Higgs inflation:

with a Gauss-Bonnet term in Einstein Frame

@ Workshop on Cosmology and Quantum Space Time (CQUeST 2023) 임채호 교수님 추모 학회*, July 31—August 04, 2023*

Gansukh Tumurtushaa(JejuNU)

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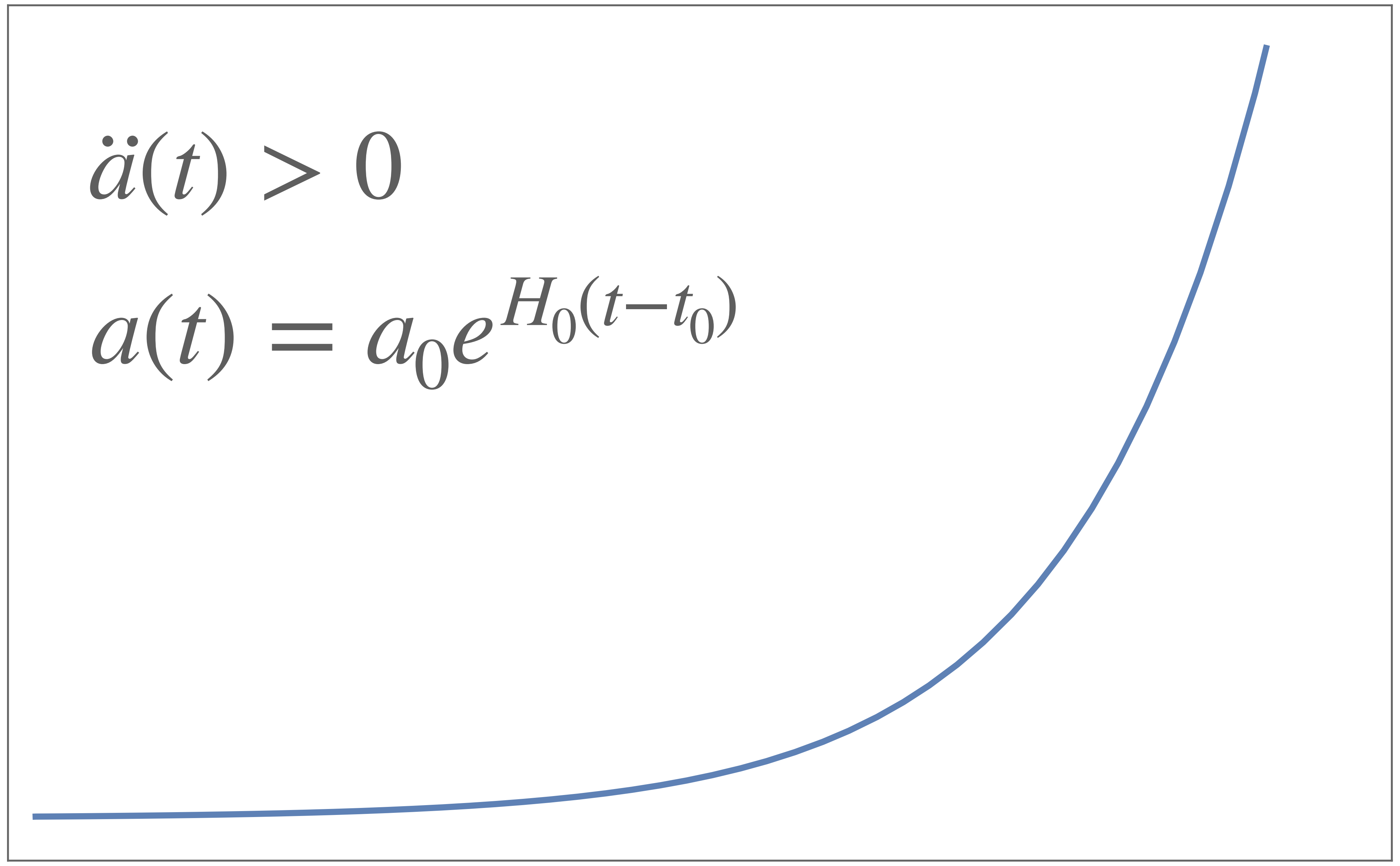
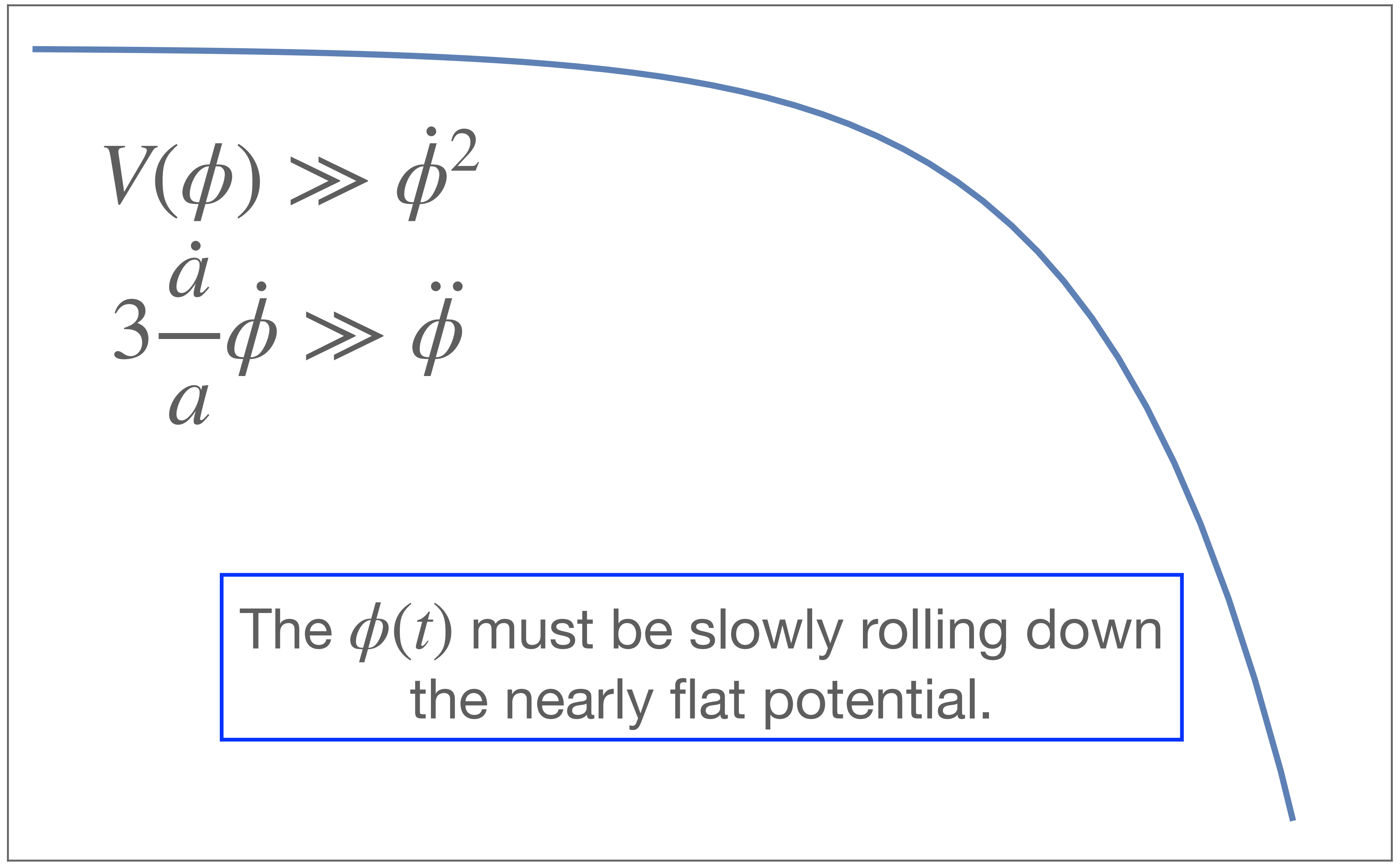
Inflation

an idea of accelerated "exponential" expansion of the early universe driven by the so-called "inflaton" field ϕ



$$\ddot{a}(t) > 0$$

$$a(t) = a_0 e^{H_0(t-t_0)}$$

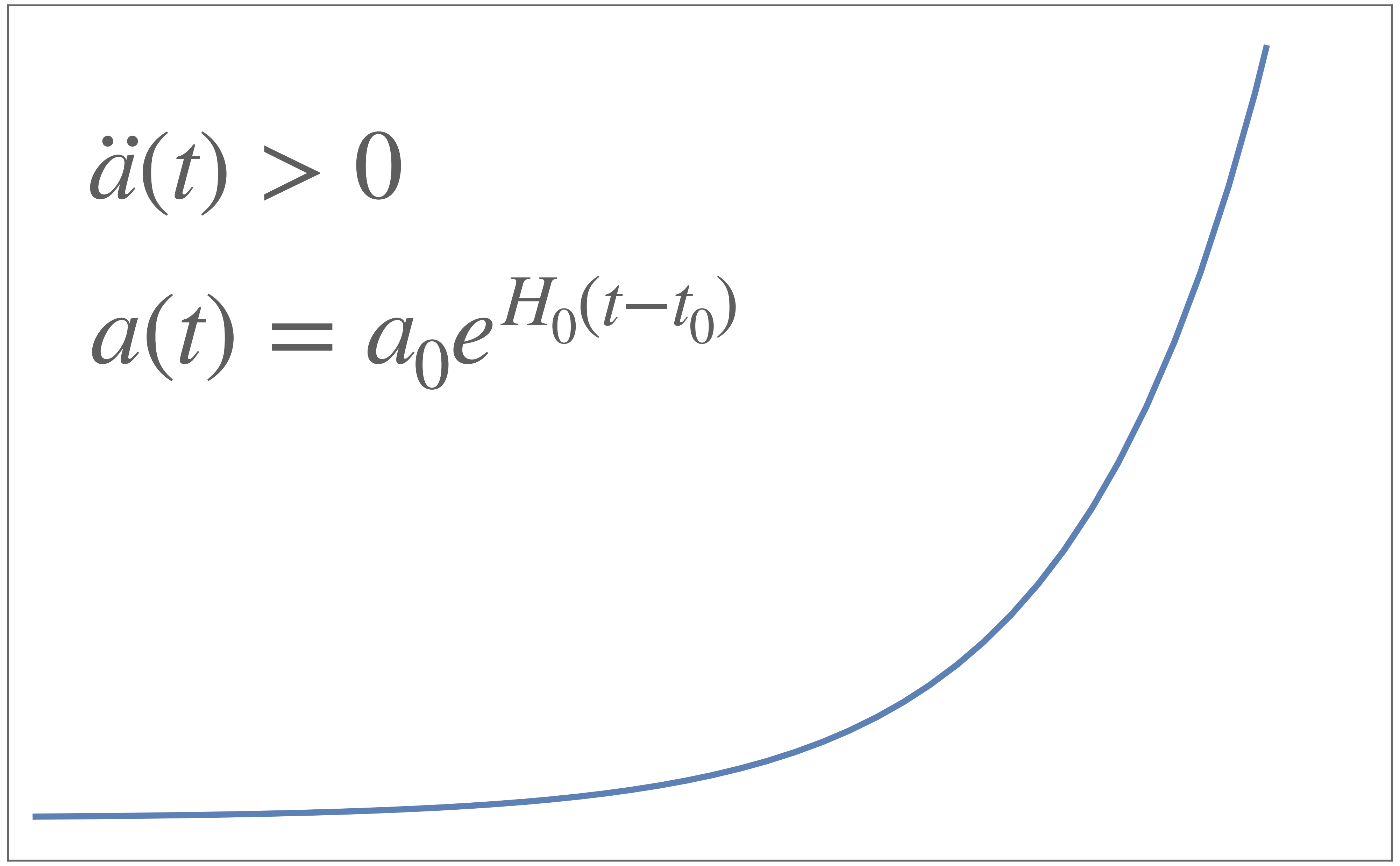


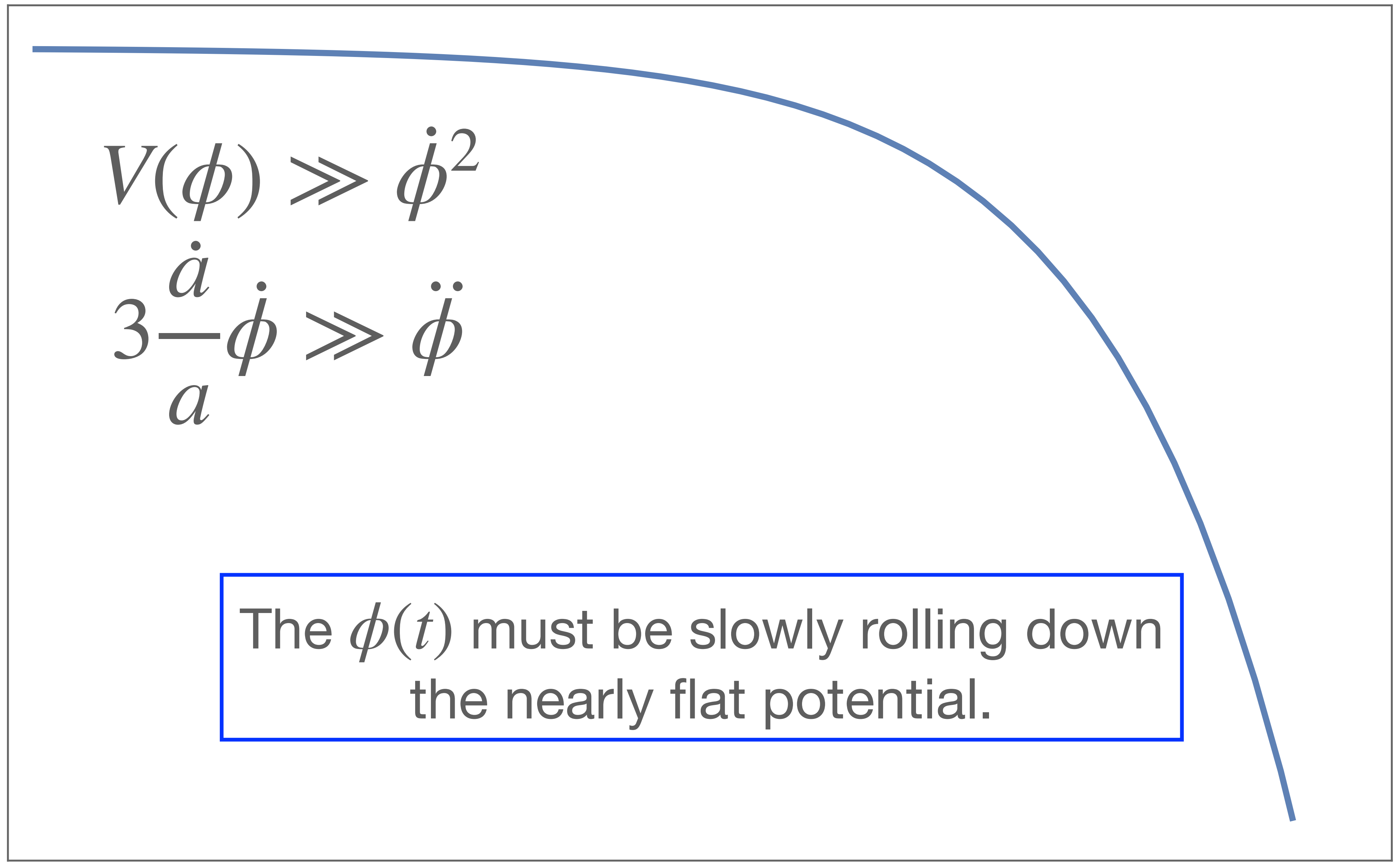
$$V(\phi) \gg \dot{\phi}^2$$

$$3\frac{\dot{a}}{a}\dot{\phi} \gg \ddot{\phi}$$

Inflation

an idea of accelerated "exponential" expansion of the early universe driven by the so-called "inflaton" field ϕ





Higgs Inflation

identifies Standard Model "Higgs" field as the inflaton field ϕ.

Higgs Inflation

with non-minimal coupling

Phys. Lett. B 659 (2008), 703-706
Phys. Lett. B 675 (2009), 88-92
JHEP 01 (2011), 016
Phys. Rev. D 39 (1989), 399-404
Phys. Rev. D 41 (1990), 1783-1791
Phys. Rev. D 86 (2012)

The action in the "Jordan" Frame:

$$S^J = \int d^4x \sqrt{-g^J} \left[\frac{M_p^2}{2} \Omega^2(\phi) R^J - \frac{1}{2} g^J_{ab} \nabla^a \phi \nabla^b \phi - \frac{1}{4} \lambda (\phi^2 - \nu^2)^2 \right] ,$$

where the Higgs field is "*non-minimally*" coupled to gravity via

$$\Omega^2(\phi) = 1 + \frac{\sigma}{M_p^2} \phi^2 .$$

* σ non-minimal coupling constant

Phys. Lett. B 659 (2008), 703-706
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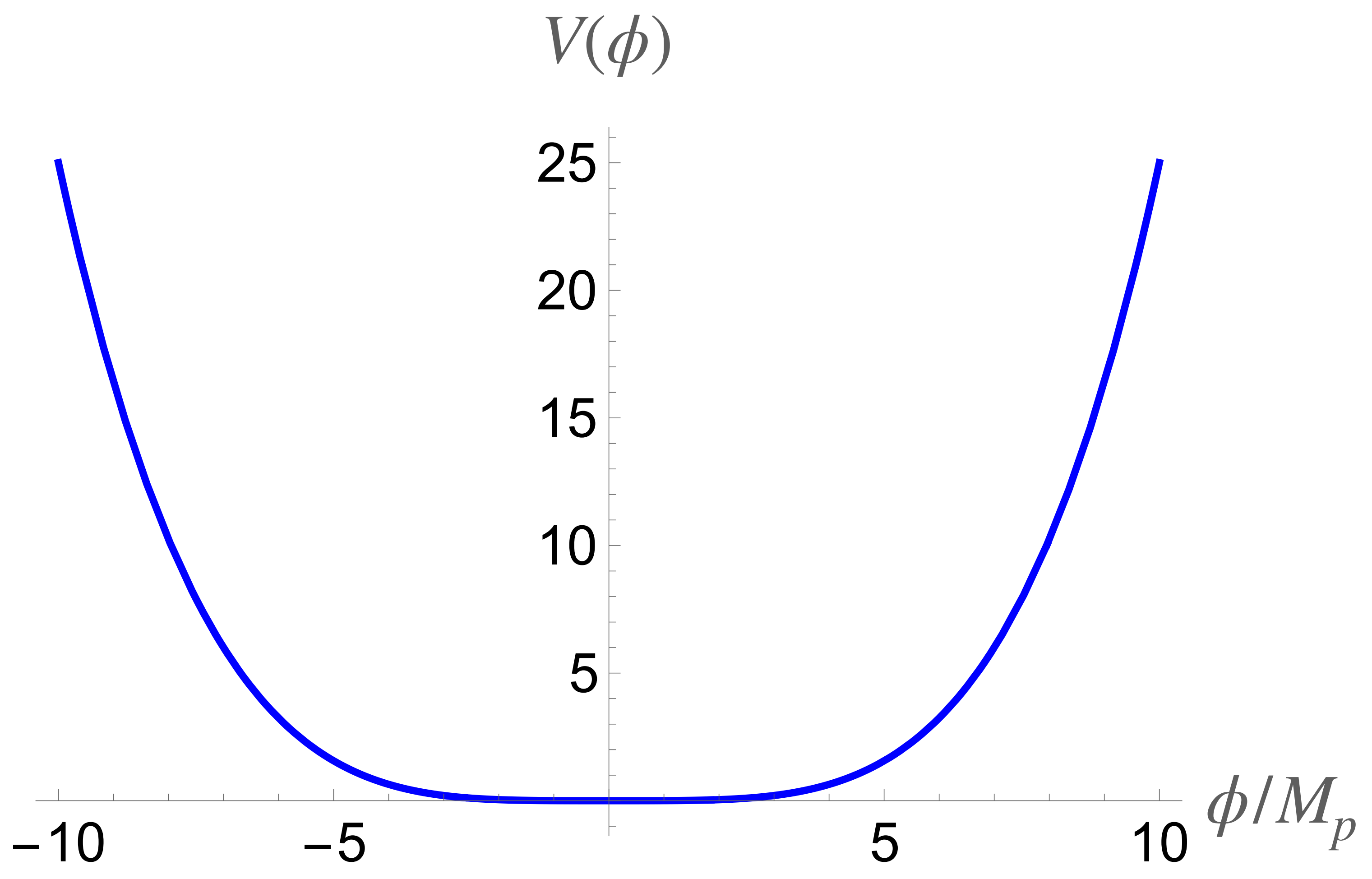

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where the Higgs field is "*non-minimally*" coupled to gravity via

$$\Omega^2(\phi) = 1 + \frac{\sigma}{M_p^2} \phi^2 .$$

One can move from the Jordan frame to the Einstein frame by using the conformal transformation (CT).

* Einstein frame: the ϕ is "*minimally*" coupled to gravity R.

Higgs Inflation

in the Einstein frame

Under the conformal transformation,

$$g^J_{ab} = \Omega^{-2} g_{ab}, \quad \text{where} \quad \Omega^2(\phi) = 1 + \frac{\sigma}{M_p^2}\phi^2,$$

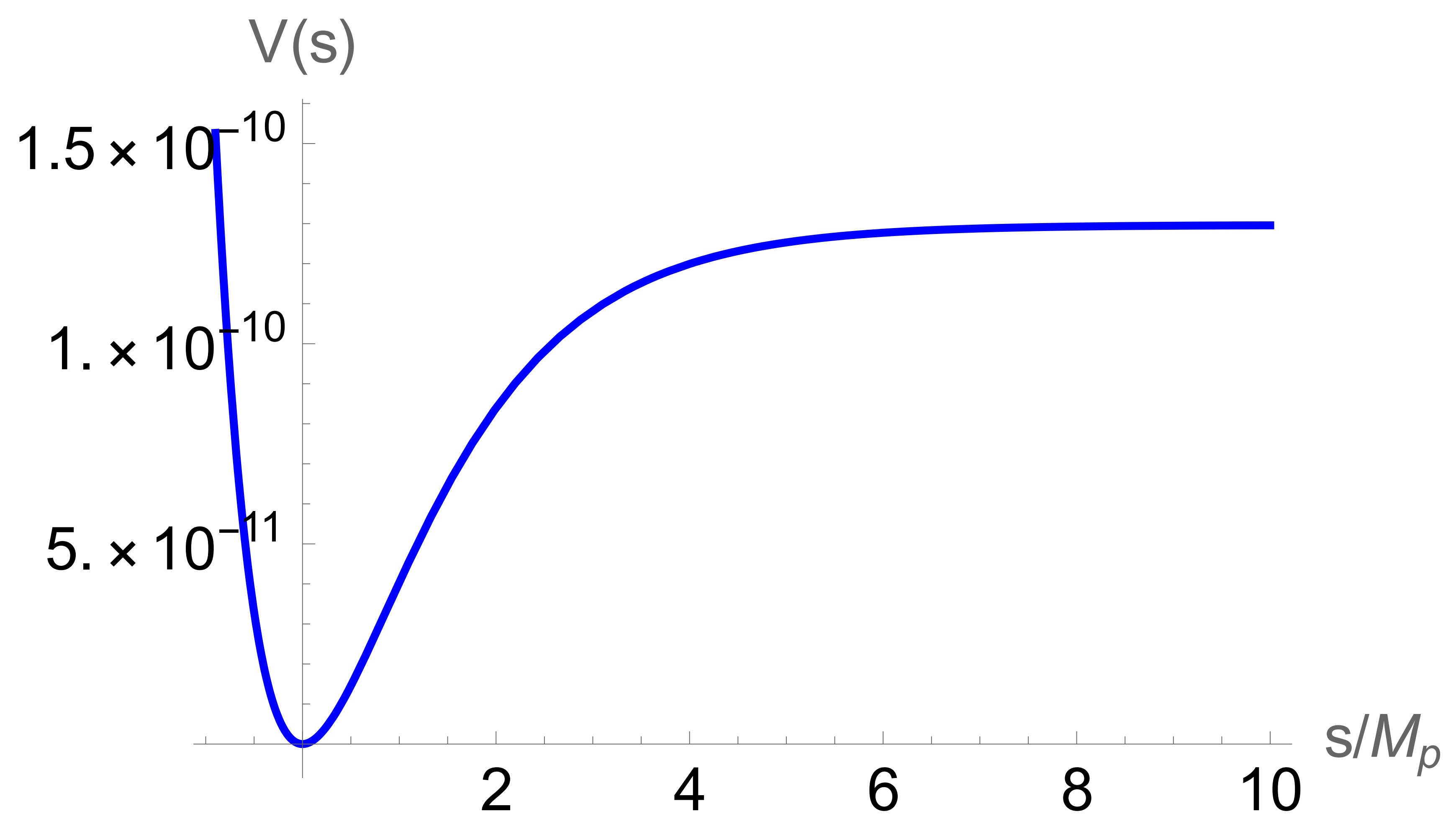
the "Einstein" frame action reads,

$$S = \int d^4x \sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}g^{ab}\nabla_a s \nabla_b s - \frac{\lambda M_p^4}{4\sigma^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{s}{M_p}}\right)^2\right],$$

where $s/M_p \equiv \sqrt{3/2}\,\ln \Omega^2$ is the canonically normalized scalar field.

Higgs Inflation

in the Einstein frame



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with $V(s) \equiv \frac{\lambda M_p^4}{4\sigma^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{s}{M_p}}\right)^2$

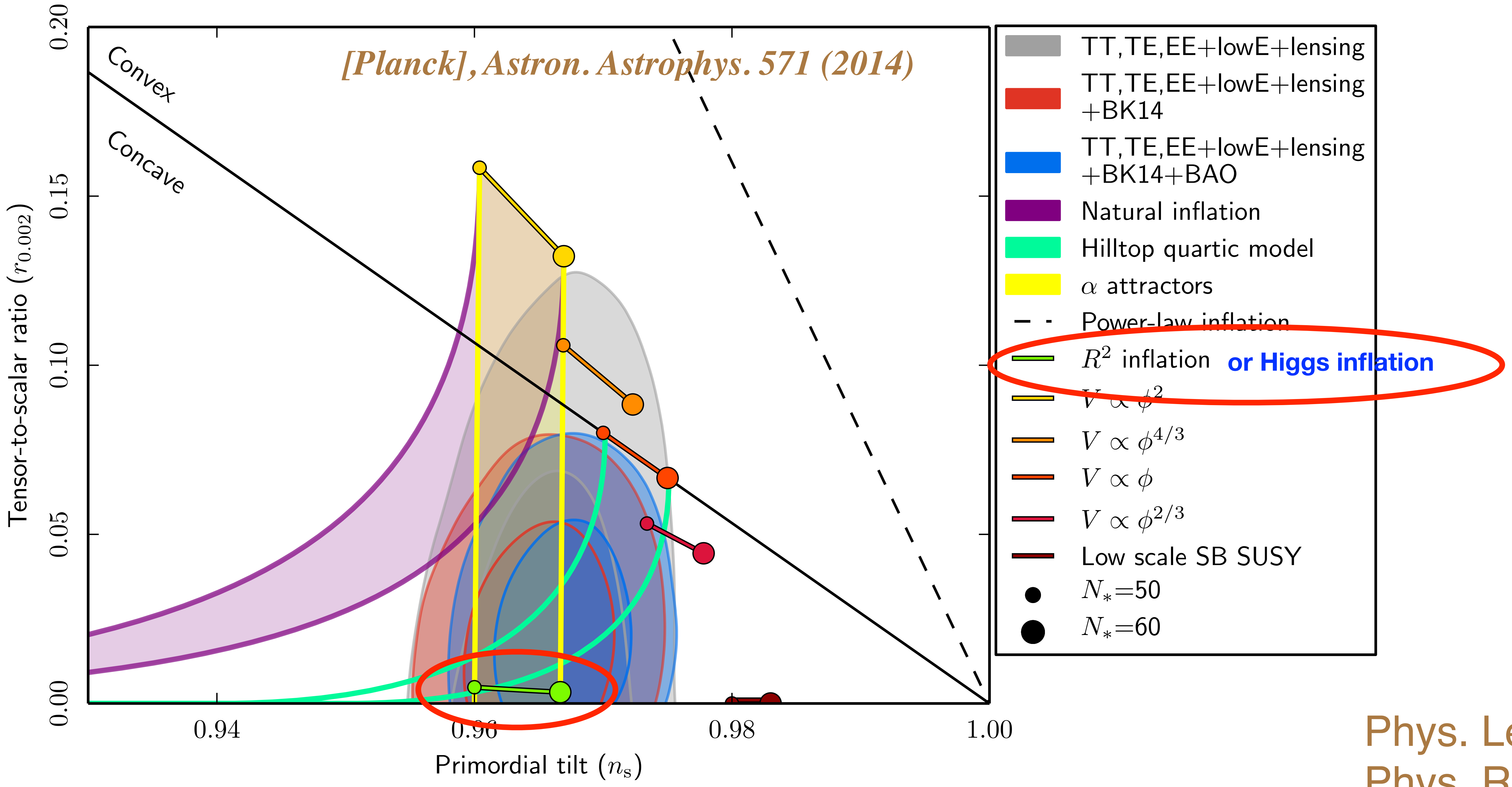
where $s/M_p \equiv \sqrt{3/2}\ln\Omega^2$ is the canonically normalized scalar field.

Higgs Inflation

in the Einstein frame

$$V(s) = \frac{\lambda M_p^4}{4\sigma^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{s}{M_p}}\right)^2$$

To be consistent with the CMB observation: $\lambda/\sigma^2 \simeq 5 \times 10^{-10}$



Phys. Lett. B 659 (2008) 703,
Phys. Rev. D 92 (2015) no.8, 083512
JHEP 02 (2021) 109

- indicating $\sigma \gg 1$ (unless the λ is tiny) $\Longrightarrow$ causes a strong coupling problem, where the model loses perturbative unitarity at a scale M_p/σ, well below the Planck scale.

While it is the most minimalistic and experimentally compatible,
it is feasible to expect additional interactions to be present.

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Jordan frame

The action in the Jordan Frame:

Phys. Rev. D 93 (2016) no.6, 063519
Phys. Rev. D 107 (2023) no.6, 06350

$$S^J = \int d^4x\sqrt{-g^J}\left[\frac{M_p^2}{2}\Omega^2(\phi)R^J - \frac{1}{2}g^J_{ab}\nabla^a\phi\,\nabla^b\phi - V(\phi) + \omega(\phi)R_{GB}^{2^J}\right] ,$$

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where $R^2_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the Gauss-Bonnet combination and $\omega(\phi)$ is the coupling function, while

$$\Omega^2(\phi) = 1 + \frac{\sigma}{M_p^2}\phi^2\,, \quad V(\phi) = \frac{\lambda}{4}(\phi^2 - \nu^2)^2\,.$$

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$$\Omega^2(\phi) = 1 + \frac{\sigma}{M_p^2}\phi^2\,, \quad V(\phi) = \frac{\lambda}{4}(\phi^2 - \nu^2)^2\,.$$

* Arpan's talk discussed the $\sigma = 0$ case.

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where $R^2_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the Gauss-Bonnet combination and $\omega(\phi)$ is the coupling function, while

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Phys. Rev. D 93 (2016) no.6, 063519

To allow much of this analysis to be confidently performed in the Jordan frame, as the conformal transformation of the Gauss-Bonnet term is rather complicated, we first rediscovered the known analytical predictions for standard Higgs inflation to leading order in slow-roll without appealing to a conformal transformation to the Einstein frame as is usually done.

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Jordan frame

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$$\Omega^2(\phi) = 1 + \frac{\sigma}{M_p^2}\phi^2\,, \quad V(\phi) = \frac{\lambda}{4}(\phi^2 - \nu^2)^2\,.$$

"We are interested in to see how this action change under the CT and what consequent dynamics would be apparent in the Einstein frame that otherwise does not come into sight in the Jordan frame"

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Jordan frame

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We already know how this part transforms under the conformal transformation.

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}g^{ab}\nabla_a s\nabla_b s - \frac{\lambda M_p^4}{4\sigma^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{s}{M_p}}\right)^2\right]$$

Higgs Inflation

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We do not know how this term transforms under the conformal transformation.

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Jordan frame

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Phys. Rev. D 107 (2023) no.6, 06350

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We already know how this part transforms under the conformal transformation.

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}g^{ab}\nabla_a s\nabla_b s - \frac{\lambda M_p^4}{4\sigma^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{s}{M_p}}\right)^2\right]$$

We do not know how this term transforms under the conformal transformation.

Let us focus on this term…

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

The Gauss-Bonnet part of the action reads:

$$S^J = \int d^4x\sqrt{-g^J}\omega(\phi)R^{2^J}_{GB}.$$

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

The Gauss-Bonnet part of the action reads:

$$S^J = \int d^4x\sqrt{-g^J}\,\omega(\phi)R^{2^J}_{GB}.$$

Under the $g^J_{ab} = \Omega^{-2}g_{ab}$, the Gauss-Bonnet term changes as

$$R^{2^J}_{GB} = \Omega^4\Big[R^2_{GB} - 8\Omega^{-1}G_{ab}\nabla^a\nabla^b\Omega - 4R\Omega^{-2}\nabla_a\Omega\nabla^a\Omega$$
$$+8\Omega^{-2}\left(\nabla_a\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\nabla_a\Omega\nabla^b\nabla^a\Omega\right) - 24\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega + 24\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\Big].$$

Grav. Cosmol. 10, 305 (2004)

* $R^2_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$

** $G_{ab} \equiv R_{ab} - g_{ab}R/2$

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Under the $g^J_{ab} = \Omega^{-2}g_{ab}$, the Gauss-Bonnet term changes as

$$R^{2^J}_{GB} = \Omega^4\left[R^2_{GB} - 8\Omega^{-1}G_{ab}\nabla^a\nabla^b\Omega - 4R\Omega^{-2}\nabla_a\Omega\nabla^a\Omega \right.$$
$$\left. +8\Omega^{-2}\left(\nabla_a\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\nabla_a\Omega\nabla^b\nabla^a\Omega\right) - 24\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega + 24\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\right].$$

Grav. Cosmol. 10, 305 (2004)

Consequently, the action becomes

$$S = \int d^4x\sqrt{-g}\,\omega(\phi)\left[R^2_{GB} - 8\Omega^{-1}G_{ab}\nabla^a\nabla^b\Omega - 4R\Omega^{-2}\nabla_a\Omega\nabla^a\Omega + 8\Omega^{-2}\left(\nabla_a\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\nabla_a\Omega\nabla^b\nabla^a\Omega\right) - 24\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega + 24\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\right].$$

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

$$S = \int d^4x\sqrt{-g}\,\omega\left[R_{GB}^2 - 8\Omega^{-1}G_{ab}\nabla^a\nabla^b\Omega - 4R\Omega^{-2}\nabla_a\Omega\nabla^a\Omega + 8\Omega^{-2}\left(\nabla_a\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\nabla_a\Omega\nabla^b\nabla^a\Omega\right) - 24\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega + 24\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\right].$$

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What happens if ω=*const.*?

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

$$S = \int d^4x \sqrt{-g}\, \omega \left[R_{GB}^2 - 8\Omega^{-1} G_{ab} \nabla^a \nabla^b \Omega - 4R\Omega^{-2} \nabla_a \Omega \nabla^a \Omega + 8\Omega^{-2} \left(\nabla_a \nabla^a \Omega \nabla_b \nabla^b \Omega - \nabla_b \nabla_a \Omega \nabla^b \nabla^a \Omega \right) - 24\Omega^{-3} \nabla_a \Omega \nabla^a \Omega \nabla_b \nabla^b \Omega + 24\Omega^{-4} \left(\nabla_a \Omega \nabla^a \Omega \right)^2 \right] .$$

What happens if ω=*const.*?

Do we expect to see any nontrivial effects of the “Gauss-Bonnet contributions” in the Einstein frame?

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

$$S = \int d^4x\sqrt{-g}\,\omega\left[R^2_{GB} - 8\Omega^{-1}G_{ab}\nabla^a\nabla^b\Omega - 4R\Omega^{-2}\nabla_a\Omega\nabla^a\Omega + 8\Omega^{-2}\left(\nabla_a\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\nabla_a\Omega\nabla^b\nabla^a\Omega\right) - 24\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega + 24\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\right].$$

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Do we expect to see any nontrivial effects of the "Gauss-Bonnet contributions" in the Einstein frame?

i. When ω=*const.*, we obtain

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame when $\omega = const.$

$$S = \omega \int d^4x\sqrt{-g}\,\left[R^2_{GB} - 8\Omega^{-1}G_{ab}\nabla^a\nabla^b\Omega - 4R\Omega^{-2}\nabla_a\Omega\nabla^a\Omega + 8\Omega^{-2}\left(\nabla_a\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\nabla_a\Omega\nabla^b\nabla^a\Omega\right) - 24\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega + 24\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\right].$$

What happens if ω=*const.*?

Do we expect to see any nontrivial effects of the "Gauss-Bonnet contributions" in the Einstein frame?

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* $$-8\omega\int d^4x\sqrt{-g}\Omega^{-1}G_{ab}\nabla^a\nabla^b\Omega = -8\omega\int d^4x\sqrt{-g}\Omega^{-2}G_{ab}\nabla^a\Omega\nabla^b\Omega$$

* $$8\omega\int d^4x\sqrt{-g}\,\Omega^{-2}\left(\nabla_a\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\nabla_a\Omega\nabla^b\nabla^a\Omega\right)$$

$$= 8\omega\int d^4x\sqrt{-g}\,\left(\Omega^{-2}R^{ab}\nabla_a\Omega\nabla_b\Omega + 2\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega - 2\Omega^{-3}\nabla_a\Omega\nabla_b\Omega\nabla^a\nabla^b\Omega\right)$$

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$$S = \omega \int d^4x\sqrt{-g}\ \left[R^2_{GB} - 8\Omega^{-1}G_{ab}\nabla^a\nabla^b\Omega - 4R\Omega^{-2}\nabla_a\Omega\nabla^a\Omega + 8\Omega^{-2}\left(\nabla_a\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\nabla_a\Omega\nabla^b\nabla^a\Omega\right) - 24\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega + 24\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\right] .$$

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* $8\omega\int d^4x\sqrt{-g}\ \Omega^{-2}\left(\nabla_a\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\nabla_a\Omega\nabla^b\nabla^a\Omega\right)$

$$= 8\omega\int d^4x\sqrt{-g}\ \left(\Omega^{-2}R^{ab}\nabla_a\Omega\nabla_b\Omega + 2\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega - 2\Omega^{-3}\nabla_a\Omega\nabla_b\Omega\nabla^a\nabla^b\Omega\right)$$

To obtain this, we used

$$\nabla_a\left[\Omega^{-2}\left(\nabla_b\nabla^b\Omega\nabla^a\Omega - \frac{1}{2}\nabla^a(\nabla\Omega)^2\right)\right] = \Omega^{-2}\left[(\nabla_a\nabla^a\Omega)^2 - (\nabla_a\nabla_b\Omega)^2\right] - R^{ab}\Omega^{-2}\nabla_a\Omega\nabla_b\Omega - 2\Omega^{-3}\nabla_a\Omega\left(\nabla_b\nabla^b\Omega\nabla^a\Omega - \frac{1}{2}\nabla^a(\nabla\Omega)^2\right)$$

$$[\nabla_a, \nabla_b]V^c = -V^dR^c{}_{dba}$$

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame when $\omega = const.$

$$S = \omega \int d^4x\sqrt{-g}\,\left[R_{GB}^2 - 8\Omega^{-1}G_{ab}\nabla^a\nabla^b\Omega - 4R\Omega^{-2}\nabla_a\Omega\nabla^a\Omega + 8\Omega^{-2}\left(\nabla_a\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\nabla_a\Omega\nabla^b\nabla^a\Omega\right) - 24\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega + 24\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\right].$$

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Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame when $\omega = const.$

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⇩

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Higgs Inflation

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Higgs Inflation

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What happens if ω=*const.*? **No dynamical contributions!**

Do we expect to see any nontrivial effects of the "Gauss-Bonnet contributions" in the Einstein frame?

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⇩

$$S = \omega \int d^4x\sqrt{-g}\,\left[R^2_{GB} - 16\nabla_a\ln\Omega\nabla_b\ln\Omega\left(\Omega^{-1}\nabla^a\nabla^b\Omega\right) - 8\nabla_a\ln\Omega\nabla^a\ln\Omega\left(\Omega^{-1}\nabla_b\nabla^b\Omega\right) + 24\left(\nabla_a\ln\Omega\nabla^a\ln\Omega\right)^2\right],$$

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The Gauss-Bonnet coupling must be a function of a scalar field, i.e., $\omega(\phi)$*!*

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

$$S=\int d^4x\sqrt{-g}\,\omega(\phi)\Big[R^2_{GB}-8\Omega^{-1}G_{ab}\nabla^a\nabla^b\Omega-4R\Omega^{-2}\nabla_a\Omega\nabla^a\Omega+8\Omega^{-2}\left(\nabla_a\nabla^a\Omega\nabla_b\nabla^b\Omega-\nabla_b\nabla_a\Omega\nabla^b\nabla^a\Omega\right)-24\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega+24\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\Big]\,.$$

$\Longrightarrow$ **This time $\omega(\phi)$ is a function of a scalar field ϕ.**

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

$$S = \int d^4x\sqrt{-g}\,\omega(\phi)\Big[R^2_{GB} - 8\Omega^{-1}G_{ab}\nabla^a\nabla^b\Omega - 4R\Omega^{-2}\nabla_a\Omega\nabla^a\Omega + 8\Omega^{-2}\left(\nabla_a\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\nabla_a\Omega\nabla^b\nabla^a\Omega\right) - 24\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega + 24\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\Big]\,.$$

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** $G_{ab} \equiv R_{ab} - g_{ab}R/2$

Higgs Inflation

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$$=8\int d^4x\sqrt{-g}\left[\omega\Omega^{-2}R_{ab}\nabla^a\Omega\nabla^b\Omega-\omega\Omega^{-2}\left(\omega^{-1}\nabla_a\omega-2\Omega^{-1}\nabla_a\Omega\right)\left(\nabla^a\Omega\nabla_b\nabla^b\Omega-\nabla_b\Omega\nabla^a\nabla^b\Omega\right)\right]$$

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$$= 8\int d^4x\sqrt{-g}\left[\omega\Omega^{-2}R_{ab}\nabla^a\Omega\nabla^b\Omega - \omega\Omega^{-2}\left(\omega^{-1}\nabla_a\omega - 2\Omega^{-1}\nabla_a\Omega\right)\left(\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\Omega\nabla^a\nabla^b\Omega\right)\right]$$

$$S = \int d^4x\sqrt{-g}\left[\omega R^2_{GB} + 8\Omega^{-1}G_{ab}\nabla^a\omega\nabla^b\Omega - 8\omega\Omega^{-2}\left(\omega^{-1}\nabla_a\omega - 2\Omega^{-1}\nabla_a\Omega\right)\left(\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\Omega\nabla^a\nabla^b\Omega\right) - 24\omega\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega + 24\omega\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\right]$$

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* $-8\int d^4x\sqrt{-g}\,\omega\left[\Omega^{-1}G_{ab}\nabla^a\nabla^b\Omega + \frac{1}{2}g_{ab}R\Omega^{-2}\nabla^a\Omega\nabla^b\Omega\right] = \int d^4x\sqrt{-g}\left[\omega\Omega^{-2}R_{ab}\nabla^a\Omega\nabla^b\Omega - \Omega^{-1}G_{ab}\nabla^a\omega\nabla^b\Omega\right]$

* $8\int d^4x\sqrt{-g}\,\omega\Omega^{-2}\left(\nabla_a\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\nabla_a\Omega\nabla^b\nabla^a\Omega\right)$

$$= 8\int d^4x\sqrt{-g}\left[\omega\Omega^{-2}R_{ab}\nabla^a\Omega\nabla^b\Omega - \omega\Omega^{-2}\left(\omega^{-1}\nabla_a\omega - 2\Omega^{-1}\nabla_a\Omega\right)\left(\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\Omega\nabla^a\nabla^b\Omega\right)\right]$$

$$S = \int d^4x\sqrt{-g}\left[\omega R_{GB}^2 + 8\Omega^{-1}G_{ab}\nabla^a\omega\nabla^b\Omega - 8\omega\Omega^{-2}\left(\omega^{-1}\nabla_a\omega - 2\Omega^{-1}\nabla_a\Omega\right)\left(\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\Omega\nabla^a\nabla^b\Omega\right) - 24\omega\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega + 24\omega\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\right]$$

If $\omega = \alpha\Omega^2$, $\left(\omega^{-1}\nabla_a\omega - 2\Omega^{-1}\nabla_a\Omega\right) = 0.$

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

$$S = \int d^4x\sqrt{-g}\,\omega(\phi)\Big[R^2_{GB} - 8\Omega^{-1}G_{ab}\nabla^a\nabla^b\Omega - 4R\Omega^{-2}\nabla_a\Omega\nabla^a\Omega + 8\Omega^{-2}\left(\nabla_a\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\nabla_a\Omega\nabla^b\nabla^a\Omega\right) - 24\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega + 24\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\Big]\,.$$

* $$-8\int d^4x\sqrt{-g}\,\omega\left[\Omega^{-1}G_{ab}\nabla^a\nabla^b\Omega + \frac{1}{2}g_{ab}R\Omega^{-2}\nabla^a\Omega\nabla^b\Omega\right] = \int d^4x\sqrt{-g}\left[\omega\Omega^{-2}R_{ab}\nabla^a\Omega\nabla^b\Omega - \Omega^{-1}G_{ab}\nabla^a\omega\nabla^b\Omega\right]$$

* $$8\int d^4x\sqrt{-g}\,\omega\Omega^{-2}\left(\nabla_a\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\nabla_a\Omega\nabla^b\nabla^a\Omega\right)$$

$$= 8\int d^4x\sqrt{-g}\left[\omega\Omega^{-2}R_{ab}\nabla^a\Omega\nabla^b\Omega - \omega\Omega^{-2}\left(\omega^{-1}\nabla_a\omega - 2\Omega^{-1}\nabla_a\Omega\right)\left(\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\Omega\nabla^a\nabla^b\Omega\right)\right]$$

$$S = \int d^4x\sqrt{-g}\left[\omega R^2_{GB} + 8\Omega^{-1}G_{ab}\nabla^a\omega\nabla^b\Omega - 8\omega\Omega^{-2}\left(\omega^{-1}\nabla_a\omega - 2\Omega^{-1}\nabla_a\Omega\right)\left(\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\Omega\nabla^a\nabla^b\Omega\right) - 24\omega\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega + 24\omega\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\right]$$

If $\omega = \alpha\Omega^2$, $\left(\omega^{-1}\nabla_a\omega - 2\Omega^{-1}\nabla_a\Omega\right) = 0.$

$$S = \int d^4x\sqrt{-g}\,\alpha\Omega^2\left[R^2_{GB} + 16\Omega^{-2}G_{ab}\nabla^a\Omega\nabla^b\Omega - 24\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega + 24\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\right]$$

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

$$S = \int d^4x\sqrt{-g}\,\omega(\phi)\Big[R^2_{GB} - 8\Omega^{-1}G_{ab}\nabla^a\nabla^b\Omega - 4R\Omega^{-2}\nabla_a\Omega\nabla^a\Omega + 8\Omega^{-2}\left(\nabla_a\nabla^a\Omega\nabla_b\nabla^b\Omega - \nabla_b\nabla_a\Omega\nabla^b\nabla^a\Omega\right) - 24\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega + 24\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\Big]\,.$$

$$S = \int d^4x\sqrt{-g}\alpha\Omega^2\left[R^2_{GB} + 16G_{ab}\nabla^a\ln\Omega\,\nabla^b\ln\Omega - 24\nabla_a\ln\Omega\,\nabla^a\ln\Omega\left(\Omega^{-1}\nabla_b\nabla^b\Omega\right) + 24\left(\nabla_a\ln\Omega\,\nabla^a\ln\Omega\right)^2\right]$$

$$S = \int d^4x\sqrt{-g}\alpha\Omega^2\left[R^2_{GB} + 16\Omega^{-2}G_{ab}\nabla^a\Omega\nabla^b\Omega - 24\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega + 24\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\right]$$

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

$$S=\int d^4x\sqrt{-g}\,\omega(\phi)\Big[R_{GB}^2-8\Omega^{-1}G_{ab}\nabla^a\nabla^b\Omega-4R\Omega^{-2}\nabla_a\Omega\nabla^a\Omega+8\Omega^{-2}\left(\nabla_a\nabla^a\Omega\nabla_b\nabla^b\Omega-\nabla_b\nabla_a\Omega\nabla^b\nabla^a\Omega\right)-24\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega+24\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\Big]\,.$$

$$S=\int d^4x\sqrt{-g}\alpha\Omega^2\left[R_{GB}^2+16G_{ab}\nabla^a\ln\Omega\nabla^b\ln\Omega-24\nabla_a\ln\Omega\nabla^a\ln\Omega\nabla_b\nabla^b\ln\Omega\right]$$

Using $\Omega^{-1}\nabla^a\nabla^b\Omega=\nabla^a\nabla^b\ln\Omega+\nabla^a\ln\Omega\nabla^b\ln\Omega$

$$S=\int d^4x\sqrt{-g}\alpha\Omega^2\left[R_{GB}^2+16G_{ab}\nabla^a\ln\Omega\nabla^b\ln\Omega-24\nabla_a\ln\Omega\nabla^a\ln\Omega\left(\Omega^{-1}\nabla_b\nabla^b\Omega\right)+24\left(\nabla_a\ln\Omega\nabla^a\ln\Omega\right)^2\right]$$

$$S=\int d^4x\sqrt{-g}\alpha\Omega^2\left[R_{GB}^2+16\Omega^{-2}G_{ab}\nabla^a\Omega\nabla^b\Omega-24\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega+24\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\right]$$

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

$$S=\int d^4x\sqrt{-g}\,\omega(\phi)\Big[R_{GB}^2-8\Omega^{-1}G_{ab}\nabla^a\nabla^b\Omega-4R\Omega^{-2}\nabla_a\Omega\nabla^a\Omega+8\Omega^{-2}\left(\nabla_a\nabla^a\Omega\nabla_b\nabla^b\Omega-\nabla_b\nabla_a\Omega\nabla^b\nabla^a\Omega\right)-24\Omega^{-3}\nabla_a\Omega\nabla^a\Omega\nabla_b\nabla^b\Omega+24\Omega^{-4}\left(\nabla_a\Omega\nabla^a\Omega\right)^2\Big]$$

$$\omega=\alpha\Omega^2$$

$$S=\int d^4x\sqrt{-g}\times\alpha e^{\sqrt{\frac{2}{3}}\frac{s}{M_p}}\left[R_{GB}^2+\frac{8}{3M_p^2}G_{ab}\nabla^a s\nabla^b s-\frac{1}{M_p^3}\sqrt{\frac{8}{3}}\nabla^b\nabla_b s\nabla_a s\nabla^a s\right]$$

arXiv:2308.00897

by defining $s/M_p\equiv\sqrt{3/2}\ln\Omega^2$

$$S=\int d^4x\sqrt{-g}\alpha\Omega^2\left[R_{GB}^2+16G_{ab}\nabla^a\ln\Omega\nabla^b\ln\Omega-24\nabla_a\ln\Omega\nabla^a\ln\Omega\nabla_b\nabla^b\ln\Omega\right]$$

Using $\Omega^{-1}\nabla^a\nabla^b\Omega=\nabla^a\nabla^b\ln\Omega+\nabla^a\ln\Omega\nabla^b\ln\Omega$

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Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

Combining with the minimal coupling case,

arXiv:2308.00897

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}(\partial s)^2 - V(s) - \frac{1}{2}\xi(s)\left(c_1 R_{GB}^2 + \frac{c_2}{M_p^2}G_{ab}\nabla^a s\nabla^b s + \frac{c_3}{M_p^3}\nabla_a s\nabla^a s\nabla^b\nabla_b s\right)\right],$$

where $V(s) \equiv \frac{\lambda M_p^4}{4\sigma^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{s}{M_p}}\right)^2, \quad \xi(s) \equiv -2\alpha e^{\sqrt{\frac{2}{3}}\frac{s}{M_p}}, \quad c_1 = 1, \quad c_2 = 8/3, \quad c_3 = -\sqrt{8/3}.$

$\lambda/\sigma^2 \simeq 5 \times 10^{-10}$

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

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$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}(\partial s)^2 - V(s) - \frac{1}{2}\xi(s)\left(c_1 R_{GB}^2 + \frac{c_2}{M_p^2}G_{ab}\nabla^a s\nabla^b s + \frac{c_3}{M_p^3}\nabla_a s\nabla^a s\nabla^b\nabla_b s\right)\right],$$

where $V(s) \equiv \frac{\lambda M_p^4}{4\sigma^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{s}{M_p}}\right)^2$, $\xi(s) \equiv -2\alpha e^{\sqrt{\frac{2}{3}}\frac{s}{M_p}}, \quad c_1 = 1, \quad c_2 = 8/3, \quad c_3 = -\sqrt{8/3}\,.$

JAI-CHAN HWANG AND HYERIM NOH

PHYSICAL REVIEW D **71,** 063536 (2005)

F. String corrections

We consider an action in Eq. (45) with the following additional corrections in the action [20,21]:

$$L_{(c)} = -\tfrac{1}{2}\xi(\phi)[c_1 R_{GB}^2 + c_2 G^{ab}\phi_{,a}\phi_{,b} + c_3\Box\phi\phi^{,c}\phi_{,c} + c_4(\phi^{,c}\phi_{,c})^2],$$

Higgs Inflation

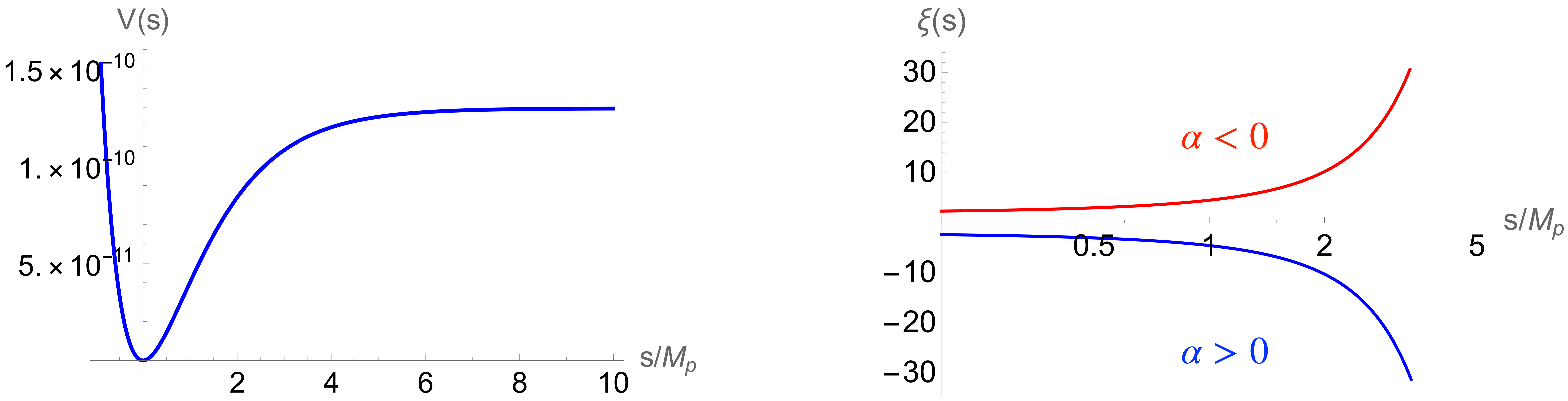
with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

Combining with the minimal coupling case,

arXiv:2308.00897

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}(\partial s)^2 - V(s) - \frac{1}{2}\xi(s)\left(c_1 R_{GB}^2 + \frac{c_2}{M_p^2}G_{ab}\nabla^a s\nabla^b s + \frac{c_3}{M_p^3}\nabla_a s\nabla^a s\nabla^b\nabla_b s\right)\right],$$

where $V(s) \equiv \frac{\lambda M_p^4}{4\sigma^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{s}{M_p}}\right)^2$, $\xi(s) \equiv -2\alpha e^{\sqrt{\frac{2}{3}}\frac{s}{M_p}}$, $c_1 = 1$, $c_2 = 8/3$, $c_3 = -\sqrt{8/3}$.



Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

Combining with the minimal coupling case,

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$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}(\partial s)^2 - V(s) - \frac{1}{2}\xi(s)\left(c_1 R_{GB}^2 + \frac{c_2}{M_p^2}G_{ab}\nabla^a s\nabla^b s + \frac{c_3}{M_p^3}\nabla_a s\nabla^a s\nabla^b\nabla_b s\right)\right],$$

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In the flat FRW universe with metric: $ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j$, the background equations of motion read:

$$3M_p^2H^2 = \frac{1}{2}\dot{s}^2 + V + 12c_1\dot{\xi}H^3 - \frac{9}{2}\frac{c_2}{M_p^2}\xi\dot{s}^2H^2 + \frac{1}{2}\frac{c_3}{M_p^3}(\dot{\xi} - 6\xi H)\dot{s}^3\,,$$

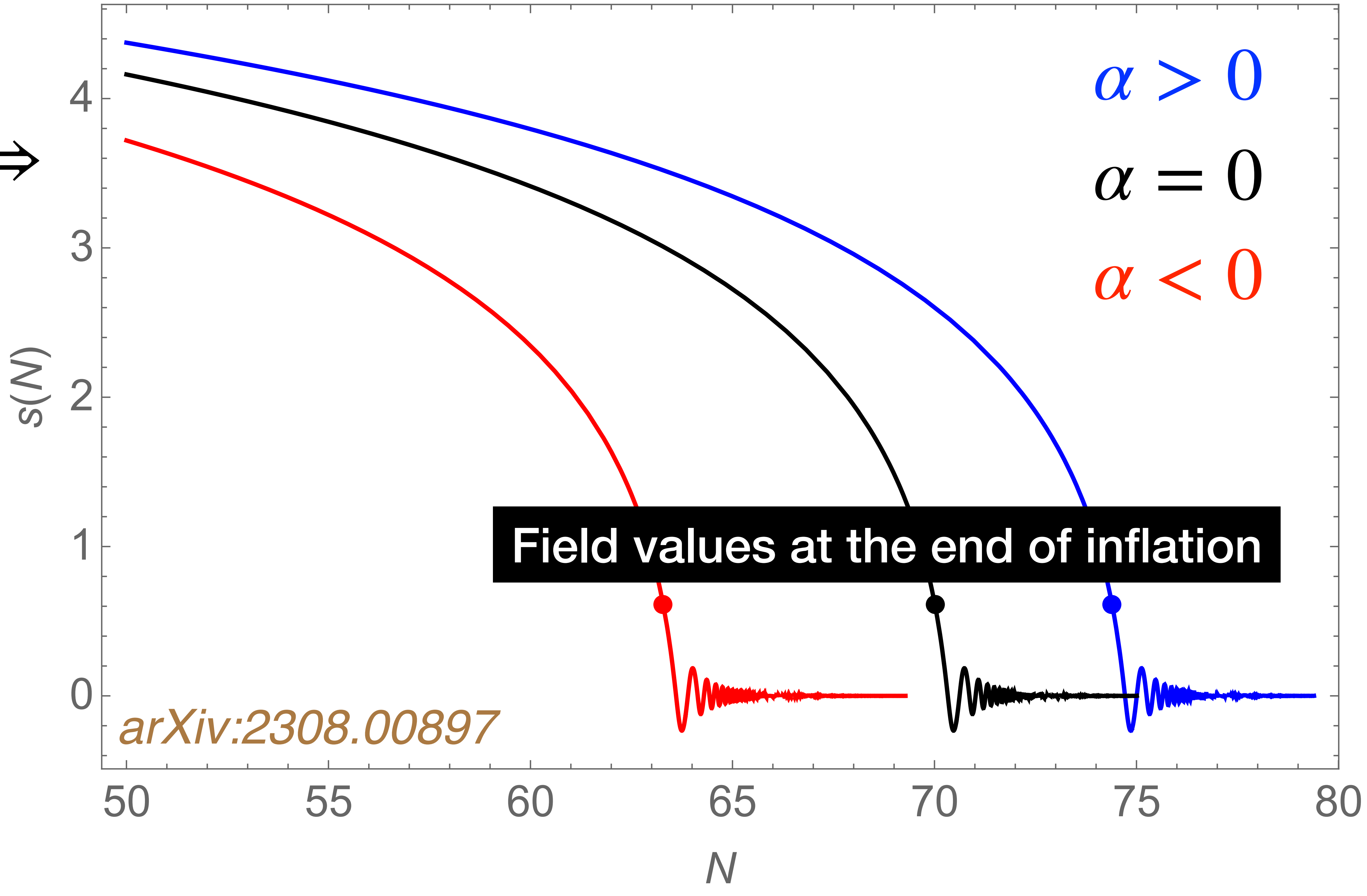
$$M_p^2(2\dot{H} + 3H^2) = -\frac{1}{2}\dot{s}^2 + V + 4c_1\left[\ddot{\xi}H^2 + 2\dot{\xi}H(\dot{H} + H^2)\right] - \frac{1}{2}\frac{c_2}{M_p^2}\dot{s}\left[\xi\dot{s}(2\dot{H} + 3H^2) + 4\xi\ddot{s}H + 2\dot{\xi}\dot{s}H\right] - \frac{1}{2}\frac{c_3}{M_p^3}\dot{s}^2(2\xi\ddot{s} + \dot{\xi}\dot{s})\,,$$

$$\ddot{s} + 3H\dot{s} + V_{,s} = -12c_1\xi_{,s}H^2(\dot{H} + H^2) + \frac{3}{2}\frac{c_2}{M_p^2}\left[H^2(\dot{\xi}\dot{s} + 2\xi\ddot{s}) + 2H\xi\dot{s}(2\dot{H} + 3H^2)\right] - \frac{1}{2}\frac{c_3}{M_p^3}\dot{s}\left[\ddot{\xi}\dot{s} + 3\dot{\xi}\ddot{s} - 6\xi(\dot{H}\dot{s} + 2H\ddot{s} + 3H^2\dot{s})\right].$$

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

The solutions to the background equations of motion: $\Longrightarrow$



$$V(s) \equiv \frac{\lambda M_p^4}{4\sigma^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{s}{M_p}}\right)^2 , \quad \xi(s) \equiv -2\alpha e^{\sqrt{\frac{2}{3}}\frac{s}{M_p}}$$

$$3M_p^2H^2 = \frac{1}{2}\dot{s}^2 + V + 12c_1\dot{\xi}H^3 - \frac{9}{2}\frac{c_2}{M_p^2}\xi\dot{s}^2H^2 + \frac{1}{2}\frac{c_3}{M_p^3}(\dot{\xi} - 6\xi H)\dot{s}^3 ,$$

$$M_p^2(2\dot{H} + 3H^2) = -\frac{1}{2}\dot{s}^2 + V + 4c_1\left[\ddot{\xi}H^2 + 2\dot{\xi}H(\dot{H} + H^2)\right] - \frac{1}{2}\frac{c_2}{M_p^2}\dot{s}\left[\xi\dot{s}(2\dot{H} + 3H^2) + 4\xi\ddot{s}H + 2\dot{\xi}\dot{s}H\right] - \frac{1}{2}\frac{c_3}{M_p^3}\dot{s}^2(2\xi\ddot{s} + \dot{\xi}\dot{s}) ,$$

$$\ddot{s} + 3H\dot{s} + V_{,s} = -12c_1\xi_{,s}H^2(\dot{H} + H^2) + \frac{3}{2}\frac{c_2}{M_p^2}\left[H^2(\dot{\xi}\dot{s} + 2\xi\ddot{s}) + 2H\xi\dot{s}(2\dot{H} + 3H^2)\right] - \frac{1}{2}\frac{c_3}{M_p^3}\dot{s}\left[\ddot{\xi}\dot{s} + 3\dot{\xi}\ddot{s} - 6\xi(\dot{H}\dot{s} + 2H\ddot{s} + 3H^2\dot{s})\right].$$

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

In the context of slow-roll inflation, *i.e.*, $V \gg \dot{s}^2$, $\ddot{s} \ll 3H\dot{s}$, and $\dot{\xi}/(2\xi H) \ll 1$, the equations of motion are simplified as

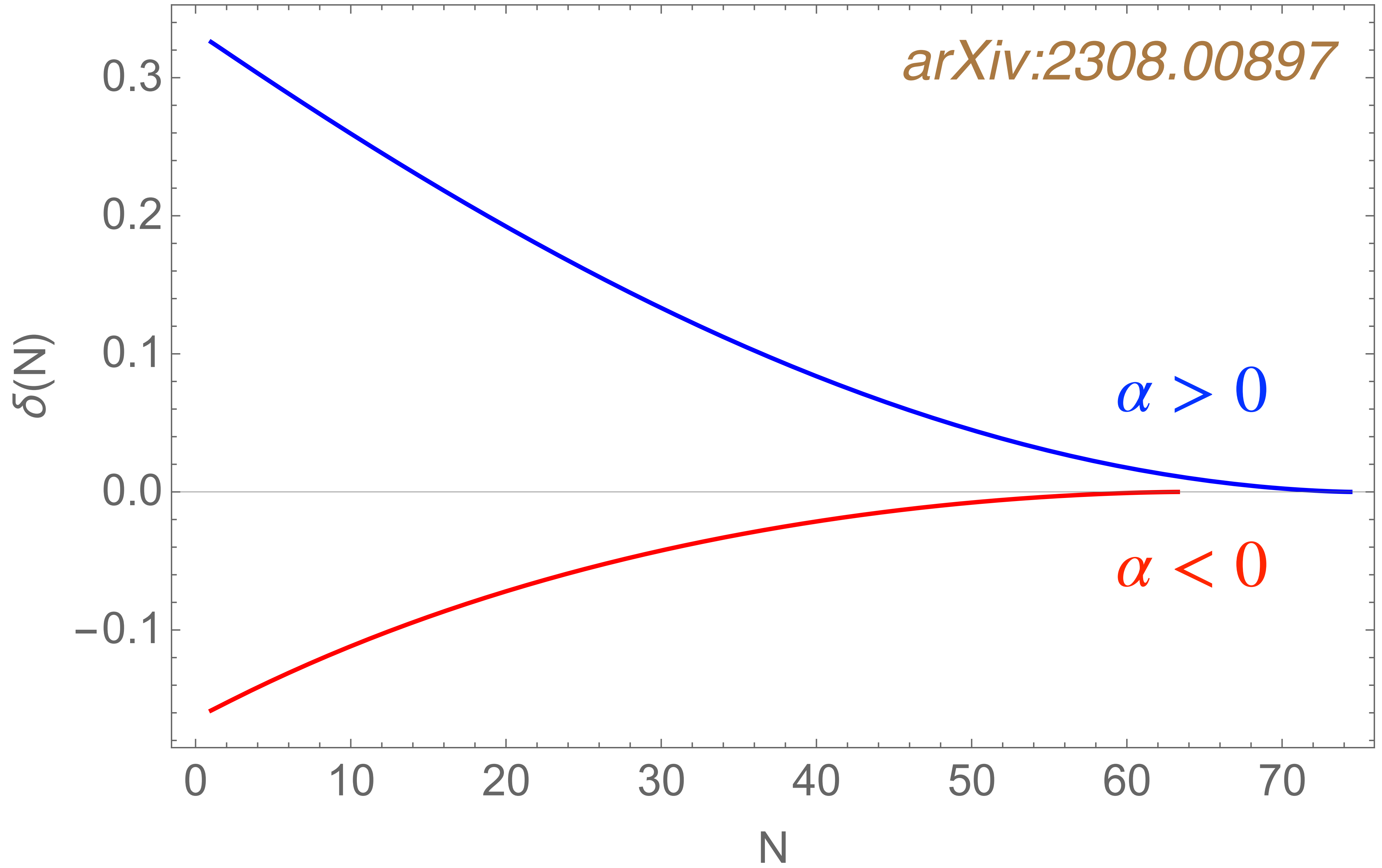
$$3M_p^2 H^2 \simeq V\,,$$

$$3H\dot{s} \simeq -V_{,s}\left[1+\delta(s)\right],$$

where $\delta(s) \equiv \dfrac{\mathscr{B} - \sqrt{\mathscr{B}^2 - 4\mathscr{A}\mathscr{C}}}{2\mathscr{A}V_{,s}} - 1\,,$

with $\mathscr{A} \equiv \dfrac{c_3}{M_p^3}\xi\,, \mathscr{B} \equiv 1 - \dfrac{3c_2}{M_p^2}\xi H^2\,, \mathscr{C} \equiv V_{,s} + 12c_1\xi_{,s}H^4.$

The number of $e-$folds: $N \equiv \displaystyle\int_{t_i}^{t_e} H dt = \int_{s_i}^{s_e} \frac{H}{\dot{s}} ds$, where $\dfrac{\dot{s}}{H} \simeq -M_p^2 \dfrac{V_{,s}}{V}(1+\delta)$,



Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

Combining with the minimal coupling case,

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}(\partial s)^2 - V(s) - \frac{1}{2}\xi(s)\left(c_1 R_{GB}^2 + \frac{c_2}{M_p^2}G_{ab}\nabla^a s\nabla^b s + \frac{c_3}{M_p^3}\nabla_a s\nabla^a s\nabla^b\nabla_b s\right)\right],$$

where $V(s) \equiv \frac{\lambda M_p^4}{4\sigma^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{s}{M_p}}\right)^2$, $\xi(s) \equiv -2\alpha e^{\sqrt{\frac{2}{3}}\frac{s}{M_p}}, \quad c_1 = 1, \quad c_2 = 8/3, \quad c_3 = -\sqrt{8/3}.$

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PHYSICAL REVIEW D **71,** 063536 (2005)

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Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

$$n_S - 1 = 2(2\epsilon_1 - \epsilon_2 - \epsilon_3)\,, \quad n_T = 2(\epsilon_1 - \epsilon_5)\,, \quad r = 16\left|\frac{1}{Q_t}\left(\frac{c_A}{c_T}\right)^3\left(\epsilon_1 - \frac{1}{4M_p^2H^2}\left(2Q_c + Q_d - HQ_e + H^2Q_f\right)\right)\right|\,.$$

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$$\epsilon_1 \equiv \frac{\dot H}{H^2}\,,\ \epsilon_2 \equiv \frac{\ddot s}{H\dot s}\,,\ \epsilon_3 \equiv \frac{\dot E}{2EH}\,,\ \epsilon_4 \equiv \frac{Q_a}{2H(2M_p^2 + Q_b)}\,,\ \epsilon_5 \equiv \frac{\dot Q_t}{2Q_tH}\,,\ \text{where } E \equiv \frac{1}{\dot s^2}\left(\dot s^2 + \frac{3Q_a^2}{2M_p^2 + Q_b} + Q_c\right),$$

$$Q_a \equiv -4c_1\dot\xi H^2 + \frac{2c_2}{M_p^2}\xi\dot s^2 H + \frac{c_3}{M_p^3}\xi\dot s^3\,, Q_b \equiv -8c_1\dot\xi H + \frac{c_2}{M_p^2}\xi\dot s^2\,, Q_c \equiv -\frac{3c_2}{M_p^2}\xi\dot s^2H^2 + \frac{2c_3}{M_p^3}\dot s^3(\dot\xi - 3\xi H),$$

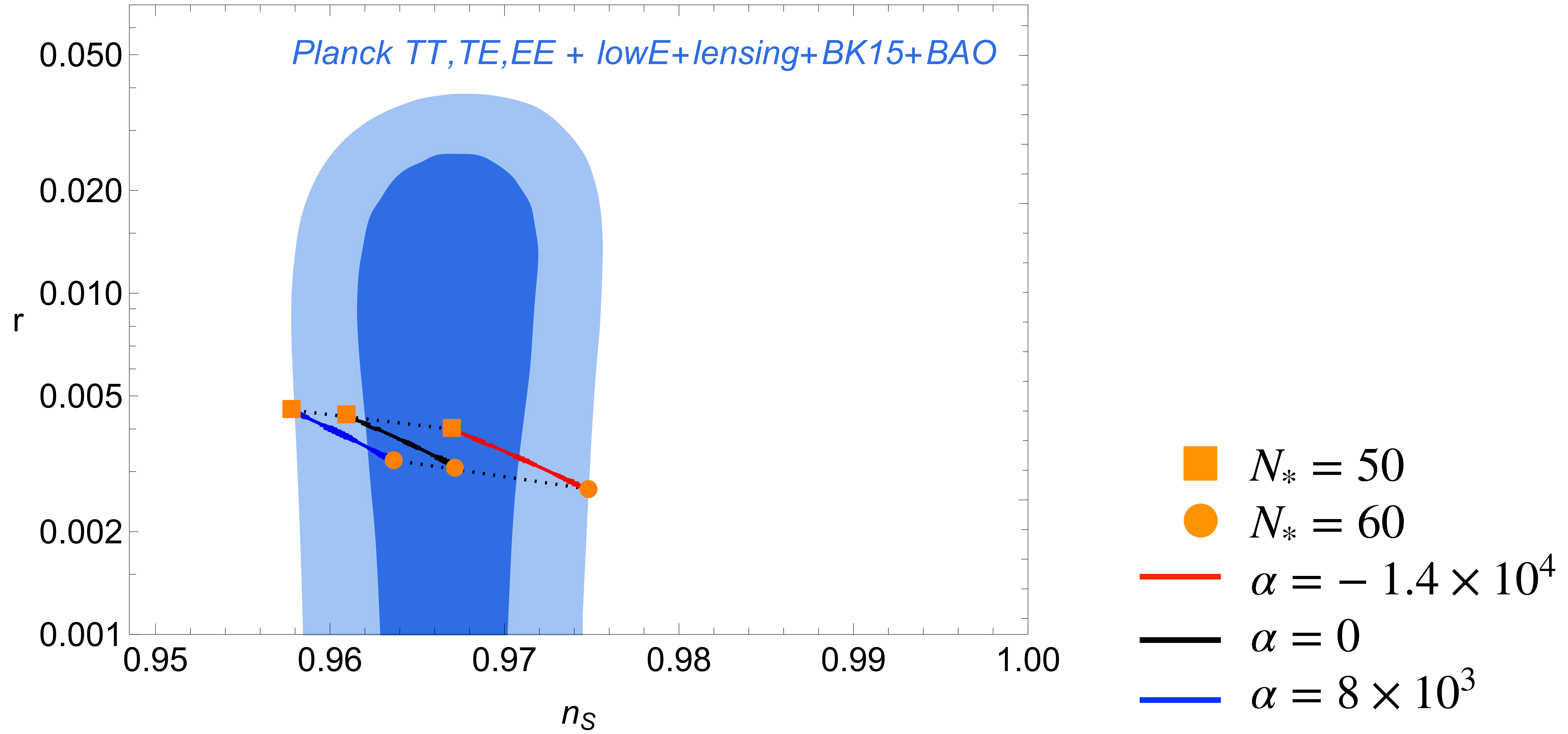
$$Q_d \equiv -\frac{2c_2}{M_p^2}\xi\dot s^2\dot H - \frac{2c_3}{M_p^3}\dot s^2(\dot\xi\dot s + \xi\ddot s - \xi\dot s H),\ Q_e \equiv -16c_1\dot\xi\dot H + \frac{2c_2}{M_p^2}\dot s(\dot\xi\dot s + 2\xi\ddot s - 2\xi\dot s H) - \frac{4c_3}{M_p^3}\xi\dot s^3\,,\ Q_f \equiv 8c_1(\ddot\xi - \dot\xi H) + \frac{2c_2}{M_p^2}\xi\dot s^2\,, Q_t \equiv 1 + \frac{Q_b}{2M_p^2},$$

$$c_A^2 \equiv 1 + \frac{Q_d + \frac{Q_a}{2M_p^2 + Q_b}Q_e + \left(\frac{Q_a}{2M_p^2 + Q_b}\right)^2 Q_f}{\dot s^2 + \frac{3Q_a^2}{2M_p^2 + Q_b} + Q_c}\,, \qquad c_T^2 \equiv 1 - \frac{Q_f}{2M_p^2 + Q_b}\,,$$

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

$$n_S - 1 = 2(2\epsilon_1 - \epsilon_2 - \epsilon_3)\,, \quad n_T = 2(\epsilon_1 - \epsilon_5)\,, \quad r = 16\left|\frac{1}{Q_t}\left(\frac{c_A}{c_T}\right)^3\left(\epsilon_1 - \frac{1}{4M_p^2H^2}\left(2Q_c + Q_d - HQ_e + H^2Q_f\right)\right)\right|\,.$$



Higgs Inflation

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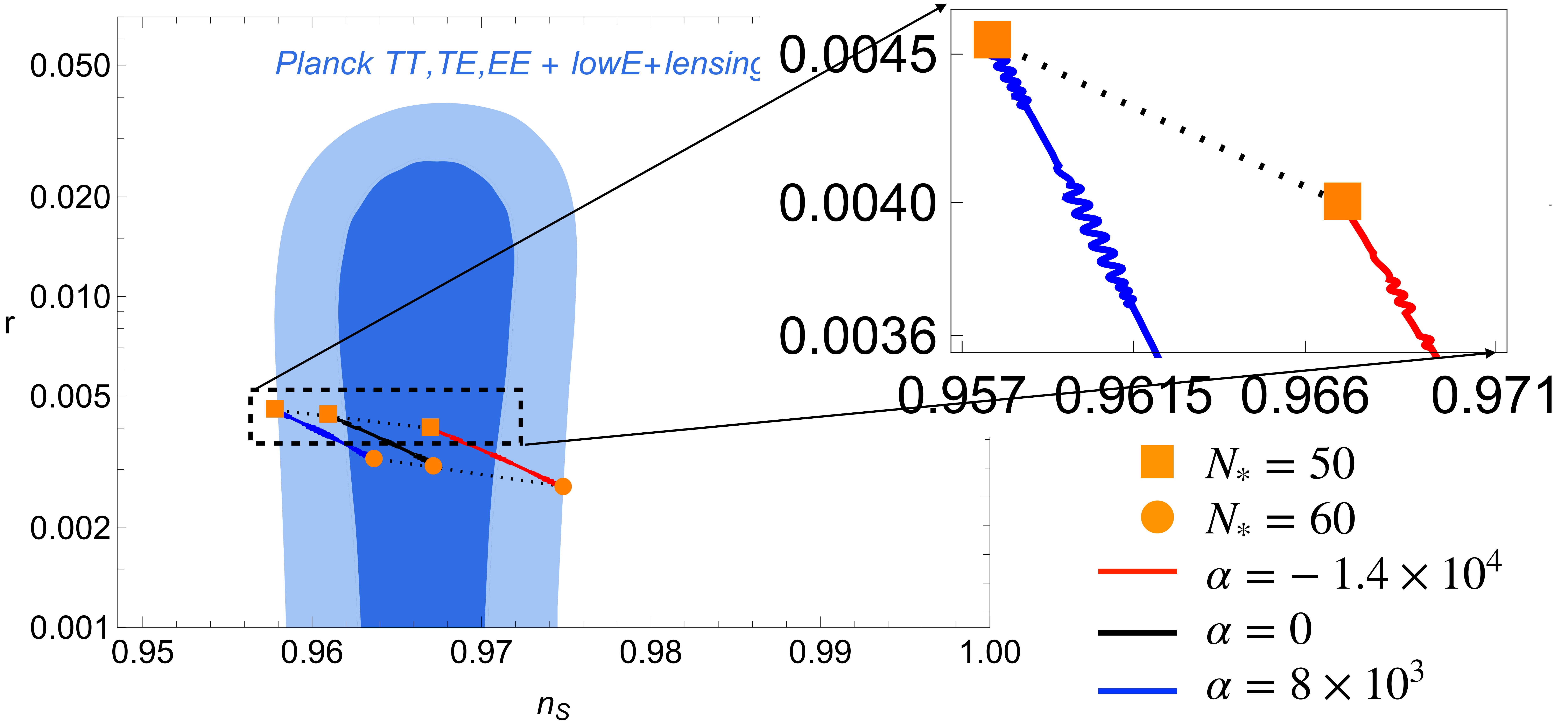
$$n_S - 1 = 2(2\epsilon_1 - \epsilon_2 - \epsilon_3)\,, \quad n_T = 2(\epsilon_1 - \epsilon_5)\,, \quad r = 16\left|\frac{1}{Q_t}\left(\frac{c_A}{c_T}\right)^3\left(\epsilon_1 - \frac{1}{4M_p^2H^2}\left(2Q_c + Q_d - HQ_e + H^2Q_f\right)\right)\right|\,.$$



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with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

$$c_A^2 \equiv 1 + \frac{Q_d + \frac{Q_a}{2M_p^2 + Q_b} Q_e + \left(\frac{Q_a}{2M_p^2 + Q_b}\right)^2 Q_f}{\dot{s}^2 + \frac{3Q_a^2}{2M_p^2 + Q_b} + Q_c}, \qquad c_T^2 \equiv 1 - \frac{Q_f}{2M_p^2 + Q_b},$$

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

$$c_A^2 \equiv 1 + \frac{Q_d + \frac{Q_a}{2M_p^2 + Q_b} Q_e + \left(\frac{Q_a}{2M_p^2 + Q_b}\right)^2 Q_f}{\dot{s}^2 + \frac{3Q_a^2}{2M_p^2 + Q_b} + Q_c}, \qquad c_T^2 \equiv 1 - \frac{Q_f}{2M_p^2 + Q_b},$$

GW170817 & GRB170817A: $-3 \times 10^{-15} \leq c_T/c_\gamma - 1 \leq 7 \times 10^{-16}$

Phys. Rev. Lett. 119, 161101 (2017)
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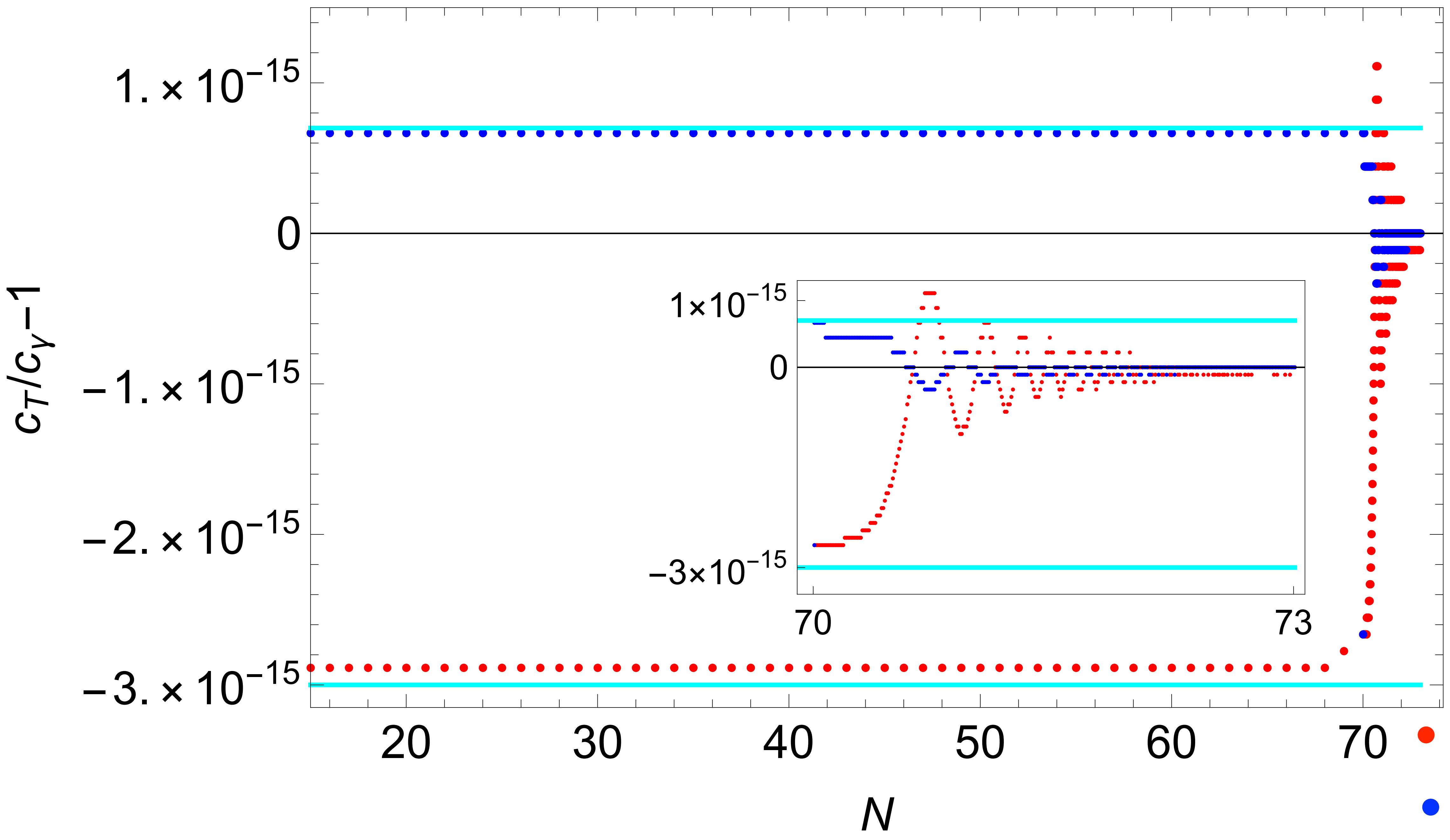

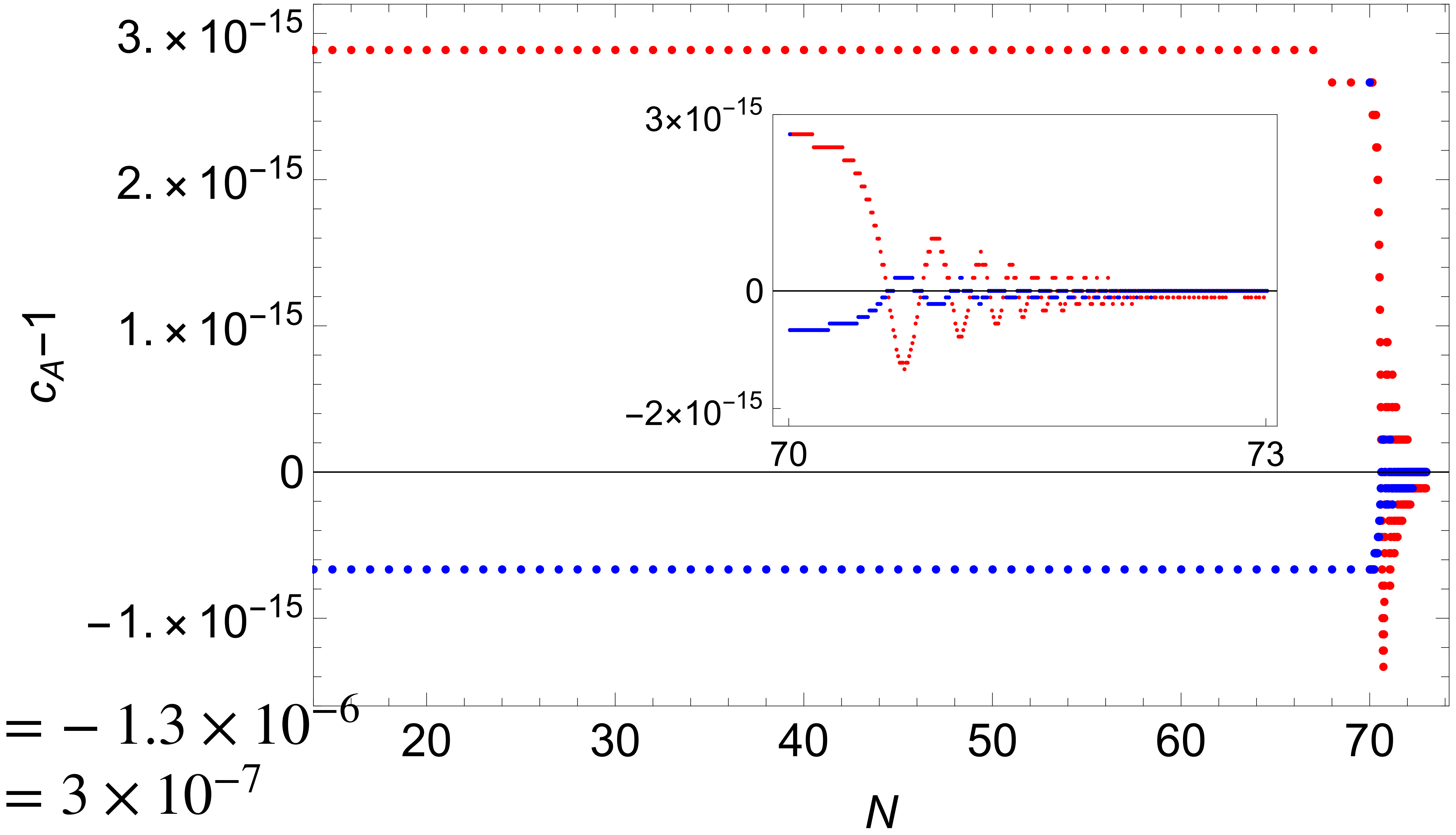

$\alpha = -1.3 \times 10^{-6}$

$\alpha = 3 \times 10^{-7}$

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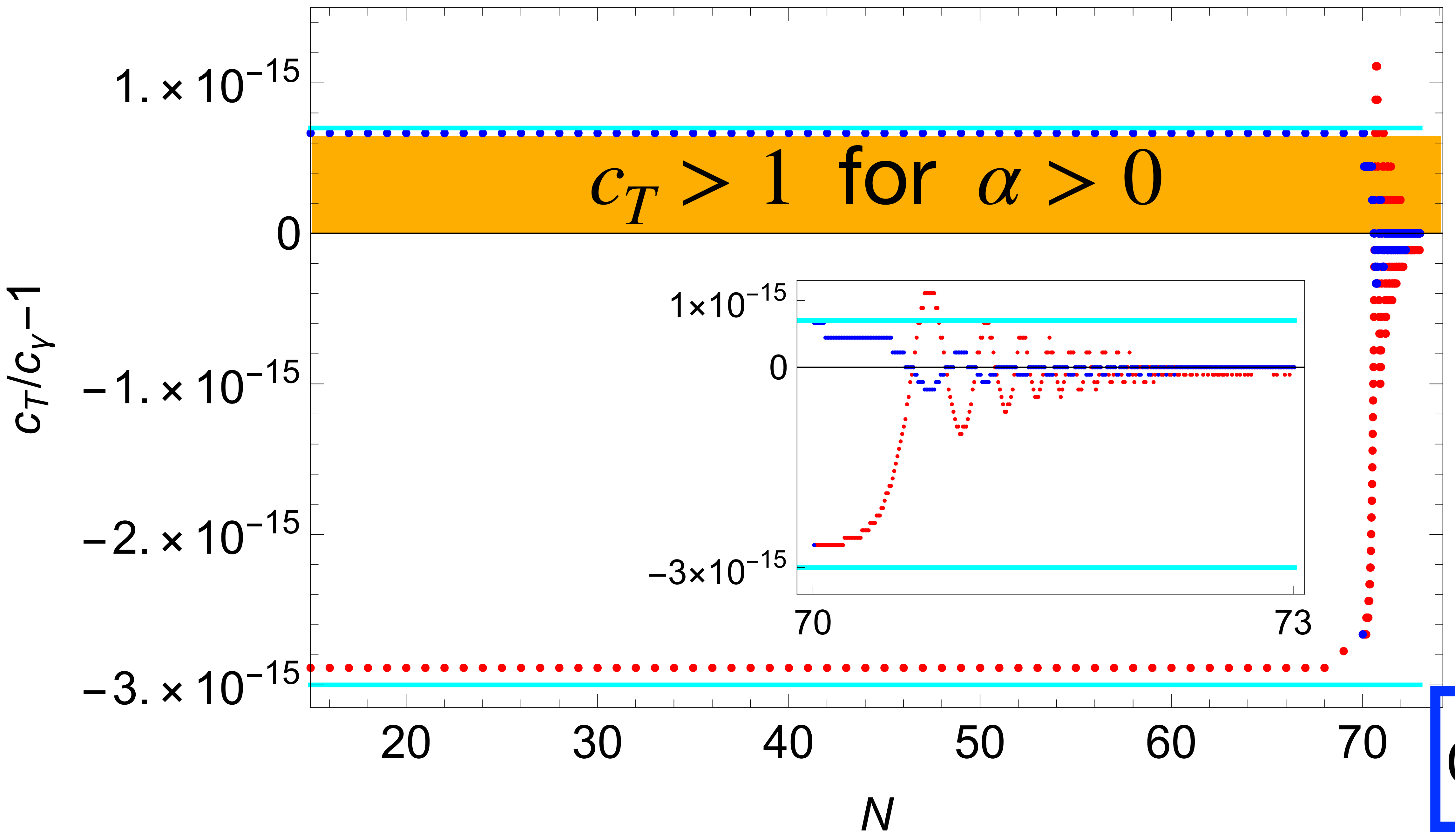
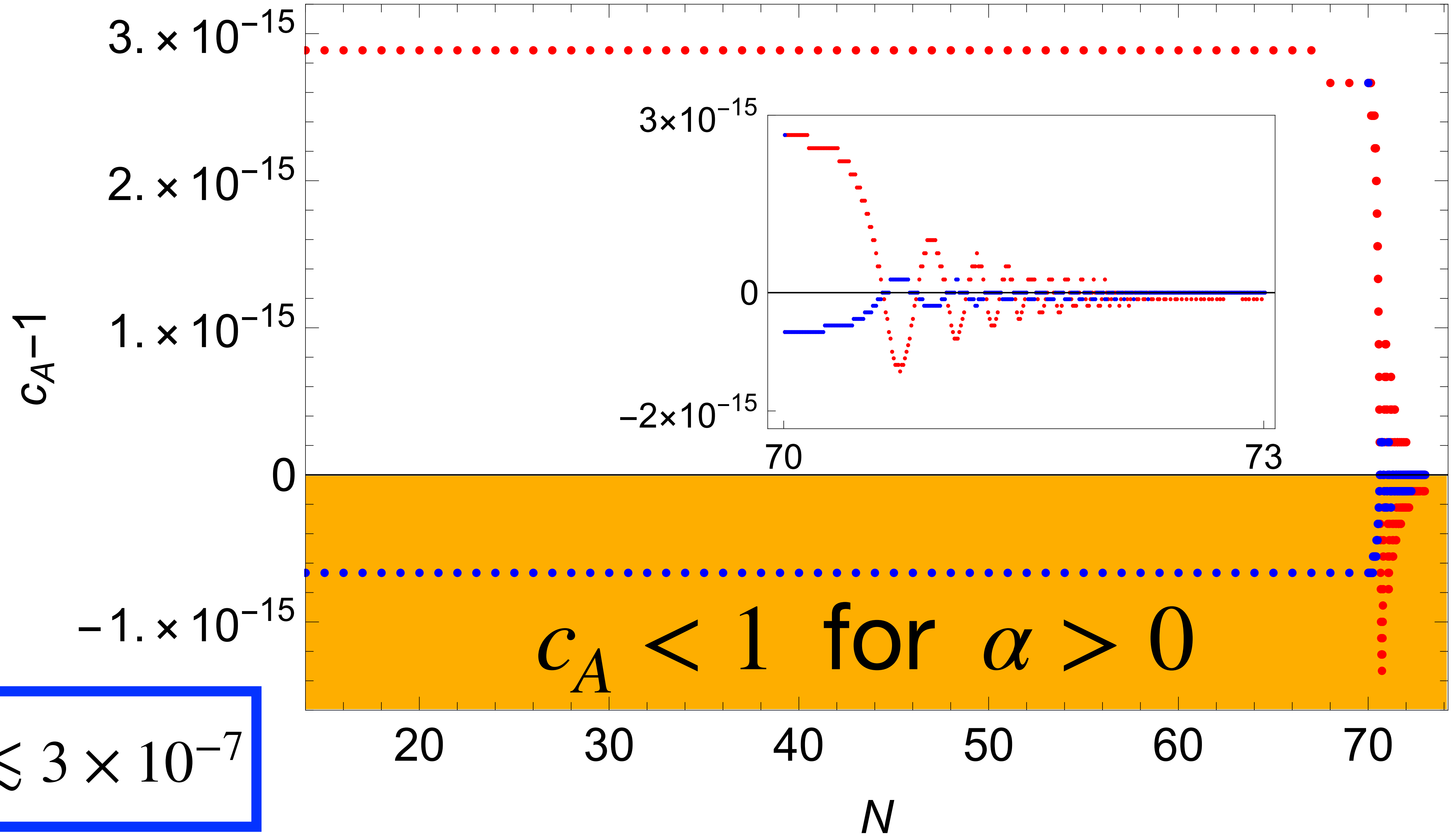


$0 < \alpha \lesssim 3\times10^{-7}$



Summary

- We studied Higgs inflation with a Gauss-Bonnet combination in the Einstein frame.
- The Gauss-Bonnet coupling must be a function of a scalar field $\omega(\phi)$ to give rise to non-trivial effect(s).
- Considering the relation $\omega(\phi) = \alpha\Omega^2(\phi)$, we put a constraint on the parameter α using the CMB data.
- The model is consistent with the CMB data for both positive and negative α.
- The propagation speed of GWs puts further constraints on the α.
- As a result, the positive values of the α is favored, i.e., $0 < \alpha \lesssim 3 \times 10^{-7}$.

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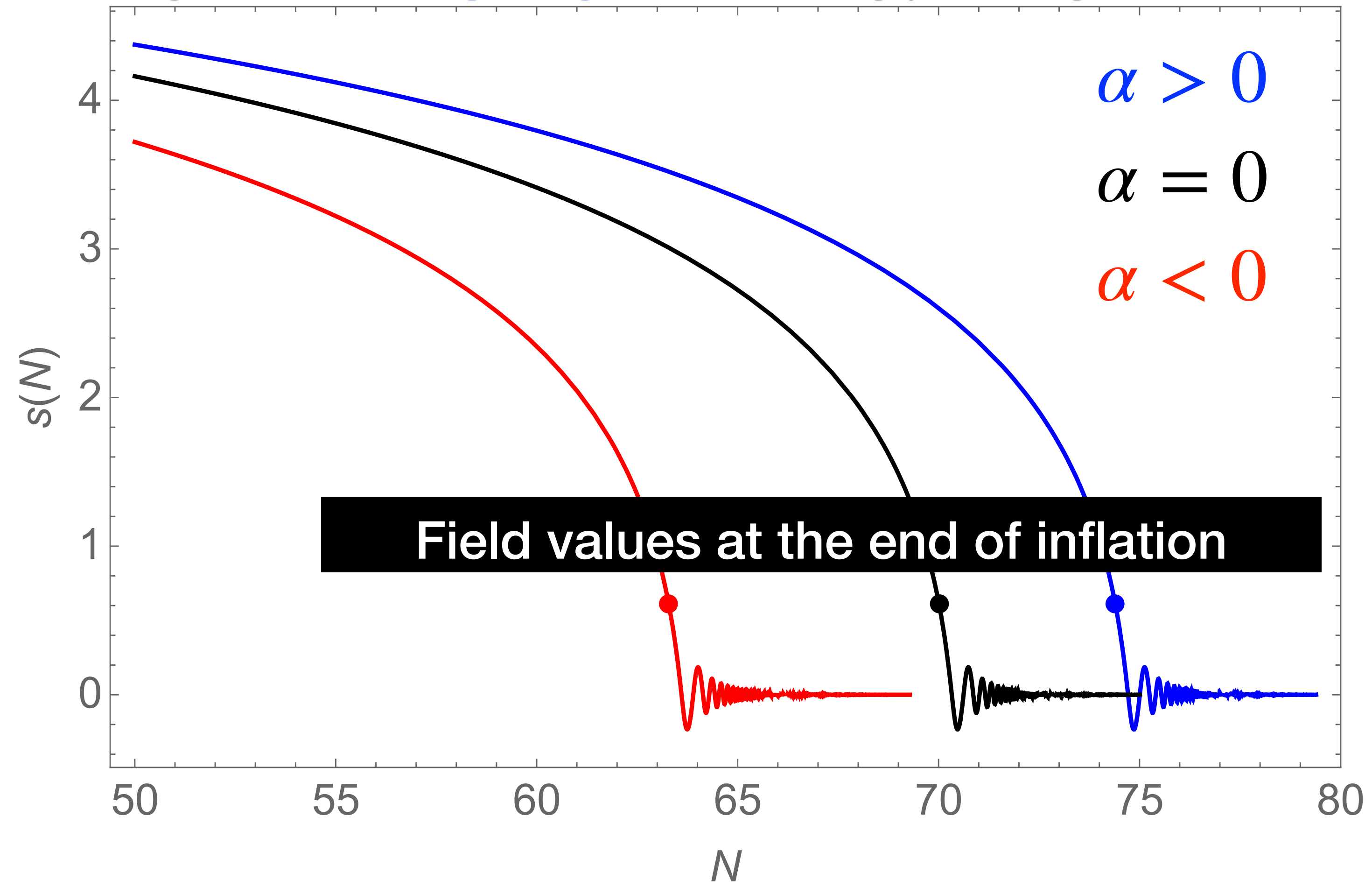
Thank you for your kind attention!

Backup Slides

Higgs Inflation

with non-minimal coupling + Gauss-Bonnet term in the Einstein frame

The solutions to the background equations of motion: $\Longrightarrow$



$$V(s) \equiv \frac{\lambda M_p^4}{4\sigma^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{s}{M_p}}\right)^2 , \quad \xi(s) \equiv -2\alpha e^{\sqrt{\frac{2}{3}}\frac{s}{M_p}}$$

$$\ddot{s} + 3H\dot{s} + V_{,s} = -12c_1\xi_{,s}H^2(\dot{H} + H^2) + \frac{3}{2}\frac{c_2}{M_p^2}\left[H^2(\dot{\xi}\dot{s} + 2\xi\ddot{s}) + 2H\xi\dot{s}(2\dot{H} + 3H^2)\right] - \frac{1}{2}\frac{c_3}{M_p^3}\dot{s}\left[\ddot{\xi}\dot{s} + 3\dot{\xi}\ddot{s} - 6\xi(\dot{H}\dot{s} + 2H\ddot{s} + 3H^2\dot{s})\right].$$

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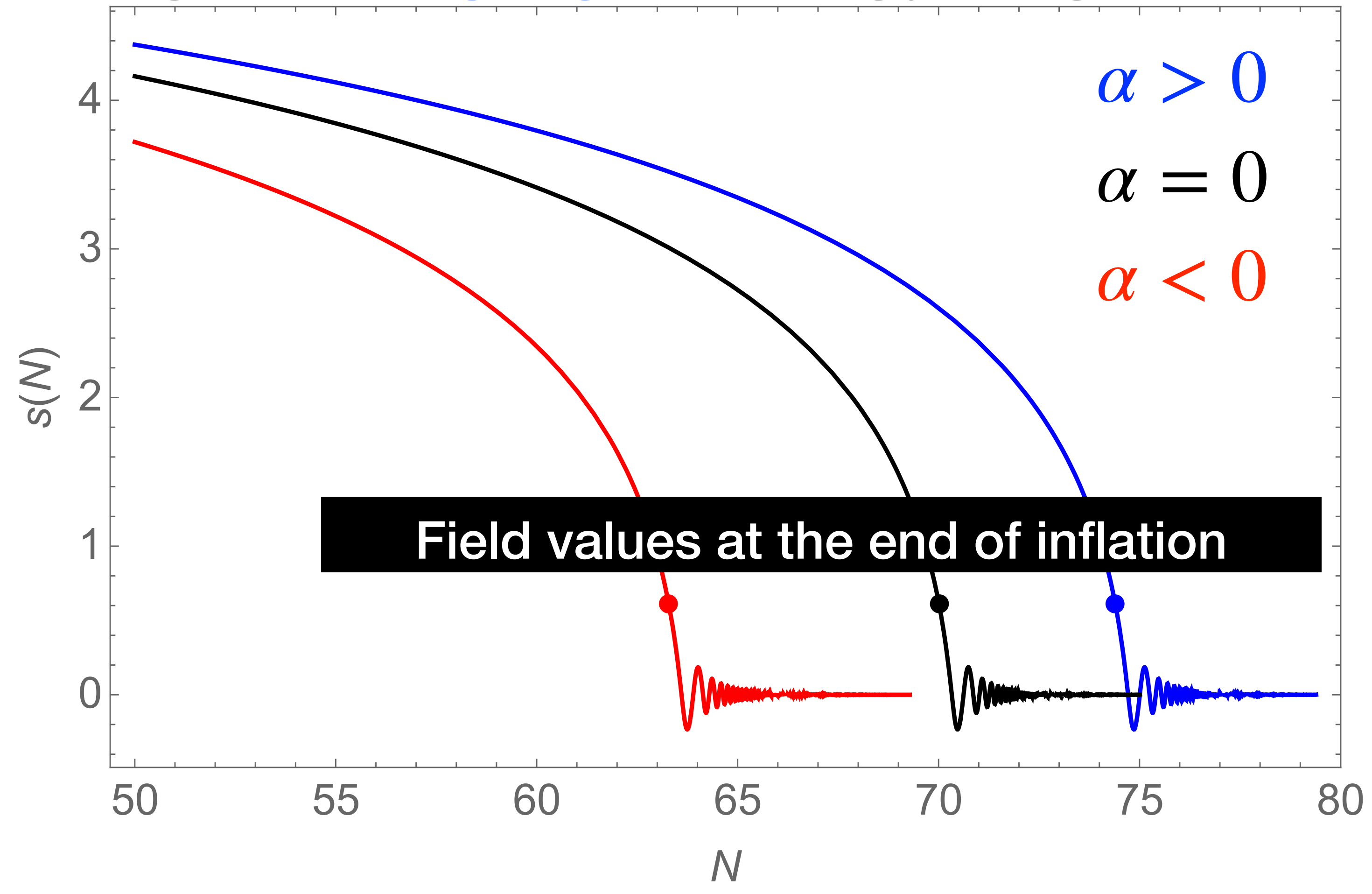

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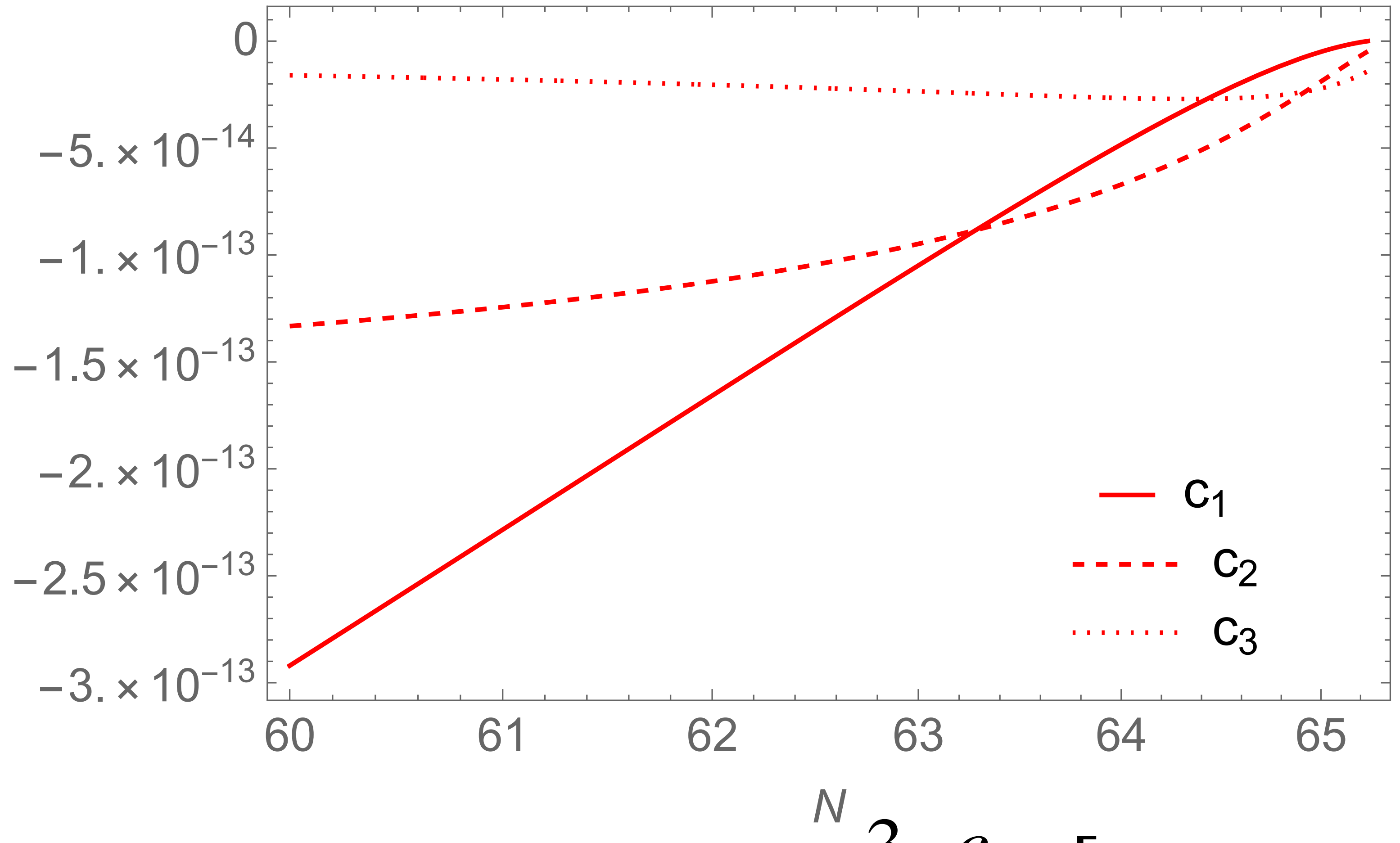

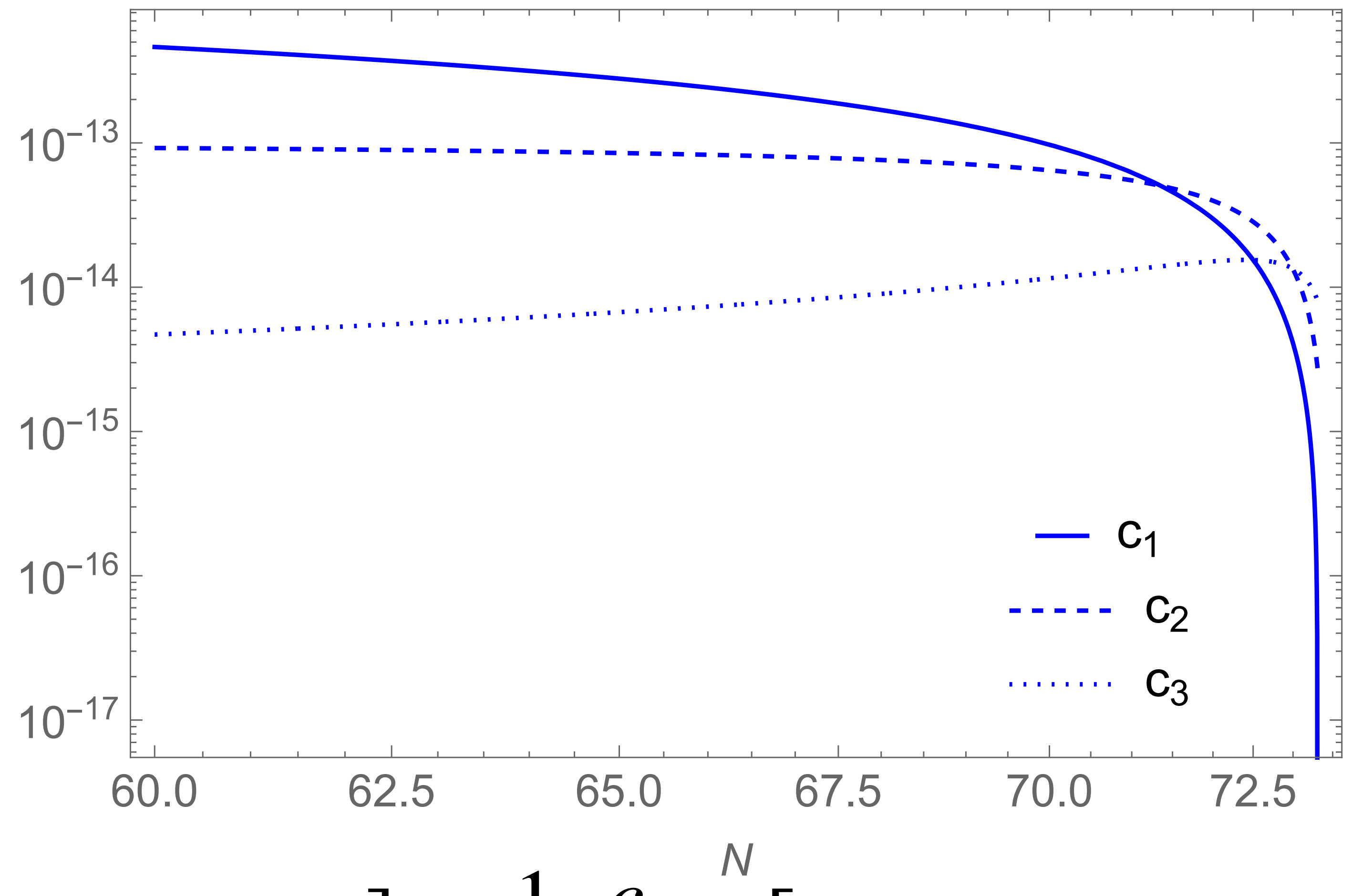

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$$\epsilon_1 \equiv \frac{\dot{H}}{H^2}\,,\ \epsilon_2 \equiv \frac{\ddot{s}}{H\dot{s}}\,,\ \epsilon_3 \equiv \frac{\dot{E}}{2EH}\,,\ \epsilon_4 \equiv \frac{Q_a}{2H(2M_p^2 + Q_b)}\,,\ \epsilon_5 \equiv \frac{\dot{Q}_t}{2Q_tH}\,,$$