory (GUT)

observation of a stochastic GW background is possible

- Experiments with potential to detect GW signals

- One of GW signals

GW produced by high-temperature first-order phase transitions (FOPT) induced by the symmetry breaking expected in GUTs

-not present in SM

Symmetry breaking

- Breaking chain pattern

$$SO(10) \xrightarrow[45\ [1]]{M_{\rm GUT}} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$\xrightarrow[126\ [2]]{M_R} SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow[10]{M_{\rm EW}} SU(3)_C \times U(1)_{\rm em}$$

Gauge coupling unification without introducing supersymmetry

Compliance with the current experimental limits on the proton decay rate

Symmetry breaking

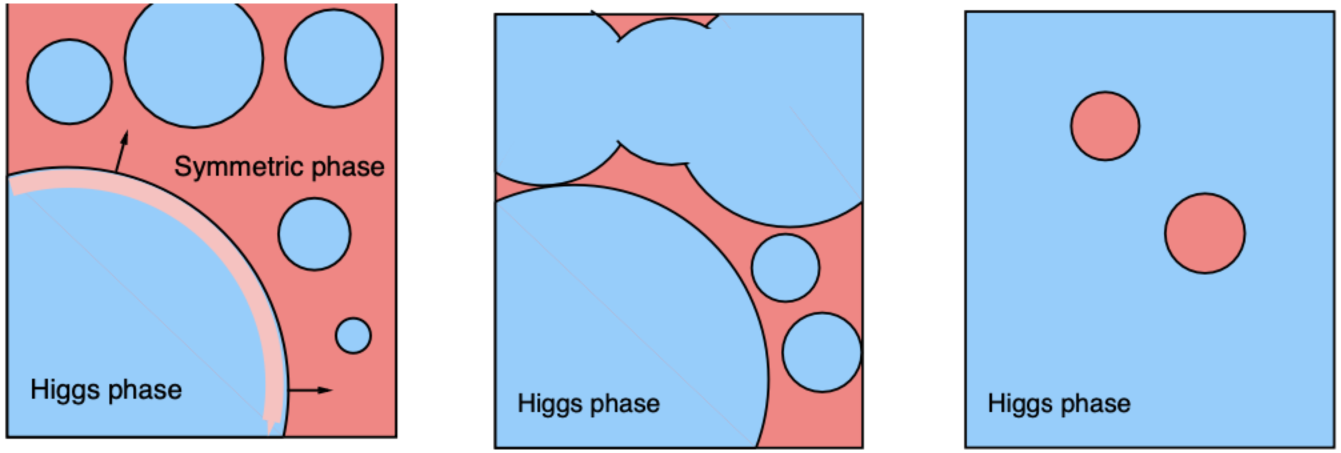
- Minimal SO(10) model – three families of fermions transforming under the representation **16**, gauge bosons, scalars transforming under **10**, **45**, and **126**

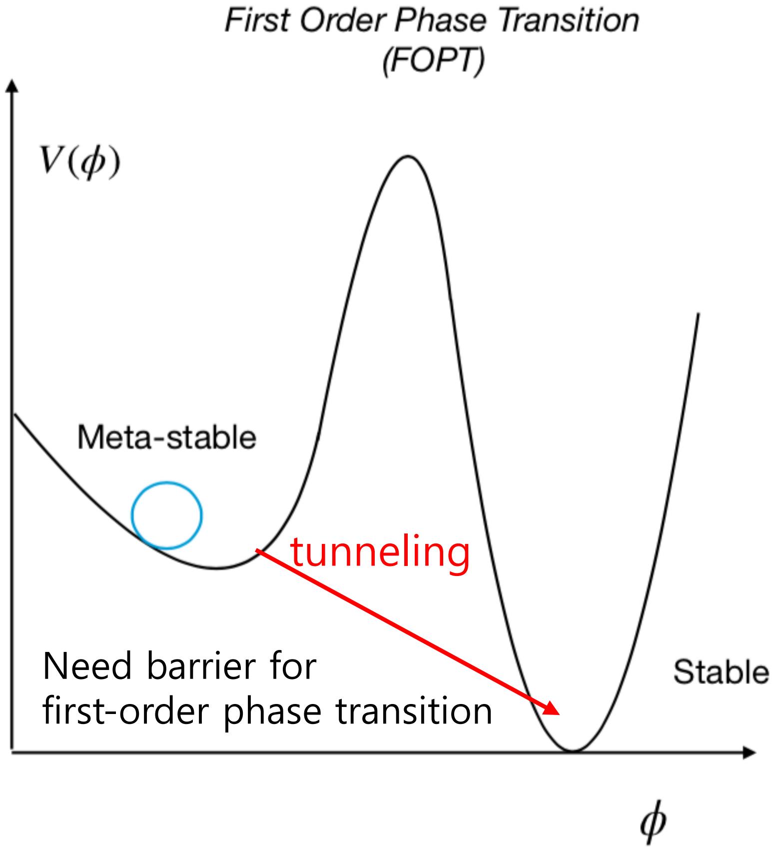
- Potential for a first-order phase transition triggered by a vev v of the G_{3221}-singlet component of the **45**

$$G_{3221} \coloneqq SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

First-order phase transition

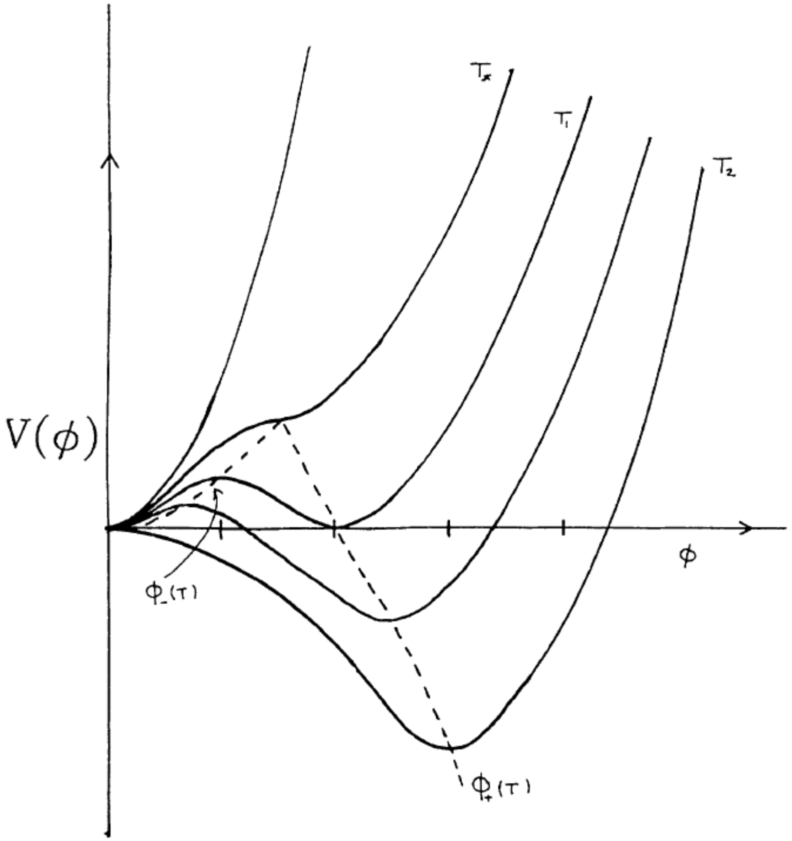
- FOPT can generate GW through nucleation, expansion, collision, and merger of bubbles



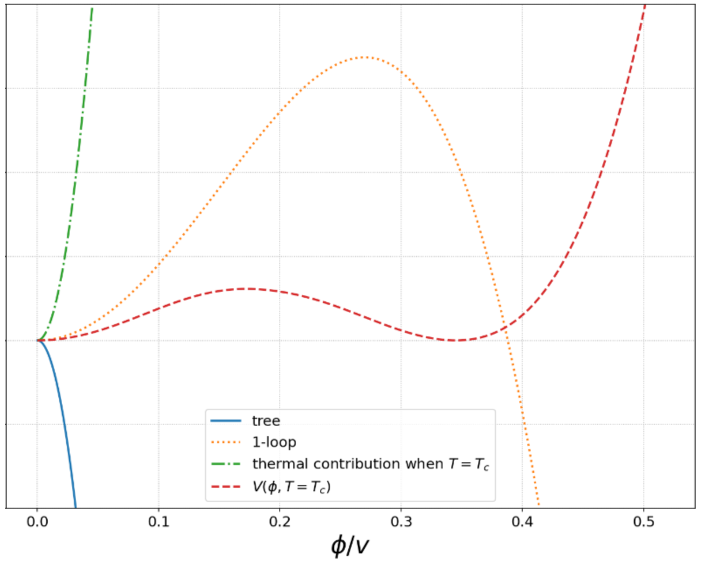


First-order phase transition

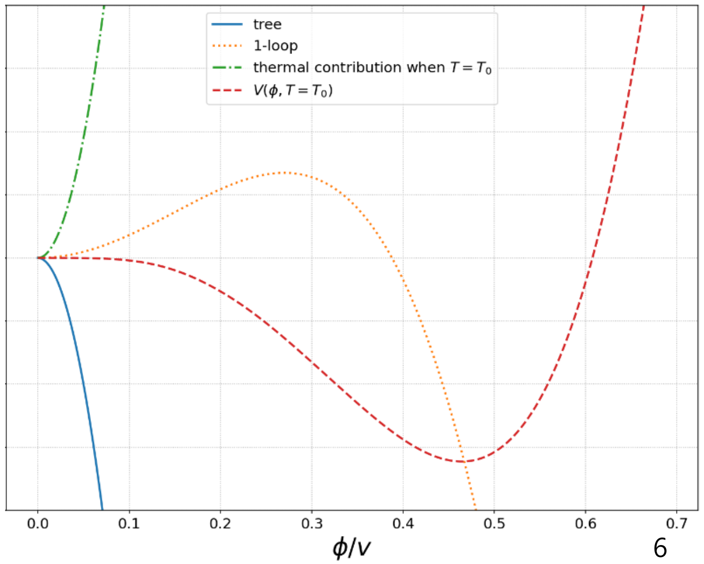
- FOPT can generate GW through nucleation, expansion, collision, and merger of bubbles



$T=T_c$(Critical temperature)



$T=T_0$ (Barrier vanishes)



First-order phase transition

- Decay rate

$$\Gamma(T) \approx T^4 \left(\frac{S_3}{2\pi T}\right)^{\frac{3}{2}} \exp(-S_3/T)$$

$S_3[\phi_c(r), T] = 4\pi \int_0^\infty r^2 dr \left[\frac{1}{2}\left(\frac{d\phi_c(r)}{dr}\right)^2 + V(\phi_c(r), T)\right]$: Action describing the bubbles forming

- For valid decay rate, the nucleation temperature is calculated by solving : $\mathrm{N}(T_n) = \left(\frac{3\bar{M}_{\mathrm{Pl}}}{\pi}\right)^4 \left(\frac{10}{g_*}\right)^2 \int_{T_n}^{T_c} \frac{dT}{T^5} \left(\left(\frac{S_3}{2\pi T}\right)^{3/2}\right) \exp(-S_3/T) \sim 1$

└Average number of bubbles nucleated per Hubble horizon ~ 1

Decay rate per commoving volume V

$\frac{\Gamma}{V} \approx T_n^4 e^{S_3(T_n)/T_n}$

Sidney Coleman. : Phys. Rev. D 16 (4 Aug. 1977), pp. 1248–1248.

A. Linde, Nucl. Phys. B216(1983) 421 : Nuclear Physics B 223.2 (1983), p. 544.

First-order phase transition

Mark Hindmarsh et al. : Phys. Rev. Lett. 112 (4 Jan. 2014), p. 041301.

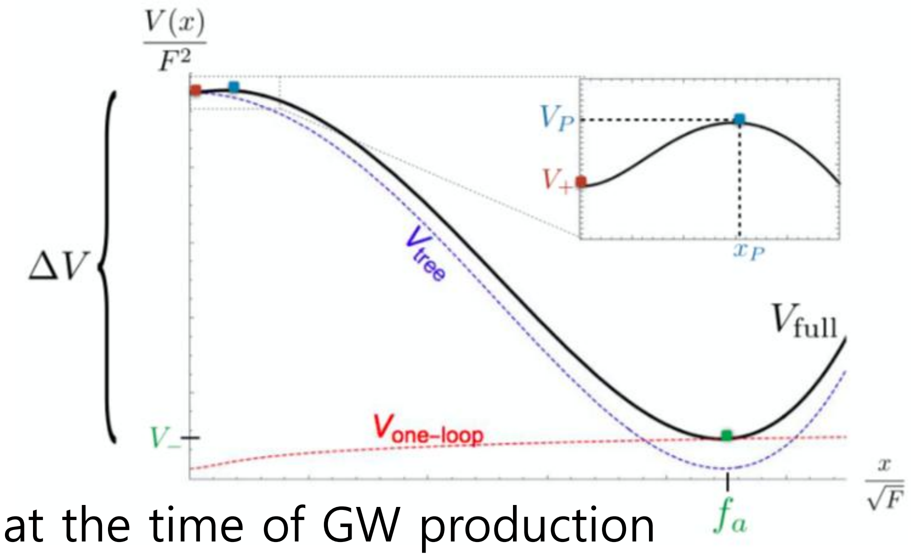
Mark Hindmarsh et al. : Phys. Rev. D 92 (12 Dec. 2015),p. 123009.

- **Transition parameters**

Strength of phase transition

$$\alpha = \frac{1}{\rho_{\rm rad}}\left[\Delta V(\phi_c, T) - \frac{T}{4}\frac{\partial \Delta V(\phi_c, T)}{\partial T}\right]\Bigg|_{T=T_n}$$

ΔV : difference between true and false vacuum



Rate at which the FOPT occurs

$$\beta = H_* \, T \frac{d}{dT}\left(\frac{S_3}{T}\right)\Bigg|_{T=T_n \approx T_*}$$

H_* : Hubble parameter at the time of GW production

a measure of the duration of the phase transition

$\beta < H_*$: Expansion of Universe is faster than bubbles

First-order phase transition

- GW signals

Red-shifted sound wave contribution to GW

$$\Omega_{\rm sw} h^2(f) = 2.65 \times 10^{-6}\, H_* \tau_{\rm sw} \left(\frac{\beta}{H_*}\right)^{-1} v_w \left(\frac{\kappa_V \alpha}{1+\alpha}\right)^2 \left(\frac{g_*}{100}\right)^{-\frac{1}{3}} \left(\frac{f}{f_{\rm sw}}\right)^3 \left(\frac{7}{4+3\,(f/f_{\rm sw})^2}\right)^{7/2}$$

$$f_{\rm sw} = 1.9 \times 10^{-5}\, \frac{1}{v_w} \left(\frac{\beta}{H_*}\right) \left(\frac{T_*}{100\,\mathrm{GeV}}\right) \left(\frac{g_*}{100}\right)^{1/6} \mathrm{Hz}$$

Need to calculate α and β from theory

g_*: degree of freedom at the time of GW production

John Ellis, Marek Lewicki, and Jos´e Miguel No. :
Journal of Cosmology and Astroparticle Physics 2019.04 (Apr. 2019), p. 003.

First-order phase transition

- GW signals

Red-shifted sound wave contribution to GW

$$\Omega_{\rm sw}h^2(f) = 2.65\times10^{-6}\,H_*\tau_{\rm sw}\left(\frac{\beta}{H_*}\right)^{-1} v_w\left(\frac{\kappa_\nu\alpha}{1+\alpha}\right)^2\left(\frac{g_*}{100}\right)^{-\frac{1}{3}}\left(\frac{f}{f_{\rm sw}}\right)^3\left(\frac{7}{4+3\,(f/f_{\rm sw})^2}\right)^{7/2}$$

$\tau_{\rm sw} = \min\left[\frac{1}{H_*},\frac{R_*}{\bar{U}_f}\right]$: time scale of the duration of phase transition where $H_*R_* = \max(v_w, c_s)\,(8\pi)^{1/3}(\beta/H_*)^{-1}$

Approximation $\bar{U}_f^2 \approx \frac{3}{4}\left(\frac{\kappa_\nu\alpha}{1+\alpha}\right)$ is restricted to $\alpha \lesssim 0.1$ $\quad \bar{U}_f \lesssim 0.05$

Chiara Caprini et al. : Journal of Cosmology and Astroparticle Physics 2016.04 (Apr. 2016), p. 001.

From numerical simulations

$$\kappa_\nu \simeq \begin{cases} \alpha(0.73+0.83\sqrt{\alpha}+\alpha)^{-1}, & v_w\sim 1 \\ v_w^{6/5}\,6.9\alpha\,(1.36-0.037\sqrt{\alpha}+\alpha)^{-1}, & v_w\ll 1 \end{cases}$$

José R. Espinosa et al. : Journal of Cosmology and Astroparticle Physics 2010.06 (June 2010),p. 028.

Effective potential

- Tree-level SO(10)-symmetric potential for the scalar **45**

$$V_0(\phi) = -\frac{\mu^2}{2}\operatorname{Tr}\phi^2 + \frac{a_0}{4}\left(\operatorname{Tr}\phi^2\right)^2 + \frac{a_2}{4}\operatorname{Tr}\phi^4$$

$$\phi = \frac{i}{\sqrt{2}}\phi_{\alpha\beta}T^{\alpha\beta}$$

- Classical field is proportional to generator U_{15} in SU(4)

$$V_0(\phi_c) = -\frac{1}{2}\mu^2\varphi_c^2 + a_0\varphi_c^4 + \frac{1}{6}a_2\varphi_c^4$$

$$\phi_c = \sqrt{2}i\,\varphi_c U_{15}$$

At the potential minimum, $\varphi_c = \underline{v \equiv \sqrt{3}\omega_{BL}} \equiv M_{\rm GUT}$

Kate řina Jarkovská et al. Phys. Rev. D 105 (9 May 2022), p. 095003

- Tree-level minimization condition

$$\mu^2 = \left(4a_0 + \frac{2}{3}a_2\right)v^2$$

Effective potential

- Calculating generator and trace in potential from $T^{\alpha\beta}$

```
In[ ]:= Gens[[1, 2]] // MatrixForm
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[ ]:= Gens[[1, 2]].Gens[[1, 2]] // MatrixForm
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[ ]:= Tr[Gens[[1, 2]].Gens[[1, 2]]]
Out[ ]= 1
```

```
In[ ]:= UGens[[15]] // MatrixForm
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 0 & \frac{\phi_c}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{\phi_c}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\phi_c}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\phi_c}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\phi_c}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\phi_c}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Effective potential

- Calculating generator and trace in potential from $T^{\alpha\beta}$

```
In[ ]:= V0 = -μ^2 / 2 Sum[ϕ[i, j] × ϕ[k, l] × Tr[Gens[[i, j]].Gens[[k, l]]], {i, 9}, {j, i + 1, 10}, {k, 1, 9}, {l, k + 1, 10}] +
  a0 Sum[ϕ[i, j] × ϕ[k, l] × Tr[Gens[[i, j]].Gens[[k, l]]], {i, 9}, {j, i + 1, 10}, {k, 1, 9}, {l, k + 1, 10}]^2 +
  a2 Sum[ϕ[i, j] × ϕ[k, l] × ϕ[m, n] × ϕ[p, q] × Tr[Gens[[i, j]].Gens[[k, l]].Gens[[m, n]].Gens[[p, q]]],
    {i, 9}, {j, i + 1, 10}, {k, 1, 9}, {l, k + 1, 10}, {m, 9}, {n, m + 1, 10}, {p, 9}, {q, p + 1, 10}]
```

```
Out[ ]= -1/2 μ^2 (ϕ[1, 2]^2 + ϕ[1, 3]^2 + ϕ[1, 4]^2 + ϕ[1, 5]^2 + ϕ[1, 6]^2 + ϕ[1, 7]^2 + ϕ[1, 8]^2 + ϕ[1, 9]^2 + ϕ[1, 10]^2 + ϕ[2, 3]^2 + ϕ[2, 4]^2 + ϕ[2, 5]^2 +
   ϕ[2, 6]^2 + ϕ[2, 7]^2 + ϕ[2, 8]^2 + ϕ[2, 9]^2 + ϕ[2, 10]^2 + ϕ[3, 4]^2 + ϕ[3, 5]^2 + ϕ[3, 6]^2 + ϕ[3, 7]^2 + ϕ[3, 8]^2 + ϕ[3, 9]^2 +
   ϕ[3, 10]^2 + ϕ[4, 5]^2 + ϕ[4, 6]^2 + ϕ[4, 7]^2 + ϕ[4, 8]^2 + ϕ[4, 9]^2 + ϕ[4, 10]^2 + ϕ[5, 6]^2 + ϕ[5, 7]^2 + ϕ[5, 8]^2 + ϕ[5, 9]^2 +
   ϕ[5, 10]^2 + ϕ[6, 7]^2 + ϕ[6, 8]^2 + ϕ[6, 9]^2 + ϕ[6, 10]^2 + ϕ[7, 8]^2 + ϕ[7, 9]^2 + ϕ[7, 10]^2 + ϕ[8, 9]^2 + ϕ[8, 10]^2 + ϕ[9, 10]^2) +
 a0 (… 1 …)^2 + a2 (1/2 ϕ[1, 2]^4 + ϕ[1, 2]^2 ϕ[1, 3]^2 + 1/2 ϕ[1, 3]^4 + … 1240 … + ϕ[8, 10]^2 ϕ[9, 10]^2 + 1/2 ϕ[9, 10]^4)
```

$$\phi_c = \sqrt{2}i\,\varphi_c U_{15}$$

$$V_0(\phi_c) = -\frac{1}{2}\mu^2\varphi_c^2 + a_0\varphi_c^4 + \frac{1}{6}a_2\varphi_c^4$$

Effective potential

Lukáš Gráf et al. Phys. Rev. D 95 (7 Apr. 2017)

- 1-loop gauge boson contribution

$$V_1^g(\phi_c) = \frac{3}{64\pi^2} \sum_{i=g1,g2} n_i\, m_i^4(\phi_c) \left(\ln \frac{m_i^2(\phi_c)}{\mu_r^2} - \frac{5}{6} \right)$$

Field-dependent mass matrix for g1, g2

$$M_g^2(\phi_c)_{(\alpha\beta)(\gamma\delta)} = \frac{g^2}{2} \mathrm{Tr}\left(\left[T^{(\alpha\beta)}, \phi_c\right] \left[T^{(\gamma\delta)}, \phi_c\right] \right)$$

$$M_{SU(4)}^2(\phi_c)_{ab} = \frac{g^2}{2} \mathrm{Tr}\left(\left[U_a, \phi_c\right] \left[U_b, \phi_c\right] \right)$$

- 1-loop scalar contribution

$$V_1^s(\phi_c) = \frac{1}{64\pi^2} \sum_{i=s1,s2,s3,\chi} n_i\, m_i^4(\phi_c) \left(\ln \frac{|m_i^2(\phi_c)|}{\mu_r^2} - \frac{3}{2} \right)$$

$$M_s^2(\phi_c)_{(\alpha\beta)(\gamma\delta)} = \left. \frac{\partial^2 V_0(\phi)}{\partial\phi_{(\alpha\beta)} \partial\phi_{(\gamma\delta)}} \right|_{\phi=\phi_c}$$

In the $\overline{\mathrm{MS}}$ renormalization scheme with renormalization scale, $\mu_r = v$

Field-dependent mass squared		Particle type	Multiplicity n_i
$m_{g1}^2(\phi_c)$	$\frac{1}{6}g^2\varphi_c^2$	Gauge boson	24
$m_{g2}^2(\phi_c)$	$\frac{2}{3}g^2\varphi_c^2$	Gauge boson	6
$m_{s1}^2(\phi_c)$	$-\mu^2 + 4a_0\varphi_c^2$	Scalar	6
$m_{s2}^2(\phi_c)$	$-\mu^2 + 12a_0\varphi_c^2 + 2a_2\varphi_c^2$	Scalar	1
$m_{s3}^2(\phi_c)$	$-\mu^2 + 4a_0\varphi_c^2 + 2a_2\varphi_c^2$	Scalar	8
$m_\chi^2(\phi_c)$	$-\mu^2 + 4a_0\varphi_c^2 + \frac{2}{3}a_2\varphi_c^2$	NGB	30

Effective potential

- 1-loop thermal contribution

$$V_{\rm th}(\phi_c, T) = \sum_{i=g,s,\chi} \frac{n_i}{2\pi^2} T^4 J_b(x) \quad , \quad x \equiv \frac{m_i(\phi_c)}{T}$$

$$J_b(x) = \Re \int_0^\infty dy\, y^2 \ln\left[1 - e^{-\sqrt{y^2+x^2}}\right]$$

"Decay of the false vacuum at finite temperature:
A. Linde, Nucl. Phys. B216(1983) 421

- Complete 1-loop effective potential

$$V(\phi_c, T) = V_0(\phi_c) + V_1^g(\phi_c) + V_1^s(\phi_c) + V_{\rm th}(\phi_c, T)$$

value of scalar field in the minimum with 1-loop level , $\phi_m \neq v$
Adjusting value of μ^2 to make the minimum with 1-loop level $\cong v$

After Newton's method iteration, $0.99v \leq \phi_m \leq 1.01v$

Effective potential

- Consideration of 2-loop correction

Estimating the uncertainty by neglecting higher loop order

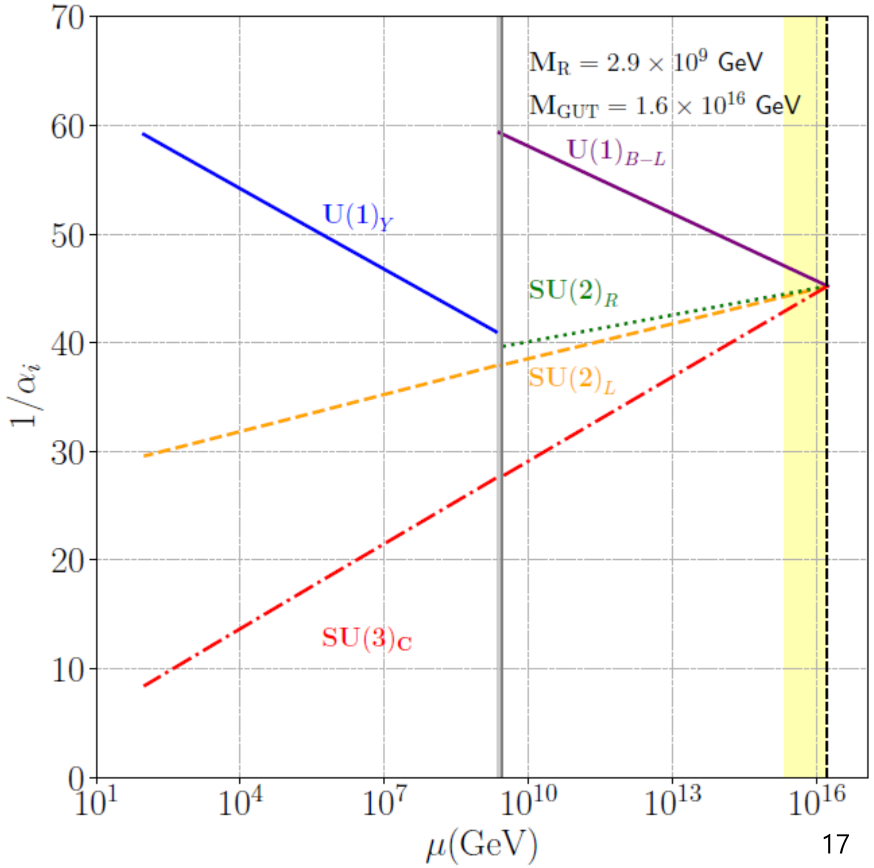
$$\Delta^{\text{2-loop}} := \left\langle \frac{V(\phi_c,0)|_{\mu_r=2v} - V(\phi_c,0)|_{\mu_r=v/2}}{\left(V(\phi_c,0)|_{\mu_r=2v} + V(\phi_c,0)|_{\mu_r=v/2} \right)/2} \right\rangle_{0<\phi_c<1.2v} < 0.5$$

Constraints on the model

- Beta function between M_{GUT} and M_R

$$dg_a/dt = g_a^3 b_a/16\pi^2 + g_a^3/(16\pi^2)^2 \left[\sum_{b=1}^{4} b_{ab} g_b^2\right]$$

$$a = 3, 2L, 2R, B-L$$



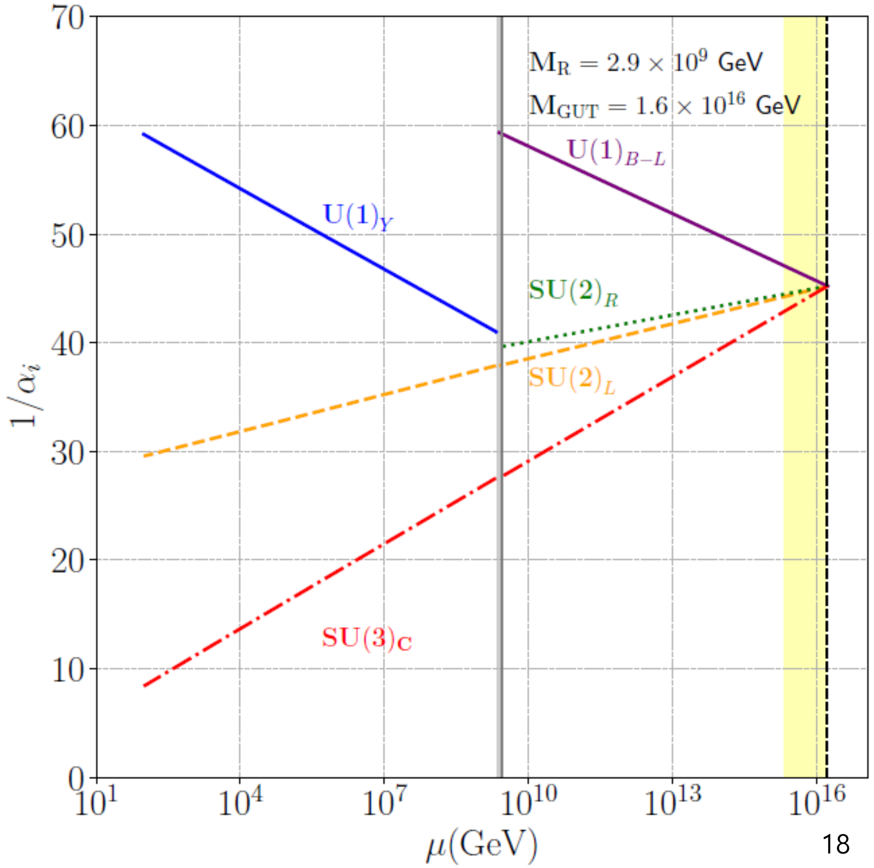
Marie E. Machacek and Michael T. Vaughn. :
Nuclear Physics B 249.1 (1985), pp. 70–92.

Constraints on the model

- Beta function between M_R and M_{EW}

$$dg_a/dt = g_a^3 b_a/16\pi^2 + g_a^3/(16\pi^2)^2 \left[\sum_{b=1}^{3} b_{ab} g_b^2 - C_a^t y_t^2\right]$$

$$M_R^{2\,\mathrm{loop}} = (2.9 \pm 1.0) \times 10^9\,\mathrm{GeV}\,,$$
$$M_{\mathrm{GUT}}^{2\,\mathrm{loop}} = (1.60 \pm 1.0) \times 10^{16}\,\mathrm{GeV}$$



Marie E. Machacek and Michael T. Vaughn. :
Nuclear Physics B 249.1 (1985), pp. 70–92.

Constraints on the model

- Gauge coupling transformation in breaking

$$b_a^{(1)} = \begin{pmatrix} -7 \\ -3 \\ -7/3 \\ 11/2 \end{pmatrix}, \quad b_{ab}^{(2)} = \begin{pmatrix} -26 & 9/2 & 9/2 & 1/2 \\ 12 & 8 & 3 & 3/2 \\ 12 & 3 & 80/3 & 27/2 \\ 12 & 9/2 & 81/3 & 61/2 \end{pmatrix}$$

Yann Mambrini et al. : Phys. Rev. D 91 (9 May 2015), p. 095010.

$$g_2^{SM}(M_R) = g_{2L}^{G_{3221}}(M_R),$$

$$g_1^{SM}(M_R) = \left[\frac{3}{5}\frac{1}{{g_{2R}^{G_{3221}}}^2(M_R)} + \frac{2}{5}\frac{1}{{g_{B-L}^{G_{3221}}}^2(M_R)}\right]^{-1/2}$$

$$g_3^{SM}(M_R) = g_3^{G_{3221}}(M_R),$$

Joydeep Chakrabortty, Rinku Maji, and Stephen F. King. : Phys. Rev.D 99 (9 May 2019), p. 095008.

Constraints on the model

- Avoiding negative mass-squared tachyons

$$a_0 \in (0.0, 0.2) \quad , \quad a_2 \in (-0.05, -0.01)$$

Kate řina Jarkovská et al. : Phys. Rev. D 105 (9 May 2022), p. 095003

- Proton decay

$$\Gamma(p \to \pi^0 e^+)$$

For the minimal G3221 model, $(2.4 \pm 1.6) \times 10^{36}$ years

Eung Jin Chun and L. Velasco-Sevilla. : Phys. Rev. D 106 (3 Aug. 2022), p. 035008.

Current experimental bound of $\tau(p \to \pi^0 e^+) > 2.34 \times 10^{34}$ years

A. Takenaka et al : Phys. Rev. D 102.11(2020), p. 112011.

Projected bound of 7.8×10^{34} years

Hyper-Kamiokande Proto-Collaboration et al. Hyper-Kamiokande Design Report.2018. arXiv: 1805.04163 [physics.ins-det].

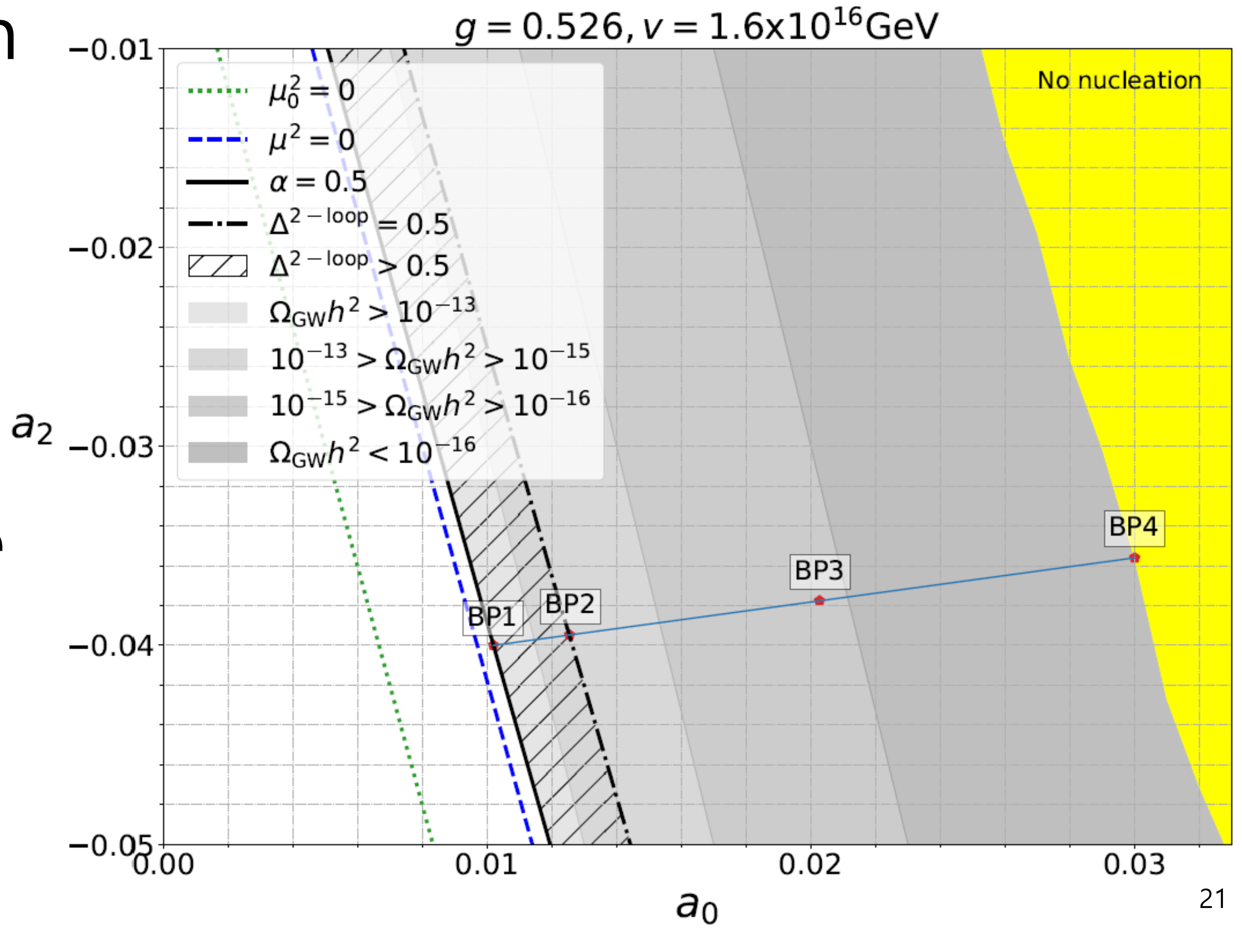
Result

- GW parameter space for a_0 and a_2

GW signal approximately depends on linear combination of a_0 and a_2

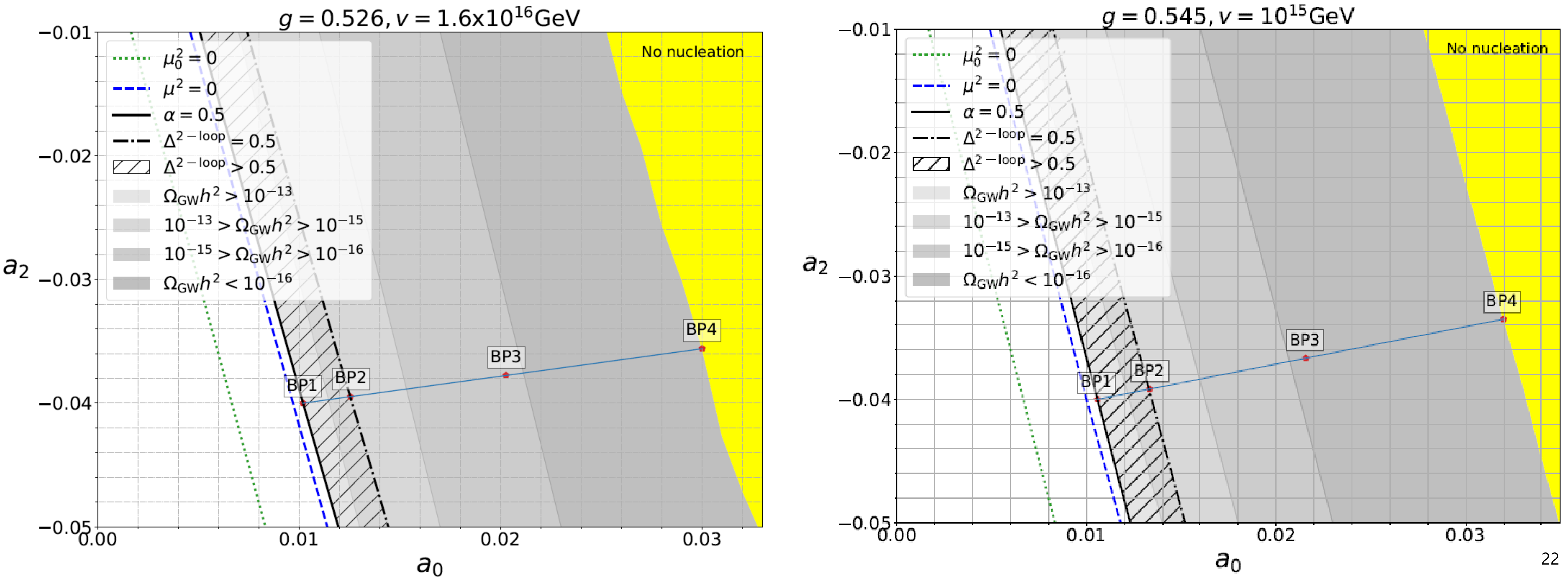
Numerical problem when $T_0 \to T_c$

Region between BP1 and BP2 can be unstable for higher order



Result

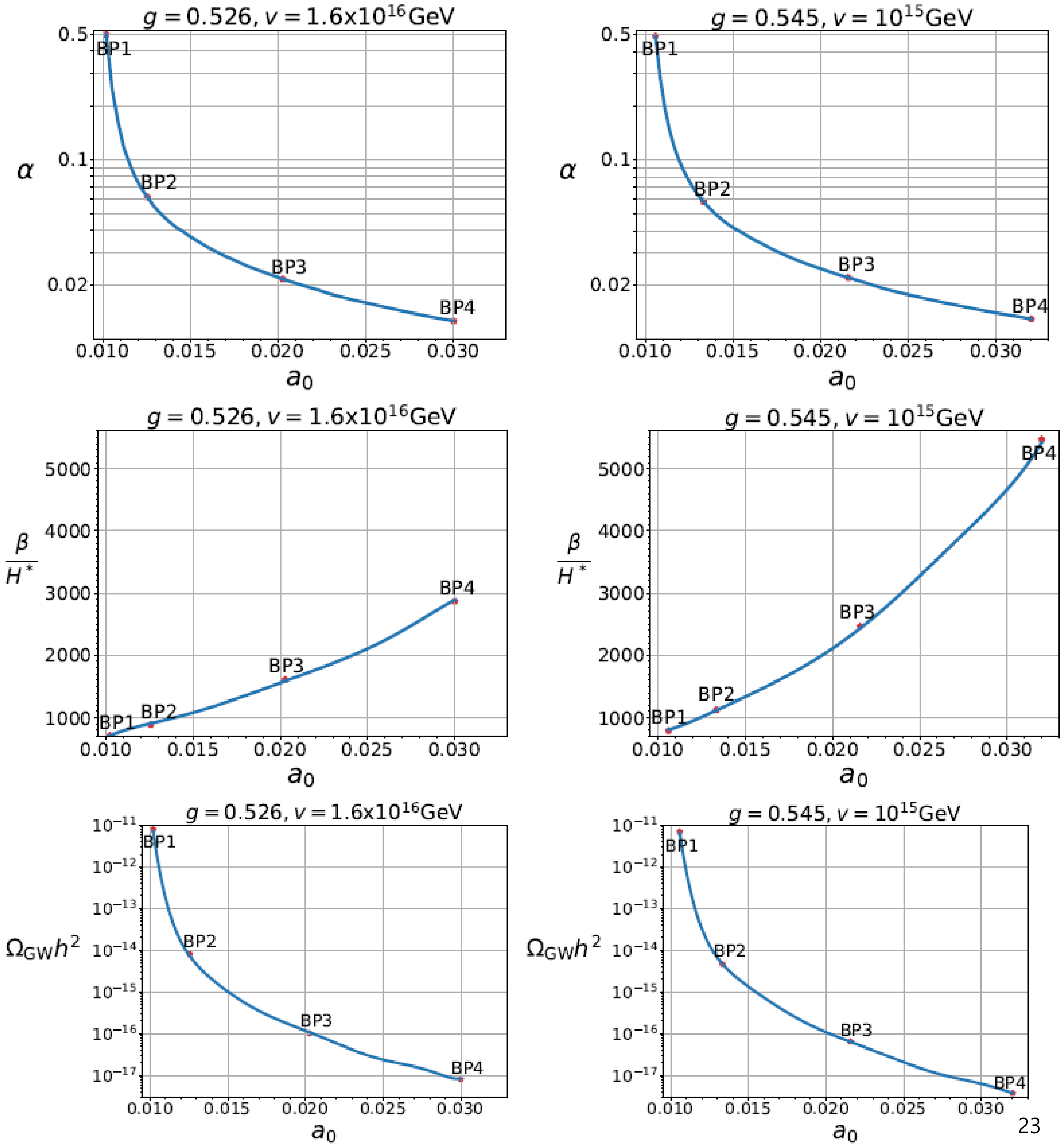
Similar behavior between two scales



Result

Behavior of PT parameters through the BP line

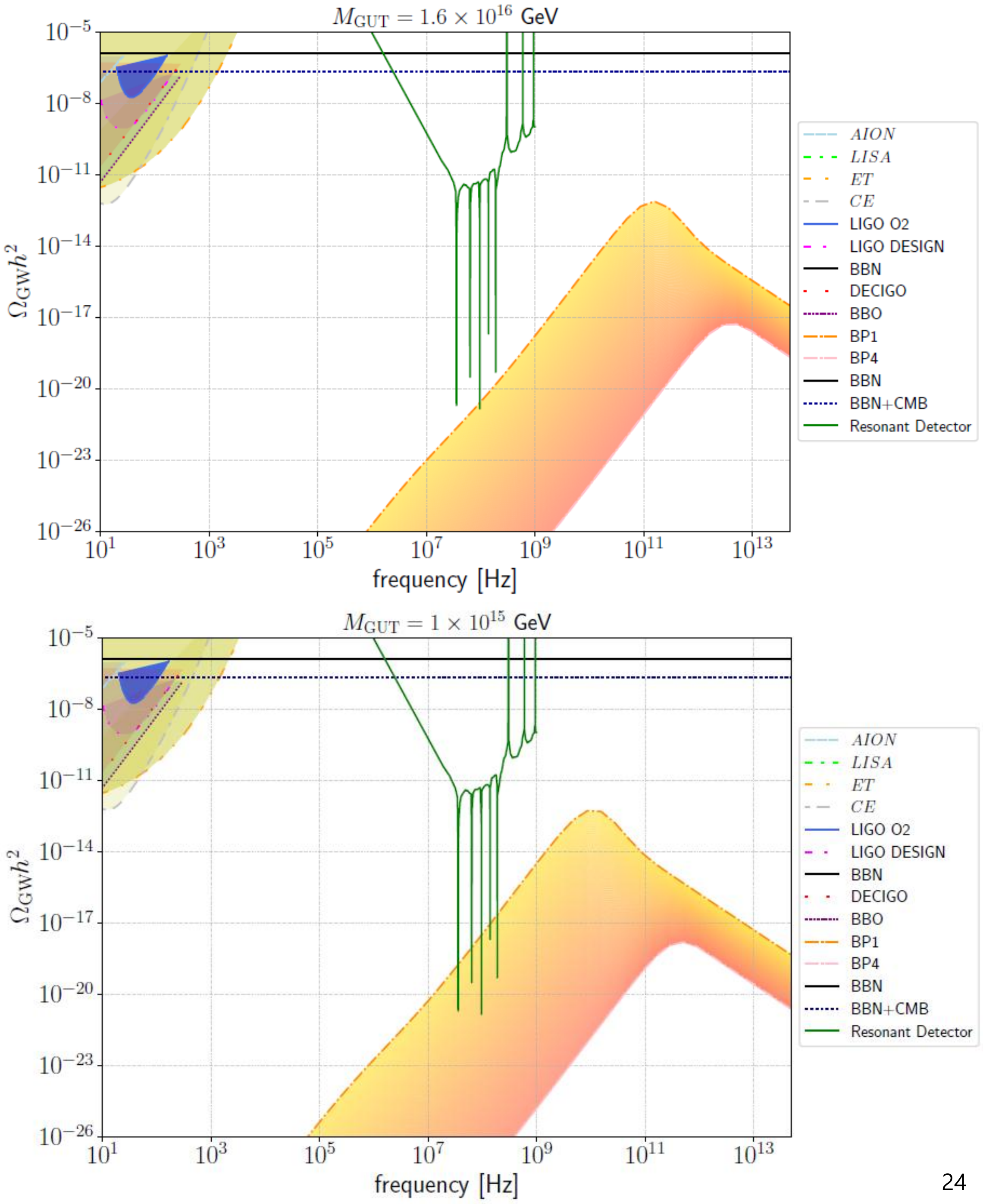
$\Omega_{\mathrm{GW}} h^2$ is proportional to $\alpha^2/(\beta/H_*)$



Result

Peak frequency can be changed as scale changes

With resonant detector there is possibility to detect signals in lower scale than inflation scale



Conclusion

- Effective potential including the one-loop and thermal corrections with compact analytic expression

-with $\mathrm{CosmoTransitions}$

- Gauge coupling unification in this scenario, between 10^{15} and 10^{16} GeV

- FOPT takes place in a significant part of the parameter space allowed by the absence of tachyons and by proton decay

- GW signal peaks at 10^{10} to 10^{11} Hz

Conclusion

- GW signal from FOPT would be the smoking gun of new physics

- Strongest signal where the accuracy of the computation is limited by large contributions of higher loop orders

- For future work, improve the calculation of the effective potential beyond one-loop precision

Thank You

$$\Omega_{\rm turb}h^2(f) =$$

$$3.35\times10^{-4}\left(\frac{\beta}{H_*}\right)^{-1} v_w \left(\frac{\epsilon\,\kappa_\nu\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{g_*}{100}\right)^{-\frac{1}{3}}\frac{(f/f_{\rm turb})^3\,(1+f/f_{\rm turb})^{-\frac{11}{3}}}{1+8\pi\frac{f}{h_*}}$$

$$h_* = 16.5\frac{T_*}{10^8\,{\rm GeV}}\left(\frac{g_*}{100}\right)^{1/6}{\rm Hz}$$

$$f_{\rm turb} = 2.7\times10^{-5}\,\frac{1}{v_w}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}{\rm Hz}$$