Exploring the complementarity of WIMP direct detection and capture in celestial bodies using WimPyDD and WimPyC

Sunghyun Kang

Based on

"WimPyC: an extension module of WimPyDD for the calculation of WIMP capture in celestial bodies", Sunghyun Kang, S.S., Gaurav Tomar, arXiv: 2510.21185

&

"WimPyDD: An object–oriented Python code for the calculation of WIMP direct detection signals", Injun Jeong, Sunghyun Kang, S.S., Gaurav Tomar, Comput.Phys.Commun. 276 (2022), 108342, arXiv: 2106.06207

SOGANG UNIVERSITY

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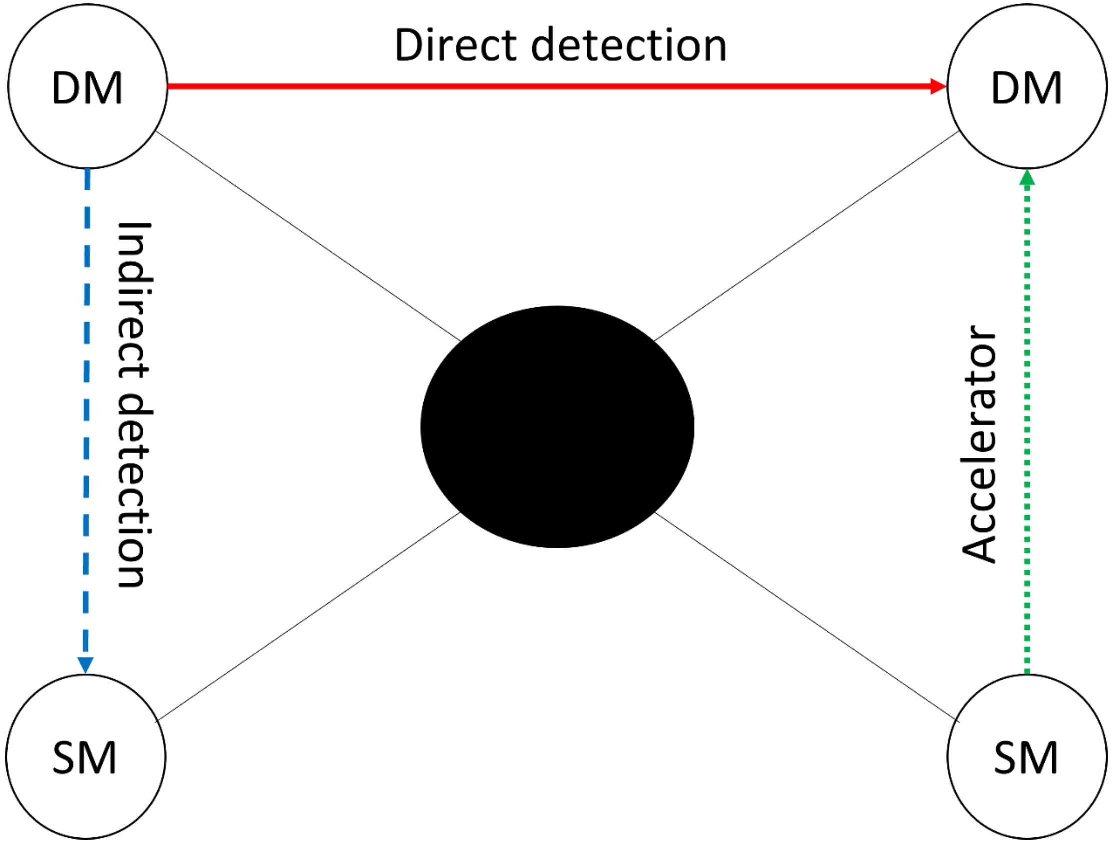
CQUeST
Center for Quantum SpaceTime

Dark Matter and WIMPs

- Many evidences of Dark Matter
  - Galaxy rotational curve
  - CMB
  - Lensing effect

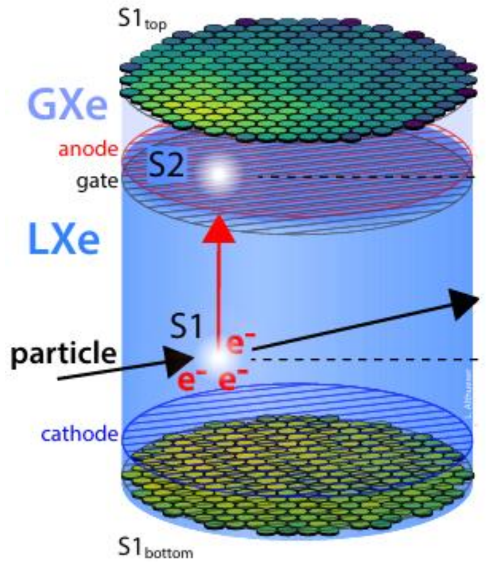
- Many candidates
  - Neutrino
  - Cold Dark Matter (CDM)
  - Weakly Interacting Massive Particle (WIMP)
  - Weak-type interaction
    - no electric charge, no color
  - Mass range in GeV-TeV range
  - WIMP miracle
    - correct relic abundance is obtained at WIMP $< \sigma v > = weak\ scale$
    - most extensions of SM are proposed independently at that scale.

Detection strategies



- Direct detection: DM interacts with SM particles (left to right)
- Indirect detection: DM annihilation (top to bottom)
- Accelerator: DM creation (bottom to top)

Direct Detection (DD)



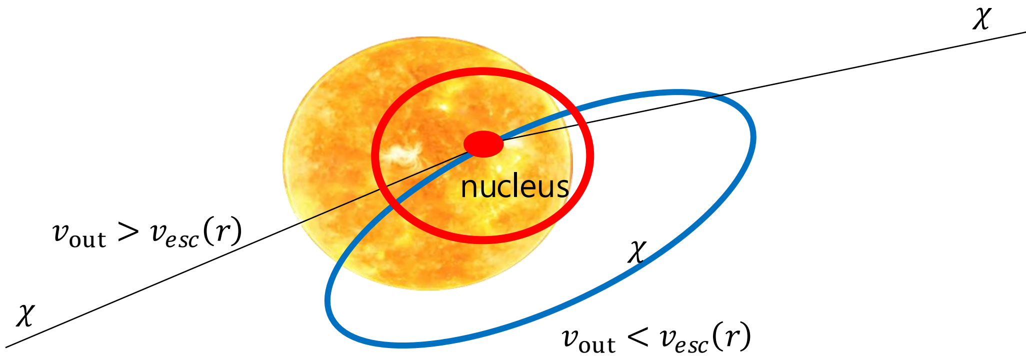
XENON-nT

- The signals are WIMP-nucleus recoil events

- Low probability requires high exposure
- Underground to avoid background

- Depend on features of targets and experimental set-ups
- Different nuclear targets and background subtraction:
  - COSINE, LZ, XENON-nT, ANAIS, DAMA, PandaX-4T, PICO-60 and ect.

Indirect Detection (NT)



- Capture rate in the celestial body

- WIMP scatters off nucleus at distance r inside celestial body
  - same interaction probed by DD

- If its outgoing speed v_{out} is below the escape velocity $v_{esc}(r)$, it gets locked into gravitationally bound orbit and keeps scattering again and again

- Capture process is favored for low (even vanishing) WIMP speeds

Non-Relativistic Effective Theory (NREFT)

- WIMP is slow, so that the recoil events are non-relativistic
- NREFT provides a general and efficient way to characterize results with mass of WIMP and coupling constants

Operators spin up to 1/2

- $\mathcal{H} = \Sigma_{i=1}^{N}\left(c_i^n \mathcal{O}_i^n + c_i^p \mathcal{O}_i^p\right)$
- Non-relativistic process
  - all operators must be invariant by Galilean transformations ($v \sim 10^{-3}c$ in galactic halo)
- Building operators using: $i\dfrac{\vec{q}}{m_N},\ \vec{v}^{\perp},\ \vec{S}_\chi,\ \vec{S}_N$

$$\mathcal{O}_1 = 1_\chi 1_N; \quad \mathcal{O}_2 = (v^\perp)^2; \quad \mathcal{O}_3 = i\vec{S}_N \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp)$$

$$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N; \quad \mathcal{O}_5 = i\vec{S}_\chi \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp); \quad \mathcal{O}_6 = (\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$$

$$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^\perp; \quad \mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp; \quad \mathcal{O}_9 = i\vec{S}_\chi \cdot (\vec{S}_N \times \frac{\vec{q}}{m_N})$$

$$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N}; \quad \mathcal{O}_{11} = i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}; \quad \mathcal{O}_{12} = \vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^\perp)$$

$$\mathcal{O}_{13} = i(\vec{S}_\chi \cdot \vec{v}^\perp)(\vec{S}_N \cdot \frac{\vec{q}}{m_N}); \quad \mathcal{O}_{14} = i(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \vec{v}^\perp)$$

$$\mathcal{O}_{15} = -(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})((\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N}),$$

Non-Relativistic Effective Theory (NREFT)

- Scattering amplitude:
$$\frac{1}{2j_\chi+1}\frac{1}{2j_N+1}\Sigma_{spins}|M|^2 \equiv \Sigma_k \Sigma_{\tau=0,1}\Sigma_{\tau'=0,1}\mathcal{R}_k^{\tau\tau'}\left(\vec{v}_T^{\perp\,2}, \frac{\vec{q}^{\,2}}{m_N^2}, \left\{c_i^\tau, c_j^{\tau'}\right\}\right)W_k^{\tau\tau'}(y)$$

- $\mathcal{R}_k^{\tau\tau'}$: WIMP response function
  - Velocity dependence: $\mathcal{R}_k^{\tau\tau'} = \mathcal{R}_{k,0}^{\tau\tau'} + \mathcal{R}_{k,1}^{\tau\tau'}(v^2 - v_{min}^2)$

- $W_k^{\tau\tau'}$: nuclear response function
  - $y = (qb/2)^\wedge 2$
  - b: harmonic oscillator size parameter
  - $k = M,\ \Delta,\ \Sigma',\ \Sigma'',\ \tilde{\Phi}'$ and Φ''
  - M: standard spin-independent interaction
  - $\Sigma' + \Sigma''$: standard spin-dependent interaction

Non-Relativistic Effective Theory (NREFT)

- Scattering amplitude:
$$\frac{1}{2j_\chi+1}\frac{1}{2j_N+1}\Sigma_{spins}|M|^2 \equiv \Sigma_k \Sigma_{\tau=0,1}\Sigma_{\tau'=0,1} R_k^{\tau\tau'}\left(\vec{v}_T^{\perp\,2}, \frac{\vec{q}^{\,2}}{m_N^2}, \left\{c_i^\tau, c_j^{\tau'}\right\}\right) W_k^{\tau\tau'}(y)$$

- Differential cross section : $\frac{d\sigma}{dE_R} = \frac{1}{10^6}\frac{2m_N}{4\pi}\frac{c^2}{v^2}\left[\frac{1}{2j_\chi+1}\frac{1}{2j_N+1}\Sigma_{spin}|M|^2\right]$

- Differential rate : $\frac{dR}{dE_R} = N_T\int_{v_{min}}^{v_{esc}}\frac{\rho_\chi}{m_\chi}\, v\,\frac{d\sigma}{dE_R}\, f(v)\, dv$

- With $E_R = \frac{\mu_{\chi N}^2\, v^2}{m_N}$, $\quad v_{min} = \frac{1}{\sqrt{2m_N E_R}}\left|\frac{m_N E_R}{\mu_{\chi N}} + \delta\right|$

DD event rate

- DD event rate

$$\mathrm{R_{DD}} = M\tau_{exp}\frac{\rho_\chi}{m_\chi}\int du\, f(u)u\, \Sigma_T\, N_T \int_{E_{R,th}}^{E_R^{max}} dE_R\, \zeta_{exp}\frac{d\sigma_T}{dE_R}$$

  - $M\tau_{exp}$: exposure
  - $E_{R,th}$: experimental energy threshold
  - ζ_{exp} : experimental features such as quenching, resolution, efficiency, etc.

- $R_{DD} = \int_{u_{min}^{DD}}^{u_{esc}} f(u)\, H_{DD}(u)$

$$H_{DD}(u) = u\, M\tau_{exp}\frac{\rho_\chi}{m_\chi}\ \Sigma_T\, N_T \int_{E_{R,th}}^{2\mu_{\chi T}^2 u^2/m_T} dE_R\, \zeta_{exp}\frac{d\sigma_T}{dE_R}$$

Capture rate

- Capture rate

$$C_\odot = \frac{\rho_\chi}{m_\chi} \int du\, f(u) \frac{1}{u} \int_0^{R_\odot} dr\, 4\pi r^2\, w^2\ \Sigma_T\, \rho_T(r)\, \Theta\left(u_T^{C-max} - u\right) \int_{m_\chi u^2/2}^{2\mu_{\chi T}^2 w^2/m_T} dE_R \frac{d\sigma_T}{dE_R}$$

  - ρ_T: the number of density of target
  - r: distance from the center of the Sun for Standard Solar Model AGSS09ph
  - u: DM velocity asymptotically far away from the Sun
  - $v_{esc}(r)$: escape velocity at distance r
  - $w^2(r) = u^2 + v_{esc}^2(r)$

Capture rate

- Neutrino Telescope (NT):
  - the neutrino flux from the annihilation of WIMPs captured in the Sun
  - DM annihilations into $b\bar{b}$

- with assumption of equilibrium between capture and annihilation:
  $\Gamma_{\odot} = C_{\odot}/2$

- $C_{\odot} = \int_0^{u_{max}^C} du\, f(u)\, H_C(u)$

  $H_C = \frac{\rho_\chi}{m_\chi}\frac{1}{u} \int_0^{R_\odot} dr\, 4\pi r^2\, w^2\; \Sigma_T\, \rho_T(r)\, \Theta\left(u_{max,T}^C - u\right) \int_{E_{min}^C}^{E_{max}^C} dE_R \frac{d\sigma_T}{dE_R}$

  - $u_{max,T}^C = v_{esc}(r)\sqrt{\frac{4m_\chi m_T}{(m_\chi - m_T)^2}}$: maximum WIMP speed for capture possible

Event rate

- Event rate:

velocity distribution

$$R \sim \int dv\, H(v) f(v) \sim \int dv\, \mathcal{R}(v) \eta(v)$$

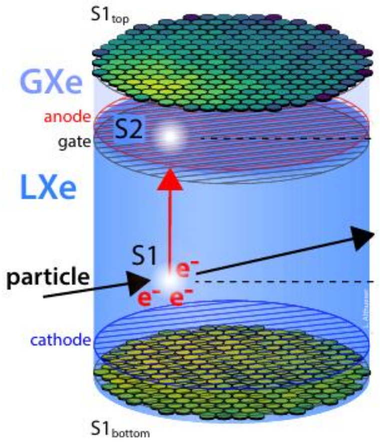
interaction

- Two parts of interaction and velocity distribution
  - needs to avoid uncertainty
  - interaction: include all possible interaction types
  - velocity distribution: halo independent approaches

- Model independent method: the most general scenarios
  - exploiting the complementarity between DD and capture
  - various targets and different WIMP speed range

Complementarity between Direct Detection and Capture in the Sun: Interaction-Independent bounds

$$R \sim \int dv\, H(v)\, f(v) \sim \int dv\, \mathcal{R}(v)\, \eta(v)$$



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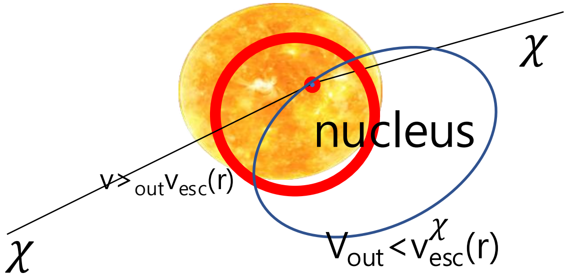

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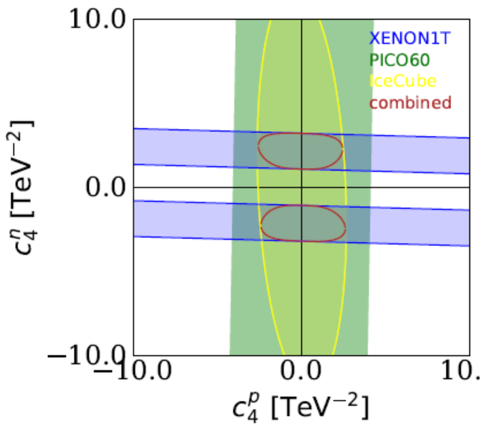


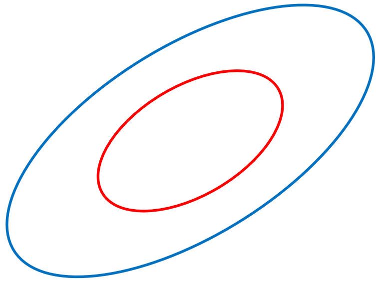
Bracketing DD experiments

- Hamiltonian with contact and long-range interaction:
  - $\mathcal{H} = \Sigma_{\tau=0,1}\Sigma_{i=1}^{15} \left(c_i^\tau + \frac{\alpha_i^\tau}{q^2}\right)\mathcal{O}_i t^\tau = \Sigma_\tau \Sigma_i c_i^\tau \mathcal{O}_i t^\tau + \Sigma_\tau \Sigma_i \alpha_i^\tau \frac{\mathcal{O}_i}{q^2} t^\tau$
  - c: coupling for contact interaction
  - α: coupling for long-range interaction
- In NREFT expected signals are quadratic forms in the couplings
- Experimental bounds single up allowed parameter space inside multi-dimensional ellipsoids:
  - $R_{Exp} = c^T \cdot A_{Exp} \cdot c = \Sigma_{i,j}\ c_i \left(A_{Exp}\right)_{ij} c_j < 1$
  - $c = (c_1, c_3, c_4, \ldots, c_{14}, c_{15}, \alpha_1, \alpha_3, \alpha_4, \ldots, \alpha_{14}, \alpha_{15})$: couplings vector
  - A_{Exp}: matrix encoding all the information

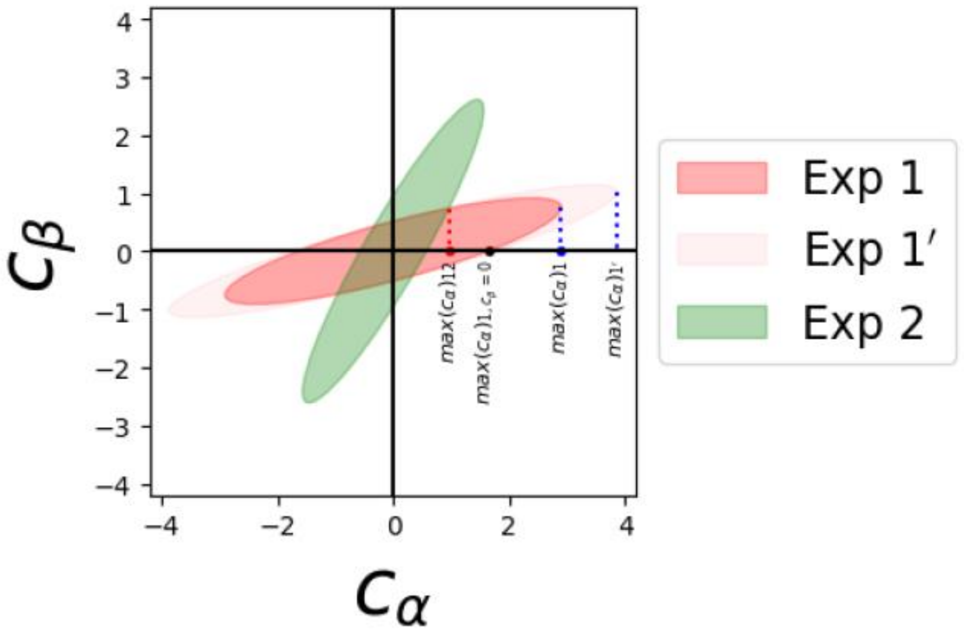
S. Kang, I. Jeong, S. Scopel "Bracketing the direct detection exclusion plot for a WIMP of spin one half in non-relativistic effective theory", j.astropartphys.2023.102854 (arXiv:2209.03646)

Bracketing DD experiments

- Allowed region is an intersection of ellipsoids
- When the ellipsoid of one experiment is totally inside of the others, only that experiment decide the bound



- single constraint:
  - $\max(c_\alpha)_{1,c_\beta=0} = \sqrt{1/\left(A_{Exp1}\right)_{\alpha\alpha}}$
  - $\max(c_\alpha)_1 = \sqrt{\left(A_{Exp1}^{-1}\right)_{\alpha\alpha}}$



A. Brenner, G. Herrera, A. Ibarra, S. Kang, S.S. and G. Tomar, "Complementarity of experiments in probing the non-relativistic effective theory of dark matter-nucleon interactions", JCAP06(2022)026 (arXiv: 2203.04210)

Bracketing DD experiments

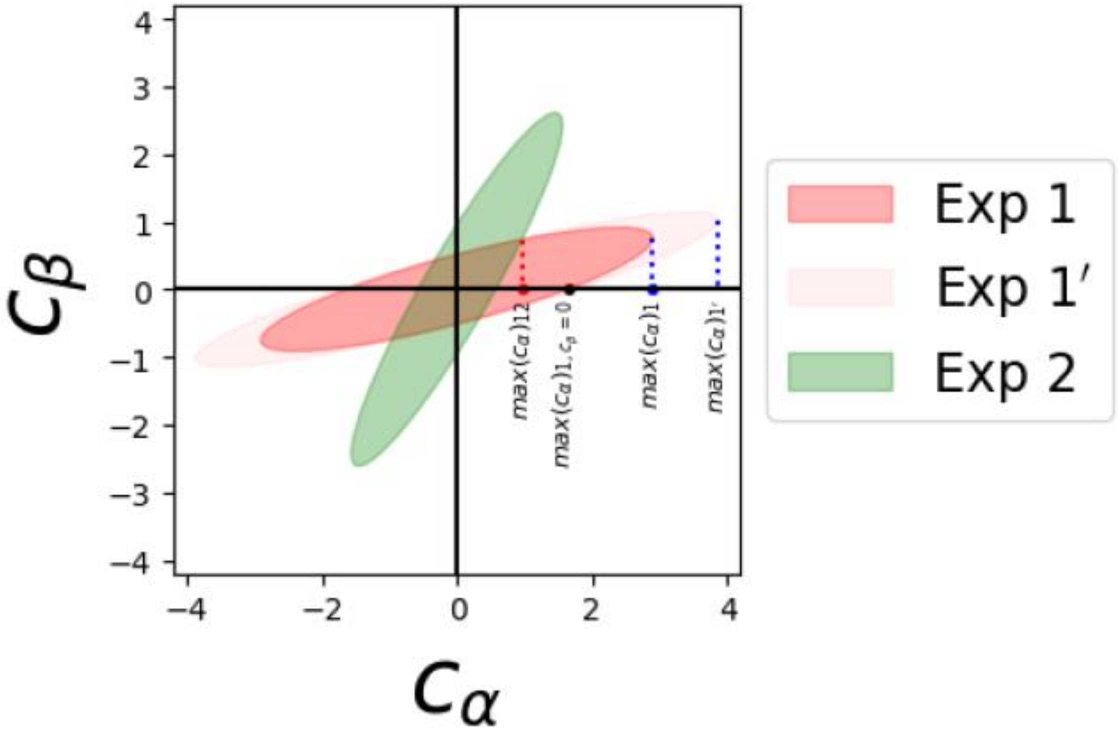
- combining different experiments provides complementarity

$$\boldsymbol{c}^t \cdot B \cdot \boldsymbol{c} \leq (\max(c_\alpha))^2, \quad B = \begin{pmatrix} 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 1 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \end{pmatrix}$$

- Linear matrix inequality

$$\xi_i \geq 0, \quad \sum_{i=1}^{n} \xi_i \leq 1,$$

$$\sum_{i=1}^{n} \xi_i A_k - \frac{B}{max(c_\alpha)^2} \quad \text{is a positive matrix.}$$



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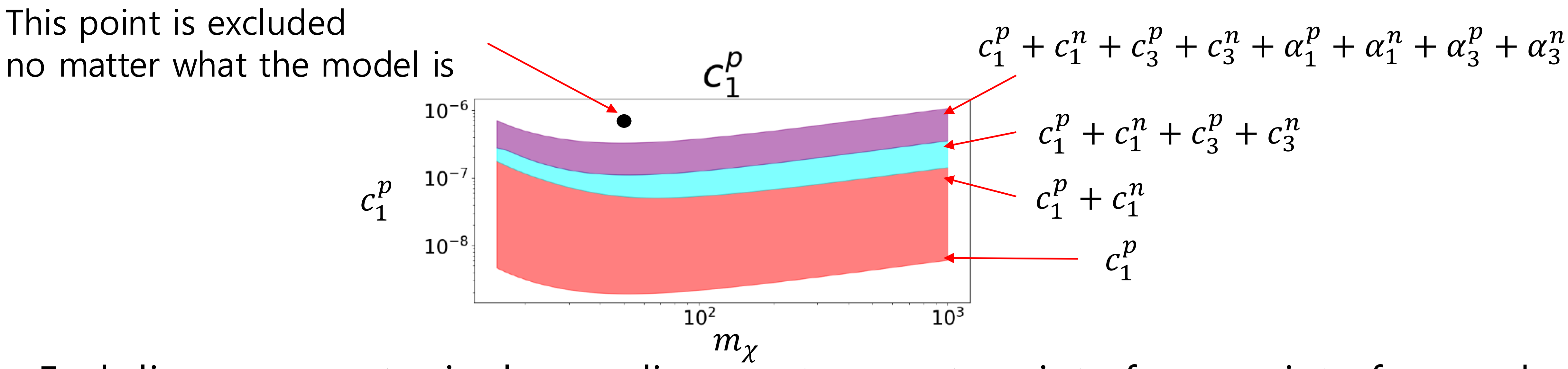
Bracketing: LMI exclusion bands



- Each line represents single coupling, proton-neutron interference, interference between only contact (or long) range interaction(if exist), interference between contact and long-range interaction
- Subspaces
  - $[c_1, c_3, \alpha_1, \alpha_3]$, $[c_{11}, c_{12}, c_{15}, \alpha_{11}, \alpha_{12}, \alpha_{15}]$
  - $[c_4, c_5, c_6, \alpha_4, \alpha_5, \alpha_6]$, $[c_8, c_9, \alpha_8, \alpha_9]$
  - $[c_7, \alpha_7]$, $[c_{10}, \alpha_{10}], [c_{13}, \alpha_{13}], [c_{14}, \alpha_{14}]$

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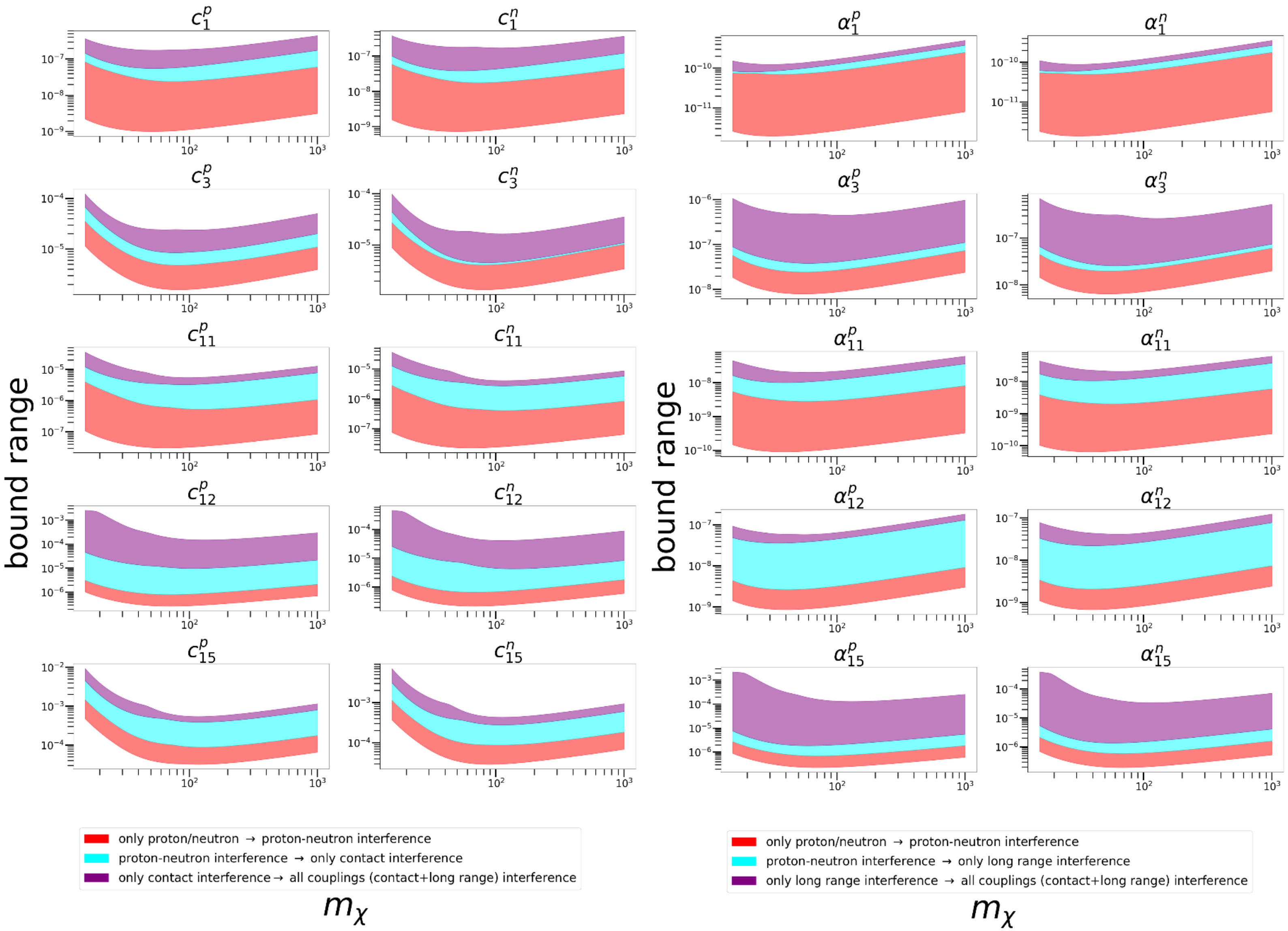

- Spin-Independent
- $[c_1, c_3, \alpha_1, \alpha_3]$, $[c_{11}, c_{12}, c_{15}, \alpha_{11}, \alpha_{12}, \alpha_{15}]$
- M and Φ''

- Favor heavy targets
  - mostly LZ and PandaX-4T

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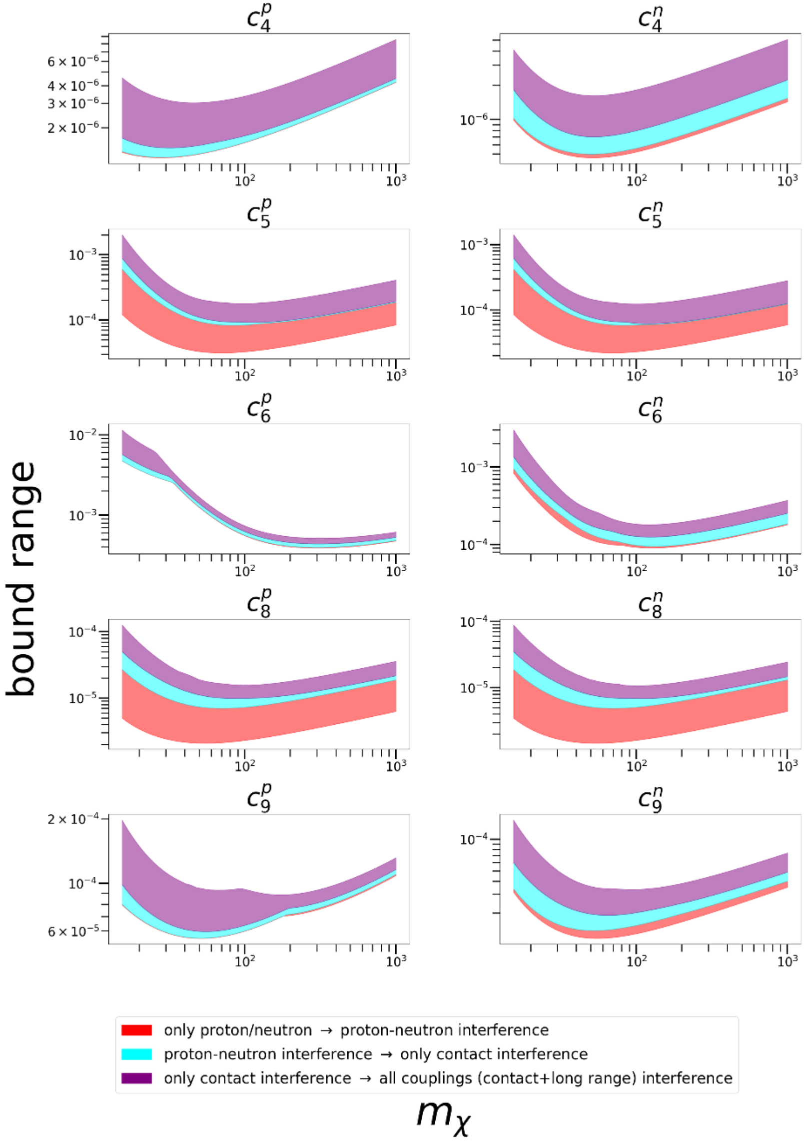

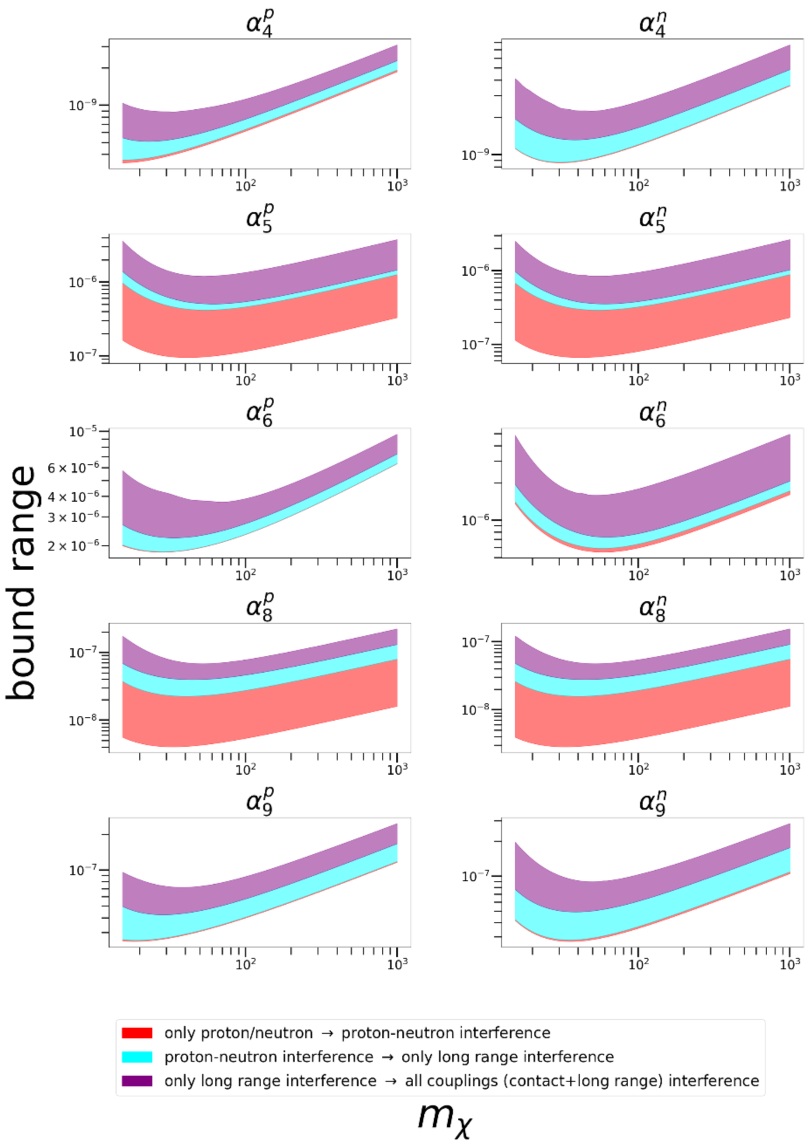

- Spin-Dependent interfering operators
- $[c_4, c_5, c_6, \alpha_4, \alpha_5, \alpha_6]$, $[c_8, c_9, \alpha_8, \alpha_9]$
- Σ'' and Σ' and Δ

- combining proton-odd targets and neutron-odd targets are effective
  - reduces band width

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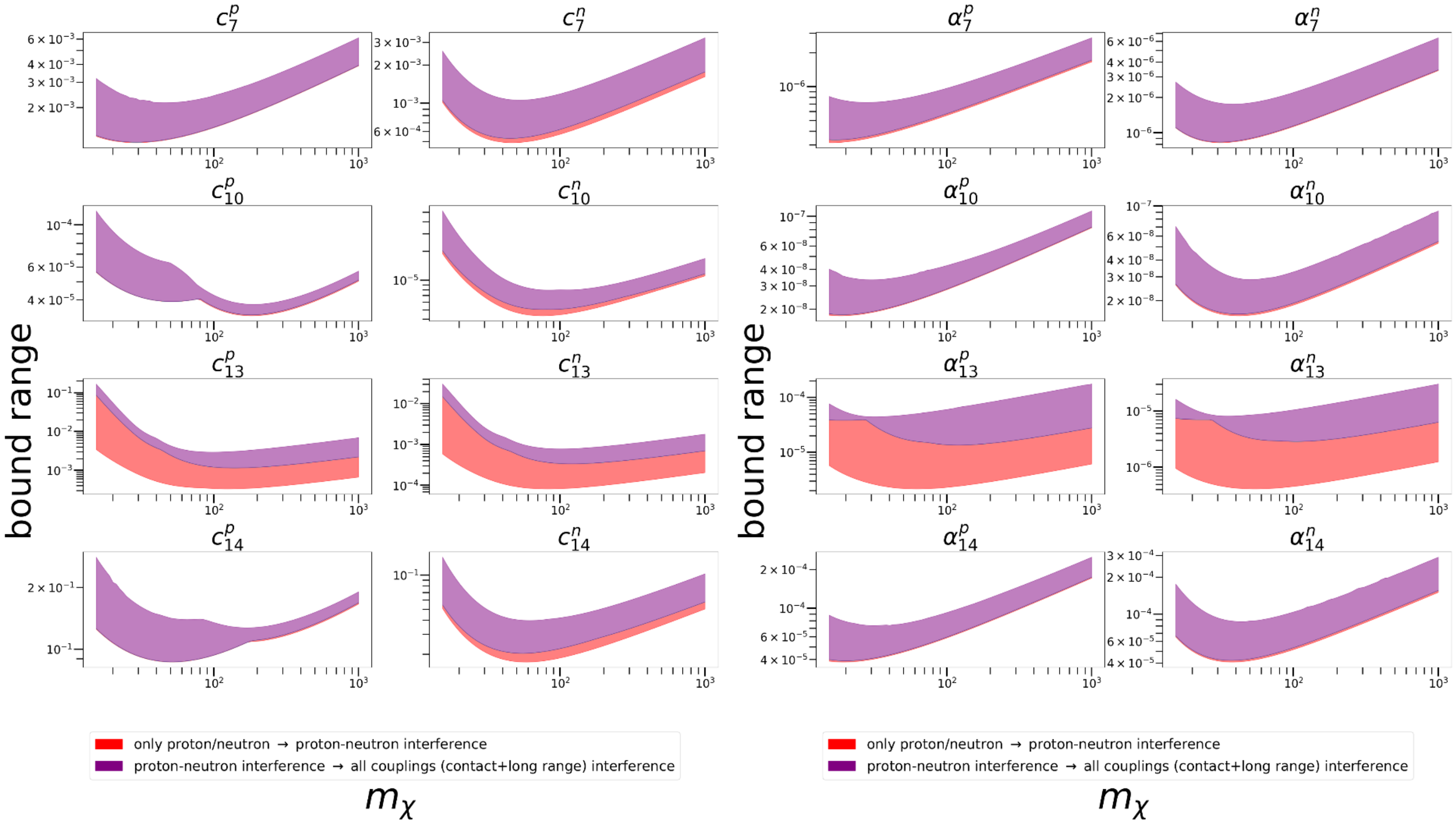

- SD interactions and non-interfering operators
- $[c_7, \alpha_7]$, $[c_{10}, \alpha_{10}]$, $[c_{13}, \alpha_{13}], [c_{14}, \alpha_{14}]$
- Only two bands

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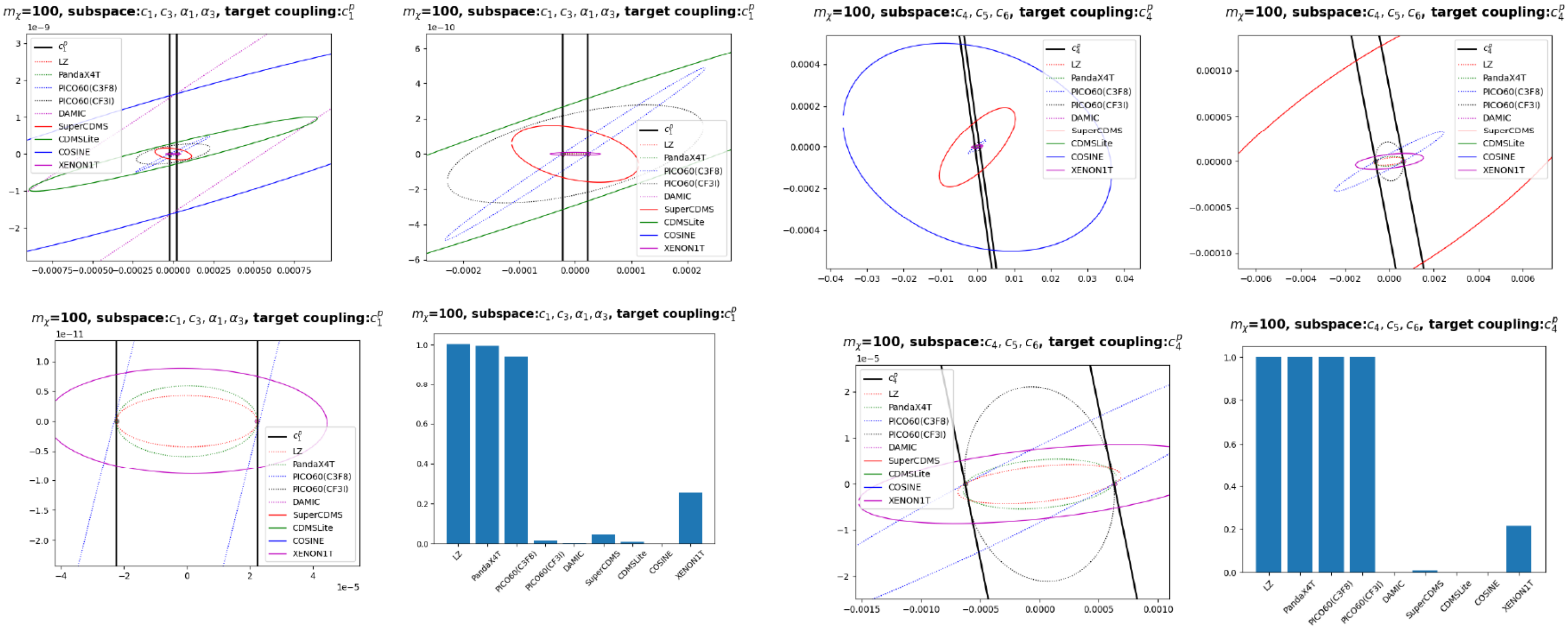

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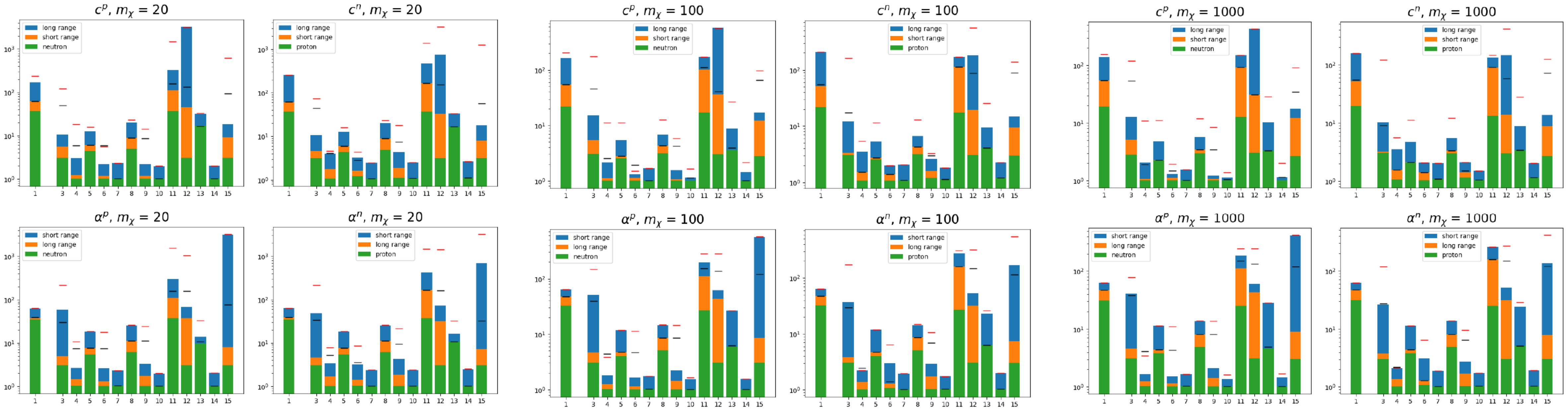
Bracketing: relaxation factors



- Except SI(1, 3, 11, 12, 15), our results have stable sensitivity
- due to the complementarity of proton-odd and neutron-odd targets

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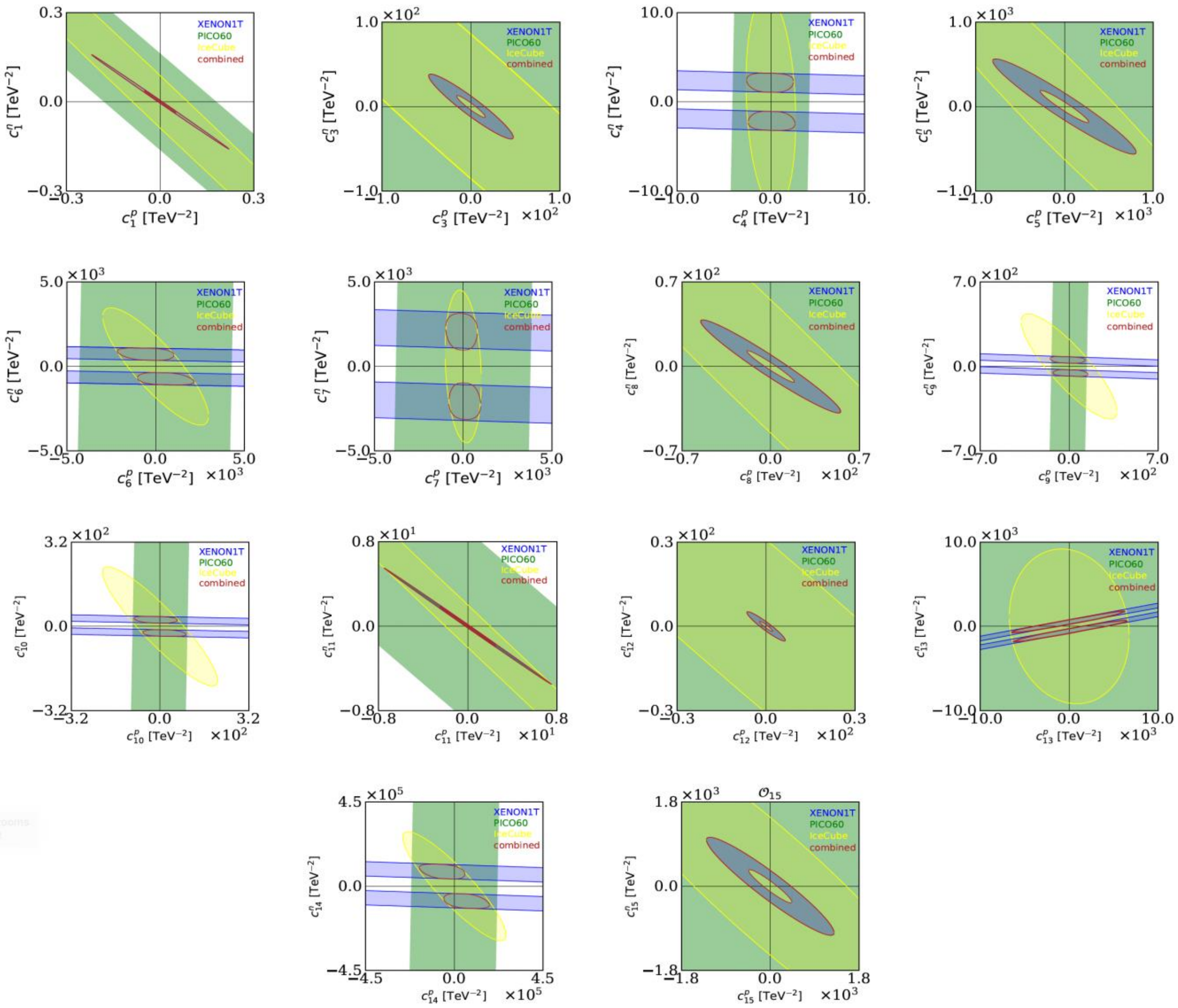
Bracketing DD experiments and capture

- Some of DD exclusion plots have Flat directions
- The Sun has many different targets not included in DD
  → more stable

- Adding new targets with different flat directions may be important rather than improving each experimental sensitivity



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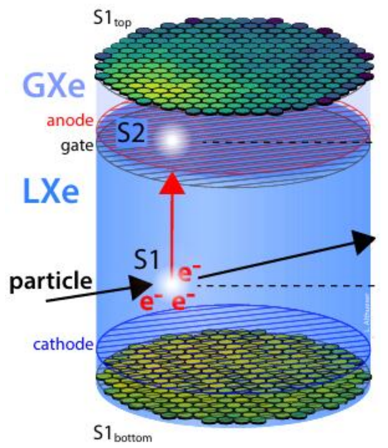
Complementarity between Direct Detection and Capture in the Sun: Halo-Independent bounds

$$R \sim \int dv\, H(v)\, f(v) \sim \int dv\, \mathcal{R}(v)\, \eta(v)$$



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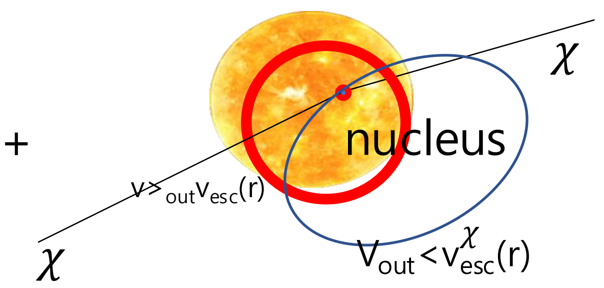

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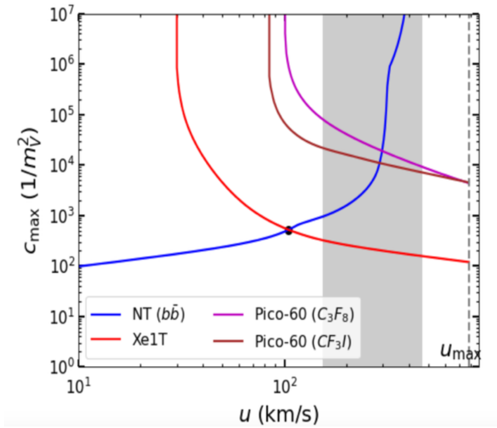
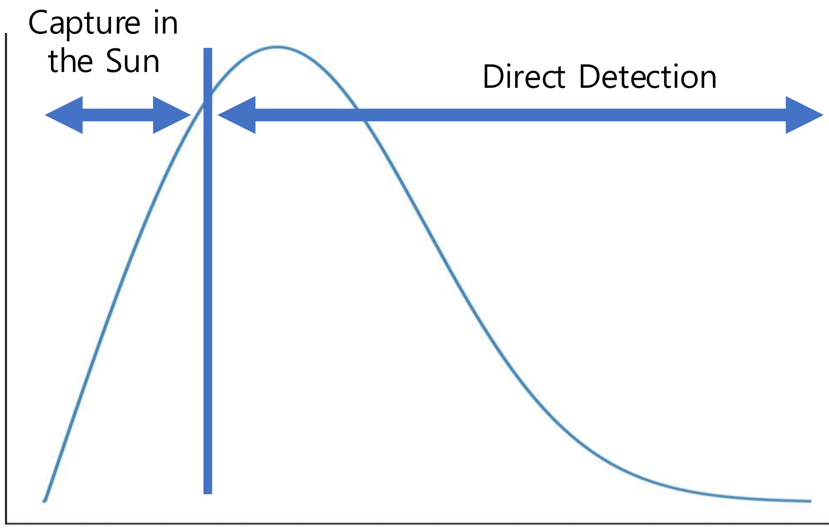


Halo independent approach

- Halo independent approach with arbitrary speed distribution, $f(u)$
  - The only constraint: $\int_{u=0}^{u_{max}} f(u)\, du = 1$

- Direct detection experiments have a threshold $u > u_{th}^{DD}$
  - Due to the energy threshold of experimental detectors

- Capture in the Sun is favored for low WIMP speeds
  - $u < u_{max,T}^{C}$



- In order to cover full speed range: combine DD and capture

F. Ferrer, A. Ibarra, S. Wild "A novel approach to derive halo-independent limits on dark matter properties", JCAP09(2015)052

Halo independent approach

- Considering one effective coupling (c_i) at a time:
  - $R_{exp}\left(c_i^2\right) = \int du\, f(u)\, H_{exp}\left(c_i^2, u\right) \leq R_{max}$
  - R_{max} : corresponding maximum experimental bound

- Using relation : $H\left(c_i^2, u\right) = c_i^2 H(c_i = 1, u)$
  - $H\left(c_{i,max}^2(u), u\right) = R_{max}$
  - $c_{i,max}^2(u) = \frac{R_{max}}{H(c_i=1,u)}$
  - $c_{i,max}(u)$: upper limit on c_i at single speed stream u

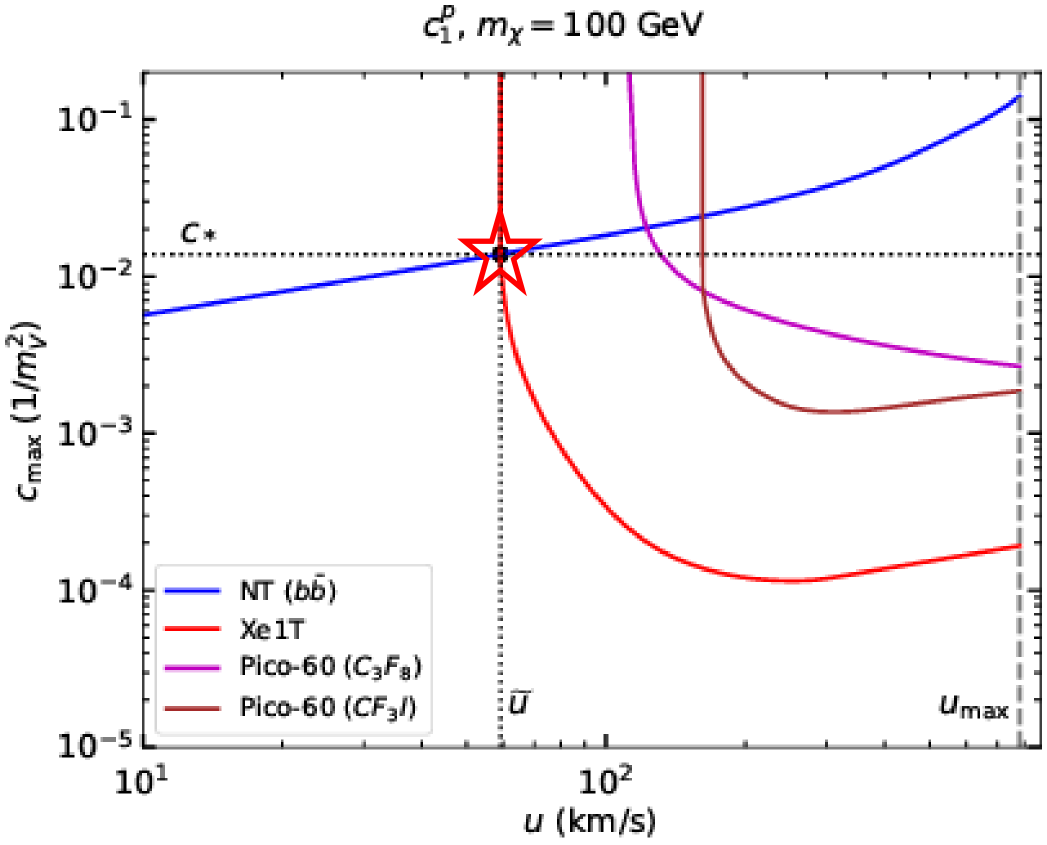
- upper limit on c_i over whole streams:

$$c_i^2 \leq \left[\int_0^{u_{max}} du\, \frac{f(u)}{c_{i,max}^2(u)}\right]^{-1}$$

F. Ferrer, A. Ibarra, S. Wild "A novel approach to derive halo-independent limits on dark matter properties", JCAP09(2015)052

Halo independent approach

- $c_* = c_{max}^{NT}(\tilde{u}) = c_{max}^{DD}(\tilde{u})$: halo independent limit
- $\tilde{u}$: intersection speed of NT and DD



S. Kang, A. Kar, S. Scopel, JCAP03(2023)011

- To cover whole speed range, one may combine DD and NT
  - $u_T^{C-max} = v_{esc}(r)\sqrt{\frac{4m_\chi m_T}{(m_\chi - m_T)^2}}$
  - $\left(u_{th}^{DD}\right)^2 = \frac{m_T}{2\mu_{\chi T}^2} E_{R,th}$

F. Ferrer, A. Ibarra, S. Wild "A novel approach to derive halo-independent limits on dark matter properties", JCAP09(2015)052

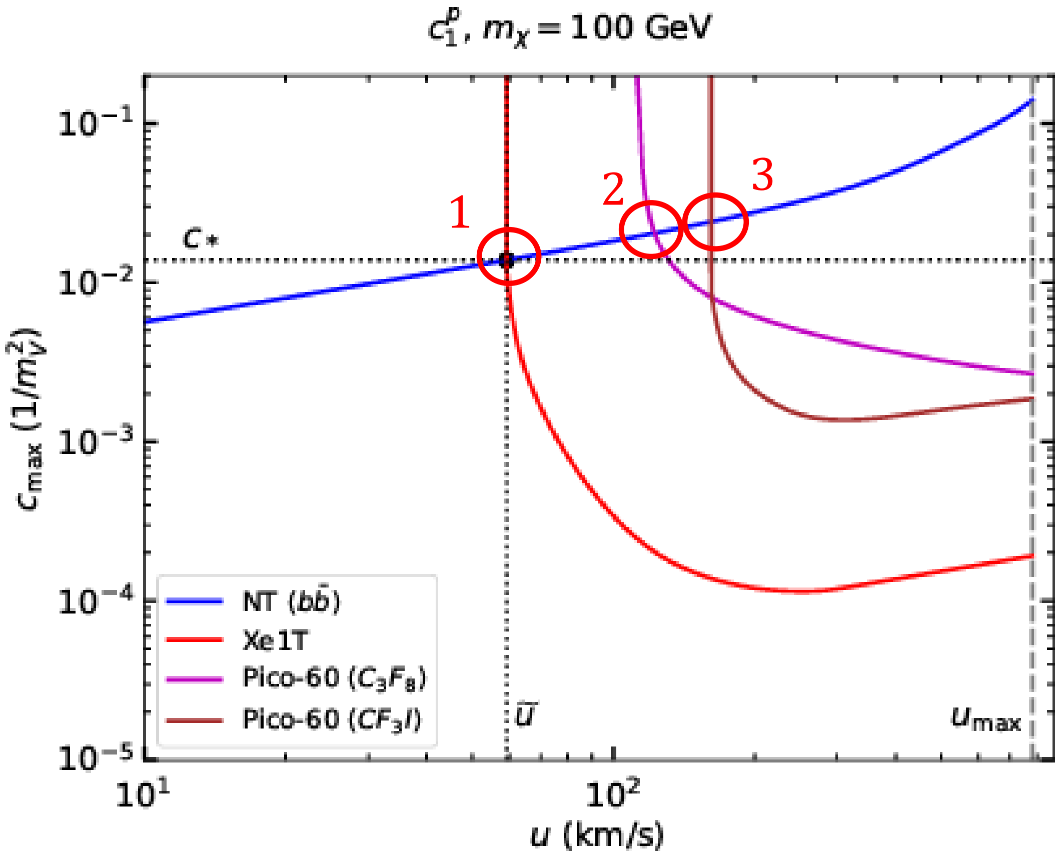
Halo independent approach case I

- Intersection:

$$(c^{NT})^2_{max}(u) \le c_*^2 \quad \text{for } 0 \le u \le \tilde{u}$$
$$(c^{DD})^2_{max}(u) \le c_*^2 \quad \text{for } \tilde{u} \le u \le u_{max}$$

$$c^2 \le c_*^2 \left[\int_0^{\tilde{u}} du\, f(u)\right]^{-1} = \frac{c_*^2}{\delta}$$
$$c^2 \le c_*^2 \left[\int_{\tilde{u}}^{u_{max}} du\, f(u)\right]^{-1} = \frac{c_*^2}{1-\delta}$$
$$\delta = \int_0^{\tilde{u}} du\, f(u)$$

$$c^2 \le 2c_*^2$$



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F. Ferrer, A. Ibarra, S. Wild "A novel approach to derive halo-independent limits on dark matter properties", JCAP09(2015)052

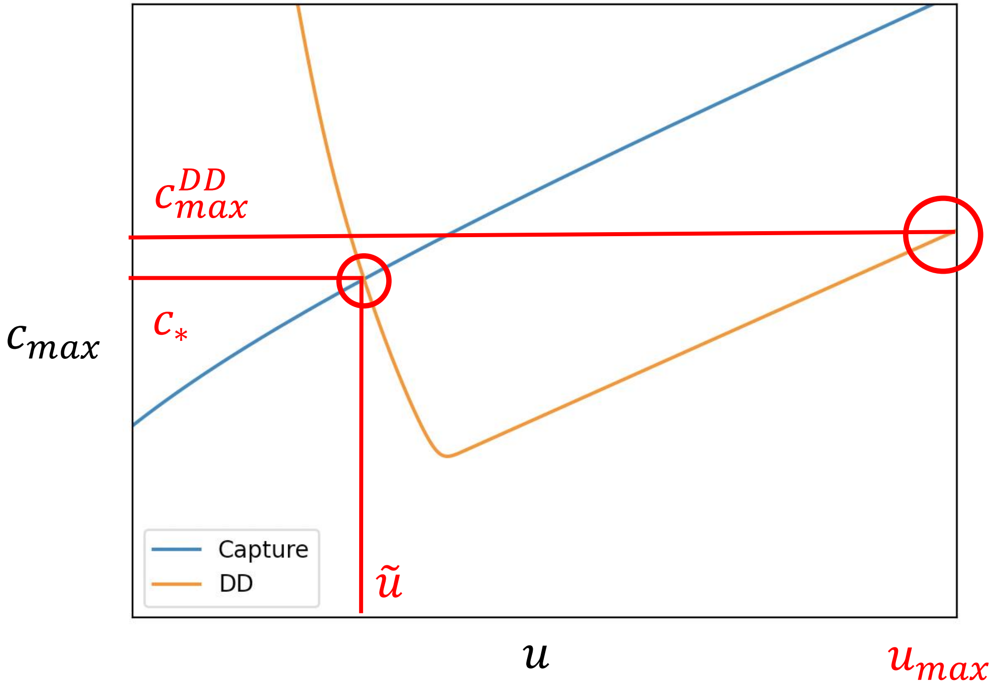
Halo independent approach case II

- If $(c^{DD})^2_{max}(u) > c_*^2$ at $u = u_{max}$:

$$c^2 \leq c_*^2 \left[\int_0^{\tilde{u}} du\, f(u)\right]^{-1} = \frac{c_*^2}{\delta}$$

$$c^2 \leq (c^{DD})^2_{max}(u_{max}) \left[\int_{\tilde{u}}^{u_{max}} du\, f(u)\right]^{-1} = \frac{(c^{DD})^2_{max}(u_{max})}{1-\delta}$$

$$c^2 \leq (c^{DD})^2_{max}(u_{max}) + c_*^2$$



- Halo independent limit may depend on u_{max}

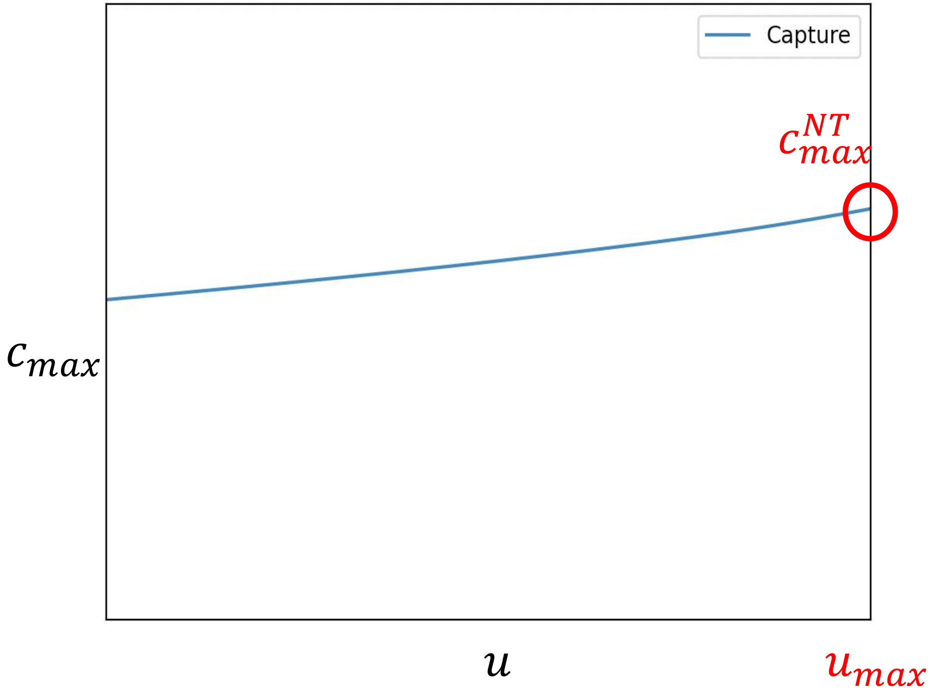
F. Ferrer, A. Ibarra, S. Wild "A novel approach to derive halo-independent limits on dark matter properties", JCAP09(2015)052

Halo independent approach case III

- If $u_{th}^{DD} > u_{max}$:

  Capture cover full speed range alone

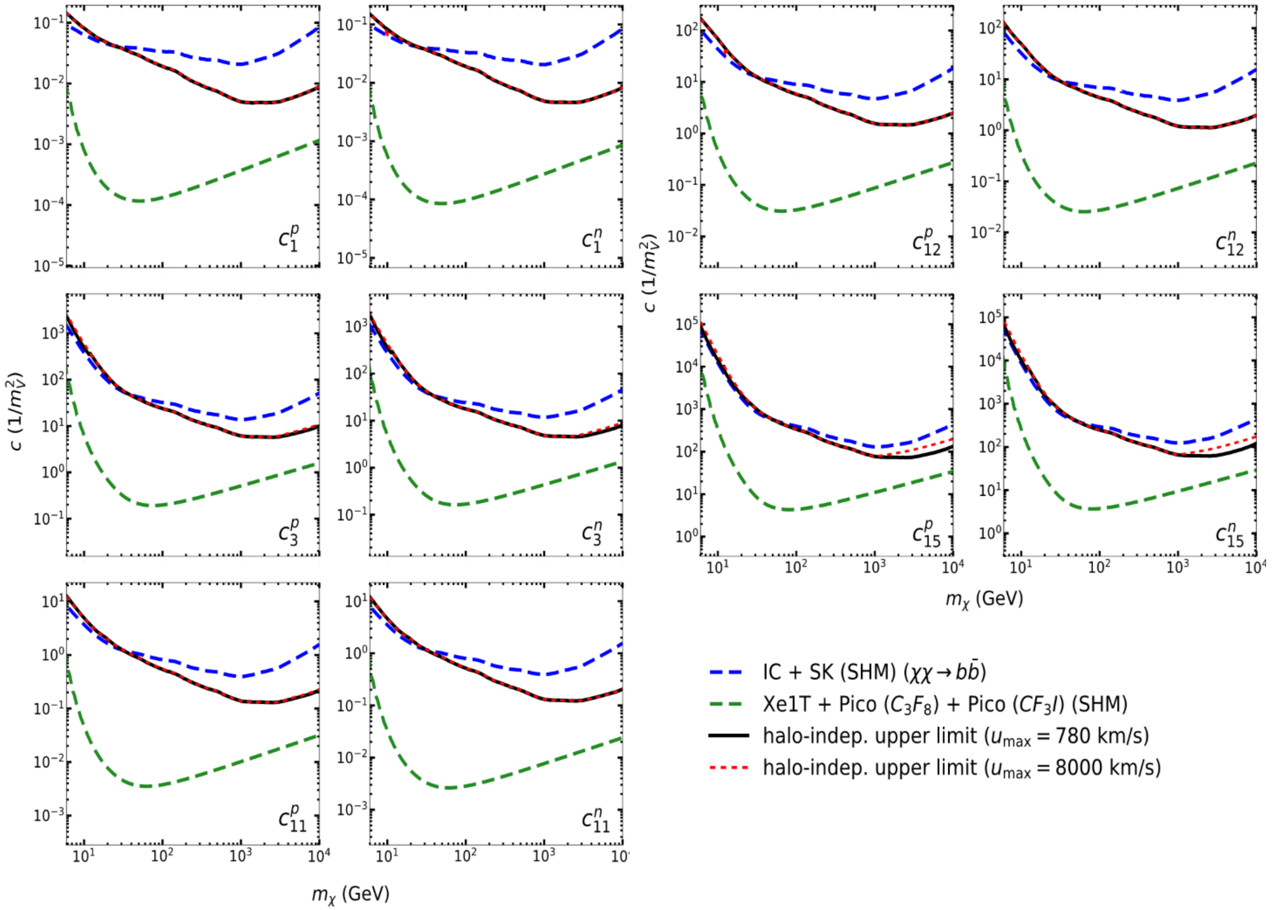
  $c^2 \leq (c^{NT})^2(u_{max})$

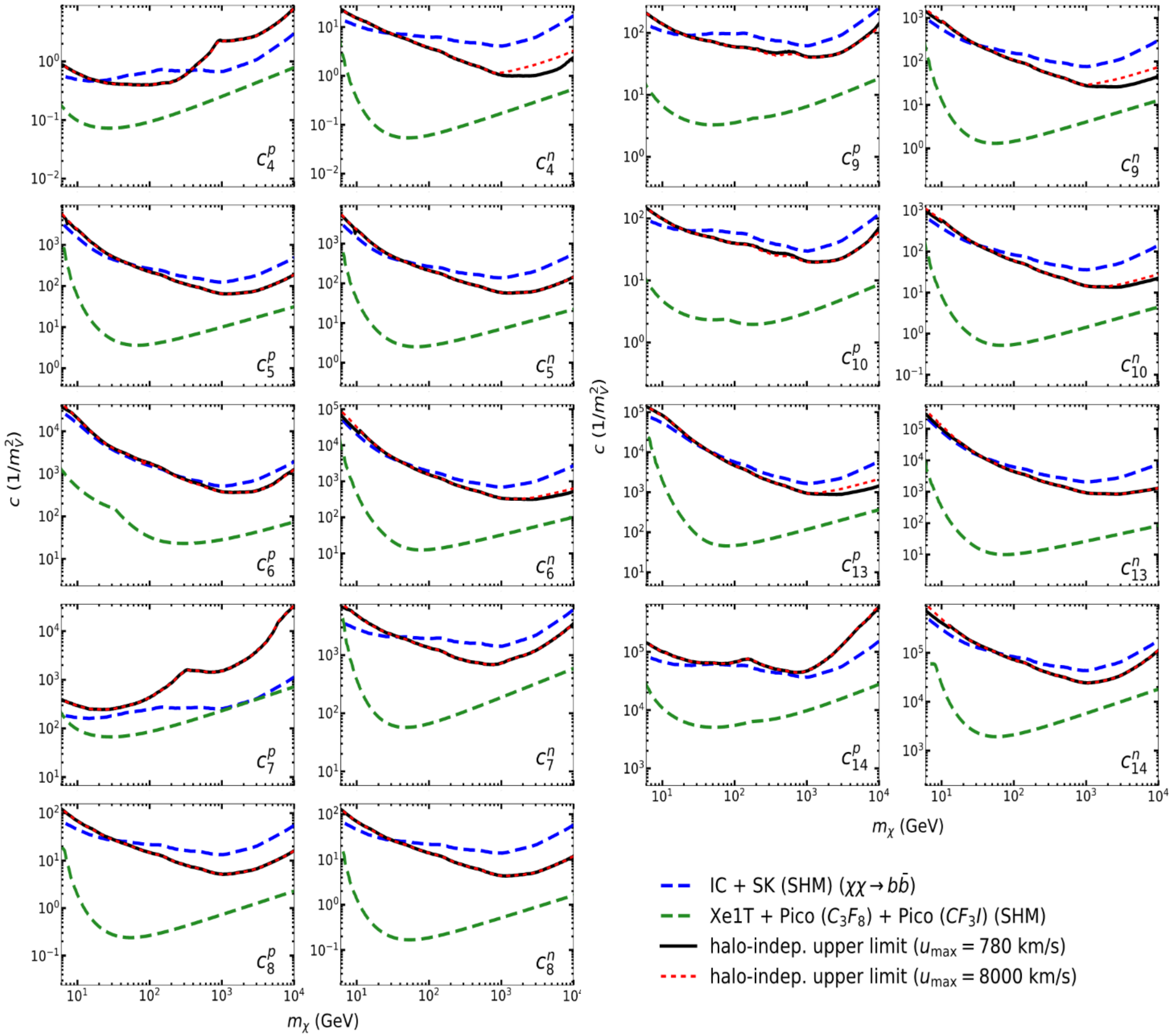


- Halo independent limit may depend on u_{max}

F. Ferrer, A. Ibarra, S. Wild "A novel approach to derive halo-independent limits on dark matter properties", JCAP09(2015)052

Halo independent approach

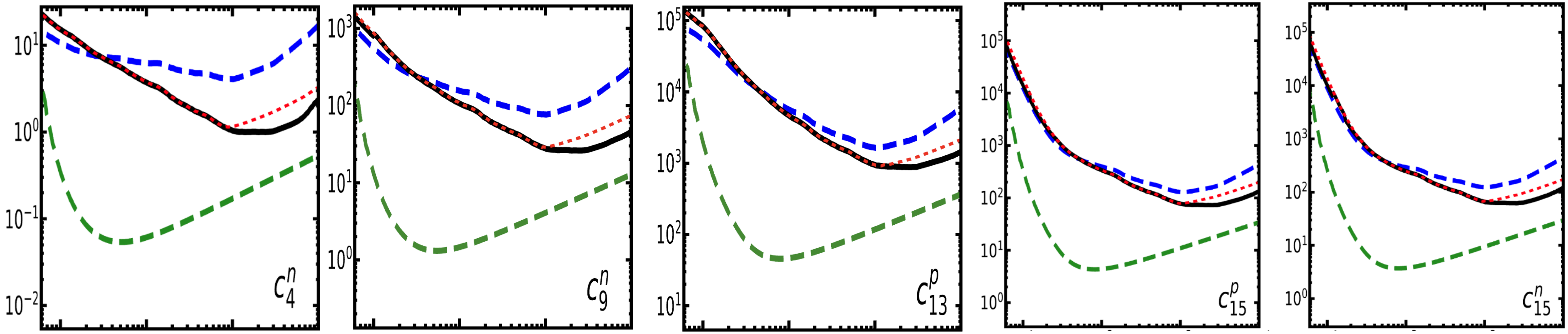




S. Kang, A. Kar, S. Scopel, "Halo-independent bounds on the non-relativistic effective theory of WIMP-nucleon scattering from direct detection and neutrino observations", JCAP03(2023)011 (arXiv:2212.05774)

Dependence on u_{max}

- Case II, $(c^{DD})^2_{max}(u) > c_*^2$ at $u = u_{max}$: $c^2 \le (c^{DD})^2_{max}(u_{max}) + c_*^2$
- Case III, $u_{th}^{DD} > u_{max}$: $c^2 \le (c^{NT})^2_{max}(u_{max})$
- u_{max}: 780 km/s → 8000 km/s
- Effect of large u_{max} is mild: factor less than 2



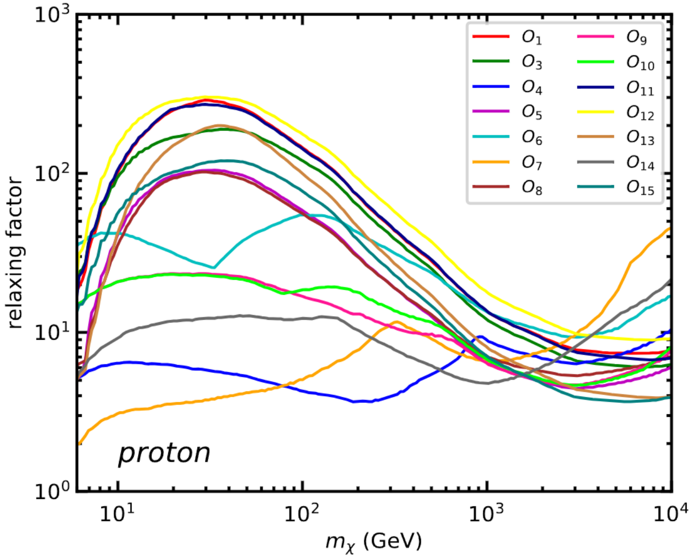
S. Kang, A. Kar, S. Scopel, "Halo-independent bounds on the non-relativistic effective theory of WIMP-nucleon scattering fro m direct detection and neutrino observations", JCAP03(2023)011 (arXiv:2212.05774)

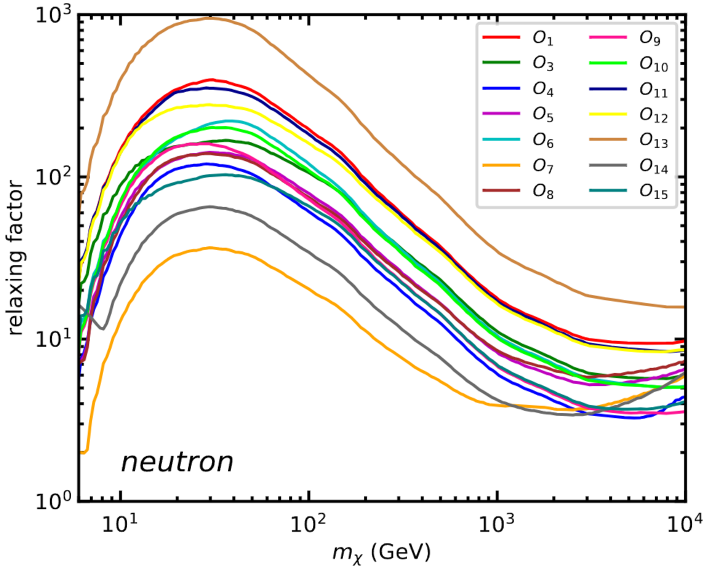
Relaxation factor

- Relaxation factor

$$r_f^2 = \frac{2c_*^2}{\left(c_{SHM}^{exp}\right)^2} = 2c_*^2 \int_0^{u_{max}} du \frac{f_M(u)}{(c^{exp})^2_{max}(u)} = 2c_*^2 < \frac{1}{(c^{exp})^2_{max}} > \cong 2c_*^2 < \frac{1}{(c^{exp})^2_{max}} >_{bulk}$$

$$\int_{bulk} du\, f_M(u) \approx 0.8$$





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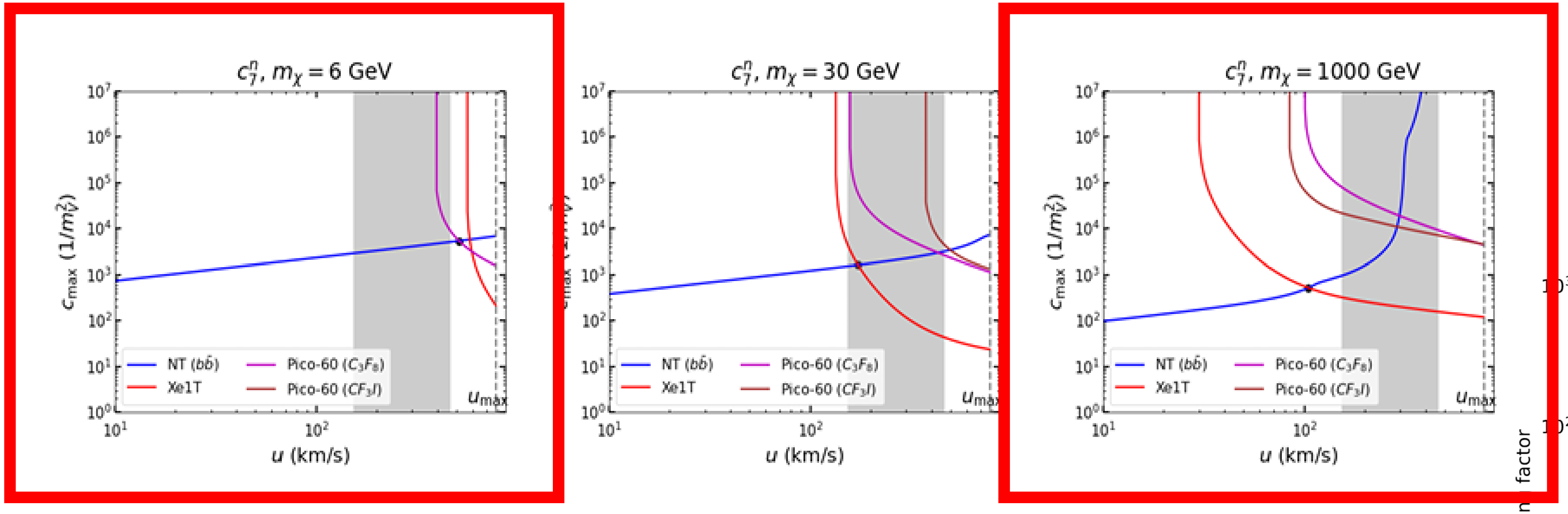

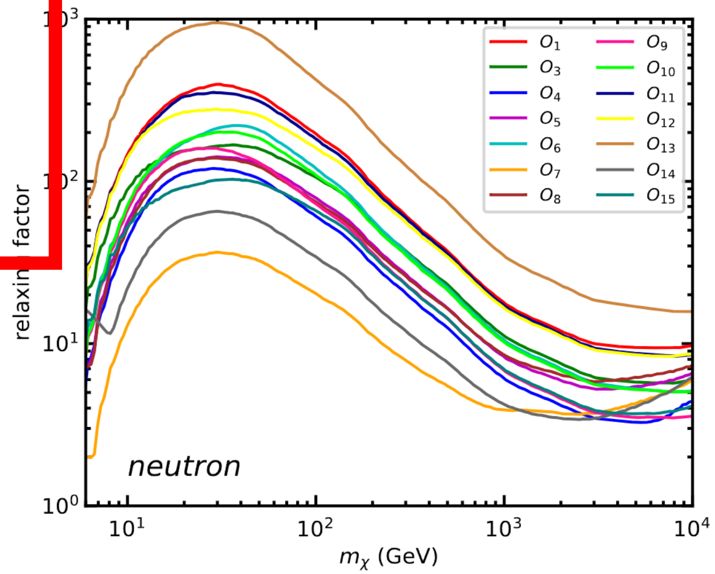

- small or large mass range
  - outside the bulk of Maxwellian
  - smooth dependence on u
- intermediate range (10 ~ 200 GeV)
  - inside the bulk of Maxwellian
  - steep dependence on u

$$r_f^2 \simeq 2c_*^2 \left\langle \frac{1}{(c^{\exp})^2_{\max}} \right\rangle_{\text{bulk}},$$

S. Kang, A. Kar, S. Scopel, “Halo-independent bounds on the non-relativistic effective theory of WIMP-nucleon scattering fro m direct detection and neutrino observations”, JCAP03(2023)011 (arXiv:2212.05774)

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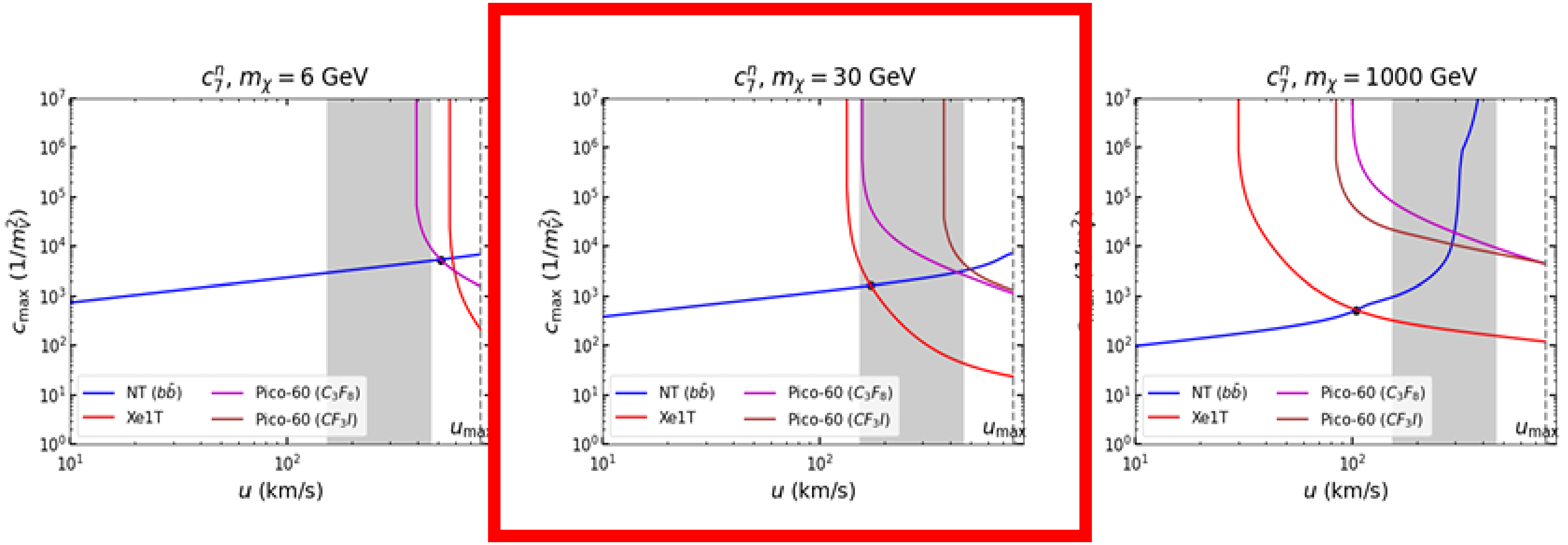

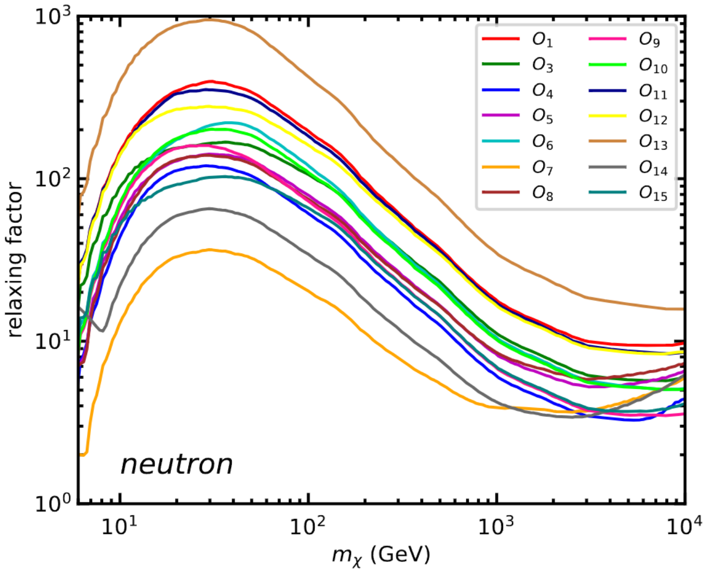

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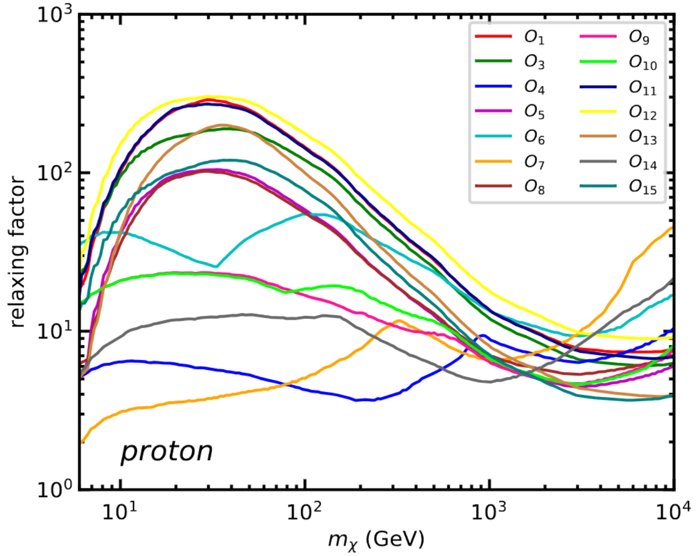
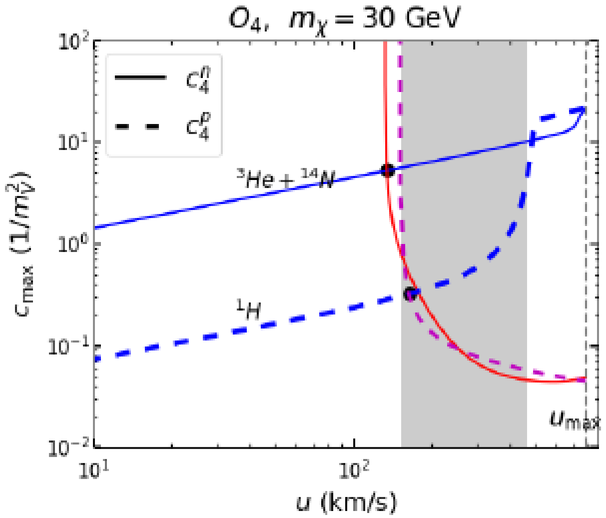
S. Kang, A. Kar, S. Scopel, "Halo-independent bounds on the non-relativistic effective theory of WIMP-nucleon scattering from direct detection and neutrino observations", JCAP03(2023)011 (arXiv:2212.05774)

Relaxing factor

- small relaxing factors
  - $O_{4,7}$: SD with no q suppression
  - $O_{9,10,14}$: SD with q^2 suppression
  - O_6 : SD with q^4 suppression



operator	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$	operator	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$
1	$M(q^0)$	-	3	$\Phi''(q^4)$	$\Sigma'(q^2)$
4	$\Sigma''(q^0)$,$\Sigma'(q^0)$	-	5	$\Delta(q^4)$	$M(q^2)$
6	$\Sigma''(q^4)$	-	7	-	$\Sigma'(q^0)$
8	$\Delta(q^2)$	$M(q^0)$	9	$\Sigma'(q^2)$	-
10	$\Sigma''(q^2)$	-	11	$M(q^2)$	-
12	$\Phi''(q^2)$,$\tilde{\Phi}'(q^2)$	$\Sigma''(q^0)$,$\Sigma'(q^0)$	13	$\tilde{\Phi}'(q^4)$	$\Sigma''(q^2)$
14	-	$\Sigma'(q^2)$	15	$\Phi''(q^6)$	$\Sigma'(q^4)$

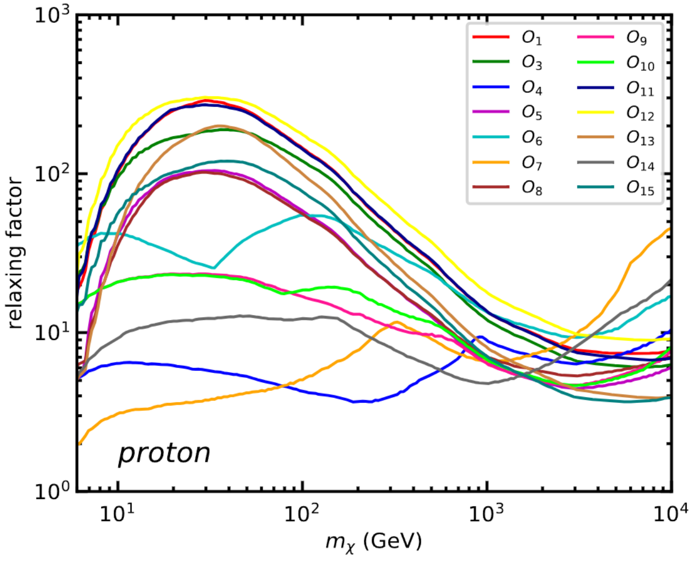


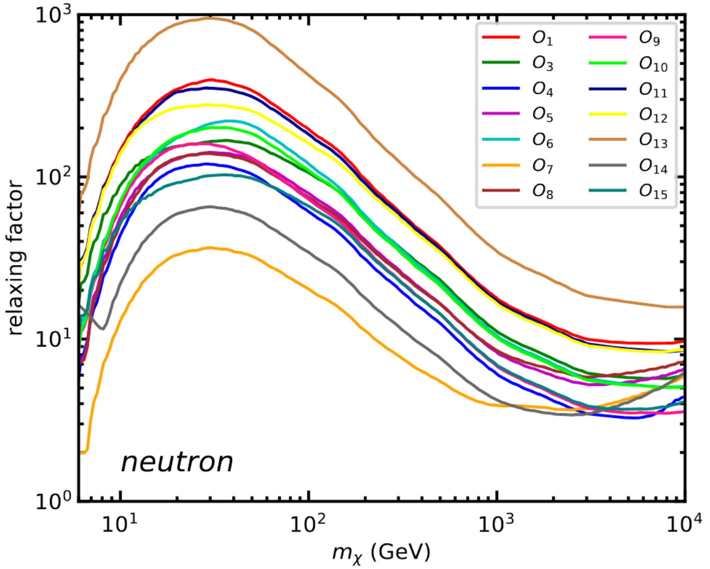
S. Kang, A. Kar, S. Scopel, "Halo-independent bounds on the non-relativistic effective theory of WIMP-nucleon scattering from direct detection and neutrino observations", JCAP03(2023)011 (arXiv:2212.05774)

Relaxation factor

- High relaxation factor:
  the halo-independent method can weaken the bound

$$r_f^2 = \frac{2c_*^2}{\left(c_{SHM}^{exp}\right)^2} = 2c_*^2 \int_0^{u_{max}} du \frac{f_M(u)}{(c^{exp})^2_{max}(u)} = 2c_*^2 < \frac{1}{(c^{exp})^2_{max}} > \cong 2c_*^2 < \frac{1}{(c^{exp})^2_{max}} >_{bulk}$$

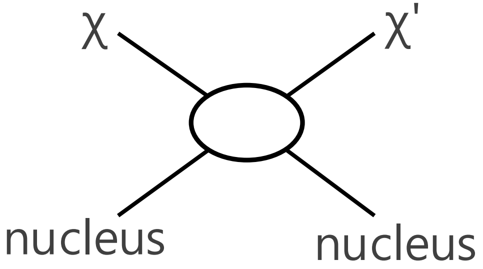




S. Kang, A. Kar, S. Scopel, “Halo-independent bounds on the non-relativistic effective theory of WIMP-nucleon scattering fro m direct detection and neutrino observations”, JCAP03(2023)011 (arXiv:2212.05774)

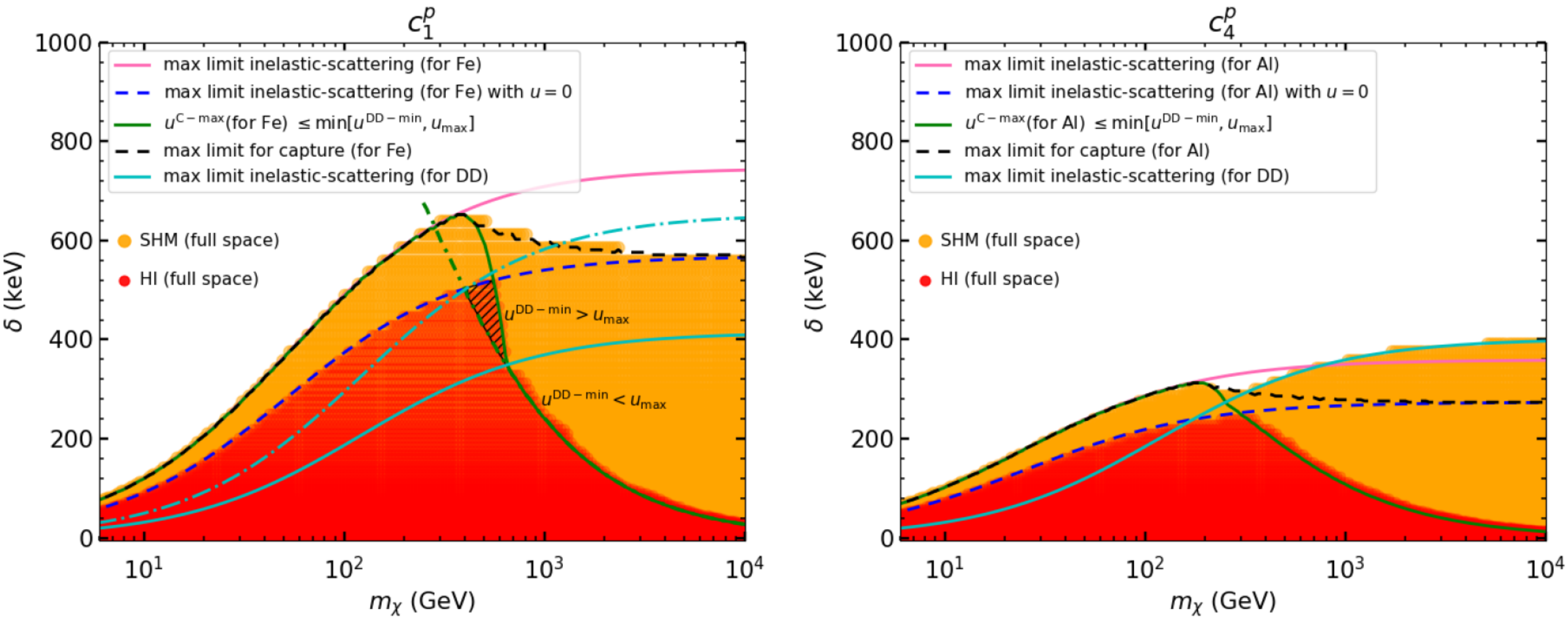
Kinematic conditions for inelastic scattering

- $u^2 + v_{esc}^2(r=0) > v_{T*}^2$
  - for inelastic scattering process to be kinematically possible
- $u > u^{DD-min}$
  - for the recoil energy to be above the DD experimental threshold
- $u < u^{C-max}$
  - for outgoing speed to be below the escape velocity in the Sun
- $u^{DD-min} < u^{C-max}$
  - for DD and capture intersect



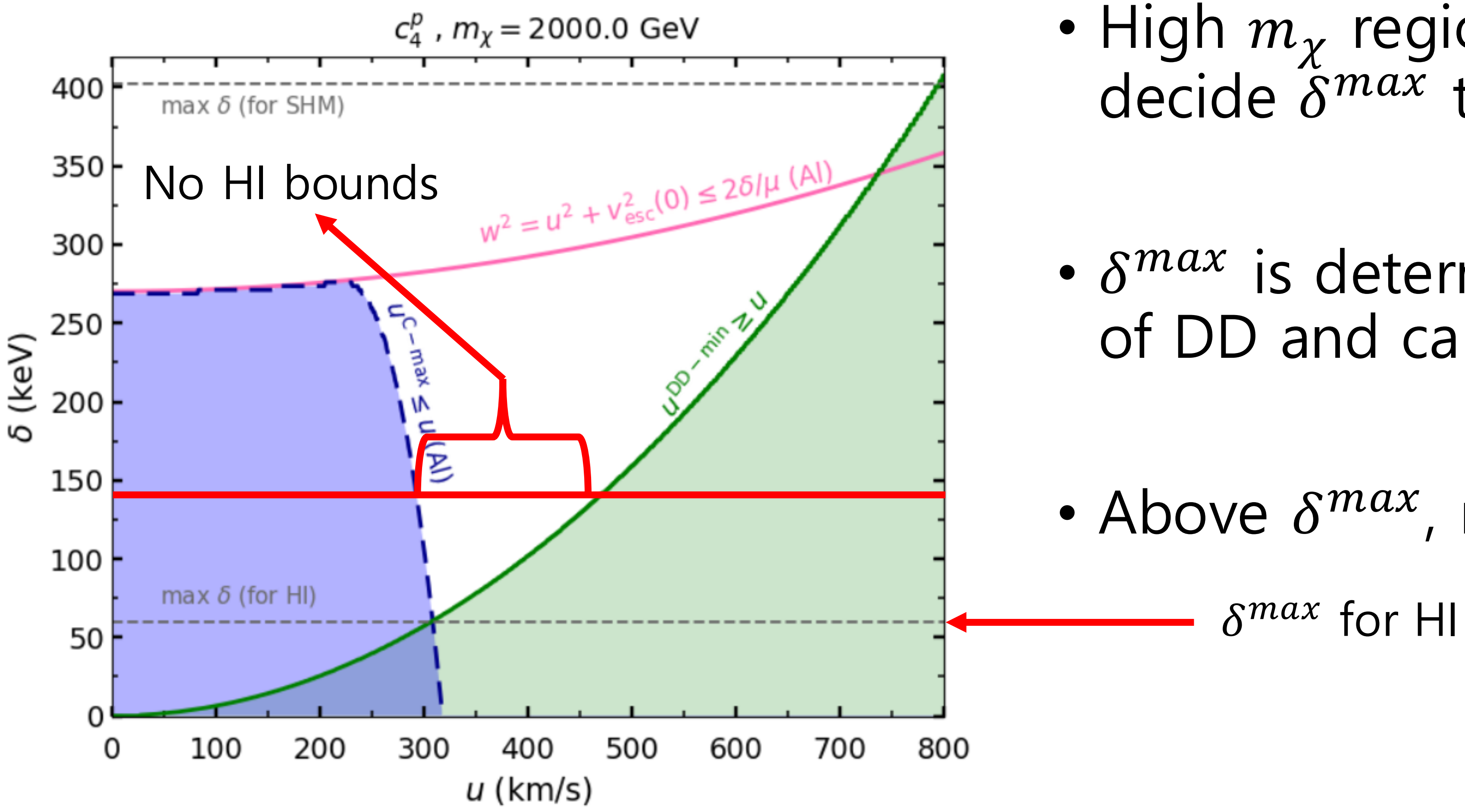
$\delta = m_{\chi'} - m_{\chi} > 0$

$m_\chi - \delta$ planes



S. Kang, A. Kar, S. Scopel, "Halo-independent bounds on Inelastic Dark Matter", JCAP11(2023)077 (arXiv:2308.13203)

Determining δ_{max}

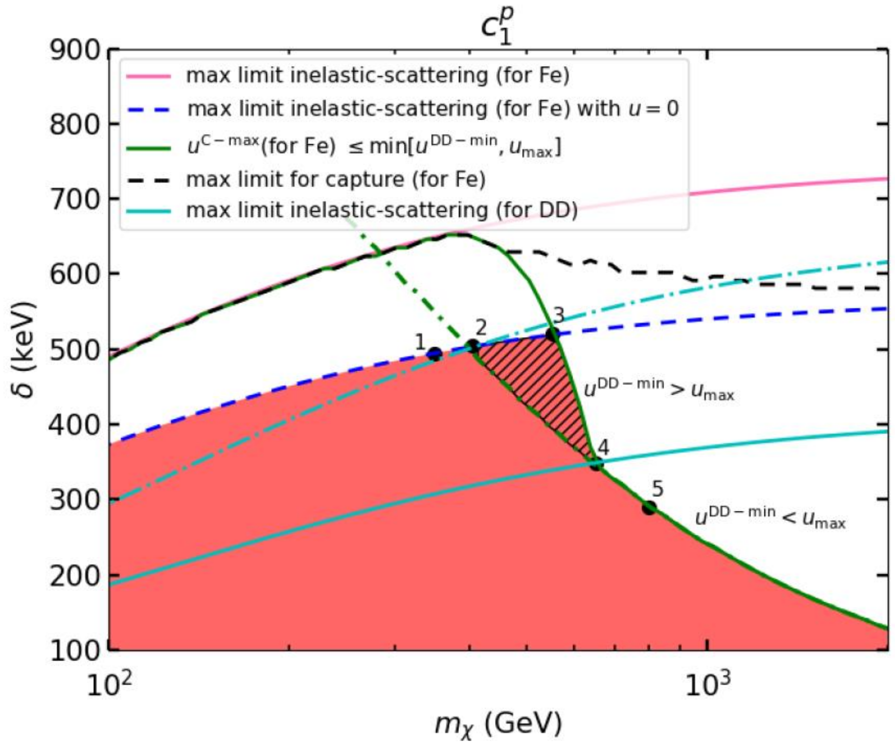


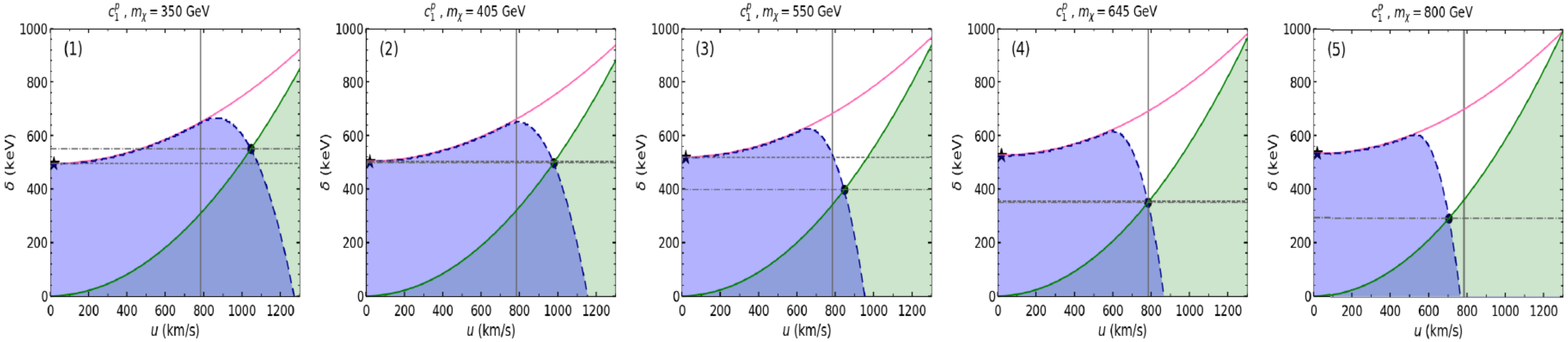
- High m_χ region, DD and capture decide δ^{max} together
- δ^{max} is determined by a combination of DD and capture
- Above δ^{max}, no HI bounds

S. Kang, A. Kar, S. Scopel, “Halo-independent bounds on Inelastic Dark Matter”, JCAP11(2023)077 (arXiv:2308.13203)

Dependence on u_{max}

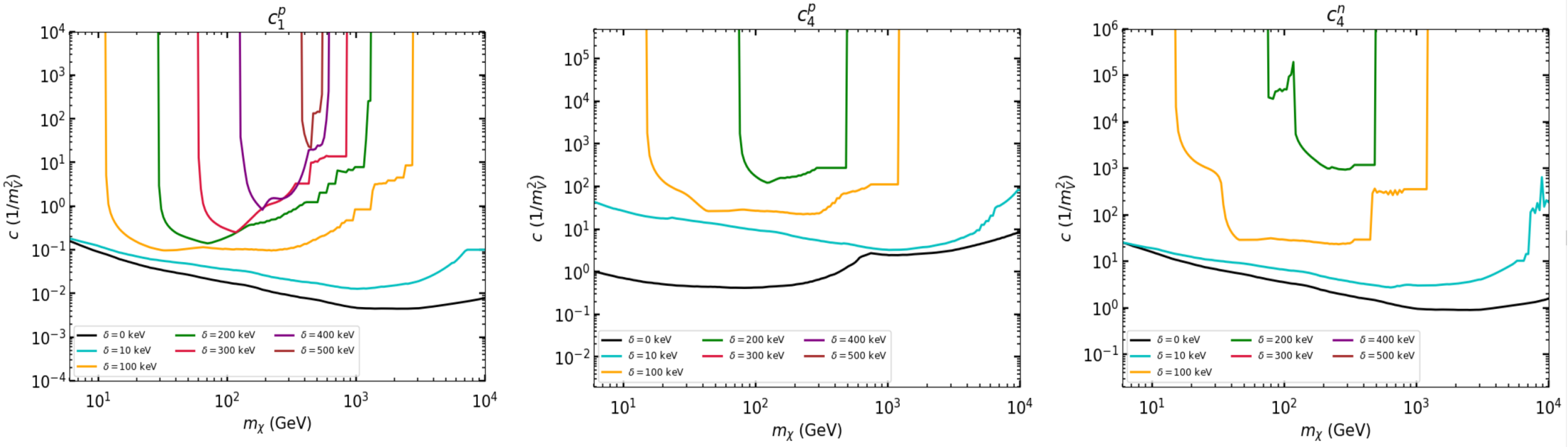
- HI might be sensitive to u_{max}
- except point (3), δ_{max} does not change even if we extend the value of u_{max}
- Only happens in SI case and δ_{max} does not decrease dramatically





S. Kang, A. Kar, S. Scopel, "Halo-independent bounds on Inelastic Dark Matter", JCAP11(2023)077 (arXiv:2308.13203)

Halo-independent exclusion plots

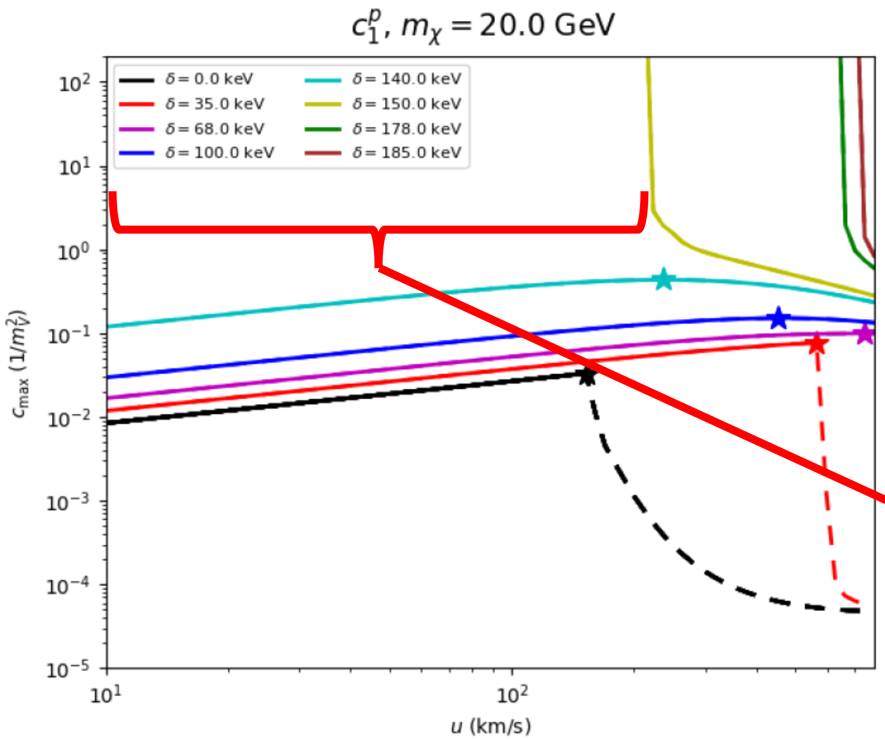


- HI exclusion plots of SI and SD couplings

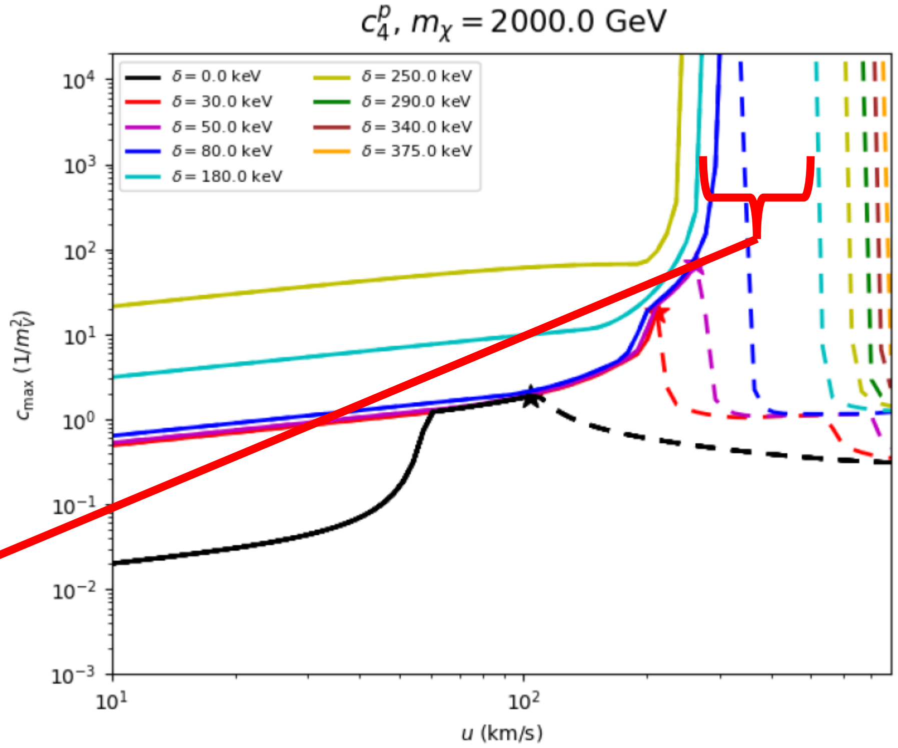
- For increasing δ the constraints get weaker and WIMP mass range for HI bounds is shrinking

S. Kang, A. Kar, S. Scopel, "Halo-independent bounds on Inelastic Dark Matter", JCAP11(2023)077 (arXiv:2308.13203)

Halo-independent exclusion plots



No HI bounds



- low m_χ: capture is kinematically impossible
- high m_χ: DD and capture doesn't intersect

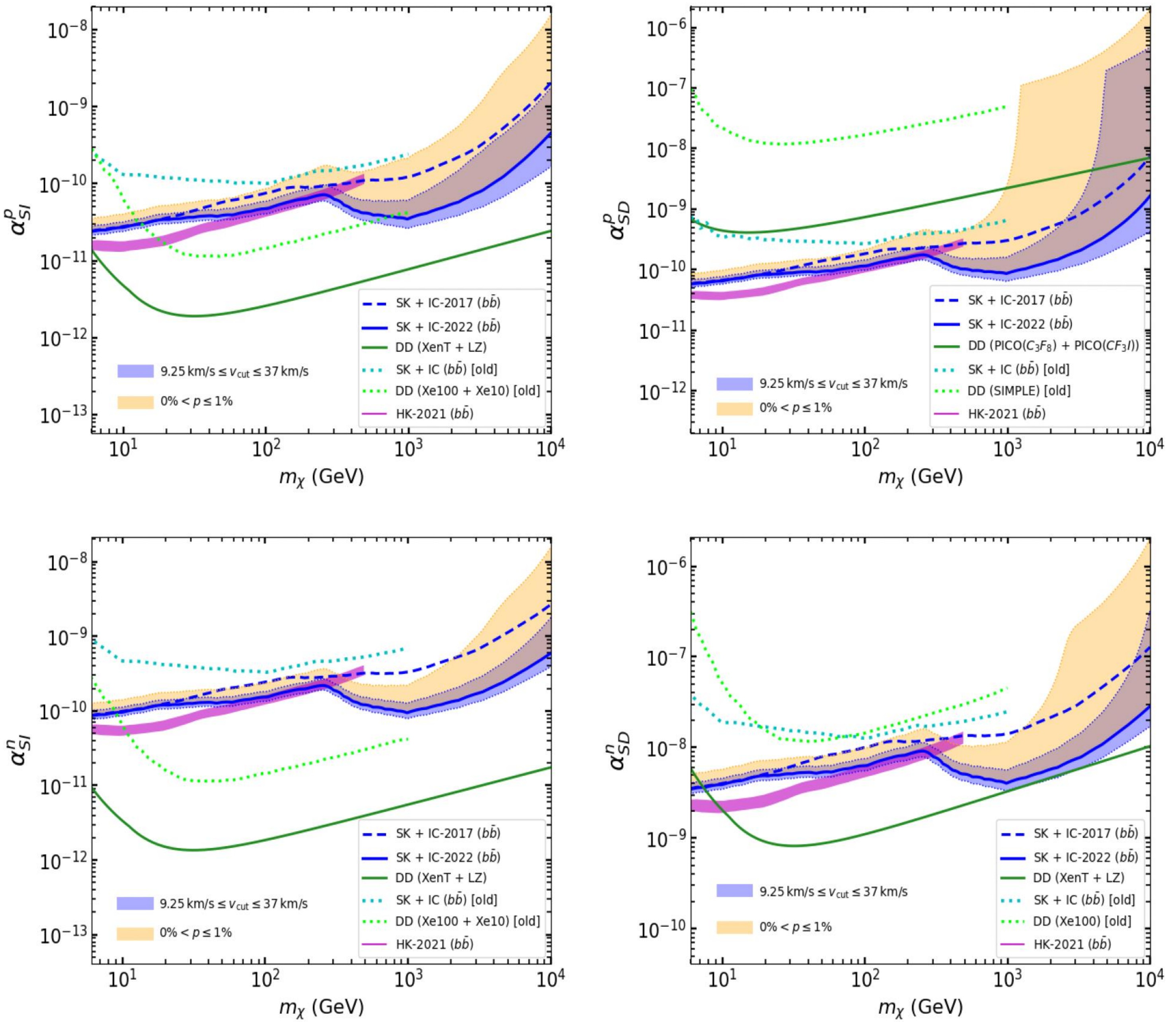
S. Kang, A. Kar, S. Scopel, "Halo-independent bounds on Inelastic Dark Matter", JCAP11(2023)077 (arXiv:2308.13203)

Bounds on α

- When $u \to 0$, $\int dE_R \frac{d\sigma}{dE_R}$ diverges
  - DD: no effect due to the $E_{R,th}$
  - NT: important

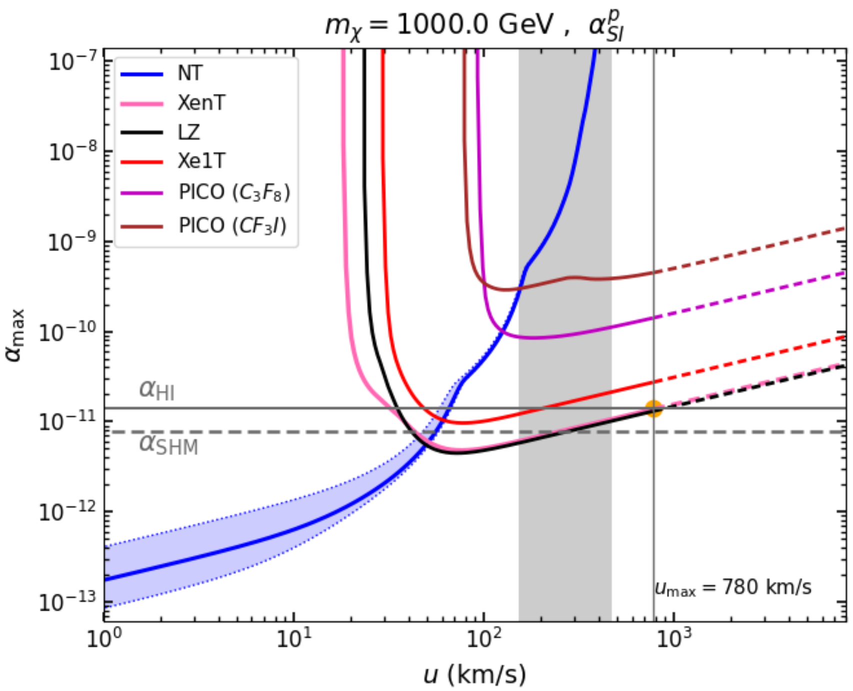
- Sensitivity on f(u)
  - removing lower tail of SHM
  - varying v_{cut}

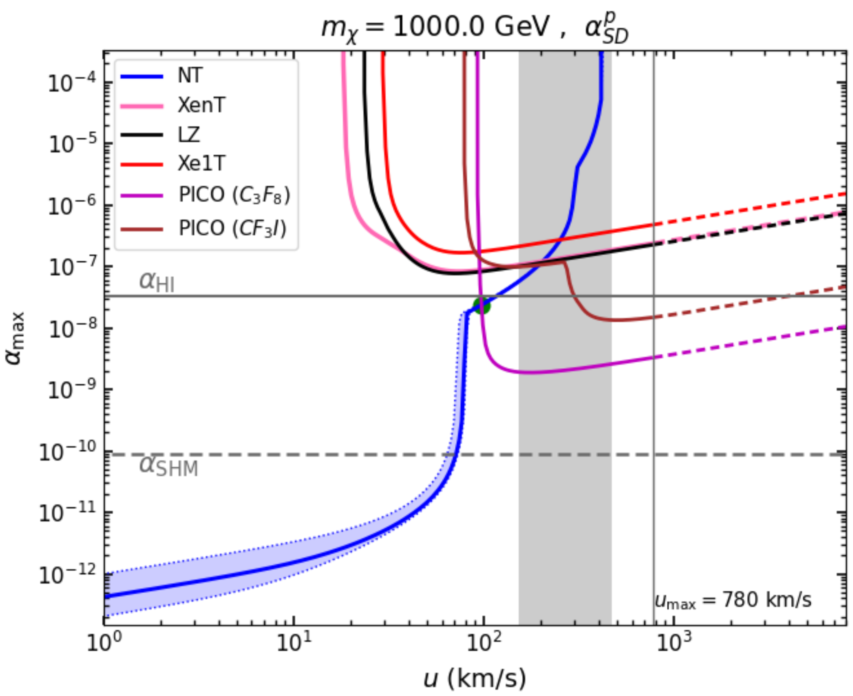
- Important at large m_χ
  - u_{max}^C is smaller than u_{max}
  - steep increasement
  - large sensitivity on f(u)



K. Choi, I. Jeong, S. K., A. Kar, S. Scopel, “Sensitivity of WIMP bounds on the velocity distribution in the limit of a massless m ediator”, JCAP01(2025)007 (arXiv:2408.09658)

HI bounds in long range interactions





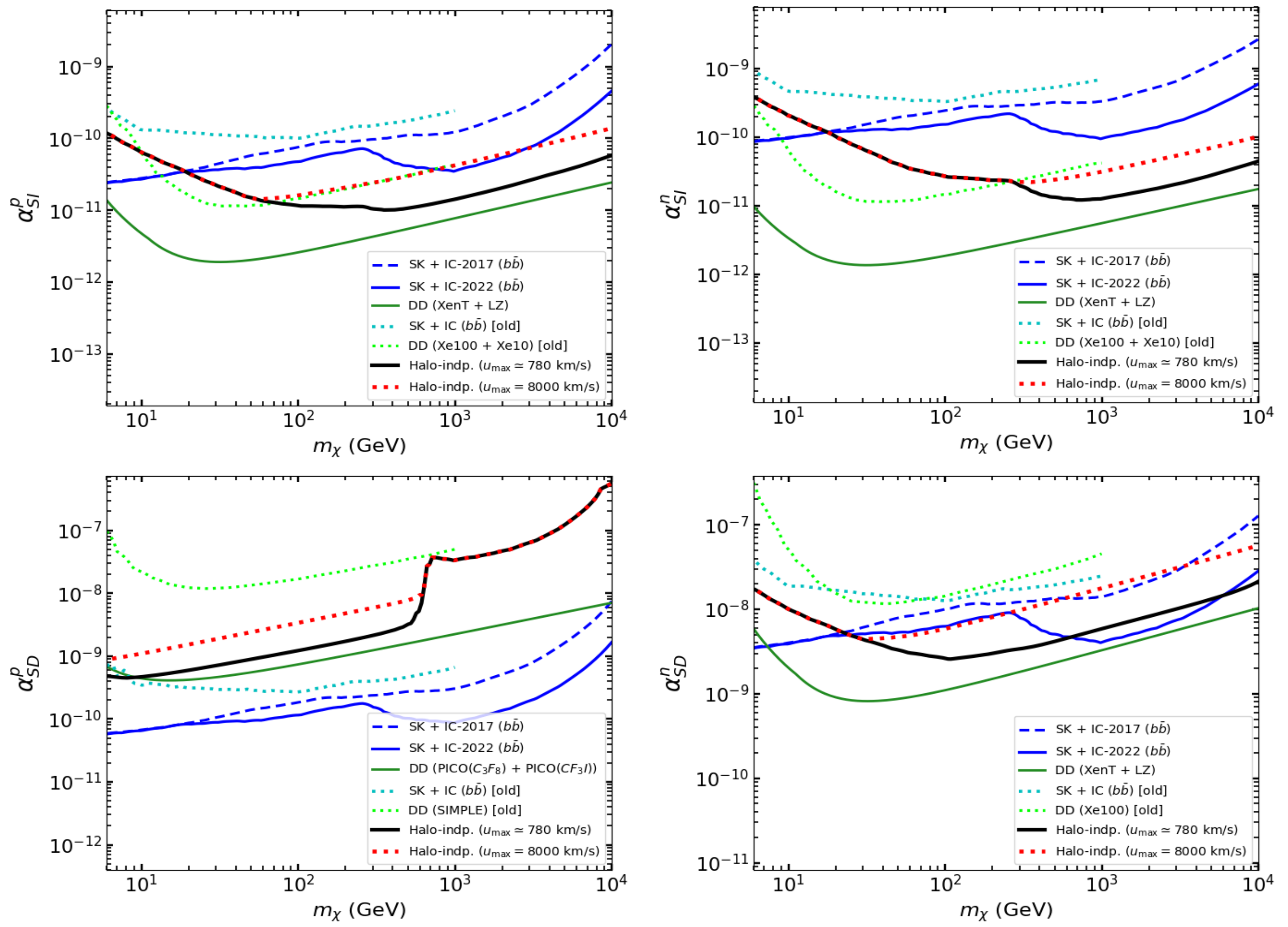
- v_{cut} does not affect much on HI bounds
- v_{cut} is sensitive at small u range

- Large u_{max} may change the type of HI
- Even if we tested unrealistic value, the effect is mild

K. Choi, I. Jeong, S. K., A. Kar, S. Scopel, "Sensitivity of WIMP bounds on the velocity distribution in the limit of a massless mediator", JCAP01(2025)007 (arXiv:2408.09658)

HI bounds in long range interactions

- Effect of large u_{max} is mild
  - factor ~ 2 or 3
- α_{max} released with larger u_{max}
  - suppression of form factor
  - finite energy bin
- Case II:
  $\alpha^2 \leq (\alpha^{DD})^2_{max}(u_{max}) + \alpha_*^2$



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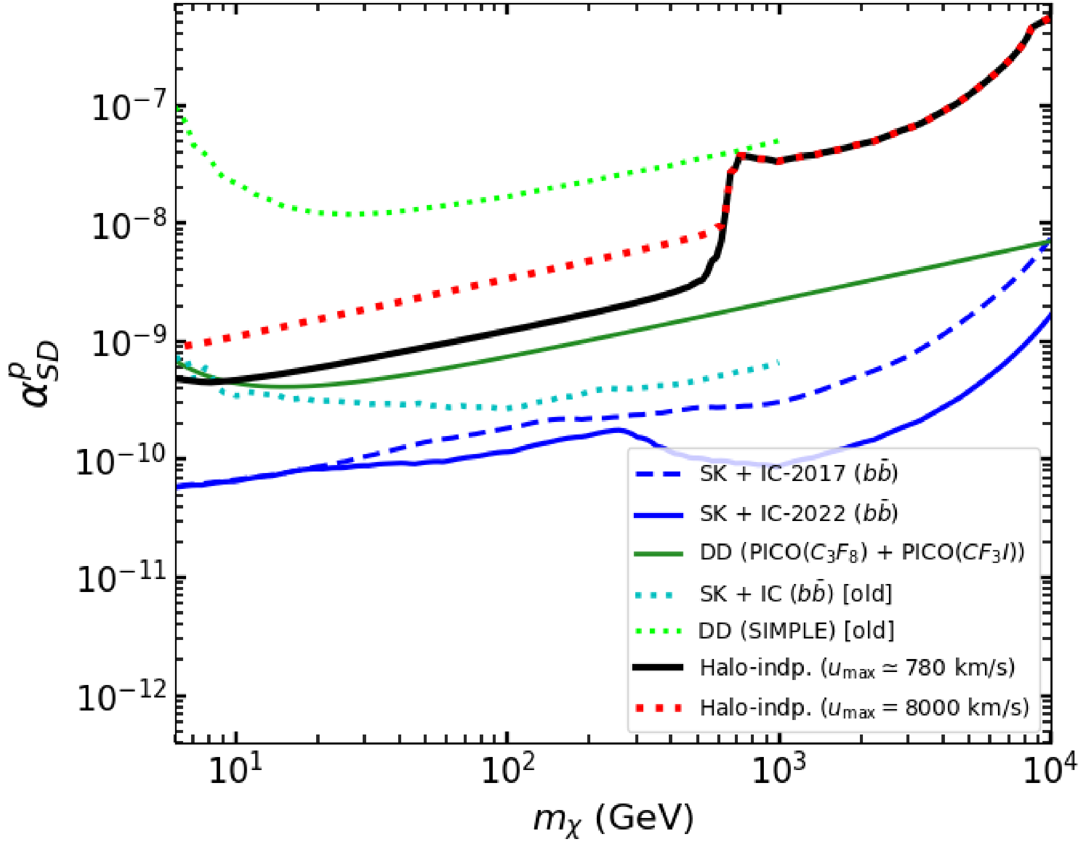

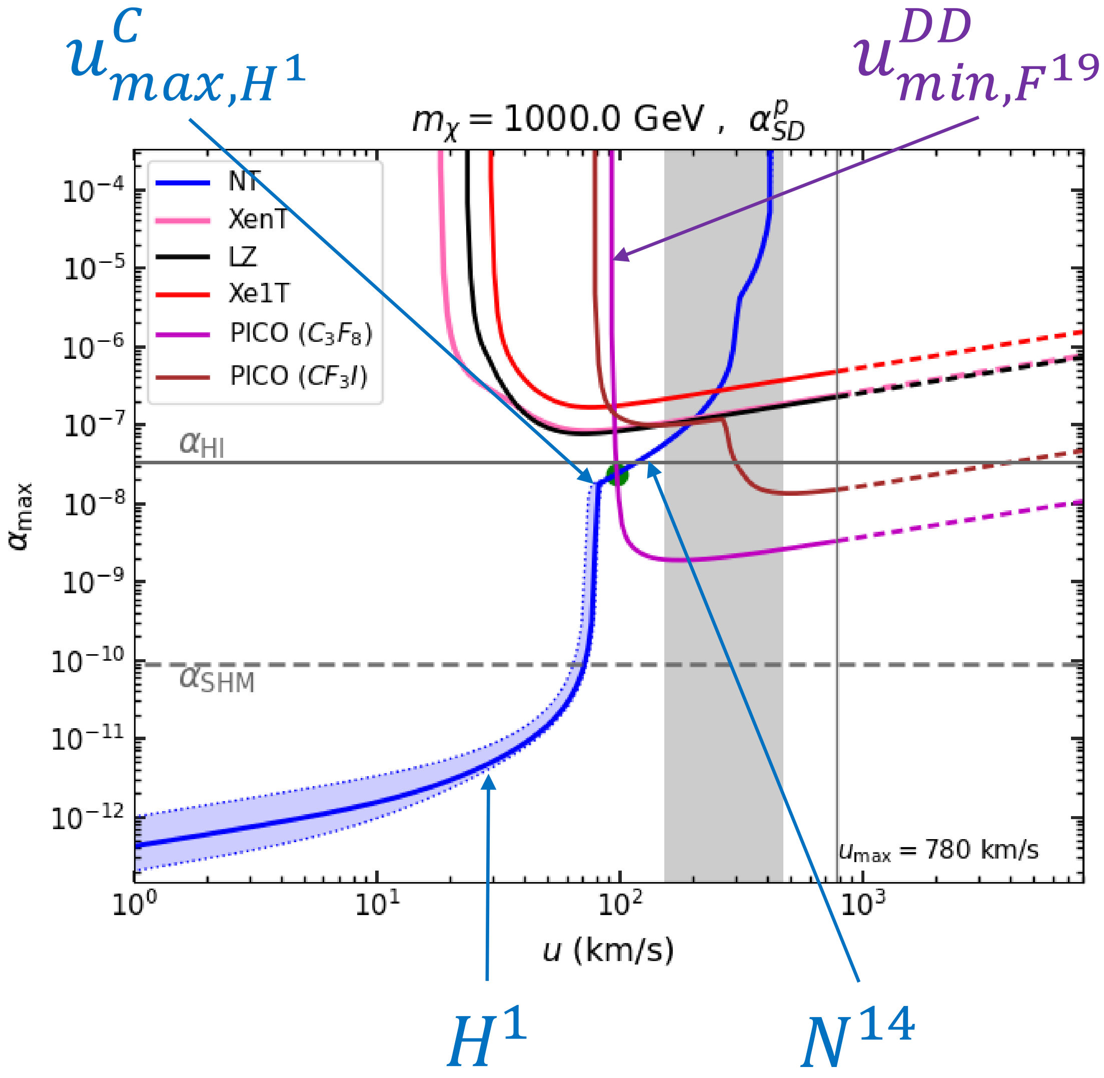

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Relaxation factor

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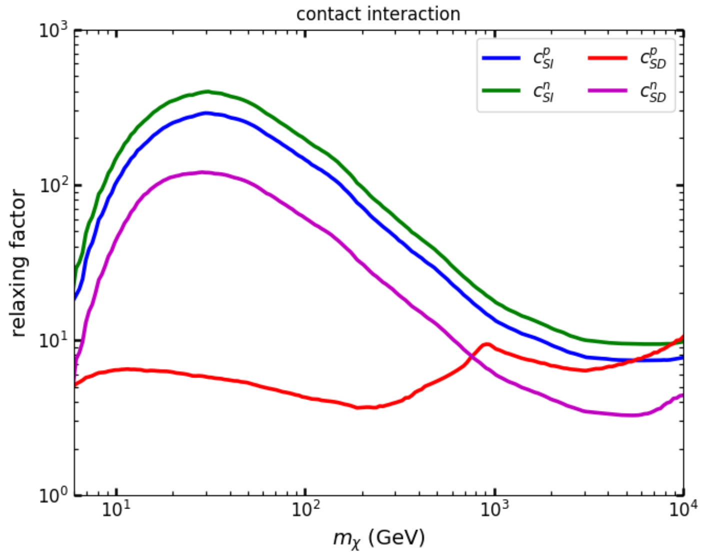
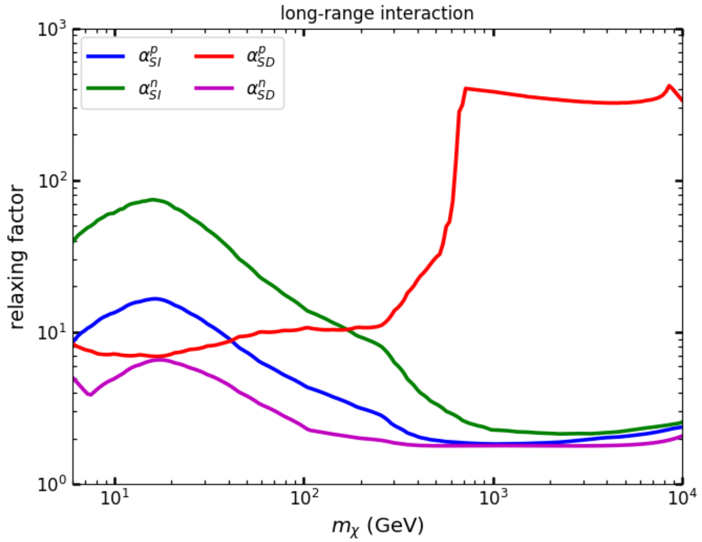
$$r_f^2 = \frac{\alpha_{HI}^2}{\left(\alpha_{SHM}^{exp}\right)^2} = \alpha_{HI}^2 \int_0^{u_{max}} du \frac{f_M(u)}{(\alpha^{exp})_{max}^2(u)} = \alpha_{HI}^2 < \frac{1}{(\alpha^{exp})_{max}^2} > \cong \alpha_{HI}^2 < \frac{1}{(\alpha^{exp})_{max}^2} >_{bulk}$$

$$\int_{bulk} du\, f_M(u) \approx 0.8$$





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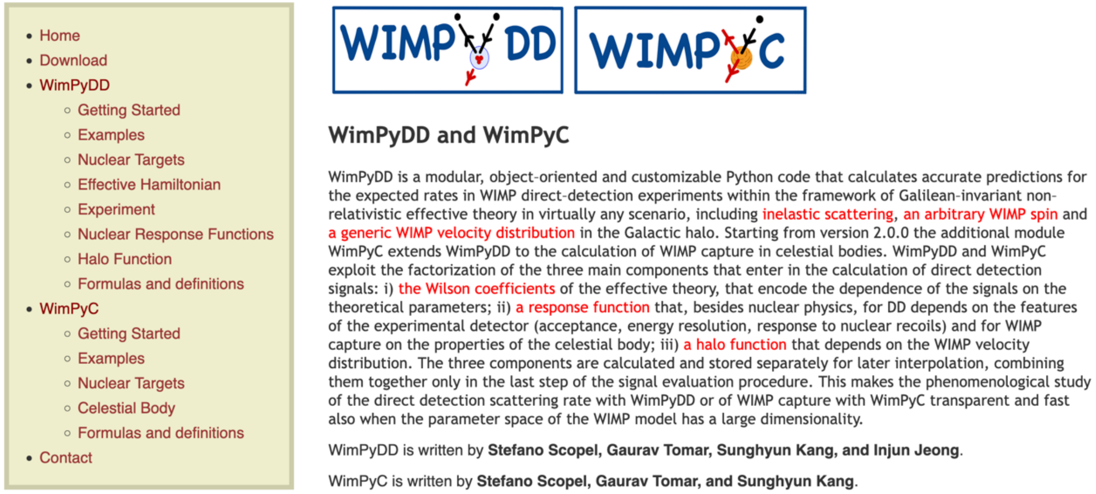
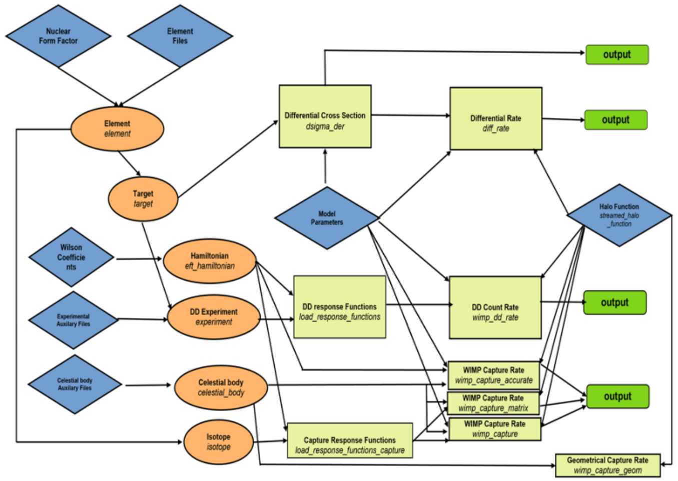
WimPyDD & WimPyC

- User-friendly Python code
  - object-oriented
  - customizable
  - portable



- Calculates expected rates in any scenarios
  - arbitrary spins
  - inelastic scattering
  - generic WIMP velocity distribution



- Published and can be downloaded:
  - https://wimpydd.hepforge.org/

I. Jeong, S. Kang, S. Scopel, G. Tomar, "WimPyDD: An object-oriented Python code for the calculation of WIMP direct detection signals", Computer Physics Communications, 2022.108342 (arXiv: 2106.06207)

WimPyDD

- Calculates DD expected rates in any scenarios

- Three main components
  - Wilson coefficients
  - response functions
  - halo function

- WimPyDD is transparent and fast, suitable for large parameter space

- **WimPyDD is used by the LZ collaboration:**
  - **Phys.Rev.Lett. 133 (2024) 22, 221801 (e-Print: 2404.17666)**
  - **Phys.Rev.D 109 (2024) 9, 092003 (e-Print: 2312.02030)**

I. Jeong, S. Kang, S. Scopel, G. Tomar, "WimPyDD: An object-oriented Python code for the calculation of WIMP direct detection signals", Computer Physics Communications, 2022.108342 (arXiv: 2106.06207)

WimPyDD

- Three main routines
  - WD.dsigma_der: calculates the differential cross section on target
  - WD.diff_rate: calculates the differential rate
    without including the response of the detector.
  - WD.wimp_dd_rate: calculates the integrated expected rates
    including the response of the detector

- Four input classes
  - element
  - target: a single element or linear combination of elements
  - eft_hamiltonian: with arbitrary combination of operators and Wilson coefficients
    - Two operator bases available: Anand et al. arXiv:1308.6288 (spin=0,1/2)
      Gondolo et al., arXiv:2008.05120 (any WIMP spin)
  - experiment: experimental set-up information

I. Jeong, S. Kang, S. Scopel, G. Tomar, "WimPyDD: An object-oriented Python code for the calculation of WIMP direct detection signals", Computer Physics Communications, 2022.108342 (arXiv: 2106.06207)

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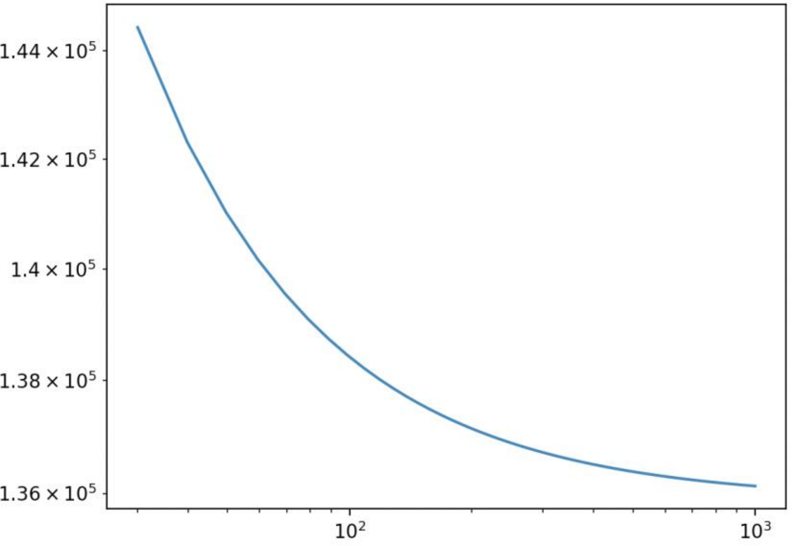
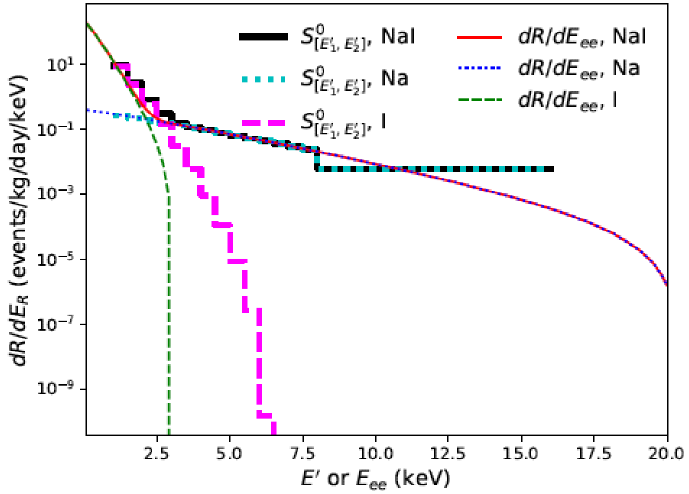
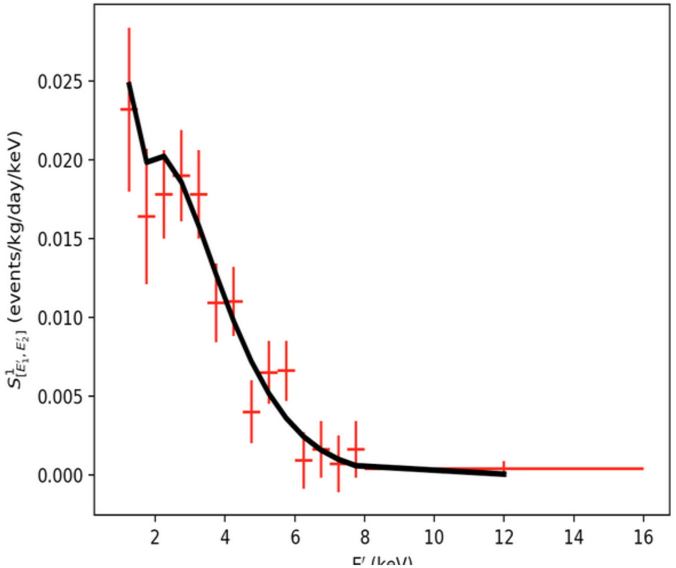






I. Jeong, S. Kang, S. Scopel, G. Tomar, "WimPyDD: An object-oriented Python code for the calculation of WIMP direct detection signals", Computer Physics Communications, 2022.108342 (arXiv: 2106.06207)

WimPyC

- Three main new routines
  - WD.wimp_capture: calculates the capture rate
  - WD.wimp_capture_matrix: calculates the matrix for the capture rate
  - WD.load_response_functions_capture: loads the response functions

- Two new classes
  - isotope: without including the response of the detector
  - celestial_body

S. Kang, S. Scopel, G. Tomar, "WimPyC: an extension module of WimPyDD for the calculation of WIMP capture in celestial bodies", arXiv: 2510.21185

WimPyC

- Set-up a new celestial body: The input string is the name of the directory WimPyDD/WIMP_Capture/Celestial_bodies/Sun containing the celestial body information

```
sun=WD.celestial_body('Sun')
```

Folder content

Name

- __init__.py
- densities.tab
- mass.tab
- radius.tab

list of isotopes

```
r_vec rho_tot 1H 4He 3He 12C 13C 14N 15N 16O 17O 18O 20Ne 23Na 24Mg 27Al 28Si P 32S Cl 40Ar K 40Ca Sc Ti V Cr Mn 56Fe Co 58Ni v_esc
0.0015 150.5 74.63614626654312 130.65634553820425 0.0017188252806313116 0.0016368611194505932 0.00037184318094558806 0.8410917473375288 0.0006597608134196186 1.3224012204963915 0.09025230786901557 0.004003887180686836 0.31107958145548975 0.00706209911106761
0.002 150.5 67.19046650038389 117.57028777936976 0.0015483371644967273 0.0014731234301690963 0.00033505793697199363 0.7572314070556714 0.0005940335665295522 1.1905085276814744 0.0810824714006352 0.003605986807137108 0.2798213836408719 0.006355372751098426
0.0025 150.5 64.52553403039796 112.84681831369802 0.001488003730392442 0.0014139956694090895 0.00032145279593987083 0.7271750222605013 0.000570069400315127 1.1433539295483293 0.07774602394978704 0.0034597190993191935 0.2686285282952369 0.006103486555925059
0.003 150.4 63.50307378922524 110.99518951225801 0.0014657444714226383 0.0013906037702674727 0.000316244070079734 0.7151408247680807 0.00056131209693806 5 1.1248047557397 0.07629879860423923 0.00340365883613578 0.2647556834925713 0.006002074779303533 0.14541
0.00525 150.25 60.90084494196752 105.90620122032831 0.0014162572661062902 0.0013274703964233994 0.0003017279427270817 0.6837446692817823 0.0005362763921841927 1.0763367890682543 0.07171985453495675 0.0032551924500586003 0.2527905942444279 0.00573832953657691
0.01325 148.6 56.574393470015075 95.25220280127195 0.001379546564346338 0.0011960111567691975 0.00027188763810846365 0.6223797558778326 0.00048798562200277505 0.9856582365842508 0.058421525870738844 0.0029613747555445767 0.22997906579777527 0.005222249178999
0.021249999999999998 145.6 56.562298882077904 90.12962342532337 0.001492511486500481 0.001134845985674977 0.00025802033746804697 0.601322616577177 0.00047127315690709086 0.961214501301532 0.04619772068847809 0.0028612400228405725 0.22215985586411946 0.005043
0.028999999999999995 141.7 57.2022180786331 84.71994121486166 0.0016719775246950605 0.0010704906443635377 0.00024347692001526147 0.5817617664042637 0.00045572175127310777 0.9397345698751723 0.03336390748624736 0.0027651621865156266 0.2146925770532356 0.00487
0.0365 137.0 58.069887496212424 78.66699479286336 0.0019191053089907528 0.0009982753141181157 5 0.0002269538573240494 0.5602196837657671 0.0004385411147387579 0.9141731090477088 0.02135085690462679 0.0026594610140918793 0.20665926682197203 0.004693936137701033
0.044 131.6 58.98064774213467 72.16520306993053 0.002239290992578019 0.0009193215105258635 0.00020904784646898013 0.537122944332282 0.0004199251325622905 0.8837753380037622 0.011581352394175065 0.002550561222403319 0.19801797311826033 0.004498560318865119
0.05150000000000001 125.79999999999998 59.75424233670353 65.42282543117136 0.002629006162939305 0.0008367785402188715 0.00019022025071556496 0.5117086085905717 0.00040025679512168213 0.8475900805513289 0.00458850493222707 0.002425080194997434 0.18845531422834
0.05899999999999998 119.80000000000001 60.31177762198074 58.72938202692113 0.003114812396257202 0.0007525245670774535 0.0001712585871126206 0.4861016089990862 0.00037969894895861284 0.8084258064797593 0.000311172899705523 0.002305201484595118 0.17925370699952
0.06649999999999999 113.60000000000001 60.555957717453595 52.25720729668912 0.003696034329423806 0.0006705248275337054 0.00015223944098350617 0.4592892565248690 7 0.0003587591144073656 0.7658579243518283 0.0 0.0021768269436432328 0.1688418484363898 0.003848
```

```
>>> print(sun)
Sun, radius 1.0 (solar units), mass 1.0 (solar units), densities of following
 targets:
['1H', '4He', '3He', '12C', '13C', '14N', '15N', '16O', '17O', '18O', '20Ne',
 '23Na', '24Mg', '27Al', '28Si', '32S', '40Ar', '40Ca', '56Fe', '58Ni']
contained in
WimPyDD/WIMP_Capture/Celestial_bodies/Sun/densities.tab.
No help provided for densities.tab content
```

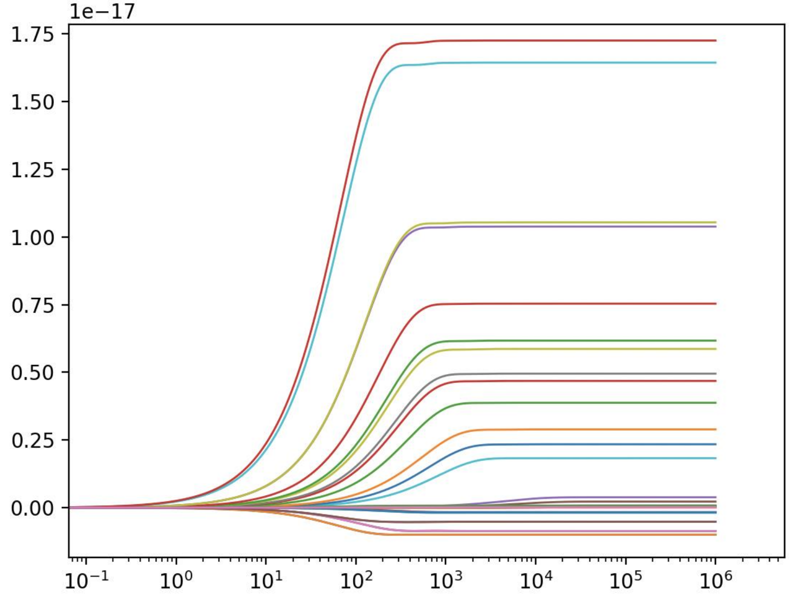
```
>>> print(sun.targets[18])
symbol 56Fe, atomic number 26, mass 52.136
Nuclear form factor:Definition from Phys.Rev.C 89 (2014) 6, 065501 (e-Print:
1308.6288[hep-ph]) used
for nuclear W functions as default
```

S. Kang, S. Scopel, G. Tomar, "WimPyC: an extension module of WimPyDD for the calculation of WIMP capture in celestial bodies", arXiv: 2510.21185

WimPyC

- Calculate response functions: Once you calculate the response functions for each isotopes you can use it for the other celestial bodies sampling in corresponding energy range.

tabulated response functions for WIMP Capture

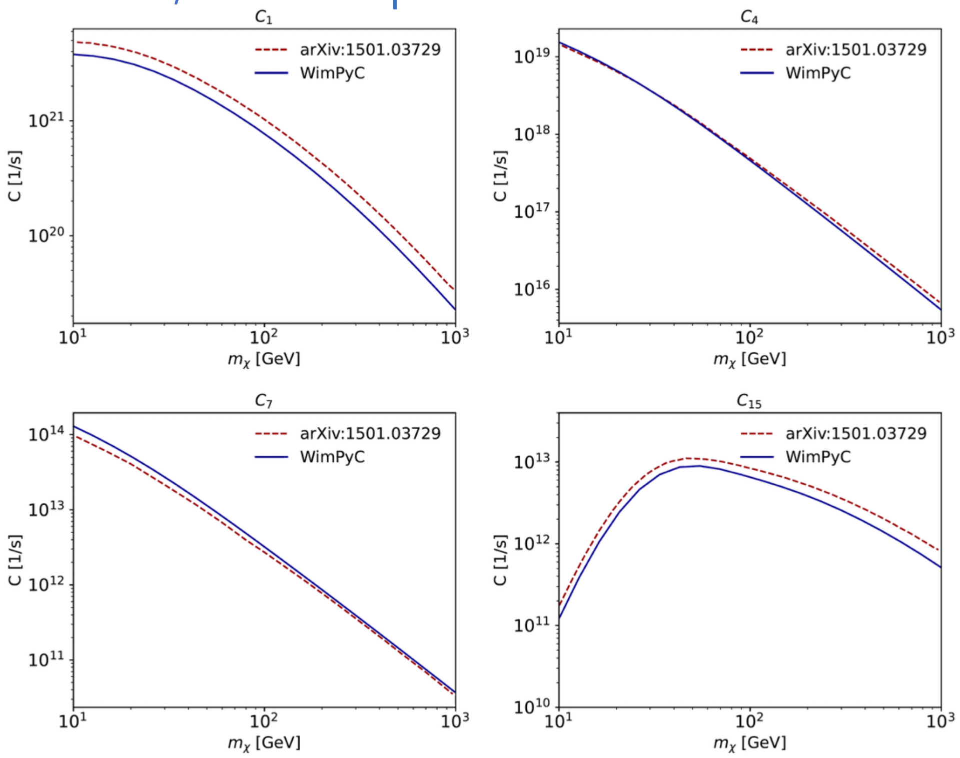


recoil energy

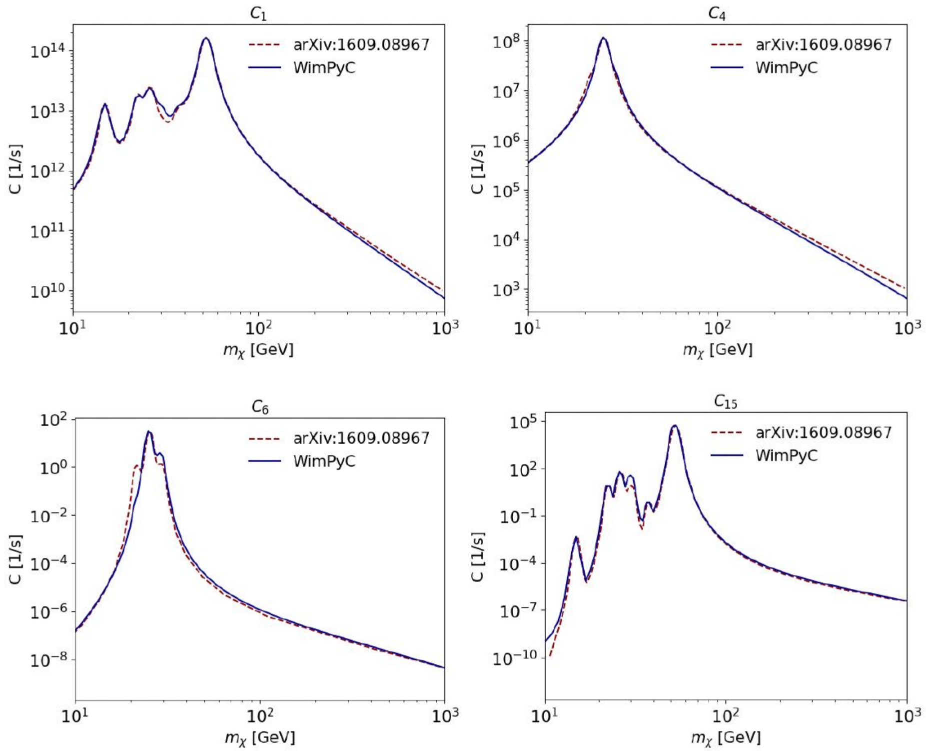
S. Kang, S. Scopel, G. Tomar, "WimPyC: an extension module of WimPyDD for the calculation of WIMP capture in celestial bodies", arXiv: 2510.21185

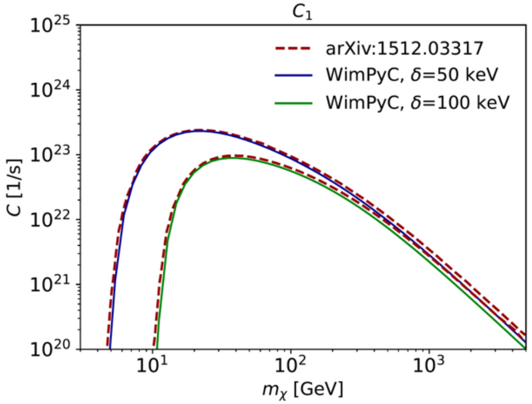
Extensive tests to validate WimPyC against published results:

Sun, NREFT operators



Earth, NREFT operators

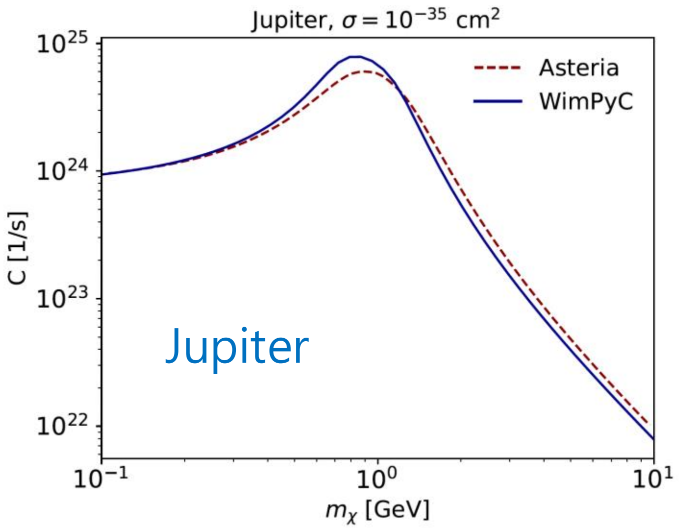


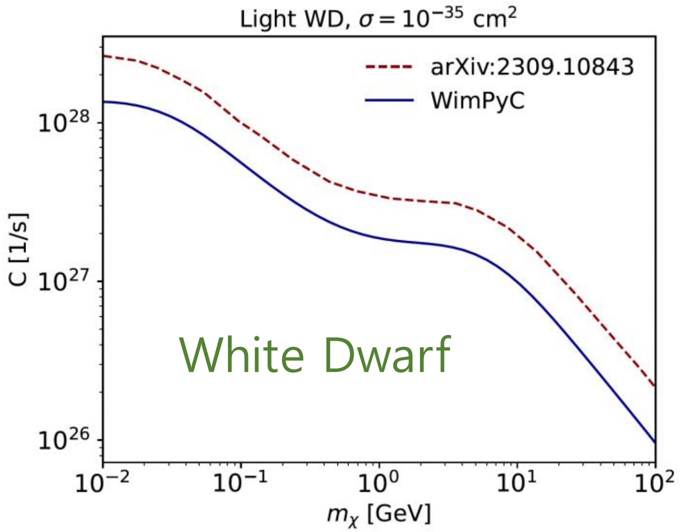


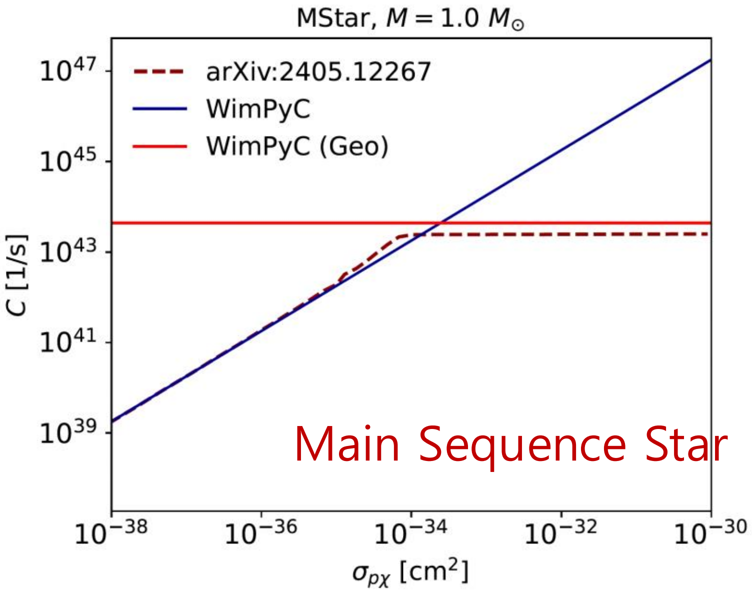
Sun, SI interaction, inelastic sc
attering

S. Kang, S. Scopel, G. Tomar, "WimPyC: an extension module of WimPyDD for the calculation of WIMP capture in celestial bodies", arXiv: 2510.21185

Extensive tests to validate WimPyC against published results:







N.B.: at high cross section optical thin regime no longer valid, multiple scatters set in (Capture rate eventually saturated by geometrical limit)

S. Kang, S. Scopel, G. Tomar, "WimPyC: an extension module of WimPyDD for the calculation of WIMP capture in celestial bodies", arXiv: 2510.21185

Match speci
DM model (
non-relativis
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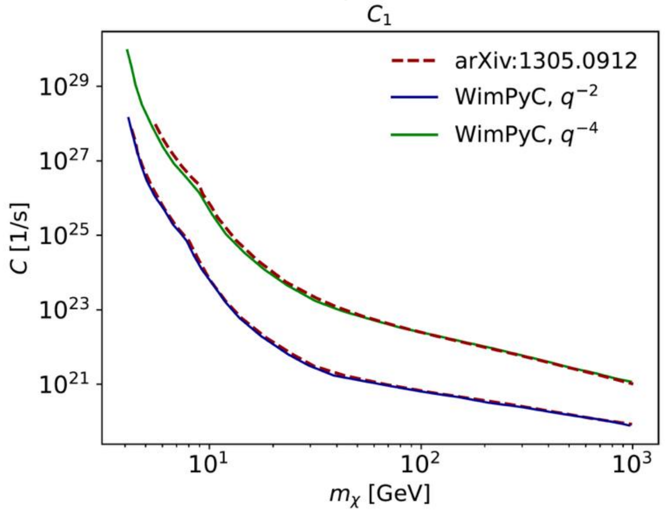
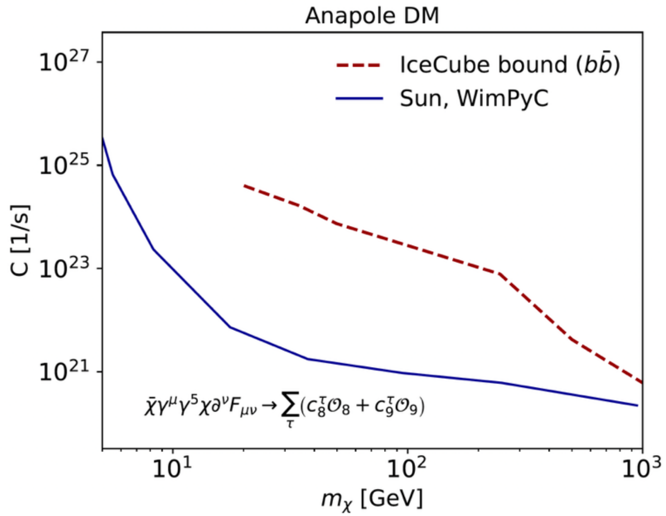
Extensive tests to validate WimPyC against published results:

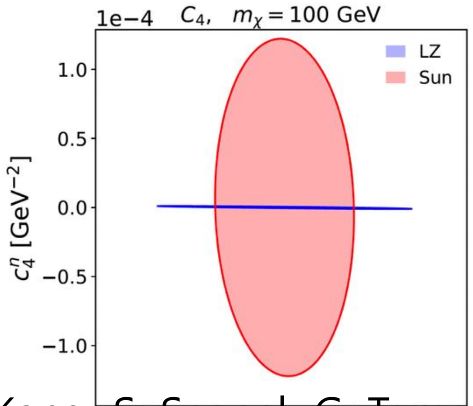
Massless propagator

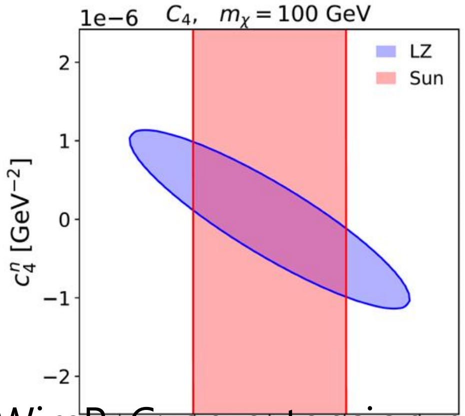


N.B.: for a massless propagator the capture rate diverges at low momentum transfer. Such events correspond to WIMPs locked on orbits of very large aphelion.
WimPyC allows to regularize the capture rate by removing orbits that cross Jupiter ("Jupiter cut").

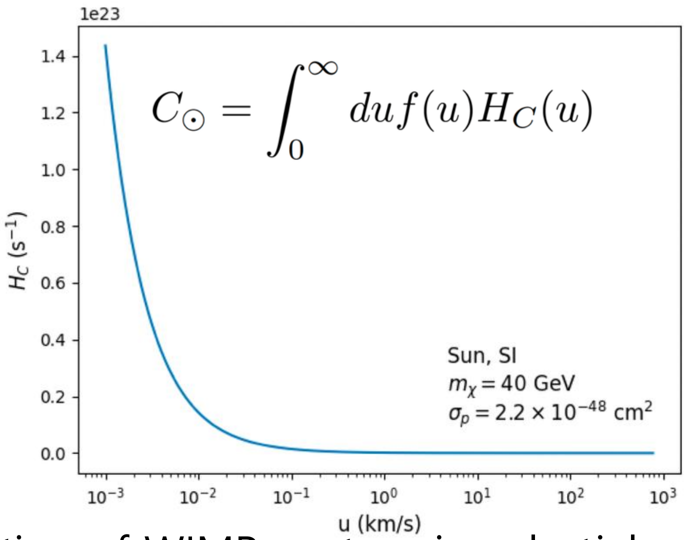


Matricial techniques using wimp_capture_matrix and wimp_dd_matrix (allowed regions inside ellipsoids)



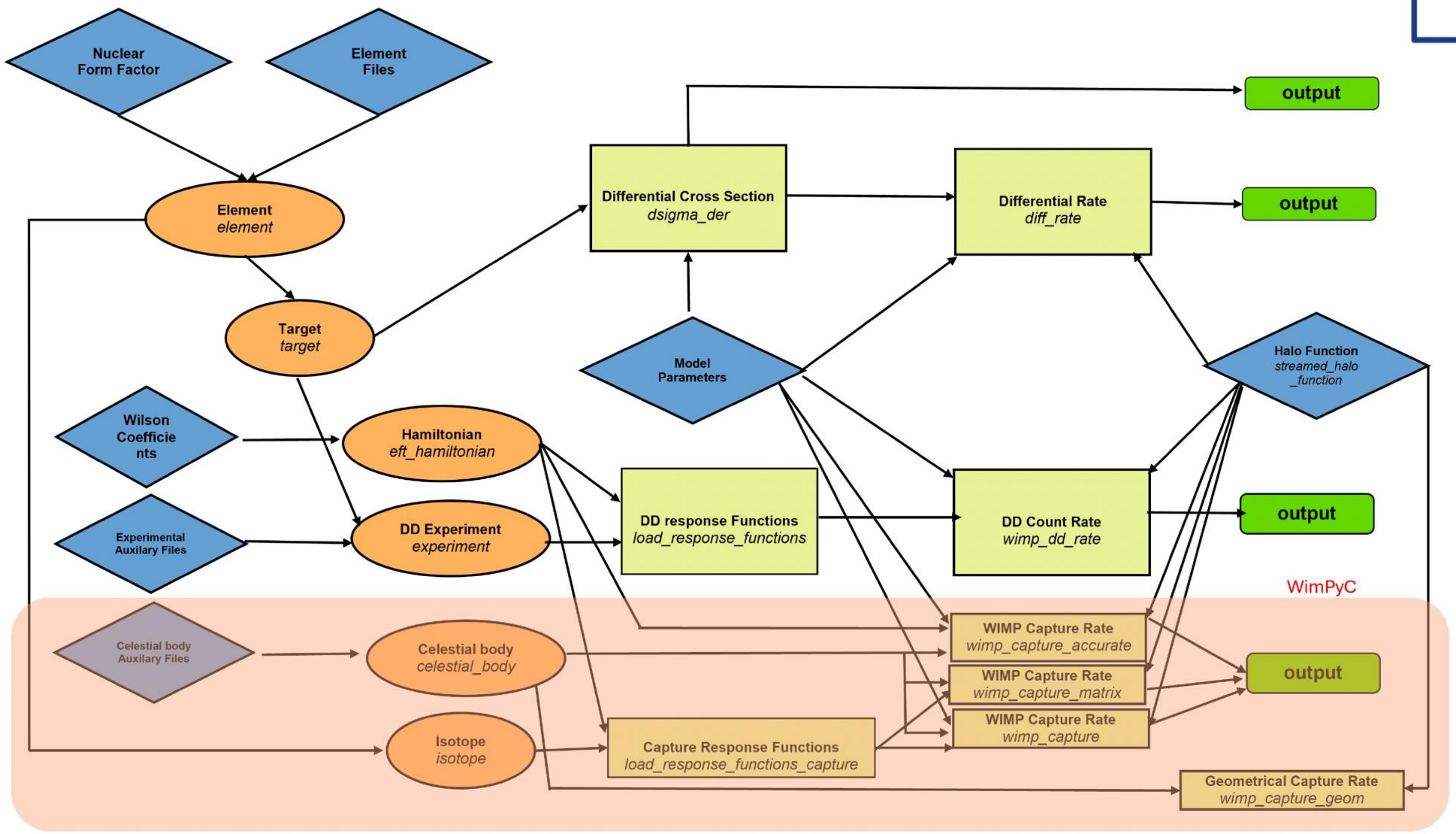


Speed response function $H_C(u)$ for halo-independent analysis



S. Kang, S. Scopel, G. Tomar, "WimPyC: an extension module of WimPyDD for the calculation of WIMP capture in celestial bodies", arXiv: 2510.21185

additional components of WimPyC module



S. Kang, S. Scopel, G. Tomar, "WimPyC: an extension module of WimPyDD for the calculation of WIMP capture in celestial bodies", arXiv: 2510.21185

WimPyC vs. other codes for WIMP capture in celestial bodies

Codes	NREFT Interactions	Arbitrary DM spin	Inelastic scattering	Velocity Distribution flexibility	Celestial Bodies	Scattering Regimes
DarkSUSY	✗ (SI/SD)	✗	✗	✓	Earth/Sun	Optically Thin
MicrOMEGAs	✗ (SI/SD)	✗	✗	✓	Earth/Sun	Optically Thin
Asteria	✗ (SI/SD)	✗	✗	✗ (Maxwellian)	Earth/Sun/ Jupiter/ BDs/…	Optically Thin/Thick
DaMaSCUS	✗ (SI/SD)	✗	✗	✗ (Maxwellian)	Earth	Optically Thick
Capt'n General	✓	✗	✗	✗ (Maxwellian)	Stars	Optically Thin
WimPyC	✓	✓	✓	✓	✓	Optically Thin

WimPyC can not be used when the speed of the WIMP is relativistic (neutron stars) and in the case of multiple scattering. And it can not calculate additional effects such as thermalization, equilibration, evaporation.

Summary

- The complementarity between WIMP direct detection and WIMP capture in the Sun allows to obtain bounds that do not depend on
  the WIMP-nucleus interaction or the WIMP velocity

- WimPyC is the extension of WimPyDD that allows to calculate WIMP capture in celestial bodies in virtually any scenario

- Using WimPyDD and WimPyC, one can easily correlate direct detection experiments and capture in celestial bodies

Back-up

Non-Relativistic Effective Theory (NREFT)

- Each operators have distinct couplings to proton and neutron:

$$\Sigma_{\alpha=p,n}\Sigma_{i=1}^{15} c_i^{\alpha}\mathcal{O}_i^{\alpha}, \qquad c_2^{\alpha} \equiv 0$$

- Equivalent form using isospin:

$$\Sigma_{i=1}^{15}\left(c_i^0 1 + c_i^1 \tau_3\right)\mathcal{O}_i = \Sigma_{\tau=0,1}\Sigma_{i=1}^{15} c_i^{\tau}\mathcal{O}_i t^{\tau}, \qquad c_2^0 = c_2^1 \equiv 0$$

$$|p\rangle = \binom{1}{0} \quad |n\rangle = \binom{0}{1} \quad 1 \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \tau_3 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$c_i^0 = \frac{1}{2}\left(c_i^p + c_i^n\right) \quad c_i^1 = \frac{1}{2}\left(c_i^p - c_i^n\right)$$

$$t^0 \equiv 1 \quad t^1 \equiv \tau_3$$

NREFT: nuclear response

- Factorize amplitude into WIMP response functions R and nuclear response functions W:

$$\frac{1}{2j_\chi+1}\frac{1}{2j_N+1}\Sigma_{spins}|M|^2 \equiv \Sigma_k \Sigma_{\tau=0,1}\Sigma_{\tau'=0,1} R_k^{\tau\tau'}\left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}, \left\{c_i^\tau, c_j^{\tau'}\right\}\right) W_k^{\tau\tau'}(y)$$

WIMP response function

Nuclear response function

- $R_k^{\tau\tau'}$: WIMP response function
- $W_k^{\tau\tau'}$: nuclear response function
  - $y = (qb/2)$^2
  - b: harmonic oscillator size parameter
- $k = M,\ \Delta,\ \Sigma',\ \Sigma'',\ \tilde{\Phi}'$ and Φ''
  - allowed responses assuming nuclear ground state is a good approximation of P and T

TABLE VII. Parity of the nucleon currents under space reflection P and time reversal T. Columns P_J and T_J list the parities of their Jth multipole moments (the notation L, TE, and TM stands for longitudinal, transverse electric, and transverse magnetic multipole, respectively). The last column lists the allowed Js in a ground state that is P and T (or CP) invariant.

X	Operator	P	T	Multipole:	P_J	T_J	Ground state
M	1	+1	+1		$(-1)^J$	$(-1)^J$	Even J
$\tilde{\Omega}$	$\vec{v}_N^\perp \cdot \vec{\sigma}_N$	−1	+1		$(-1)^{J+1}$	$(-1)^J$	Forbidden
Σ	$\vec{\sigma}_N$	+1	−1	L:	$(-1)^{J+1}$	$(-1)^{J+1}$	Odd J
				TE:	$(-1)^{J+1}$	$(-1)^{J+1}$	Odd J
				TM:	$(-1)^J$	$(-1)^{J+1}$	Forbidden
$\tilde{\Delta}$	$\vec{v}_N^\perp$	−1	−1	L:	$(-1)^J$	$(-1)^{J+1}$	Forbidden
				TE:	$(-1)^J$	$(-1)^{J+1}$	Forbidden
				TM:	$(-1)^{J+1}$	$(-1)^{J+1}$	Odd J
$\tilde{\Phi}$	$\vec{v}_N^\perp \times \vec{\sigma}_N$	−1	+1	L:	$(-1)^J$	$(-1)^J$	Even J
				TE:	$(-1)^J$	$(-1)^J$	Even J
				TM:	$(-1)^{J+1}$	$(-1)^J$	Forbidden

W.C. Haxton, N.Anand and A. L.Fitzpatrick, Weakly interacting massive particle-nucleus elastic scattering response, Phys. Rev., 2014
R. Catena and B. Schwabe, Form factors for dark matter captured by the Sun in effective theories, JCAP, 2015

NREFT: nuclear response

- Multipole expansions for nuclear electroweak response
- In elastic transitions the contributing multipoles are restricted by the known approximate good parity and T of nuclear ground states

TABLE VII. Parity of the nucleon currents under space reflection P and time reversal T. Columns P_J and T_J list the parities of their Jth multipole moments (the notation L, TE, and TM stands for longitudinal, transverse electric, and transverse magnetic multipole, respectively). The last column lists the allowed Js in a ground state that is P and T (or CP) invariant.

X	Operator	P	T	Multipole:	P_J	T_J	Ground state
M	1	+1	+1		$(-1)^J$	$(-1)^J$	Even J
$\tilde{\Omega}$	$\vec{v}_N^+ \cdot \vec{\sigma}_N$	−1	+1		$(-1)^{J+1}$	$(-1)^J$	Forbidden
Σ	$\vec{\sigma}_N$	+1	−1	L:	$(-1)^{J+1}$	$(-1)^{J+1}$	Odd J
				TE:	$(-1)^{J+1}$	$(-1)^{J+1}$	Odd J
				TM:	$(-1)^J$	$(-1)^{J+1}$	Forbidden
$\tilde{\Delta}$	$\vec{v}_N^+$	−1	−1	L:	$(-1)^J$	$(-1)^{J+1}$	Forbidden
				TE:	$(-1)^J$	$(-1)^{J+1}$	Forbidden
				TM:	$(-1)^{J+1}$	$(-1)^{J+1}$	Odd J
$\tilde{\Phi}$	$\vec{v}_N^+ \times \vec{\sigma}_N$	−1	+1	L:	$(-1)^J$	$(-1)^J$	Even J
				TE:	$(-1)^J$	$(-1)^J$	Even J
				TM:	$(-1)^{J+1}$	$(-1)^J$	Forbidden

NREFT: WIMP response functions

WIMP response functions

$$R_M^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = c_1^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left[\frac{q^2}{m_N^2} v_T^{\perp 2} c_5^\tau c_5^{\tau'} + v_T^{\perp 2} c_8^\tau c_8^{\tau'} + \frac{q^2}{m_N^2} c_{11}^\tau c_{11}^{\tau'}\right],$$

$$R_{\Phi''}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \left[\frac{q^2}{4m_N^2} c_3^\tau c_3^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau'}\right)\right]\frac{q^2}{m_N^2},$$

$$R_{\Phi'' M}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \left[c_3^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right) c_{11}^{\tau'}\right]\frac{q^2}{m_N^2},$$

$$R_{\tilde{\Phi}'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \left[\frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau c_{12}^{\tau'} + \frac{q^2}{m_N^2} c_{13}^\tau c_{13}^{\tau'}\right)\right]\frac{q^2}{m_N^2},$$

$$R_{\Sigma''}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \frac{q^2}{4m_N^2} c_{10}^\tau c_{10}^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \frac{q^2}{m_N^2}(c_4^\tau c_6^{\tau'} + c_6^\tau c_4^{\tau'}) + \frac{q^4}{m_N^4} c_6^\tau c_6^{\tau'} + v_T^{\perp 2} c_{12}^\tau c_{12}^{\tau'} + \frac{q^2}{m_N^2} v_T^{\perp 2} c_{13}^\tau c_{13}^{\tau'}\right],$$

$$R_{\Sigma'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \frac{1}{8}\left[\frac{q^2}{m_N^2} v_T^{\perp 2} c_3^\tau c_3^{\tau'} + v_T^{\perp 2} c_7^\tau c_7^{\tau'}\right] + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \frac{q^2}{m_N^2} c_9^\tau c_9^{\tau'} + \frac{v_T^{\perp 2}}{2}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau'}\right) + \frac{q^2}{2m_N^2} v_T^{\perp 2} c_{14}^\tau c_{14}^{\tau'}\right],$$

$$R_{\Delta}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \frac{j_\chi(j_\chi+1)}{3}\left(\frac{q^2}{m_N^2} c_5^\tau c_5^{\tau'} + c_8^\tau c_8^{\tau'}\right)\frac{q^2}{m_N^2},$$

$$R_{\Delta\Sigma'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) = \frac{j_\chi(j_\chi+1)}{3}\left(c_5^\tau c_4^{\tau'} - c_8^\tau c_9^{\tau'}\right)\frac{q^2}{m_N^2}.$$

- c_i: coupling for i-th operator
- j_χ: spin of WIMP
- m_N: mass of nucleon
- $v_T^\perp$: WIMP incoming velocity
  - perpendicular to the direction of transferred momentum q

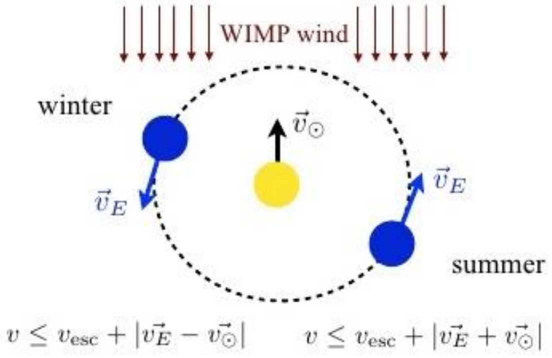
Halo functions

- Standard Halo model(SHM)
- WIMP velocity distribution as Maxwellian:

$$f_{gal}(u) = \frac{1}{\pi^{3/2} v_0^3 N_{esc}} e^{-u^2/v_0^2}\, \Theta(u_{esc} - u)$$

  - u: WIMP speed in Galactic rest frame
  - v_0: Galactic rotational velocity
  - Θ: Heaviside step function
  - u_{esc}: Escape velocity
  - $N_{esc} = \mathrm{erf}(z) - 2ze^{-z^2}/\pi^{1/2}$
  - $z^2 = u_{esc}^2/v_0^2$

Halo functions



- In laboratory frame :

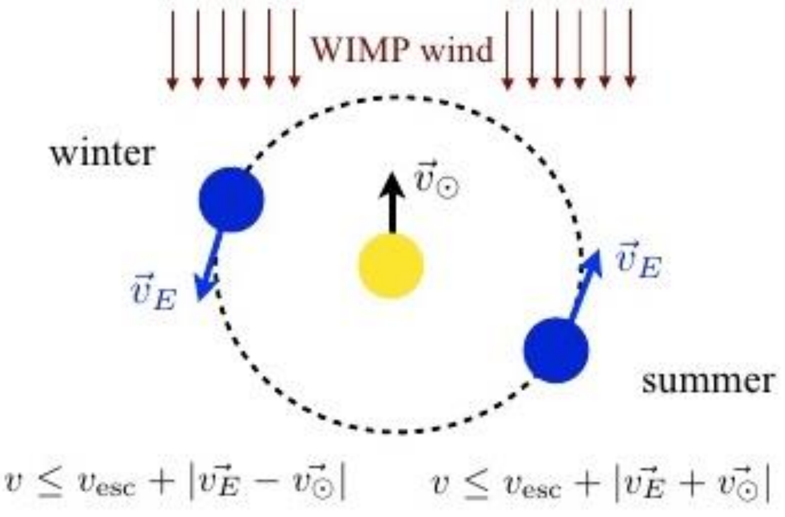
$$f(\overrightarrow{v_T}, t) = \frac{1}{N_{esc}}\left(\frac{3}{2\pi v_{rms}^2}\right)^{3/2} e^{-\frac{3\left|\overrightarrow{v_T}+\overrightarrow{v_E}\right|^2}{2v_{rms}^2}} \Theta(u_{esc} - |\overrightarrow{v_T} + \overrightarrow{v_E}(t)|)$$

  - $N_{esc} = \mathrm{erf}(z) - 2ze^{-z^2}/\pi^{1/2}$
  - $z^2 = 3u_{esc}^2/(2v_{rms}^2)$
  - $v_{rms} = \sqrt{\frac{3}{2}} v_0 \cong 270\ \mathrm{km/s}$ (assuming hydrostatic equilibrium)

- Modulation
  - $v_E(t) = \left[v_\odot^2 + v_\oplus^2 + 2v_\odot v_\oplus\, cos\gamma \cos[\omega(t-t_0)]\right]^{1/2}$
  - $cos\gamma \cong 0.49$, $v_\oplus \cong 29\ \mathrm{km/s}$, $v_\odot = v_0 + 12\ \mathrm{km/s}$, $v_0 = 220\ \mathrm{km/s}$, $u_{esc} = 550\ \mathrm{km/s}$
    - γ: elliptic angle, $v_\oplus$: Earth velocity around the Sun, $v_\odot$: velocity of the Sun
  - $\eta(t) = \eta_0 + \eta_1 \cos[\omega(t-t_0)]$

Hfunction

- Why Maxwellian? -> Violent relaxation

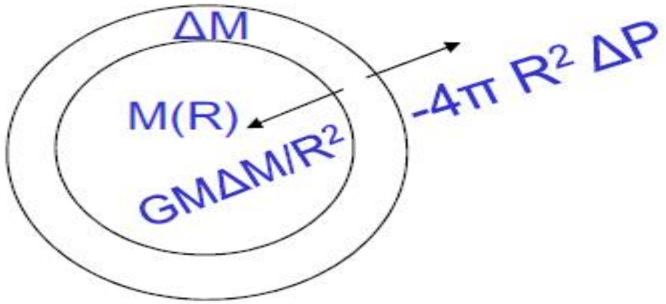


$$f_{\rm gal}(u) = \frac{1}{\pi^{3/2} v_0^3 N_{\rm esc}} e^{-u^2/v_0^2}\, \Theta(v_{\rm esc} - u)$$

$$f(\vec{v}_T, t) = \frac{1}{N}\left(\frac{3}{2\pi v_{rms}^2}\right)^{3/2} e^{-\frac{3|\vec{v}_T + \vec{v}_E|^2}{2v_{rms}^2}}\, \Theta(u_{esc} - |\vec{v}_T + \vec{v}_E(t)|)$$

$$N = \left[\mathrm{erf}(z) - \frac{2}{\sqrt{\pi}} z e^{-z^2}\right]^{-1},$$

- temperature? -> root mean square velocity
  - hydrostatic equilibrium



$$P = \frac{\rho_W}{m_W} T \qquad v_{rot}^2 = G\frac{M}{R}$$

$$\frac{3}{2} T = \frac{1}{2} m_W v_{rms}^2 \rightarrow T = \frac{m_W v_{rms}^2}{3}$$

$$dM = 4\pi R^2 \rho_W dR \rightarrow 4\pi R^2 = \frac{1}{\rho_W}\frac{dM}{dR}$$

$$-4\pi R^2\, dP = G\frac{M\, dM}{R^2} \rightarrow -\frac{1}{\rho_w}\frac{dM}{dR}\frac{m_W v_{rms}^2}{3}\frac{d\rho_W}{m_W} = \frac{v_{rot}^2}{R} dM$$

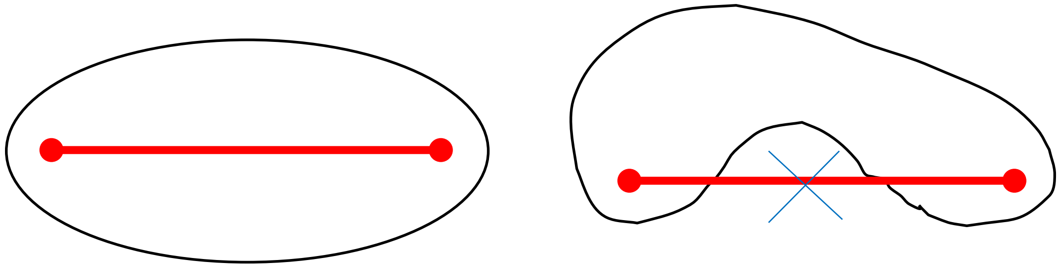
Linear Matrix Inequality (LMI)

- Linear matrix inequality
  - constrained minimization using lagrange multipliers
  - bounds are opened
  - 'Kuhn Tucker' minimization.

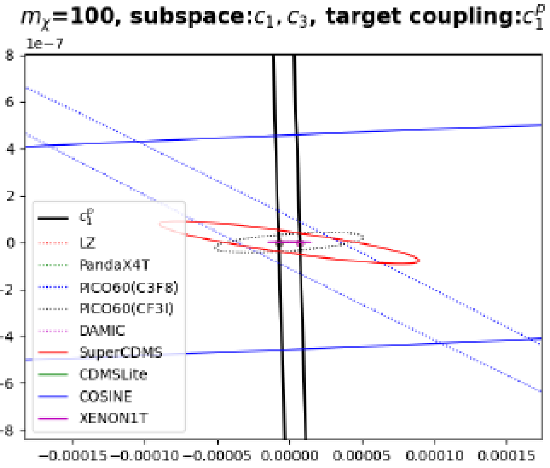
$$\xi_i \geq 0, \quad \sum_{i=1}^{n} \xi_i \leq 1,$$

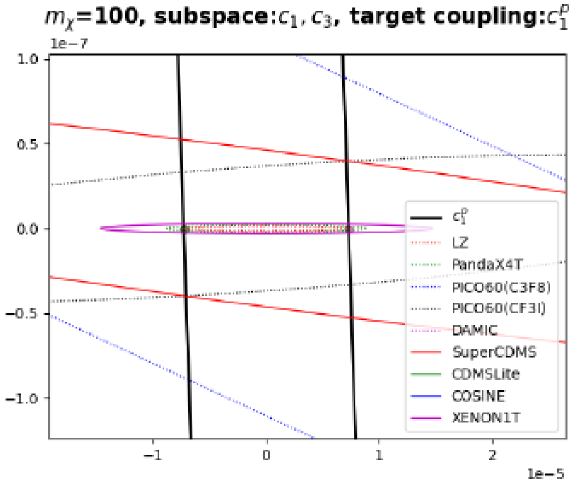
$$\sum_{i=1}^{n} \xi_i A_k - \frac{B}{max(c_\alpha)^2} \quad \text{is a positive matrix.}$$

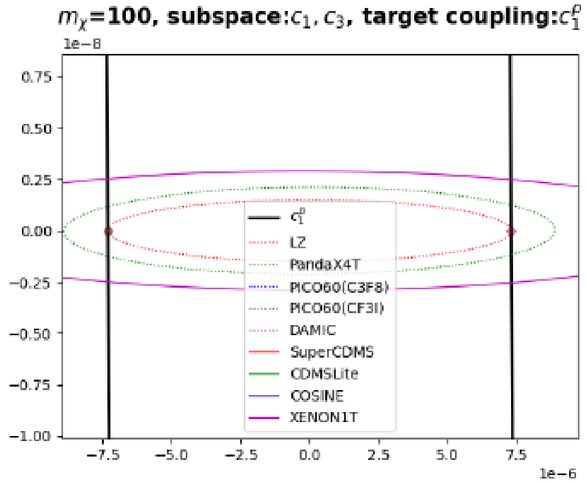
- Convex constraint
  - If two points obey the constraints, all the points joining them also obey

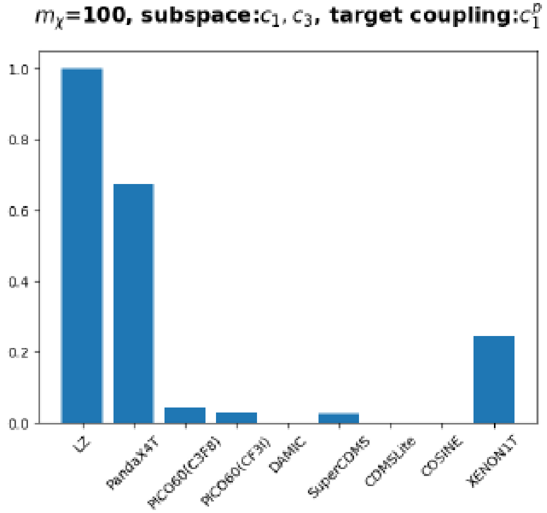


Bracketing: LMI exclusion bands







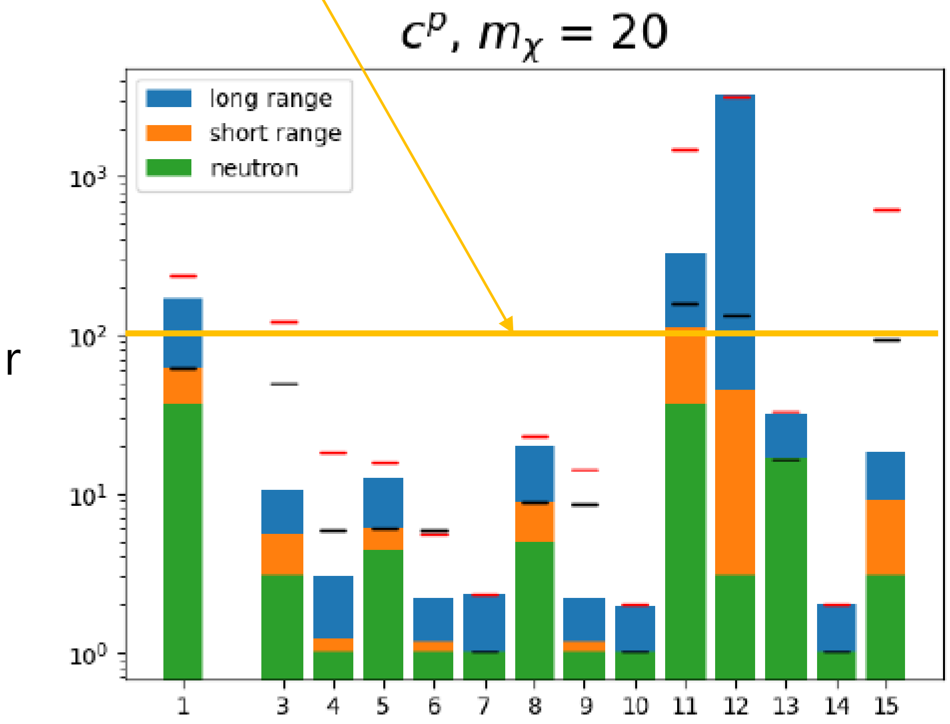


- ellipsoids for each experiments (zooming)
- for experiment saturating the bound:
    - $c^\tau A_{Exp} c = 1$

S. Kang, I. Jeong, S. Scopel Bracketing the direct detection exclusion plot for a WIMP of spin one half in non-relativistic effective theory, arXiv:2209.03646

Bracketing: relaxation factors

max$(\sqrt{\xi}) \sim 10^2$



- relaxation factor r (y-axis value):

$$c_\alpha^{most} \mathcal{M}_{\alpha\alpha} c_\alpha^{most} = \frac{1}{r^2} c_\alpha^{less} \mathcal{M}_{\alpha\alpha} c_\alpha^{less} = 1$$

- level of tuning (horizontal bar):

$$\xi \equiv \max\left(\left|[c^{less}]_i \mathcal{M}_{ij} [c^{less}]_j\right|\right) = \left|[c^{less}]_\gamma \mathcal{M}_{\gamma\delta} [c^{less}]_\delta\right| \geq r^2.$$

- degree of sensitivity:

$$|\epsilon| = 1/\xi \leq 1/r^2$$

  - sensitivity of the result to small changes of matrix element evaluation

- max$(\xi) \leq 10^4$: beyond such value numerical calculation may not be reliable

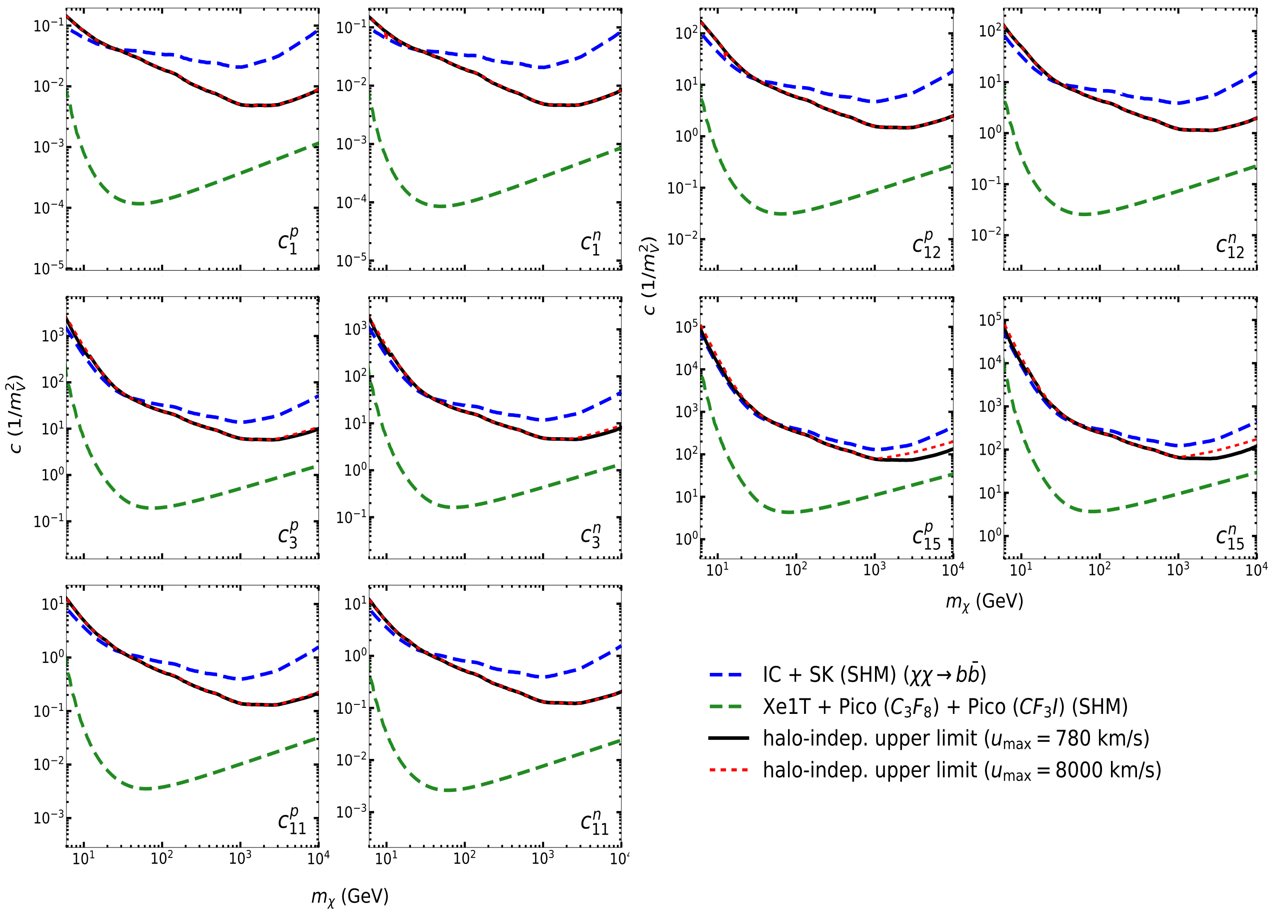
S. Kang, I. Jeong, S. Scopel Bracketing the direct detection exclusion plot for a WIMP of spin one half in non-relativistic effective theory, arXiv:2209.03646

Halo independent approach

- Halo independent approach with arbitrary speed distribution, $f(u)$
  - super position of N_s streams
  - $f(v,t) = \Sigma_k^{N_s} \lambda_k(t)\delta(v - v_k)$
  - $\eta(v,t) = \Sigma_k^{N_s} \frac{\lambda_k(t)}{v_k} \Theta(v_k - v) = \Sigma_k^{N_s} \delta\eta_k(t)\Theta(v_k - v)$
  - $\eta^{(0,1)}(v) = \Sigma_k^{N_s} \delta\eta_k^{(0,1)}(t)\Theta(v_k - v) \equiv \Sigma_k^{N_s} \frac{\lambda_k^{(0,1)}(t)}{v_k} \Theta(v_k - v)$

- Rate
  - $R_{[E_1',E_2']}(t) = \frac{\rho_\chi}{m_\chi} \Sigma_{\mathrm{k}}^{N_s} \delta\eta_k(t) \int_{v_T^*}^{v_k} \Sigma_T dv\, \mathcal{R}_{T,[E_1',E_2']}(v)$

Halo independent approach

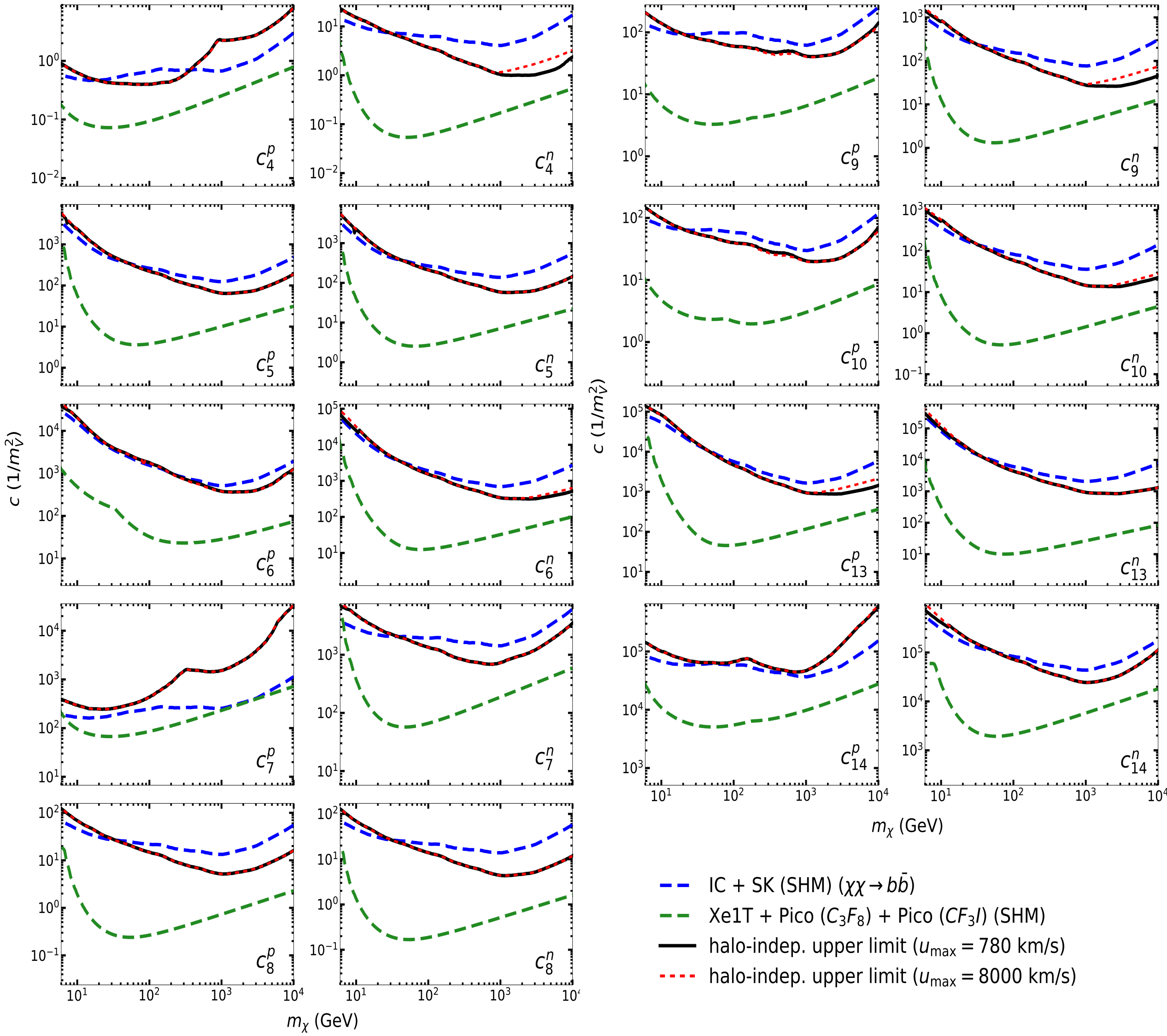


- Spin independent
  - $\mathcal{O}_{1,3,11,12,15}$
  - W_M, $W_{\Phi''}$
  - Enhanced for heavy targets

operator	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$	operator	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$
1	$M(q^0)$	-	3	$\Phi''(q^4)$	$\Sigma'(q^2)$
4	$\Sigma''(q^0),\Sigma'(q^0)$	-	5	$\Delta(q^4)$	$M(q^2)$
6	$\Sigma''(q^4)$	-	7	-	$\Sigma'(q^0)$
8	$\Delta(q^2)$	$M(q^0)$	9	$\Sigma'(q^2)$	-
10	$\Sigma''(q^2)$	-	11	$M(q^2)$	-
12	$\Phi''(q^2),\tilde{\Phi}'(q^2)$	$\Sigma''(q^0),\Sigma'(q^0)$	13	$\tilde{\Phi}'(q^4)$	$\Sigma''(q^2)$
14	-	$\Sigma'(q^2)$	15	$\Phi''(q^6)$	$\Sigma'(q^4)$

S. Kang, A. Kar, S. Scopel, Halo-independent bounds on the non-relativistic effective theory of WIMP-nucleon scattering from direct detection and neutrino observations, JCAP03(2023)011 (arXiv:2212.05774)

Halo independent approach



- Spin dependent
  - $O_{4,5,6,7,8,9,10,13,14}$
  - $W_{\Sigma''}$, $W_{\Sigma'}$: directly coupling to spin
  - W_{Δ}: related to angular momentum
  - $W_{\tilde{\Phi}'}$: spin larger than 1/2

operator	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$	operator	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$
1	$M(q^0)$	-	3	$\Phi''(q^4)$	$\Sigma'(q^2)$
4	$\Sigma''(q^0),\Sigma'(q^0)$	-	5	$\Delta(q^4)$	$M(q^2)$
6	$\Sigma''(q^4)$	-	7	-	$\Sigma'(q^0)$
8	$\Delta(q^2)$	$M(q^0)$	9	$\Sigma'(q^2)$	-
10	$\Sigma''(q^2)$	-	11	$M(q^2)$	-
12	$\Phi''(q^2),\tilde{\Phi}'(q^2)$	$\Sigma''(q^0),\Sigma'(q^0)$	13	$\tilde{\Phi}'(q^4)$	$\Sigma''(q^2)$
14	-	$\Sigma'(q^2)$	15	$\Phi''(q^6)$	$\Sigma'(q^4)$

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DD event rate (inelastic scattering)

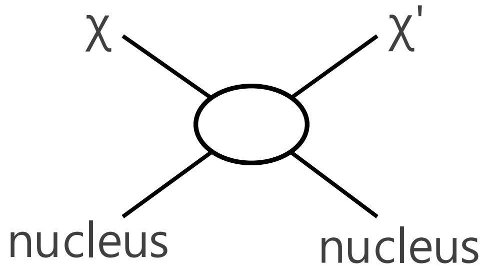
- DD event rate

$$R_{DD} = M\tau_{exp}\frac{\rho_\chi}{m_\chi}\int du\, f(u)u\, \Sigma_T\, N_T \int_{E_{R,th}}^{2\mu_{\chi T}^2 u^2/m_T} dE_R\, \zeta_{exp}\frac{d\sigma_T}{dE_R}$$

$$R_{DD} = M\tau_{exp}\frac{\rho_\chi}{m_\chi}\int du\, f(u)u\, \Sigma_T\, N_T\Theta(u^2 - v_{T*}^2) \int_{E_{min}}^{E_{max}} dE_R\, \zeta_{exp}\frac{d\sigma_T}{dE_R}$$

$$E_{max,min}(u) = \frac{\mu_{\chi T}^2 u^2}{2\, m_T}\left(1 \pm \sqrt{1 - \frac{2\,\delta}{\mu_{\chi T}\, u^2}}\right)^2$$

$$u_T^{\mathrm{DD-min}} = \begin{cases} v_{T*} = \sqrt{\dfrac{2\delta}{\mu_{\chi T}}}, & E_T^{\mathrm{th}} \le E_{T*} \\ \dfrac{1}{\sqrt{2 m_T E_T^{\mathrm{th}}}}\left(\dfrac{m_T E_T^{\mathrm{th}}}{\mu_{\chi T}} + \delta\right), & E_T^{\mathrm{th}} > E_{T*} \end{cases}$$



$$\delta = m_{\chi'} - m_\chi > 0$$

Capture rate (inelastic scattering)

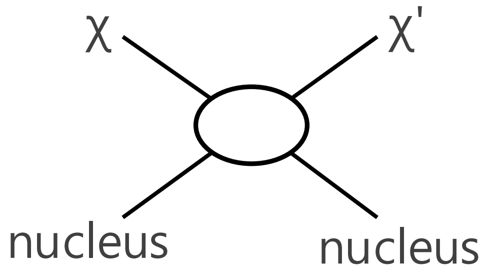
- Capture rate

$$C_{\odot} = \frac{\rho_\chi}{m_\chi} \int du \frac{f(u)}{u} \int_0^{R_\odot} dr\, 4\pi r^2\, w^2\, \Sigma_T\, \rho_T(r)\, \Theta\left(u_T^{C-max} - u\right) \int_{m_\chi u^2/2}^{2\mu_{\chi T}^2 w^2/m_T} dE_R \frac{d\sigma_T}{dE_R}$$

$$C_{\odot} = \frac{\rho_\chi}{m_\chi} \int du \frac{f(u)}{u} \int_0^{R_\odot} dr\, 4\pi r^2\, w^2\, \Sigma_T\, \rho_T(r)\, \Theta(\omega^2 - v_{T*}^2)\Theta\left(E_{max} - E_{cap}^{\chi}\right) \int_{max\left(E_{min}, E_{cap}\right)}^{E_{max}} dE_R \frac{d\sigma_T}{dE_R}$$

$$E_{max,min}(u) = \frac{1}{2}\, m_\chi \omega^2 \left(1 - \frac{\mu_{\chi T}^2}{m_T^2}\left(1 \pm \frac{m_T}{m_\chi}\sqrt{1 - \frac{v_{T*}^2}{\omega^2}}\right)^2\right)^2$$

$$E_{cap}^{\chi}(u) = \frac{1}{2} m_\chi u^2 - \delta$$



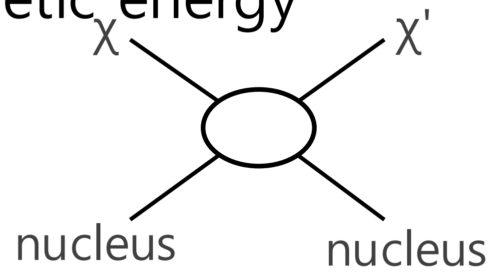
$$\delta = m_{\chi'} - m_\chi > 0$$

Capture rate (inelastic scattering)

- u_T^{C-max} is determined:

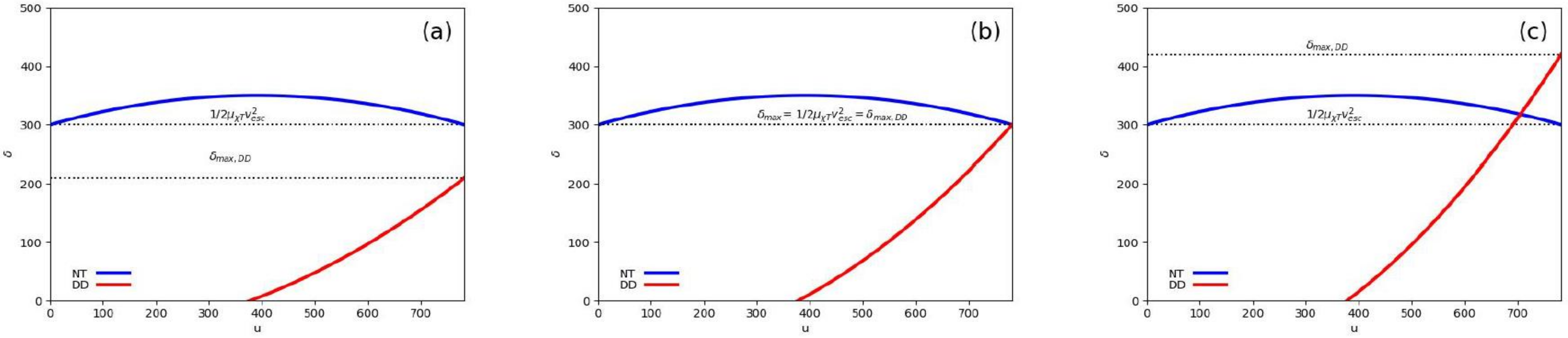
$$E_{max}^{\chi} \geq E_{cap}^{\chi} \quad \text{or} \quad \omega^2 \left[1 - \frac{\mu_{\chi T}^2}{m_T^2}\left(1 - \frac{m_T}{m_\chi}\sqrt{1 - \frac{v_{T*}^2}{\omega^2}}\right)^2\right] \geq u^2$$

- Outgoing speed of χ' is below u_{esc}
  - Heavier state quickly decays back to the lighter particles
  - Lighter particles carry away most of the energy
  - Outgoing WIMP does not receive a significant amount of kinetic energy and remains locked in a bound orbit.



$\delta = m_{\chi'} - m_\chi > 0$

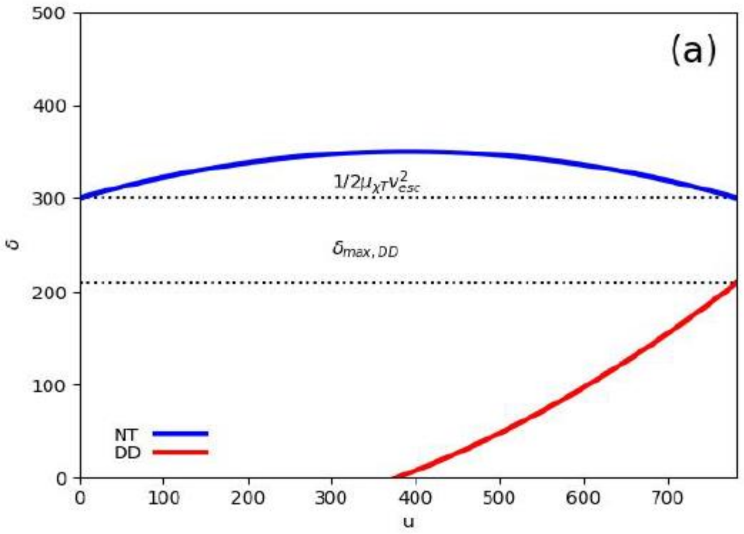
Determining δ_{max}

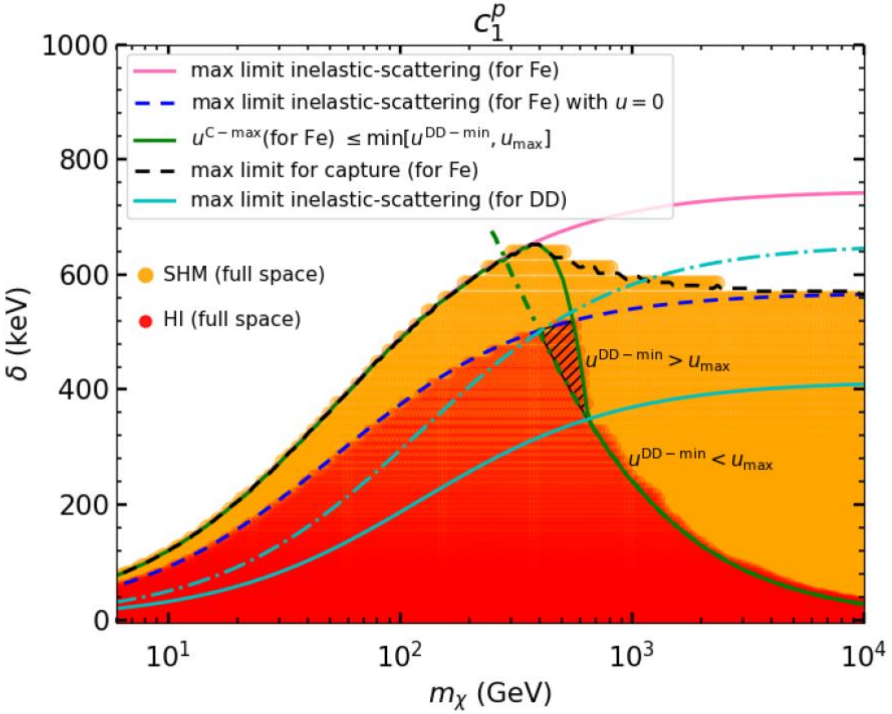


- $r = \frac{m_N}{m_T} \geq r_{min} \simeq 3.9$
  - below this value, only Capture determines δ_{max}
- SI ($T = {}^{56}Fe$, N $=$ Xe)
  - r $\simeq$ 2.3
- SD ($T = {}^{27}Al$, $N = Xe\ (I)$)
  - r $\simeq$ 4.86 (4.7)

S. Kang, A. Kar, S. Scopel, Halo-independent bounds on Inelastic Dark Matter, JCAP11(2023)077 (arXiv:2308.13203)

Determining δ_{max}





- $r = \frac{m_N}{m_T} \geq r_{min} \simeq 3.9$
  - below this value, only Capture determines δ_{max}
- SI ($T = {}^{56}Fe$, N = Xe)
  - r ≃ 2.3
  - $\delta_{max,DD} < \delta_{max}$
  - Capture alone determines δ_{max}

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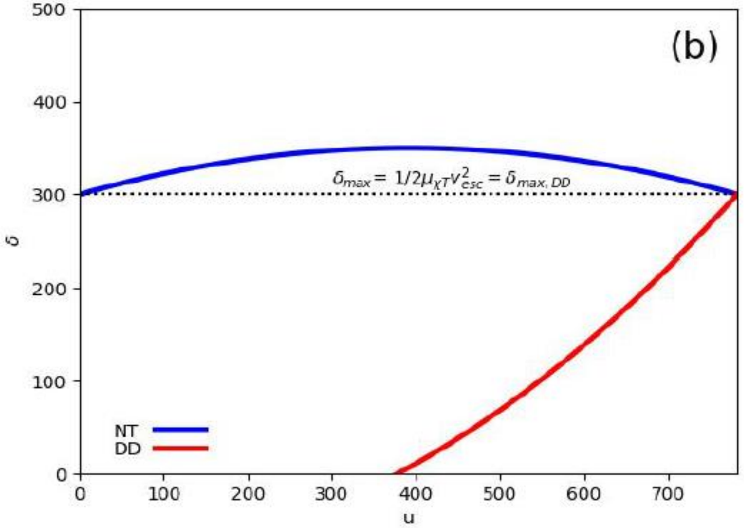
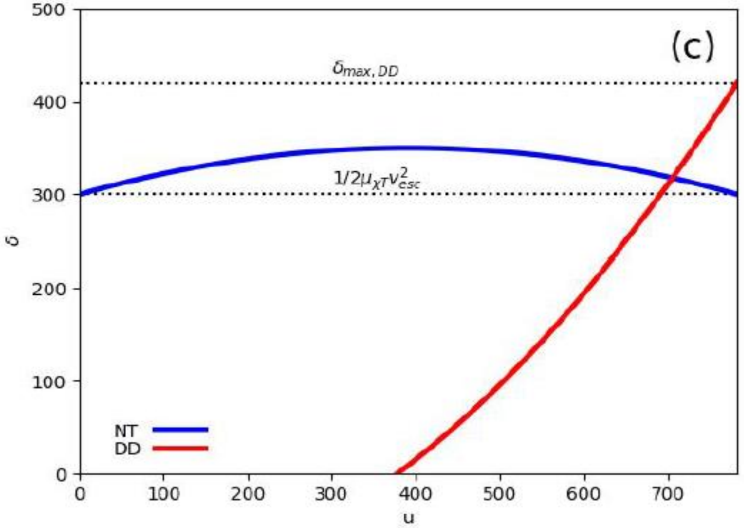
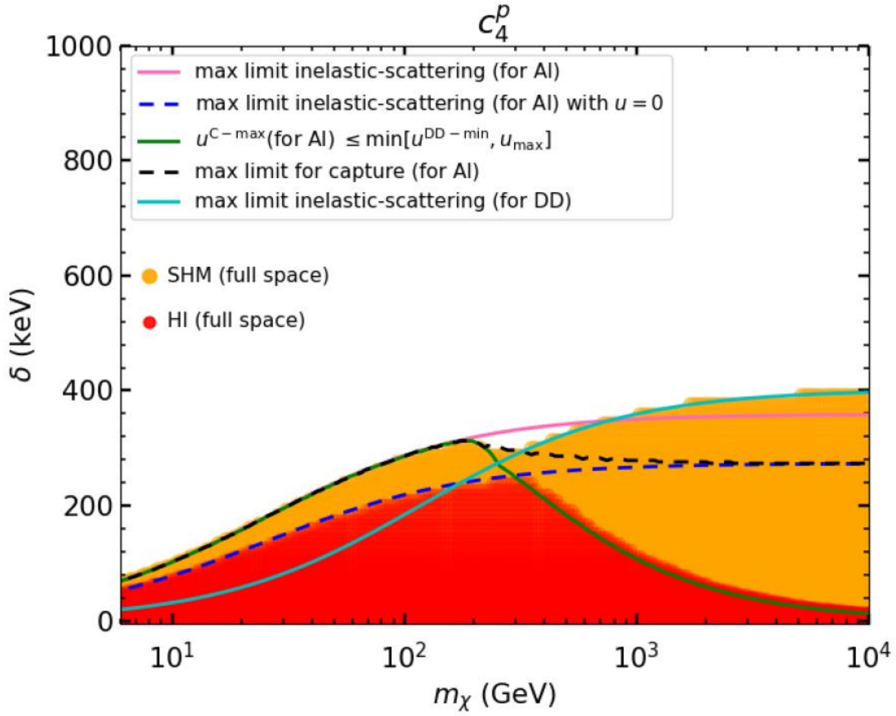
Determining δ_{max}



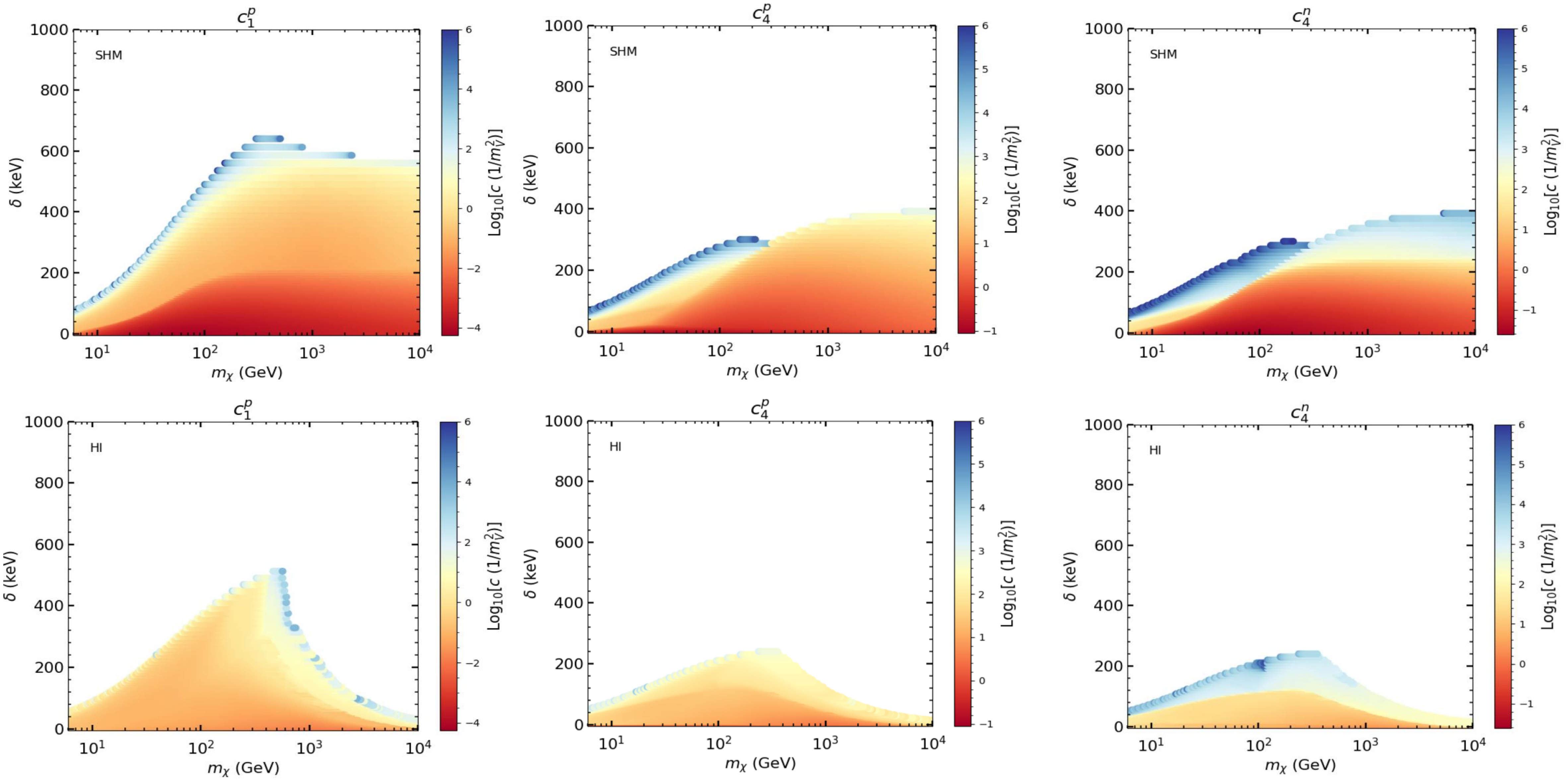


- $r = \frac{m_N}{m_T} \geq r_{min} \simeq 3.9$
  - below this value, only Capture determines δ_{max}
- SD ($T = {}^{27}Al$, $N = Xe\ (I)$)
  - r $\simeq$ 4.86 (4.7)
  - $\delta_{max,DD} > \delta_{max}$
  - δ_{max} is determined by a combination



S. Kang, A. Kar, S. Scopel, Halo-independent bounds on Inelastic Dark Matter, JCAP11(2023)077 (arXiv:2308.13203)

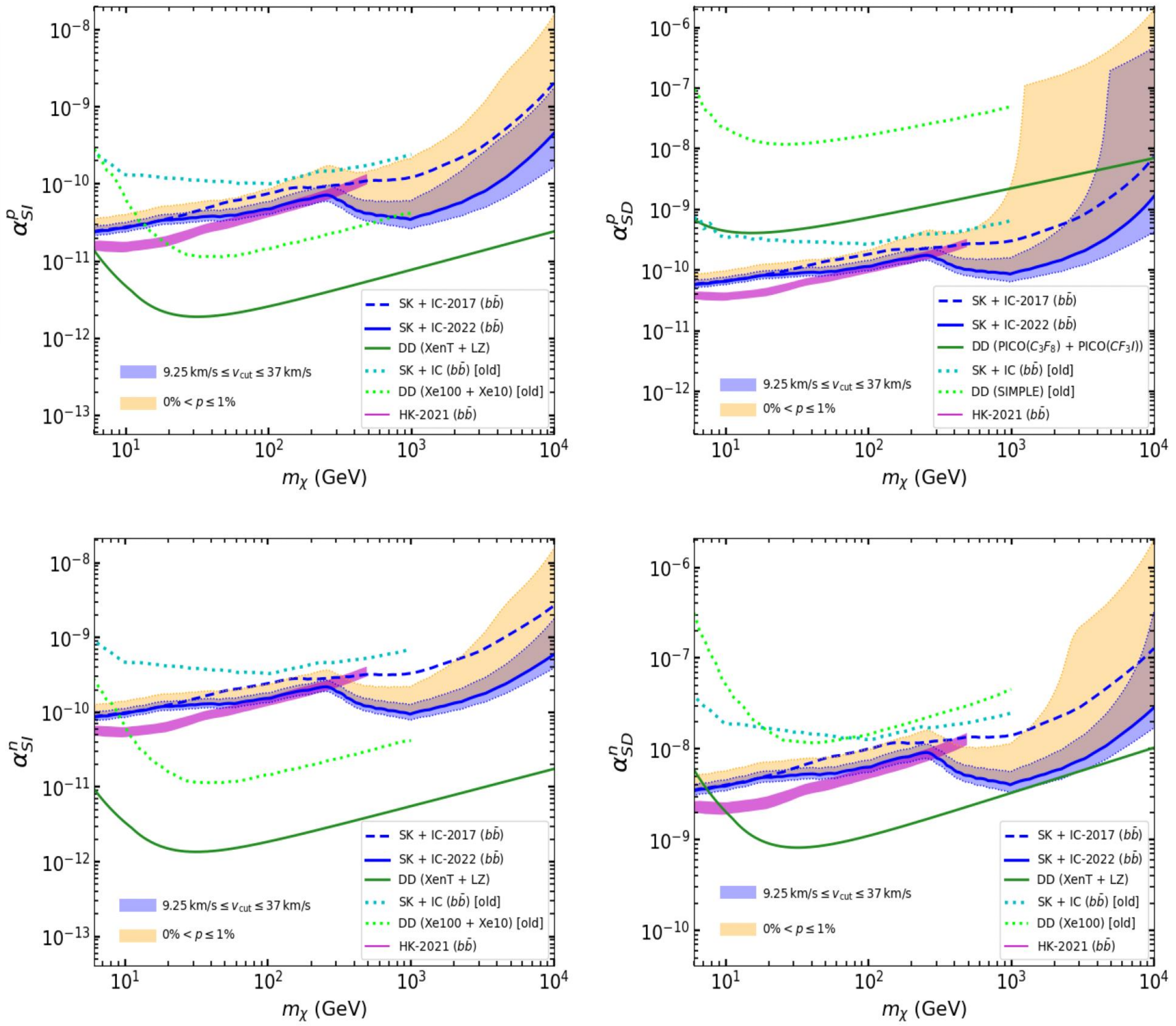
$m_\chi - \delta$ planes



S. Kang, A. Kar, S. Scopel, Halo-independent bounds on Inelastic Dark Matter, JCAP11(2023)077 (arXiv:2308.13203)

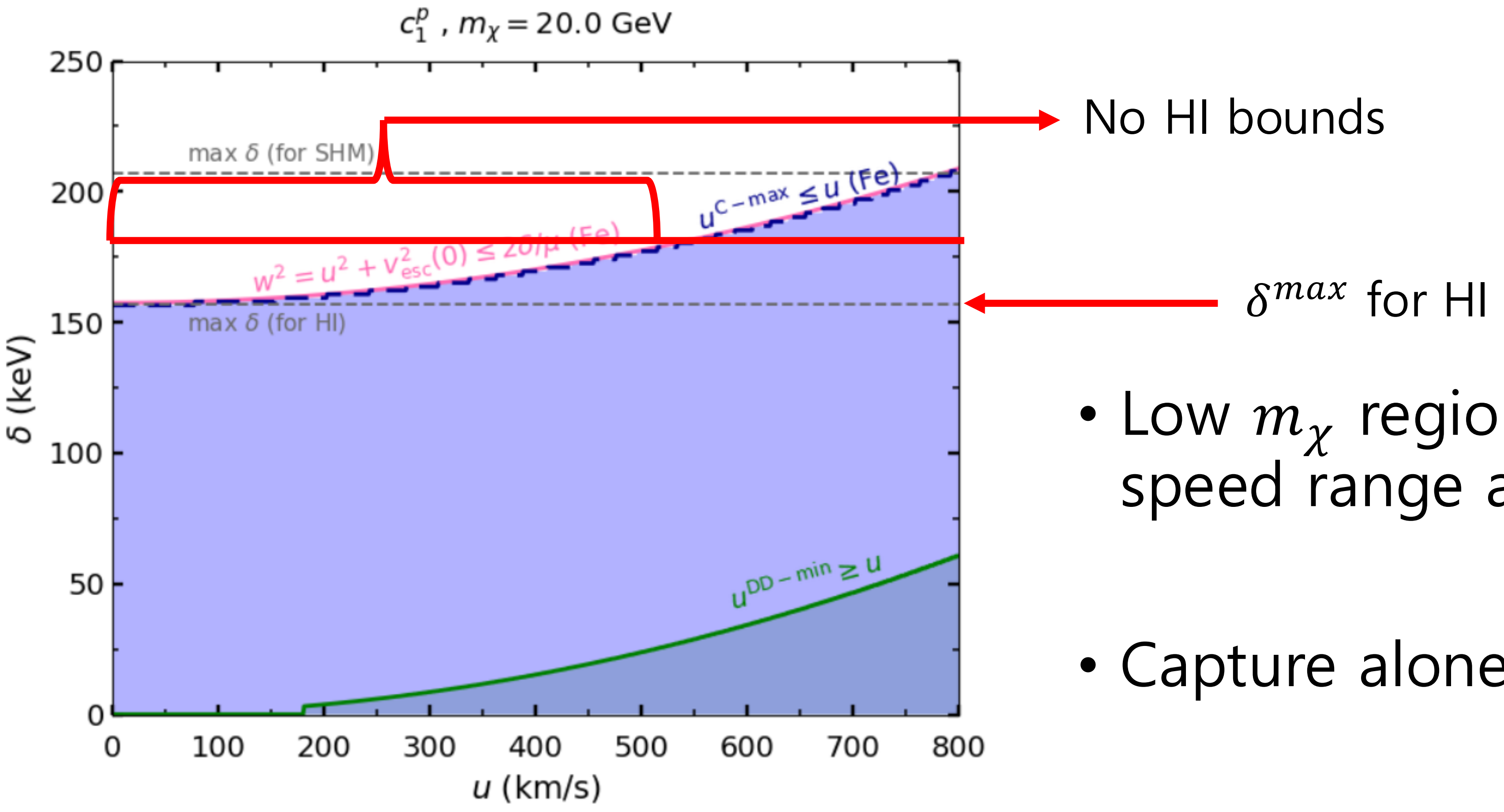
Bounds on α

- Updating bounds from SHM
- DD is more constraining than NT except a case of α_{SD}^{p} due to the large amount of H^1
  - effect of q is larger in NT



K. Choi, I. Jeong, S. K., A. Kar, S. Scopel, "Sensitivity of WIMP bounds on the velocity distribution in the limit of a massless m ediator", JCAP01(2025)007 (arXiv:2408.09658)

Determining δ_{max}



- Low m_χ region, capture can cover full speed range alone up to δ^{max}
- Capture alone can determine δ^{max}
- Above δ^{max}, no HI bounds

S. Kang, A. Kar, S. Scopel, "Halo-independent bounds on Inelastic Dark Matter", JCAP11(2023)077 (arXiv:2308.13203)

NREFT with long-range interactions

- $\mathcal{H} = \sum_{\tau=0,1} \left(\frac{\alpha_{SI}^{\tau}}{q^2+M_0^2}\ 1_\chi 1_N t^\tau + \frac{\alpha_{SD}^{\tau}}{q^2+M_0^2}\ \vec{S}_\chi \cdot \vec{S}_N t^\tau \right)$
- Scattering amplitude:

$$\frac{1}{2j_\chi+1}\frac{1}{2j_N+1}\Sigma_{spins}|M(q^2)|^2$$

$$= \frac{4\pi}{2j_N}\ [\sum_{\tau,\tau'=0,1}\{\frac{\alpha_{SI}^{\tau}\alpha_{SI}'}{q^4}\ W_{TM}^{\tau\tau'}(q) + \frac{j_\chi(j_\chi+1)}{12}\frac{\alpha_{SD}^{\tau}\alpha_{SD}'}{q^4}\ [W_{T\Sigma'}^{\tau\tau'}(q) + W_{T\Sigma''}^{\tau\tau'}(q)]\}$$

- Capture rate

$$C_\odot = \frac{\rho_\chi}{m_\chi}\int du\, f(u)\frac{1}{u}\int_0^{R_\odot} dr\, 4\pi r^2\, w^2\ \Sigma_T\, \rho_T(r)\, \Theta\left(u_T^{C-max} - u\right)\int_{E_{min}^C}^{E_{max}^C} dE_R\, \frac{d\sigma_T}{dE_R}$$

  - $C_\odot$ may diverge when $u \to 0$
  - Capture favors for low WIMP incoming speed and has no lower limit on u
  - $E_{min}^C \to \frac{1}{2}m_\chi(u^2+v_{cut}^2)$, where $v_{cut} = v_{esc}\left(r_{Sun-Jupiter}\right) \cong 18.5$ km/s