Halo-independent bounds on the non-relativistic effective theory of WIMP-nucleon scattering from direct detection and neutrino observations

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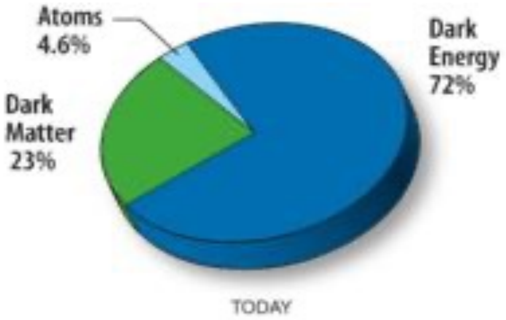
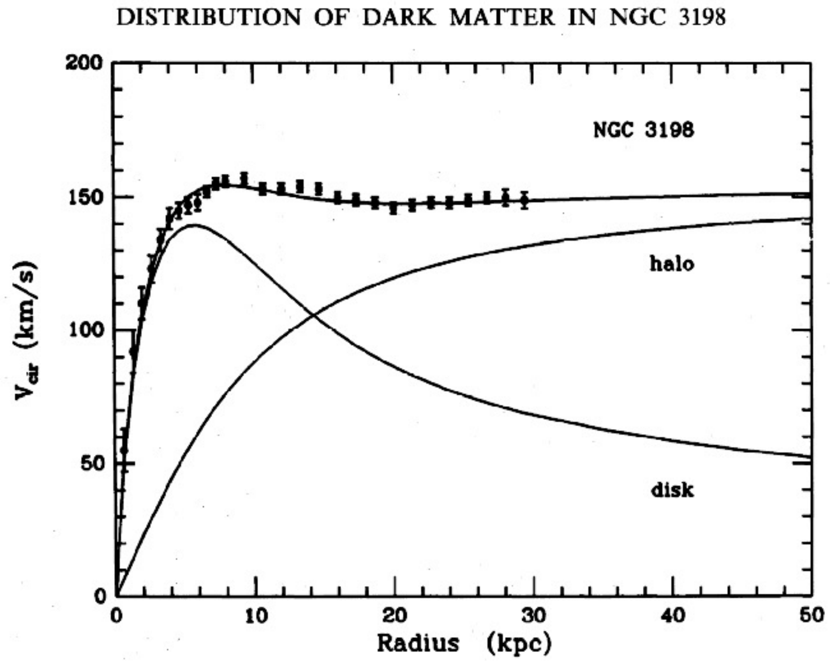
Introduction: evidences

- Dark Matter (DM)
  - 25% of mass density of the Universe
  - 'Dark': invisible

- Galaxy rotational curve
  - contrast to Kepler's law
  - The rotation curve can be used to measure the mass distribution
  - predicted more mass than the visible one



"Content of the Universe - Pie Chart" *Wilkinson Microwave Anisotropy Probe*. National Aeronautics and Space Administration. Retrieved 9 January 2018.

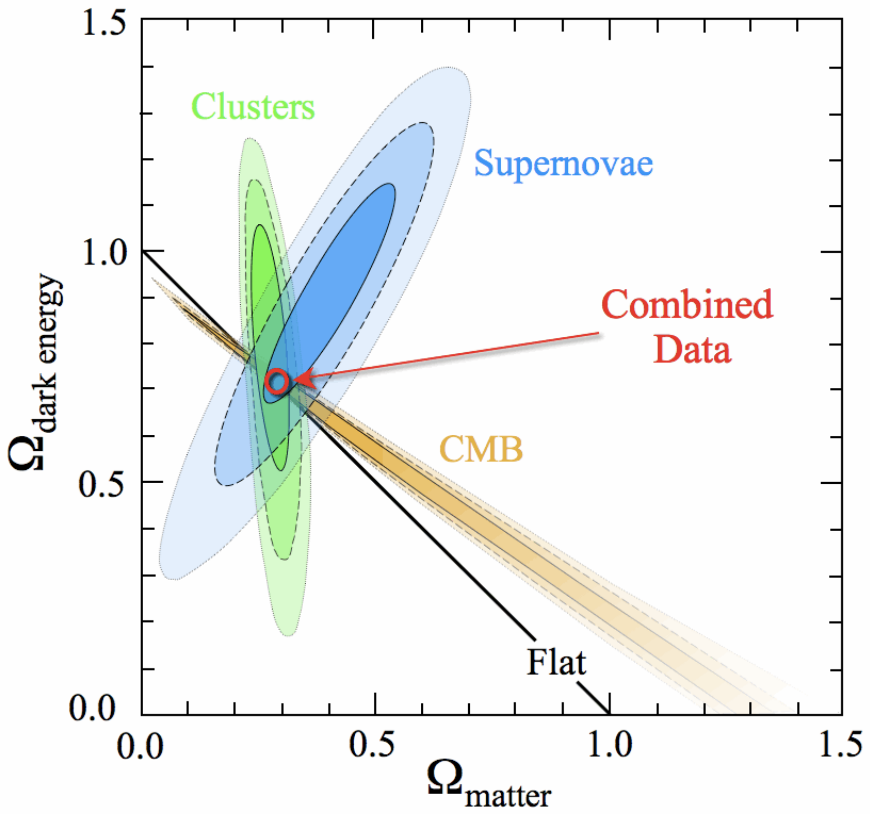


Galaxy rotational curve
K.G. Begeman et al. *Monthly Notices of the Royal Astronomical Society*, Volume 249, Issue 3, April 1991, Pages 523–537

Introduction: evidences

- The density of the Universe (ρ_c)
  - equations of state: $w_i = p_i/\rho_i$
  - Dark Energy ($w_\Lambda = -1$)
  - Dark Matter ($w_{DM} = 0$)
  - baryons ($w_b = 0$)

- Concordance model
  - $\Omega_i = \rho_i/\rho_c$
  - CMB: flat Universe → $\Sigma_i\Omega_i = \Omega_\Lambda + \Omega_{DM} + \Omega_b = 1$
  - SN: accelerated expansion → $\Omega_\Lambda \cong 0.7$
  - lensing effect of Galactic cluster → $\Omega_{DM} \cong 0.25$
  - $\Omega_b \cong 0.05$ → amount of light isotopes well explained



Concordance model
Jaan Einasto, Dark matter (arXiv:0901.0632) 2009

Introduction: candidates

- Properties
  - no interactions via EM or Strong forces
  - need to be neutral
  - Weak-type interaction

HDM	CDM
Light	Heavy
Fast	Slow
Failing to explain distribution of Galaxy at small scales	successful to explain distribution of Galaxy at small scales
Neutrions, etc.	WIMP, Axion, etc.

- Hot Dark Matter (HDM)
  - neutrinos, etc
  - Do not cluster at small scales

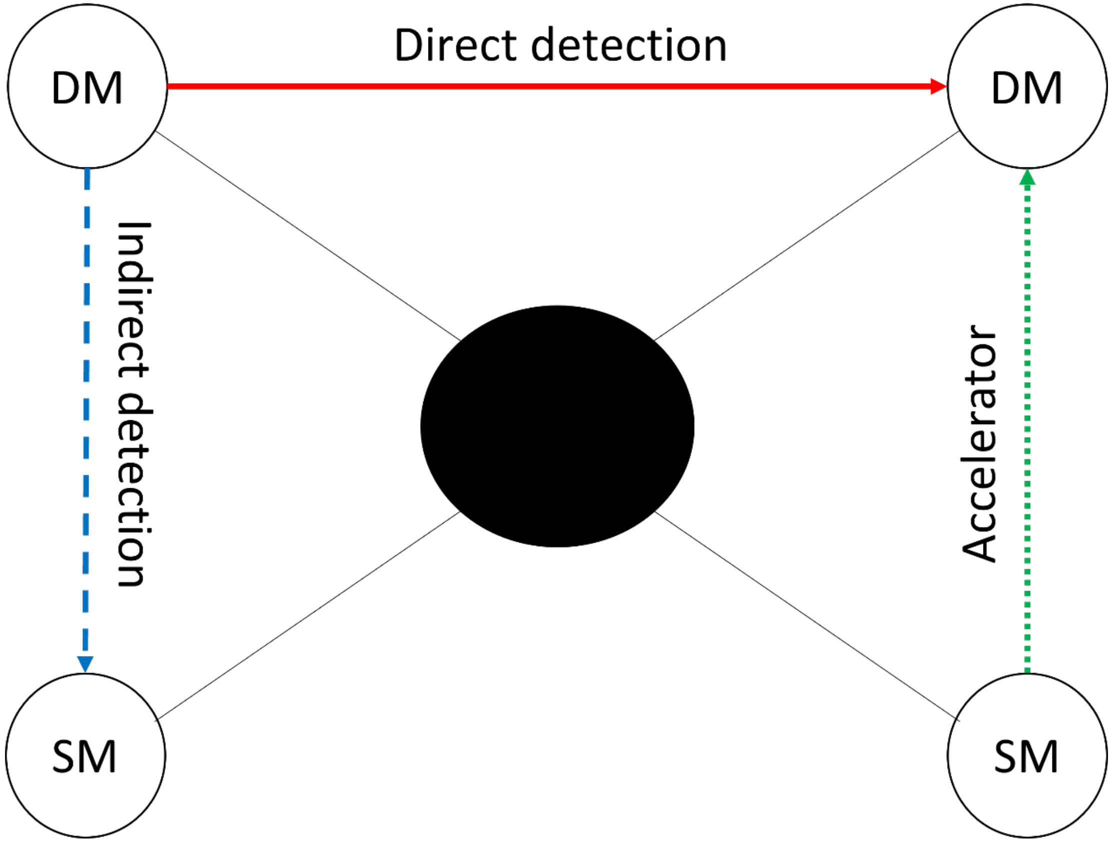
- Cold Dark Matter (CDM)
  - non-relativistic at decoupling
  - bottom-up structure formation: smaller structures formed first that merge to bigger ones

Introduction: WIMPs

Weakly Interacting Massive Particle (WIMP)

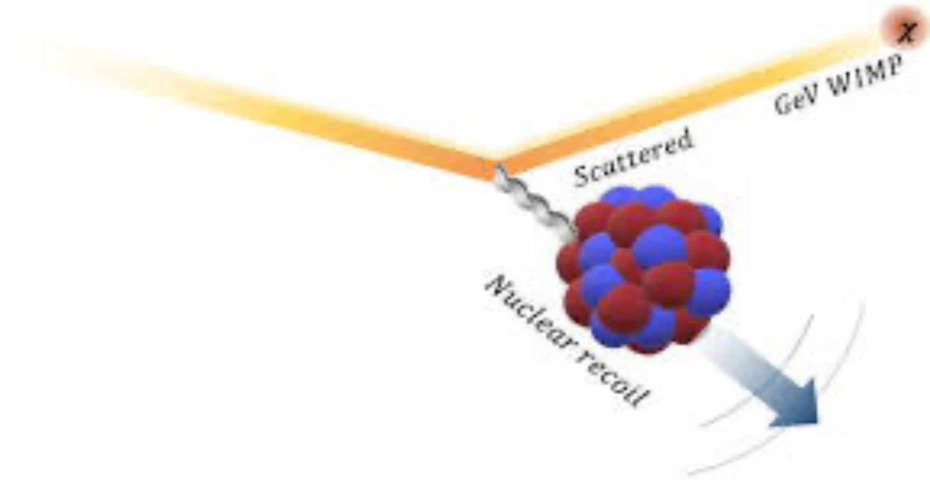
- Weak-type interaction
  - no electric charge, no color
- Mass range in GeV-TeV range
- WIMP miracle
  - correct relic abundance is obtained at WIMP $< \sigma v > = weak\ scale$
  - most extensions of SM are proposed independently at that scale.

Introduction: detection strategies



- Direct detection: DM interacts with SM particles (left to right)
- Indirect detection: DM annihilation (top to bottom)
- Accelerator: DM creation (bottom to top)

Direct Detection (DD)



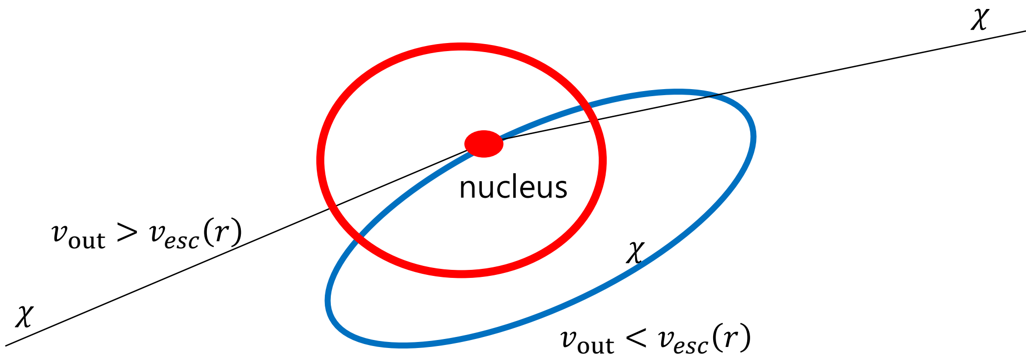
- The signals are WIMP-nucleus recoil events

- Low probability requires high exposure
- Underground to avoid background

- Direct Detection experiments depend on :
  - Detector's target (nuclear response functions)
  - Detector's response (efficiency, light/charge yield, energy resolution, exposure)

- Different nuclear targets and background subtraction:
  - Ionizators, scintillators, bubble chambers/droplet detectors and etc.
  - COSINE-100, ANAIS, DAMA, LZ, PandaX-4T, XENON-1T, PICO-60 and ect.

Indirect Detection



- WIMP scatters off nucleus at distance r inside celestial body
  - same interaction probed by DD

- If its outgoing speed v_{out} is below the escape velocity $v_{esc}(r)$, it gets locked into gravitationally bound orbit and keeps scattering again and again

- Capture process is favored for low (even vanishing) WIMP speeds

Non-Relativistic Effective Theory (NREFT)

- WIMP is slow, so that the recoil events are non-relativistic
- NREFT provides a general and efficient way to characterize results with mass of WIMP and coupling constants

Operators spin up to 1/2

- Hamiltonian: $\Sigma_{i=1}^{N}\left(c_i^n \mathcal{O}_i^n + c_i^p \mathcal{O}_i^p\right)$
- Non-relativistic process
  - all operators must be invariant by Galilean transformations ($v \sim 10^{-3}c$ in galactic halo)
- Building operators using: $i\frac{\vec{q}}{m_N},\ \vec{v}^{\perp},\ \vec{S}_\chi,\ \vec{S}_N$

$$\mathcal{O}_1 = 1_\chi 1_N; \quad \mathcal{O}_2 = (v^\perp)^2; \quad \mathcal{O}_3 = i\vec{S}_N \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp)$$

$$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N; \quad \mathcal{O}_5 = i\vec{S}_\chi \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp); \quad \mathcal{O}_6 = (\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$$

$$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^\perp; \quad \mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp; \quad \mathcal{O}_9 = i\vec{S}_\chi \cdot (\vec{S}_N \times \frac{\vec{q}}{m_N})$$

$$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N}; \quad \mathcal{O}_{11} = i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}; \quad \mathcal{O}_{12} = \vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^\perp)$$

$$\mathcal{O}_{13} = i(\vec{S}_\chi \cdot \vec{v}^\perp)(\vec{S}_N \cdot \frac{\vec{q}}{m_N}); \quad \mathcal{O}_{14} = i(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \vec{v}^\perp)$$

$$\mathcal{O}_{15} = -(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})((\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N}),$$

Non-Relativistic Effective Theory (NREFT)

- Each operators have distinct couplings to proton and neutron:

$$\Sigma_{\alpha=p,n}\Sigma_{i=1}^{15} c_i^{\alpha}\mathcal{O}_i^{\alpha}, \qquad c_2^{\alpha} \equiv 0$$

- Equivalent form using isospin:

$$\Sigma_{i=1}^{15}\left(c_i^0 1 + c_i^1 \tau_3\right)\mathcal{O}_i = \Sigma_{\tau=0,1}\Sigma_{i=1}^{15} c_i^{\tau}\mathcal{O}_i t^{\tau}, \qquad c_2^0 = c_2^1 \equiv 0$$

$$|p\rangle = \binom{1}{0} \quad |n\rangle = \binom{0}{1} \quad 1 \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \tau_3 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$c_i^0 = \frac{1}{2}\left(c_i^p + c_i^n\right) \quad c_i^1 = \frac{1}{2}\left(c_i^p - c_i^n\right)$$

$$t^0 \equiv 1 \quad t^1 \equiv \tau_3$$

Non-Relativistic Effective Theory (NREFT)

- Scattering amplitude:

$$\frac{1}{2j_\chi+1}\frac{1}{2j_N+1}\Sigma_{spins}|M|^2 \equiv \Sigma_k\Sigma_{\tau=0,1}\Sigma_{\tau'=0,1} R_k^{\tau\tau'}\left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}, \left\{c_i^\tau, c_j^{\tau'}\right\}\right) W_k^{\tau\tau'}(y)$$

WIMP response function

Nuclear response function

- $R_k^{\tau\tau'}$: WIMP response function
  - Velocity dependence: $\mathcal{R}_k^{\tau\tau'} = \mathcal{R}_{k,0}^{\tau\tau'} + \mathcal{R}_{k,1}^{\tau\tau'}(v^2 - v_{min}^2)$
- $W_k^{\tau\tau'}$: nuclear response function
  - $y = (qb/2)$^2
  - b: harmonic oscillator size parameter
- $k = M,\ \Delta,\ \Sigma',\ \Sigma'',\ \tilde{\Phi}'$ and Φ''
  - allowed responses assuming nuclear ground state is a good approximation of P and T

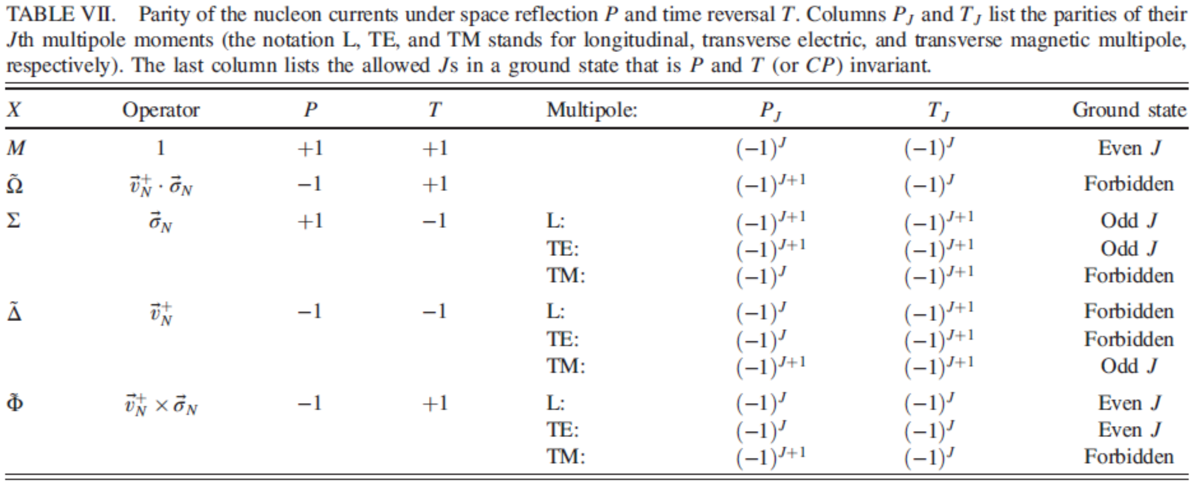


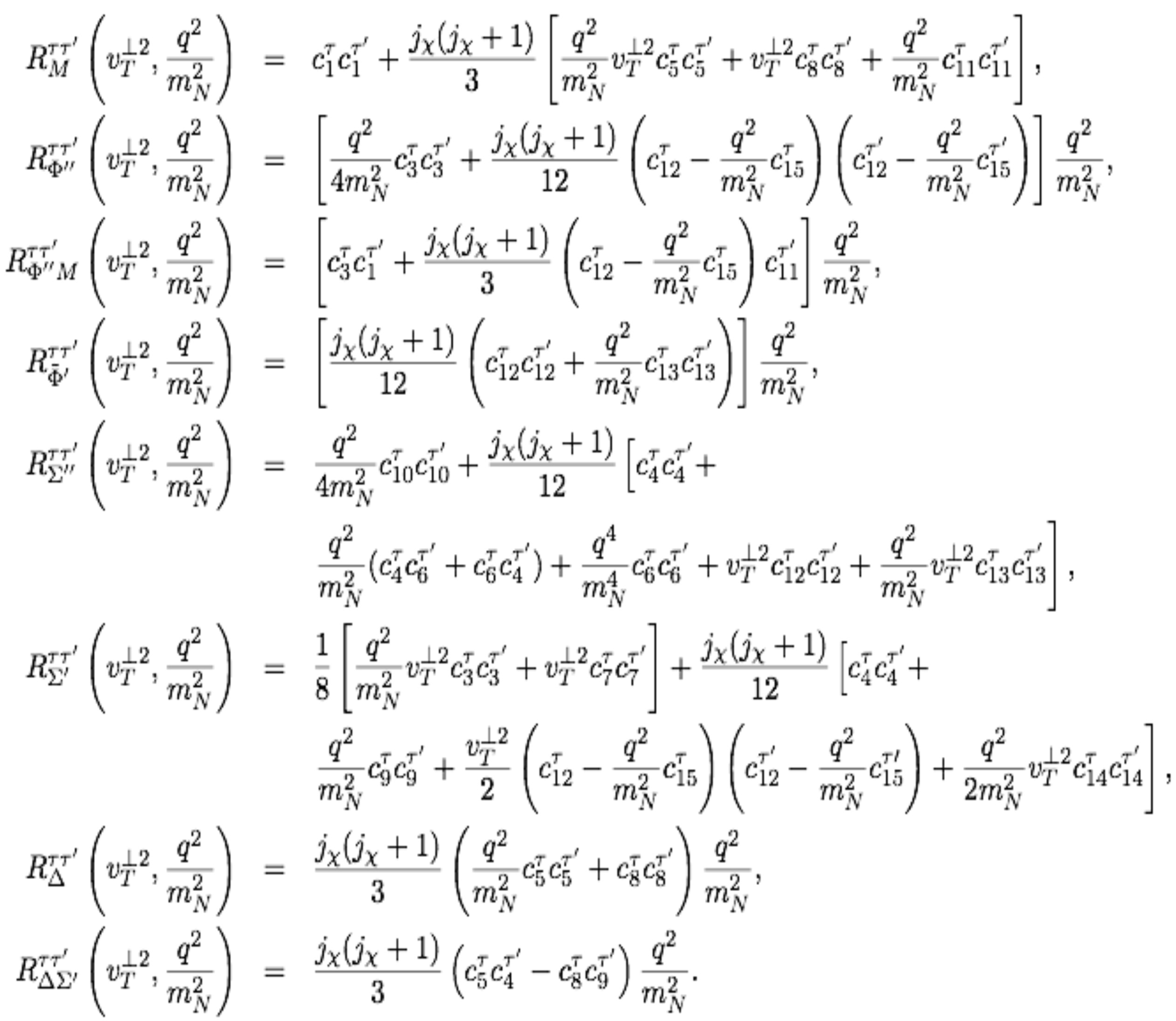
TABLE VII. Parity of the nucleon currents under space reflection P and time reversal T. Columns P_J and T_J list the parities of their Jth multipole moments (the notation L, TE, and TM stands for longitudinal, transverse electric, and transverse magnetic multipole, respectively). The last column lists the allowed Js in a ground state that is P and T (or CP) invariant.

X	Operator	P	T	Multipole:	P_J	T_J	Ground state
M	1	+1	+1		$(-1)^J$	$(-1)^J$	Even J
$\tilde{\Omega}$	$\vec{v}_N^+ \cdot \vec{\sigma}_N$	−1	+1		$(-1)^{J+1}$	$(-1)^J$	Forbidden
Σ	$\vec{\sigma}_N$	+1	−1	L:	$(-1)^{J+1}$	$(-1)^{J+1}$	Odd J
				TE:	$(-1)^{J+1}$	$(-1)^{J+1}$	Odd J
				TM:	$(-1)^J$	$(-1)^{J+1}$	Forbidden
$\tilde{\Delta}$	$\vec{v}_N^+$	−1	−1	L:	$(-1)^J$	$(-1)^{J+1}$	Forbidden
				TE:	$(-1)^J$	$(-1)^{J+1}$	Forbidden
				TM:	$(-1)^{J+1}$	$(-1)^{J+1}$	Odd J
$\tilde{\Phi}$	$\vec{v}_N^+ \times \vec{\sigma}_N$	−1	+1	L:	$(-1)^J$	$(-1)^J$	Even J
				TE:	$(-1)^J$	$(-1)^J$	Even J
				TM:	$(-1)^{J+1}$	$(-1)^J$	Forbidden

N. Anand et al. Phys.Rev.C.89.065501
R. Catena and B. Schwabe, JCAP04(2015)042

Non-Relativistic Effective Theory (NREFT)

WIMP response functions



$$
\begin{aligned}
R_M^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= c_1^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left[\frac{q^2}{m_N^2} v_T^{\perp 2} c_5^\tau c_5^{\tau'} + v_T^{\perp 2} c_8^\tau c_8^{\tau'} + \frac{q^2}{m_N^2} c_{11}^\tau c_{11}^{\tau'}\right], \\
R_{\Phi''}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \left[\frac{q^2}{4m_N^2} c_3^\tau c_3^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau'}\right)\right]\frac{q^2}{m_N^2}, \\
R_{\Phi'' M}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \left[c_3^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right) c_{11}^{\tau'}\right]\frac{q^2}{m_N^2}, \\
R_{\tilde{\Phi}'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \left[\frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau c_{12}^{\tau'} + \frac{q^2}{m_N^2} c_{13}^\tau c_{13}^{\tau'}\right)\right]\frac{q^2}{m_N^2}, \\
R_{\Sigma''}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{q^2}{4m_N^2} c_{10}^\tau c_{10}^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \right. \\
&\quad \left. \frac{q^2}{m_N^2}(c_4^\tau c_6^{\tau'} + c_6^\tau c_4^{\tau'}) + \frac{q^4}{m_N^4} c_6^\tau c_6^{\tau'} + v_T^{\perp 2} c_{12}^\tau c_{12}^{\tau'} + \frac{q^2}{m_N^2} v_T^{\perp 2} c_{13}^\tau c_{13}^{\tau'}\right], \\
R_{\Sigma'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{1}{8}\left[\frac{q^2}{m_N^2} v_T^{\perp 2} c_3^\tau c_3^{\tau'} + v_T^{\perp 2} c_7^\tau c_7^{\tau'}\right] + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \right. \\
&\quad \left. \frac{q^2}{m_N^2} c_9^\tau c_9^{\tau'} + \frac{v_T^{\perp 2}}{2}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau\prime}\right) + \frac{q^2}{2m_N^2} v_T^{\perp 2} c_{14}^\tau c_{14}^{\tau'}\right], \\
R_\Delta^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{j_\chi(j_\chi+1)}{3}\left(\frac{q^2}{m_N^2} c_5^\tau c_5^{\tau'} + c_8^\tau c_8^{\tau'}\right)\frac{q^2}{m_N^2}, \\
R_{\Delta\Sigma'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{j_\chi(j_\chi+1)}{3}\left(c_5^\tau c_4^{\tau'} - c_8^\tau c_9^{\tau'}\right)\frac{q^2}{m_N^2}.
\end{aligned}
$$

- c_i: coupling for i-th operator
- j_χ: spin of WIMP
- q: transferred momentum
- m_N: mass of nucleon
- $v_T^\perp$: WIMP incoming velocity
  - perpendicular to the direction of q

Non-Relativistic Effective Theory (NREFT)

- Scattering amplitude:
  $$\frac{1}{2j_\chi+1}\frac{1}{2j_N+1}\Sigma_{spins}|M|^2 \equiv \Sigma_k \Sigma_{\tau=0,1}\Sigma_{\tau'=0,1} R_k^{\tau\tau'}\left(\vec{v}_T^{\perp}{}^2, \frac{\vec{q}^2}{m_N^2}, \left\{c_i^\tau, c_j^{\tau'}\right\}\right) W_k^{\tau\tau'}(y)$$

- Differential cross section : $\frac{d\sigma}{dE_R} = \frac{1}{10^6}\frac{2m_N}{4\pi}\frac{c^2}{v^2}\left[\frac{1}{2j_\chi+1}\frac{1}{2j_N+1}\Sigma_{spin}|M|^2\right]$

- Differential rate : $\frac{dR}{dE_R} = N_T \int_{v_{min}}^{v_{esv}} \frac{\rho_\chi}{m_\chi}\, v\, \frac{d\sigma}{dE_R}\, f(v)\, dv$

- With $E_R = \frac{\mu_{\chi N}^2\, v^2}{m_N}$, $\quad v_{min} = \frac{1}{\sqrt{2m_N E_R}}\left|\frac{m_N E_R}{\mu_{\chi N}} + \delta\right|$

DD event rate

- DD event rate

$$\mathrm{R_{DD}} = M\tau_{exp}\frac{\rho_\chi}{m_\chi}\int du\, f(u)u\, \Sigma_T\, N_T \int_{E_{R,th}}^{2\mu_{\chi T}^2\, u^2\, /m_T} dE_R\, \zeta_{exp}\frac{d\sigma_T}{dE_R}$$

  - $M\tau_{exp}$: exposure
  - $E_{R,th}$: experimental energy threshold
  - ζ_{exp} : experimental features such as quenching, resolution, efficiency, etc.

- $R_{DD} = \int_0^{u_{esc}} f(u)\, H_{DD}(u)$

$$H_{DD}(u) = u\, M\tau_{exp}\frac{\rho_\chi}{m_\chi}\ \Sigma_T\, N_T \int_{E_{R,th}}^{2\mu_{\chi T}^2\, u^2\, /m_T} dE_R\, \zeta_{exp}\frac{d\sigma_T}{dE_R}$$

Capture rate

- Capture rate

$$C_{\odot} = \frac{\rho_\chi}{m_\chi} \int du\, f(u) \frac{1}{u} \int_0^{R\odot} dr\, 4\pi r^2\, w^2\ \Sigma_T\, \rho_T(r)\, \Theta\big(u_T^{C-max}\ - u\big) \int_{m_\chi\, u^2\ /2}^{2\mu_{\chi T}^2\, w^2\ /m_T} dE_R \frac{d\sigma_T}{dE_R}$$

  - ρ_T: the number of density of target
  - r: distance from the center of the Sun for Standard Solar Model AGSS09ph
  - u: DM velocity asymptotically far away from the Sun
  - $v_{esc}(r)$: escape velocity at distance r
  - $w^2(r) = u^2 + v_{esc}^2(r)$
- Neutrino Telescope (NT):
  - the neutrino flux from the annihilation of WIMPs captured in the Sun
  - DM annihilations into $b\bar{b}$

Capture rate

- with assumption of equilibrium between capture and annihilation: $\Gamma_{\odot} = C_{\odot}/2$

- $C_{\odot} = \int_0^{u^{c-max}} du\, f(u)\, H_C(u)$

$$H_C = \frac{\rho_\chi}{m_\chi}\frac{1}{u}\int_0^{R_\odot} dr\, 4\pi r^2\, w^2\ \Sigma_T\, \rho_T(r)\, \Theta\left(u_T^{C-max} - u\right) \int_{m_\chi u^2/2}^{2\mu_{\chi T}^2 w^2/m_T} dE_R \frac{d\sigma_T}{dE_R}$$

  - $u_T^{C-max} = v_{esc}(r)\sqrt{\frac{4m_\chi m_T}{\left(m_\chi - m_T\right)^2}}$: maximum WIMP speed for capture possible

Model independent method

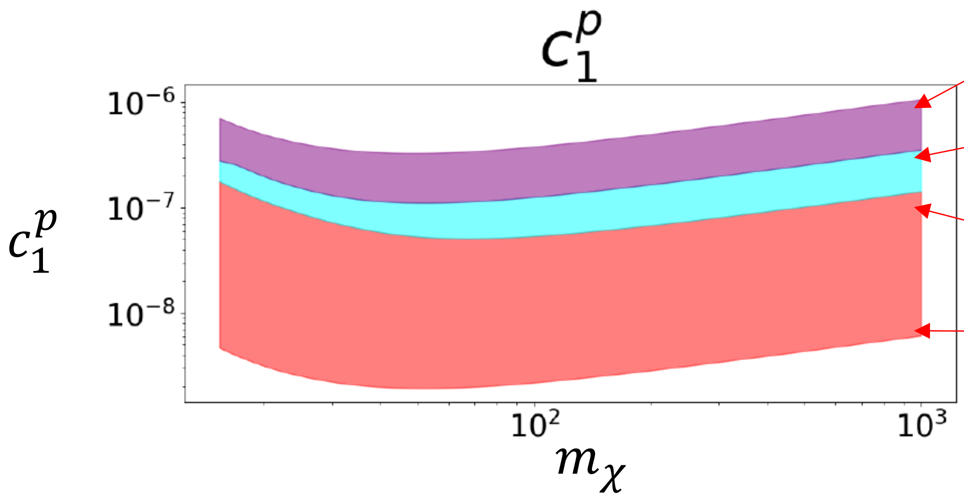
- Scattering count rate:

$$R \sim \int dv\, \underbrace{H(v)}_{\text{interaction}}\, \underbrace{f(v)}_{\text{velocity distribution}}$$

- Two parts of interaction and velocity distribution
  - needs to avoid uncertainty
  - interaction: include all possible interaction types
  - velocity distribution: halo independent approaches

- Model independent method: the most general scenarios

Bracketing DD exclusion plots



$c_1^p + c_1^n + c_3^p + c_3^n + \alpha_1^p + \alpha_1^n + \alpha_3^p + \alpha_3^n$

$c_1^p + c_1^n + c_3^p + c_3^n$

$c_1^p + c_1^n$

c_1^p

- Possible all interactions
  - Single coupling interactions
  - Interferences between p-n, operators, short-long range interactions
- Subspaces
  - $[c_1, c_3, \alpha_1, \alpha_3]$, $[c_{11}, c_{12}, c_{15}, \alpha_{11}, \alpha_{12}, \alpha_{15}]$
  - $[c_4, c_5, c_6, \alpha_4, \alpha_5, \alpha_6]$, $[c_8, c_9, \alpha_8, \alpha_9]$
  - $[c_7, \alpha_7]$, $[c_{10}, \alpha_{10}], [c_{13}, \alpha_{13}], [c_{14}, \alpha_{14}]$

S. Kang et al., astropartphys.2023.102854

Halo independent approach

- WIMP velocity distribution as Maxwellian:
  $$f_{gal}(u) = \frac{1}{\pi^{3/2} v_0^3 N_{esc}} e^{-u^2/v_0^2} \, \Theta(u_{esc} - u)$$
  - u: WIMP velocity in Galactic rest frame
  - v_0: Galactic rotational velocity
  - Θ: Heaviside step function
  - u_{esc}: Escape velocity
  - $N_{esc} = \mathrm{erf}(z) - 2ze^{-z^2}/\pi^{1/2}$
  - $z^2 = u_{esc}^2/v_0^2$

- In laboratory frame :
  $$f(v_T, t) = \frac{1}{N_{esc}} \left(\frac{3}{2\pi v_{rms}^2}\right)^{3/2} e^{-\frac{3|v_T + v_E|^2}{2v_{rms}^2}} \, \Theta(u_{esc} - |v_T + v_E(t)|)$$
  - $N_{esc} = \mathrm{erf}(z) - 2ze^{-z^2}/\pi^{1/2}$
  - $z^2 = 3u_{esc}^2/(2v_{rms}^2)$
  - $v_{rms} = \sqrt{\frac{3}{2}} v_0 \cong 270 \text{ km/s}$

Halo independent approach

- Halo independent approach with arbitrary speed distribution, $f(u)$
  - The only constraint: $\int_{u=0}^{u_{max}} f(u)\, du = 1$

- Direct detection experiments have a threshold $u > u_{th}^{DD}$
  - Due to the energy threshold of experimental detectors

- Capture in the Sun is favored for low WIMP speeds
  - $u < u_T^{C-max}$

- In order to cover full speed range: combine DD and capture

F. Ferrer et al. JCAP09(2015)052

Halo independent approach

- Considering one effective coupling (c_i) at a time:
    - $R_{exp}\left(c_i^2\right) = \int du\, f(u)\, H_{exp}\left(c_i^2, u\right) \leq R_{max}$
    - R_{max} : corresponding maximum experimental bound

- Using relation : $H\left(c_i^2, u\right) = c_i^2 H(c_i = 1, u)$
    - $H\left(c_{i,max}^2(u), u\right) = R_{max}$
    - $c_{i,max}^2(u) = \frac{R_{max}}{H(c_i=1,u)}$
    - $c_{i,max}(u)$: upper limit on c_i at single speed stream u

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Halo independent approach

- $R_{exp}\left(c_i^2\right) = \int du\, f(u)\, H_{exp}\left(c_i^2, u\right) \leq R_{max}$
  - $R_{exp} = \int_0^{u_{max}} du\, f(u)\, H\left(c_i^2, u\right)$

$$= \int_0^{u_{max}} du\, f(u) \frac{c_i^2}{c_{i,max}^2(u)}\; H\left(c_{i,max}^2(u), u\right)$$

$$= \int_0^{u_{max}} du\, f(u) \frac{c_i^2}{c_{i,max}^2(u)}\; R_{max} \leq R_{max}$$

- upper limit on c_i over whole streams:

$$c_i^2 \leq \left[\int_0^{u_{max}} du\; \frac{f(u)}{c_{i,max}^2(u)}\right]^{-1}$$

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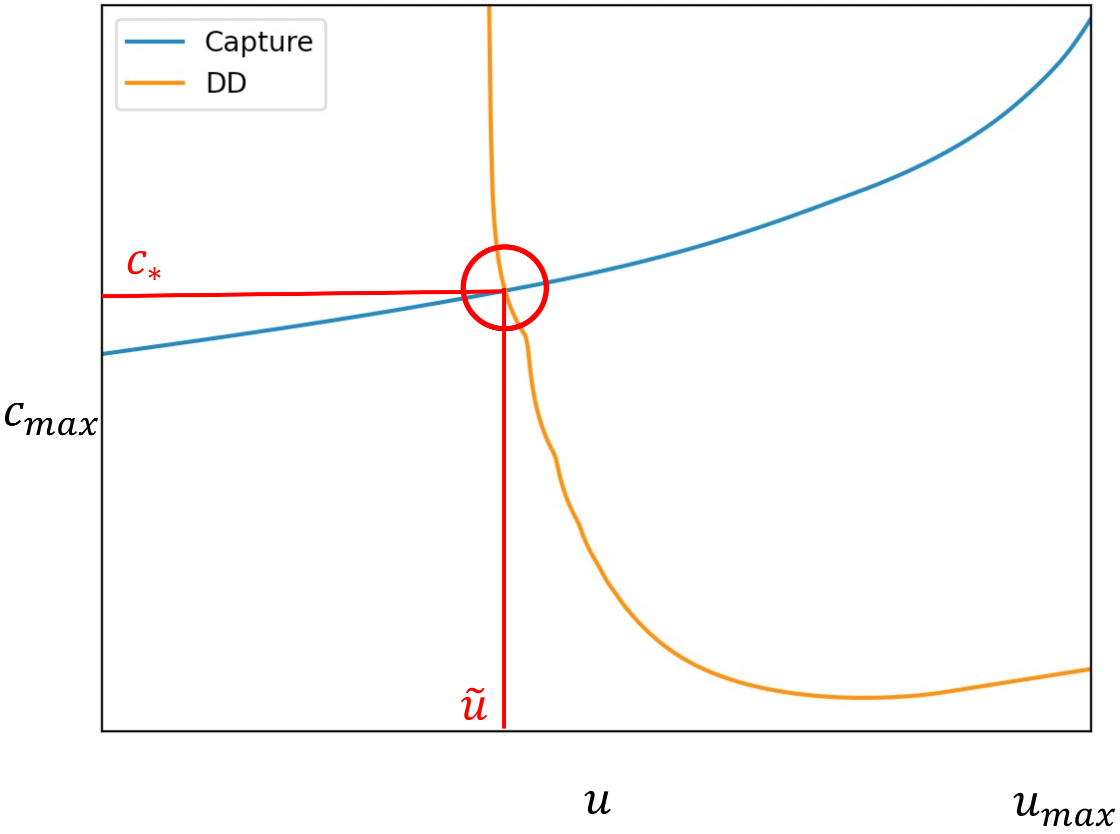
Halo independent approach

- $c_* = c_{max}^{NT}(\tilde{u}) = c_{max}^{DD}(\tilde{u})$: halo independent limit
- $\tilde{u}$: intersection speed of NT and DD

- To cover whole speed range,
  - $u_T^{C-max} > u_{th}^{DD}$
  - $u_T^{C-max} = v_{esc}(r)\sqrt{\frac{4m_\chi m_T}{(m_\chi - m_T)^2}}$
  - $\left(u_{th}^{DD}\right)^2 = \frac{m_T}{2\mu_{\chi T}^2} E_{R,th}$



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S. Kang et al. JCAP03(2023)011

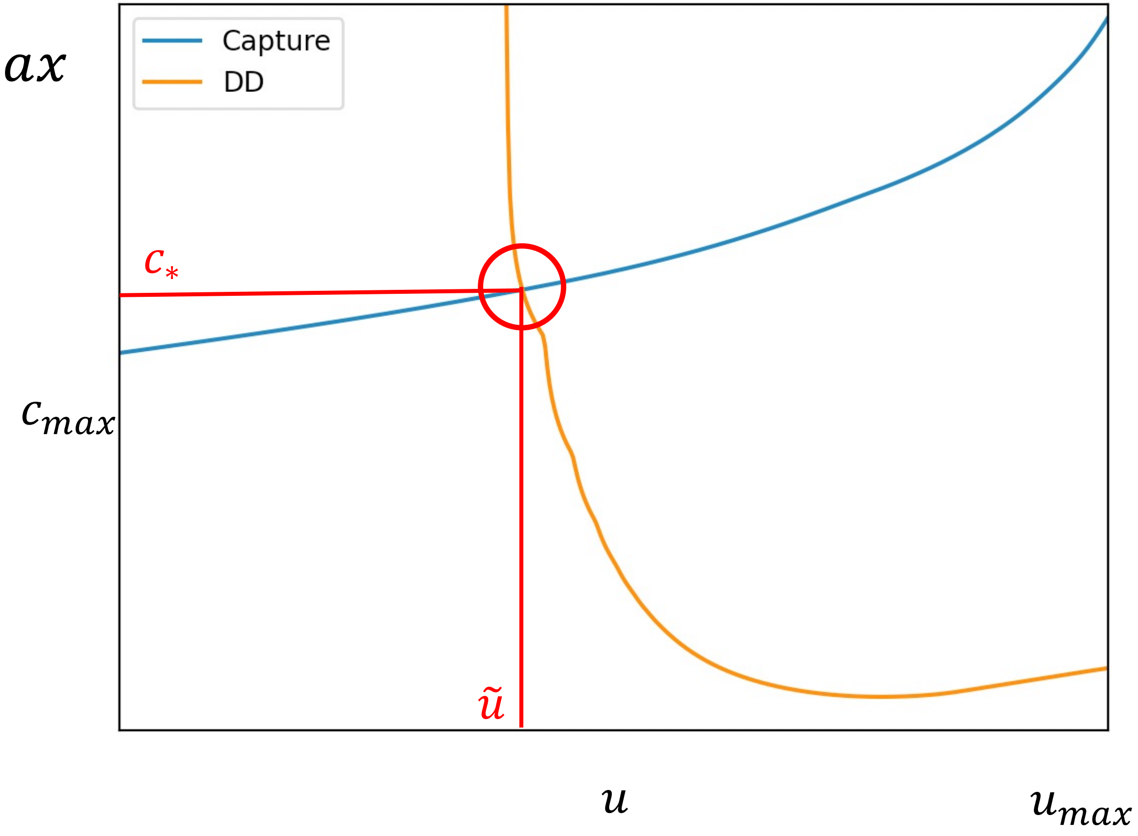
Halo independent approach

- Intersection:

$$(c^{NT})^2_{max}(u) \leq c_*^2 \quad \text{for } 0 \leq u \leq \tilde{u}$$
$$(c^{DD})^2_{max}(u) \leq c_*^2 \quad \text{for } \tilde{u} \leq u \leq u_{max}$$

$$c^2 \leq c_*^2 \left[\int_0^{\tilde{u}} du\, f(u)\right]^{-1} = \frac{c_*^2}{\delta}$$
$$c^2 \leq c_*^2 \left[\int_{\tilde{u}}^{u_{max}} du\, f(u)\right]^{-1} = \frac{c_*^2}{1-\delta}$$
$$\delta = \int_0^{\tilde{u}} du\, f(u)$$

$$c^2 \leq 2c_*^2 \quad (\delta = 1/2)$$



F. Ferrer et al. JCAP09(2015)052
S. Kang et al. JCAP03(2023)011

Halo independent approach

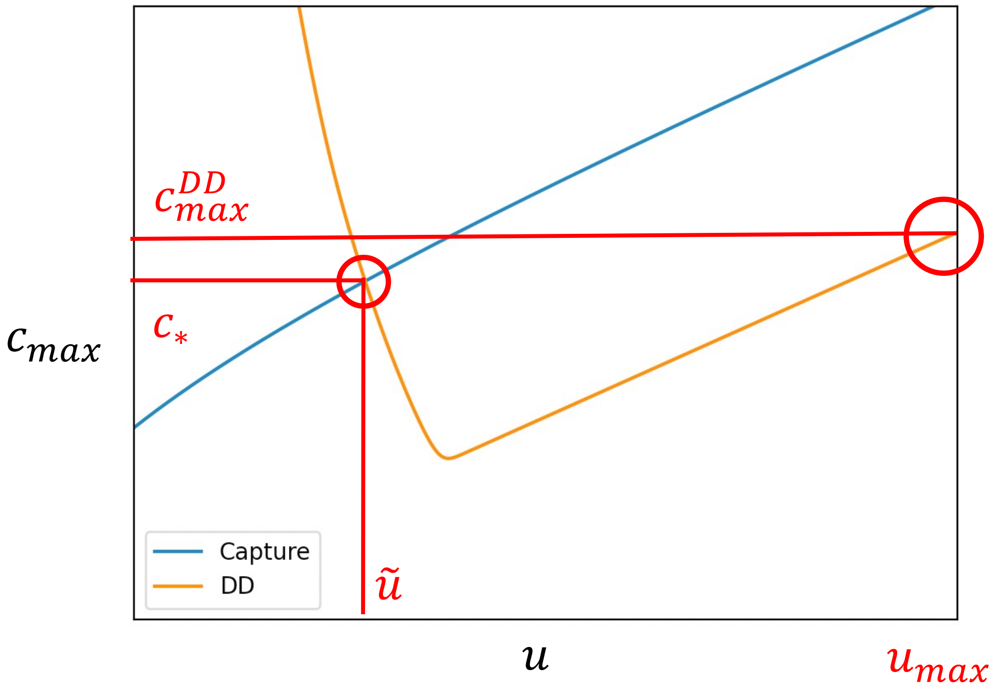
- If $(c^{DD})^2_{max}(u) > c_*^2$ at $u = u_{max}$:

$$c^2 \leq c_*^2 \left[\int_0^{\tilde{u}} du\, f(u)\right]^{-1} = \frac{c_*^2}{\delta}$$

$$c^2 \leq (c^{DD})^2_{max}(u_{max})\left[\int_{\tilde{u}}^{u_{max}} du\, f(u)\right]^{-1}$$

$$= \frac{\left(c^{DD}\right)^2_{max}(u_{max})}{1-\delta}$$

$$c^2 \leq (c^{DD})^2_{max}(u_{max}) + c_*^2$$



- Sensitive to u_{max}

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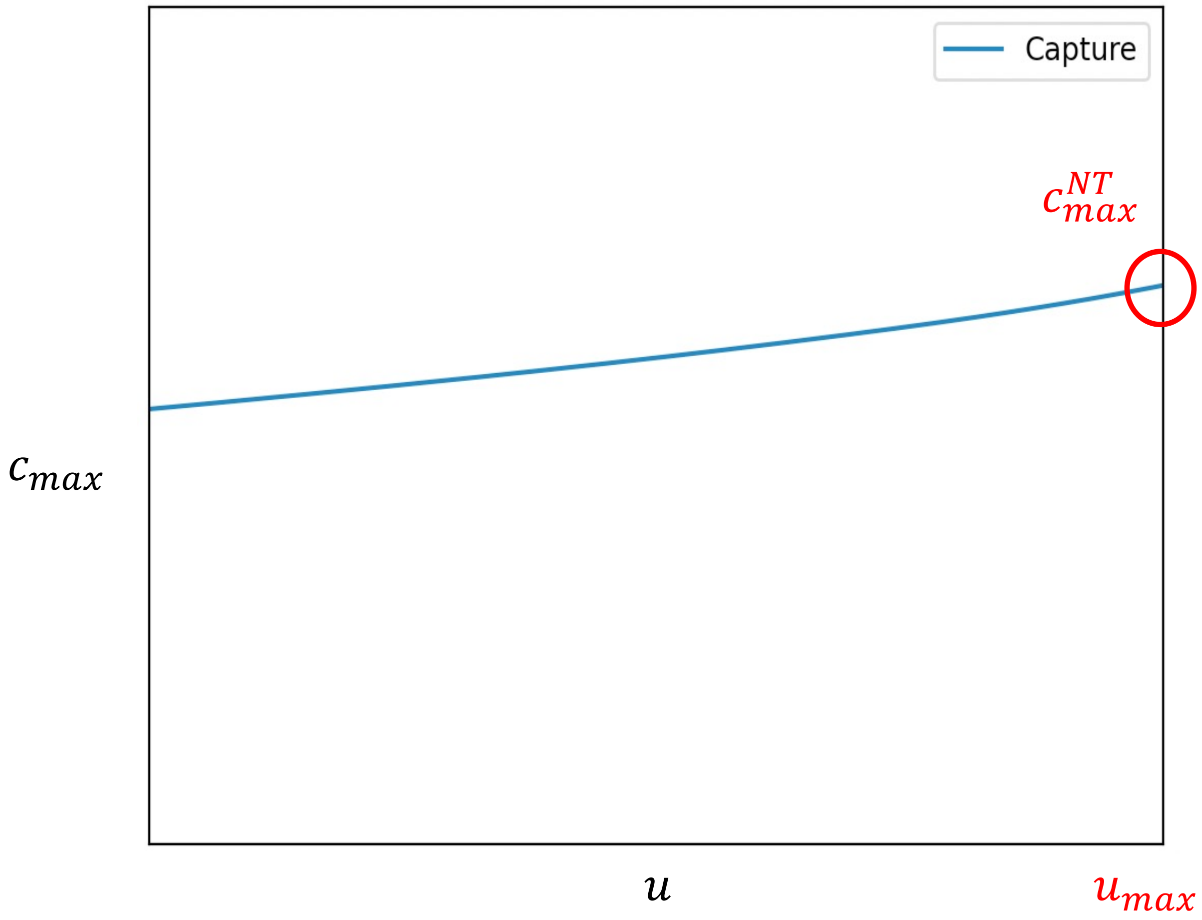
Halo independent approach

- If $u_{th}^{DD} > u_{max}$:

  $c^2 \leq (c^{NT})^2_{max}(u_{max})$

- Sensitive to u_{max}



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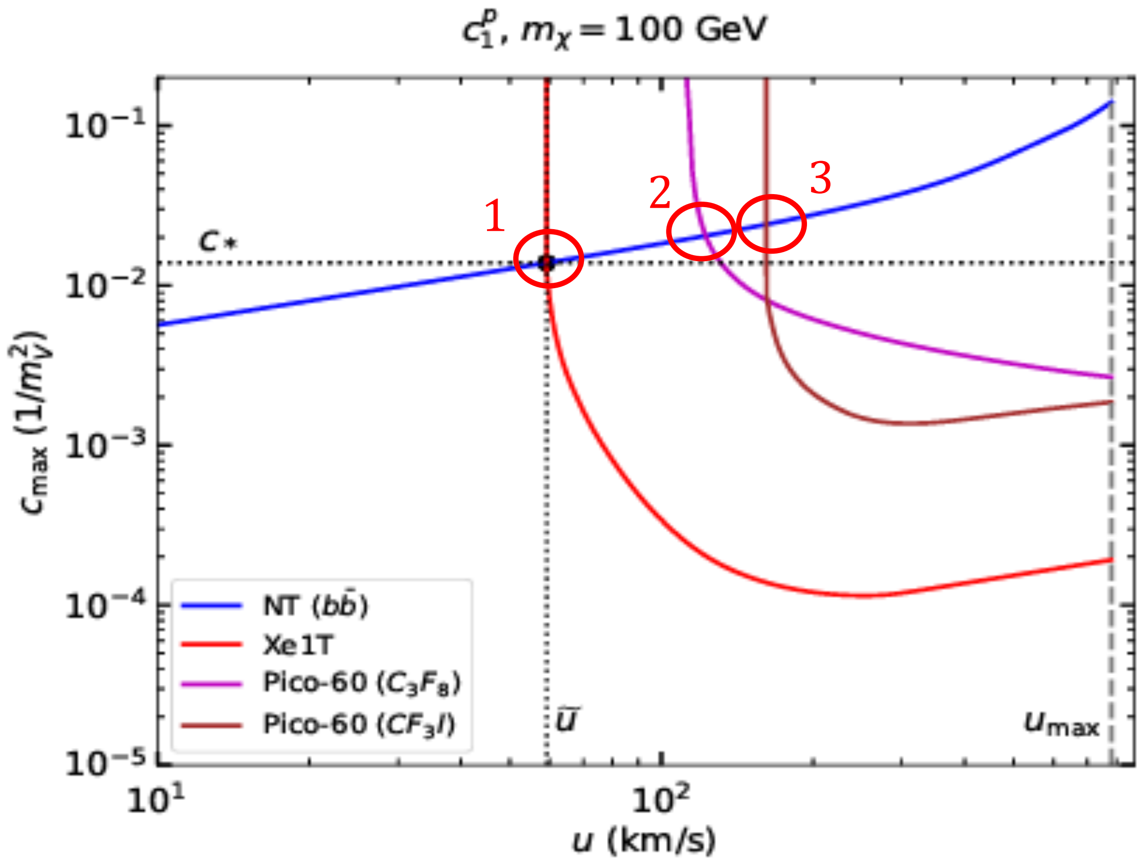
Halo independent approach

- Repeat by combining each DD to NT
  - 1: NT and XENON 1T
  - 2: NT and PICO-60(C_3F_8)
  - 3: NT and PICO-60(CF_3I)

- Halo independent limit:
  the most constraining one



S. Kang, A. Kar, S. Scopel, JCAP03(2023)011

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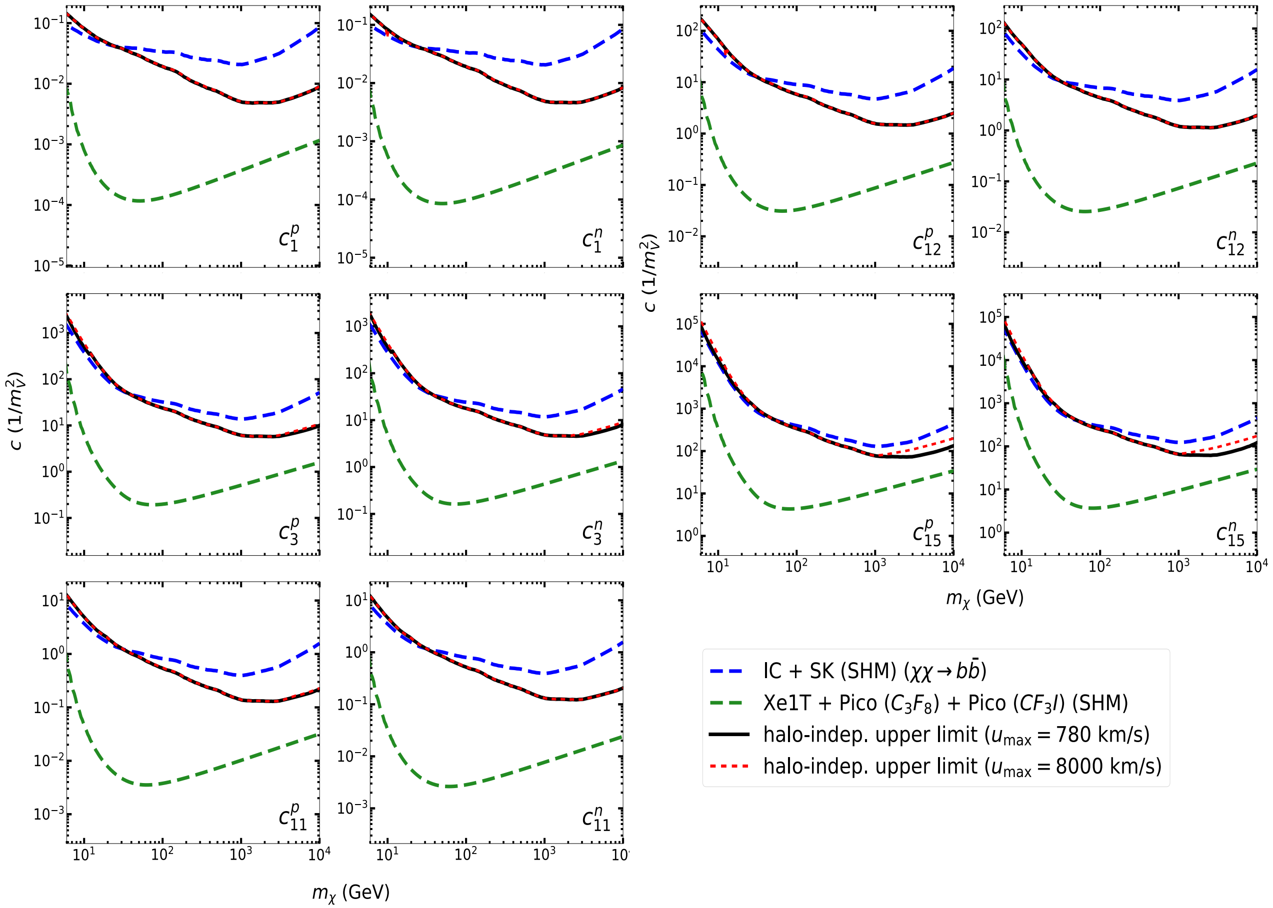
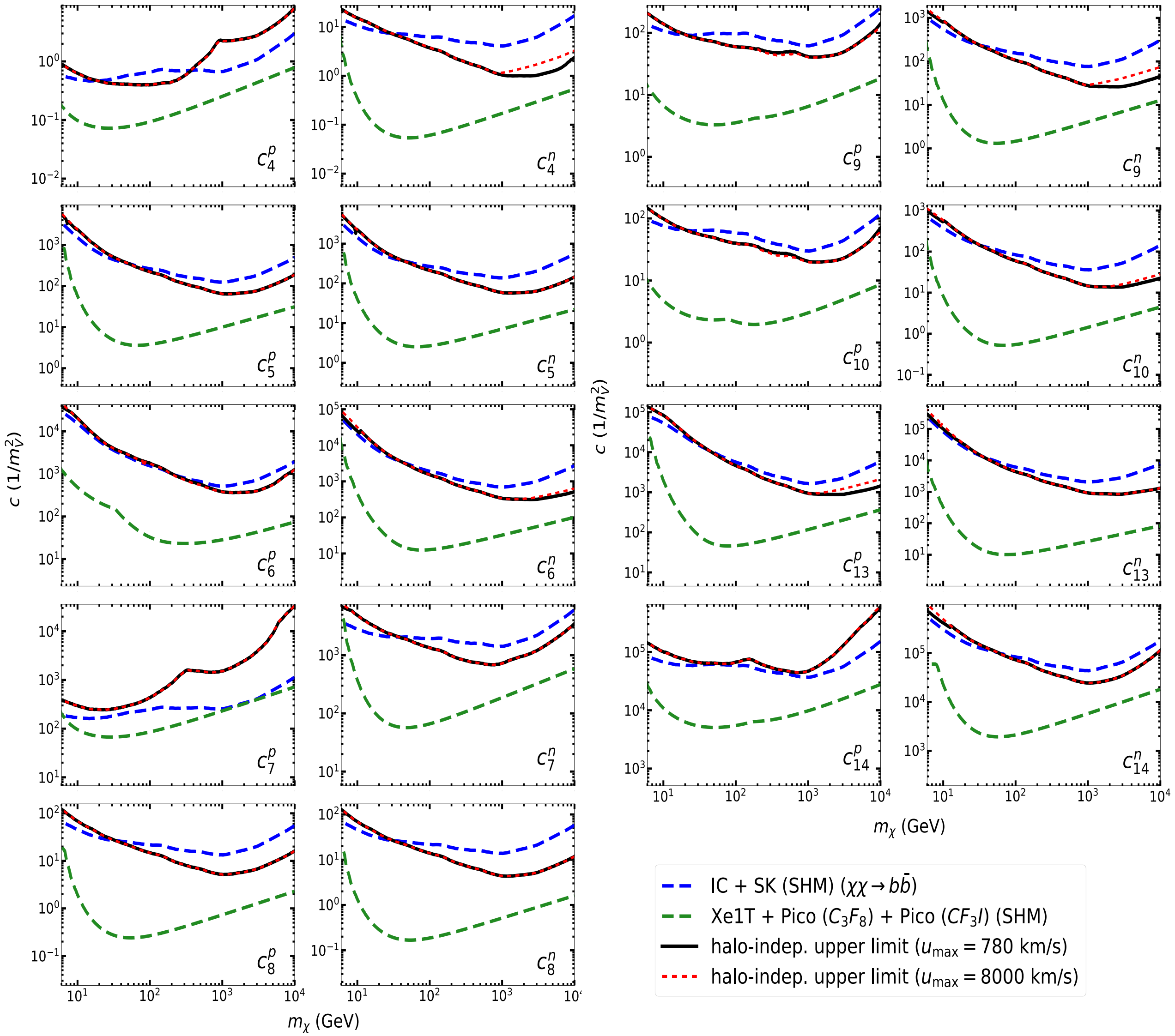
Halo independent approach





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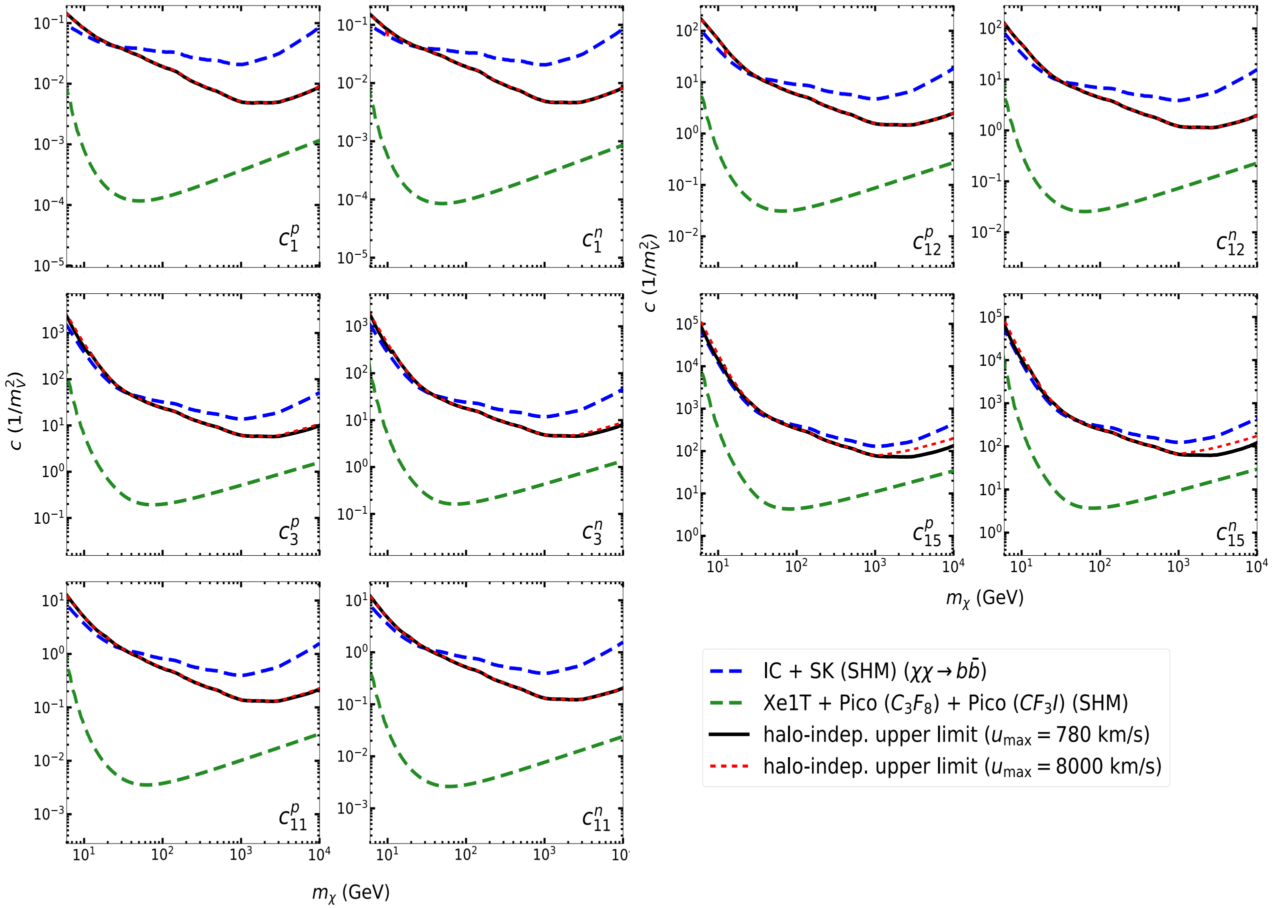
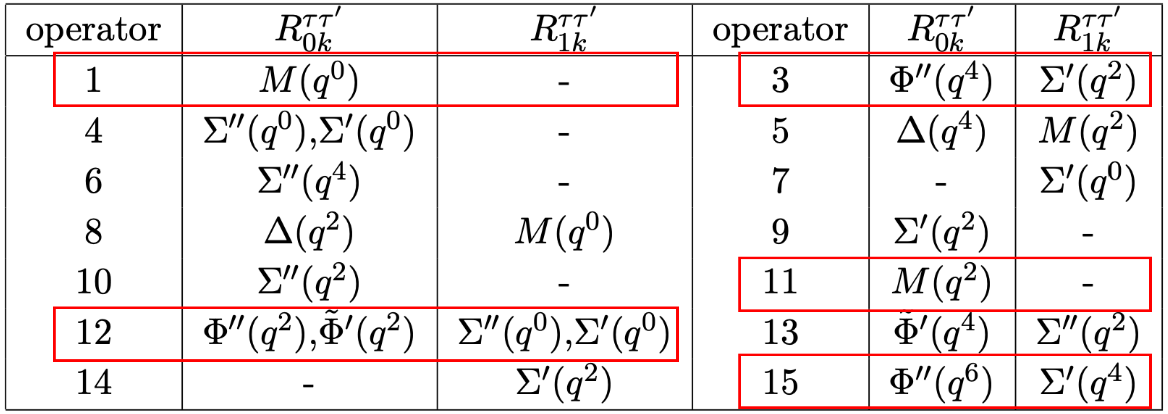
Halo independent approach



- Spin independent
  - $O_{1,3,11,12,15}$
  - W_M, $W_{\Phi''}$
  - Enhanced for heavy targets



operator	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$	operator	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$
1	$M(q^0)$	-	3	$\Phi''(q^4)$	$\Sigma'(q^2)$
4	$\Sigma''(q^0)$,$\Sigma'(q^0)$	-	5	$\Delta(q^4)$	$M(q^2)$
6	$\Sigma''(q^4)$	-	7	-	$\Sigma'(q^0)$
8	$\Delta(q^2)$	$M(q^0)$	9	$\Sigma'(q^2)$	-
10	$\Sigma''(q^2)$	-	11	$M(q^2)$	-
12	$\Phi''(q^2)$,$\tilde{\Phi}'(q^2)$	$\Sigma''(q^0)$,$\Sigma'(q^0)$	13	$\tilde{\Phi}'(q^4)$	$\Sigma''(q^2)$
14	-	$\Sigma'(q^2)$	15	$\Phi''(q^6)$	$\Sigma'(q^4)$

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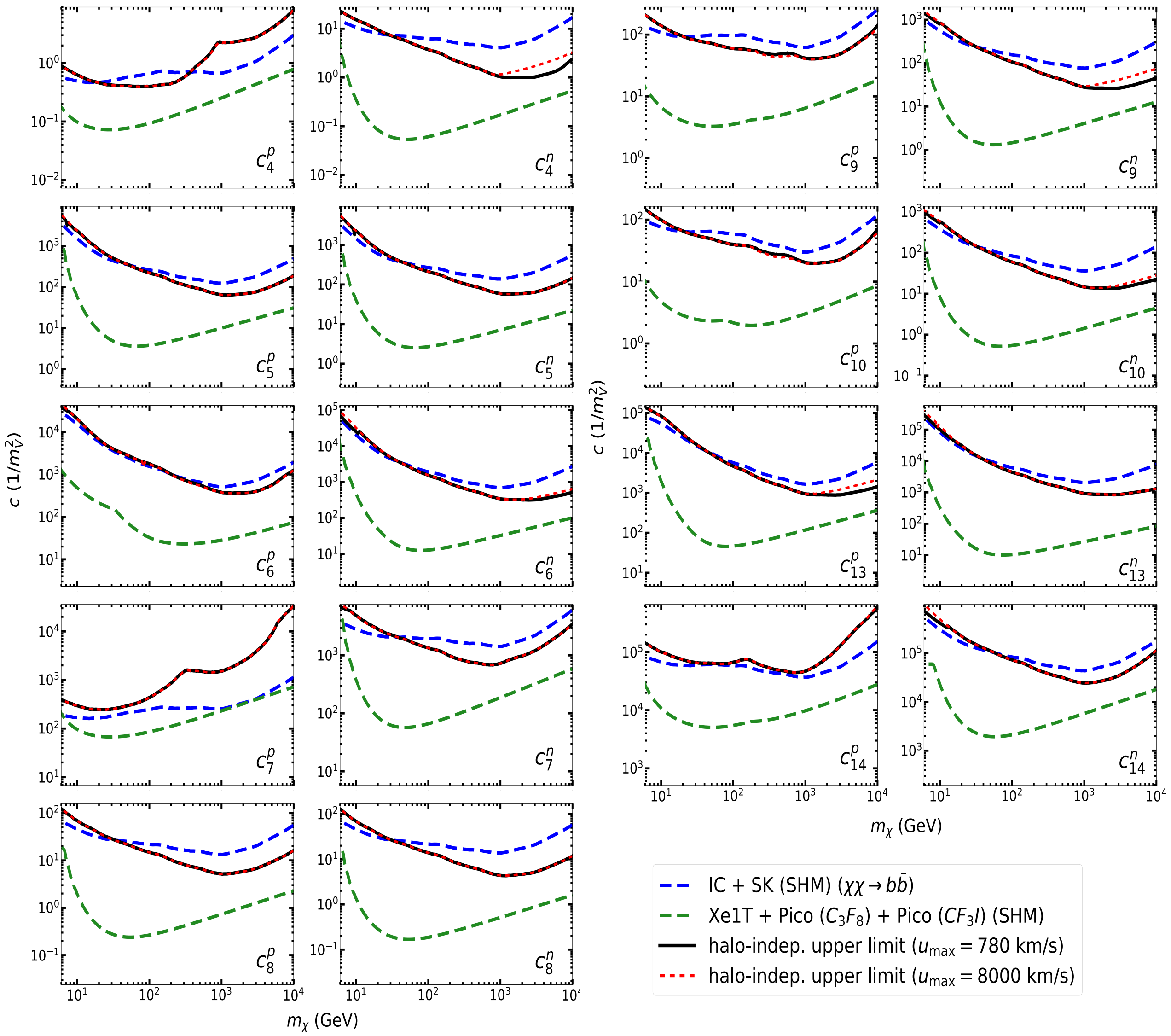
Halo independent approach



- Spin dependent
  - $O_{4,5,6,7,8,9,10,13,14}$
  - $W_{\Sigma''}$, $W_{\Sigma'}$: directly coupling to spin
  - W_{Δ}: related to angular momentum
  - $W_{\tilde{\Phi}'}$: spin larger than 1/2

operator	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$	operator	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$
1	$M(q^0)$	-	3	$\Phi''(q^4)$	$\Sigma'(q^2)$
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14	-	$\Sigma'(q^2)$	15	$\Phi''(q^6)$	$\Sigma'(q^4)$

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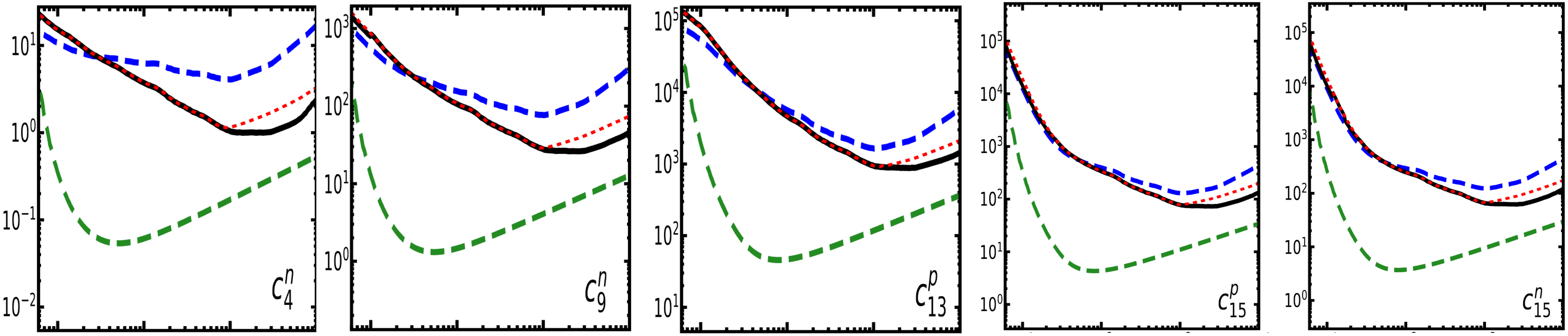
Halo independent approach

- If $(c^{DD})^2_{max}(u) > c_*^2$ at $u = u_{max}$: $c^2 \leq (c^{DD})^2_{max}(u_{max}) + c_*^2$
- If $u_{th}^{DD} > u_{max}$: $c^2 \leq (c^{NT})^2_{max}(u_{max})$
- u_{max}: 780 km/s → 8000 km/s
- Effect of large u_{max} is mild: factor less than 2



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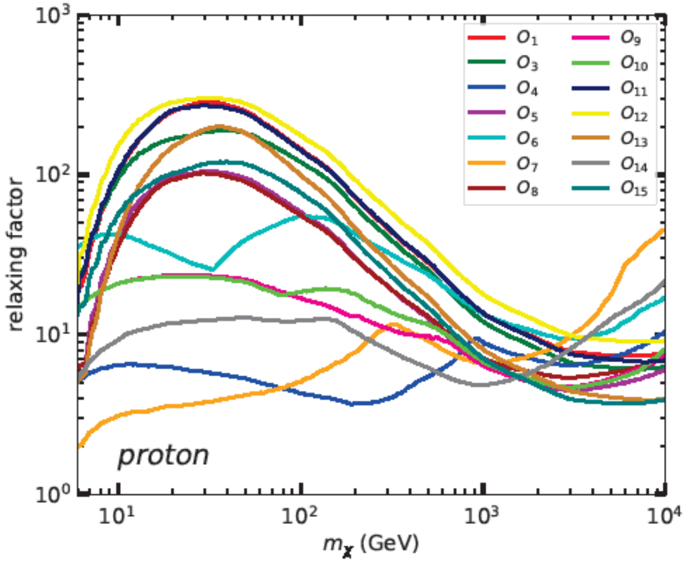
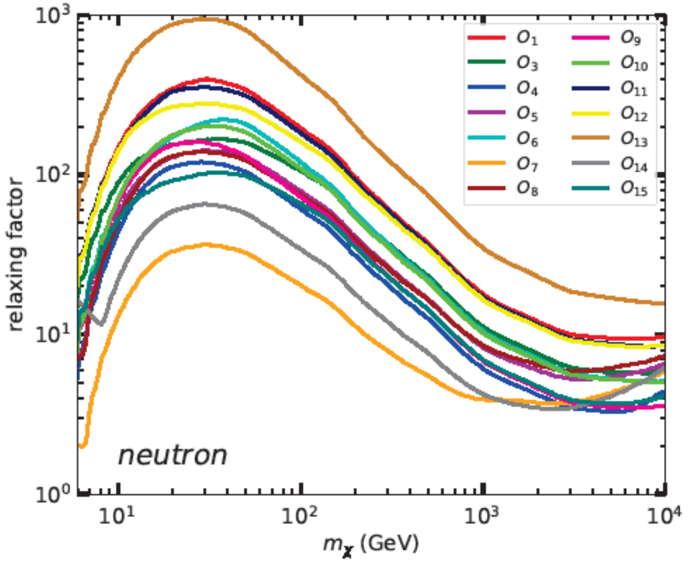
Halo independent approach

- Relaxing factor

$$r_f^2 = \frac{2c_*^2}{\left(c_{SHM}^{exp}\right)^2} = 2c_*^2 \int_0^{u_{max}} du \, \frac{f_M(u)}{(c^{exp})^2_{max}(u)} = 2c_*^2 < \frac{1}{(c^{exp})^2_{max}} > \cong 2c_*^2 < \frac{1}{(c^{exp})^2_{max}} >_{bulk}$$

$$\int_{bulk} du \, f_M(u) \approx 0.8$$





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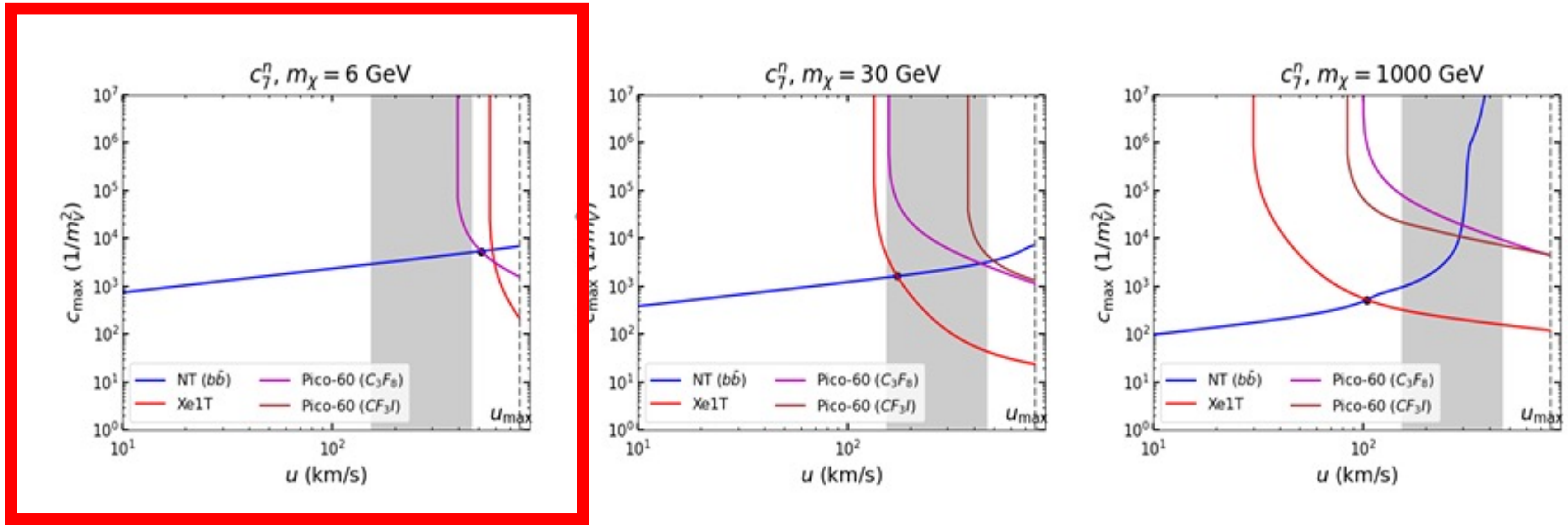
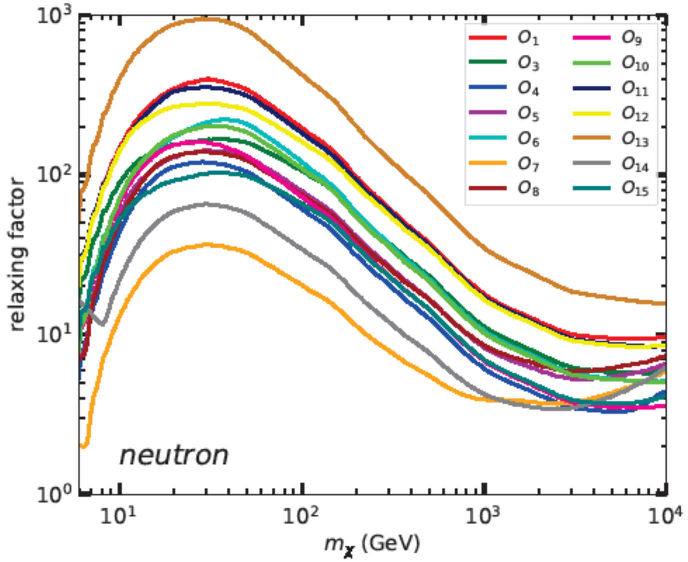
Halo independent approach



- small or large mass range
  - outside the bulk of Maxwellian
  - smooth dependence on u
- intermediate range (10 ~ 200 GeV)
  - inside the bulk of Maxwellian
  - steep dependence on u



$$r_f^2 \simeq 2c_*^2 \left\langle \frac{1}{(c^{\exp})^2_{\max}} \right\rangle_{\text{bulk}},$$

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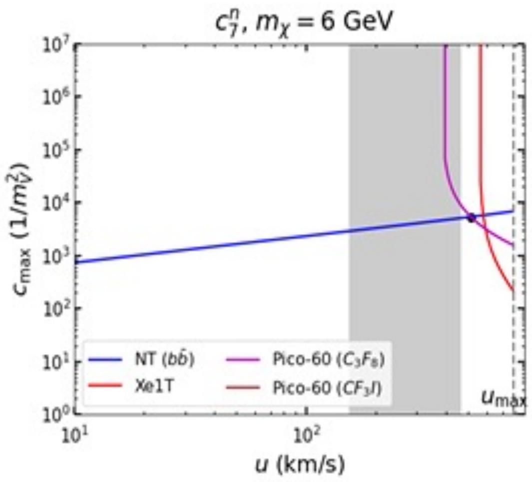
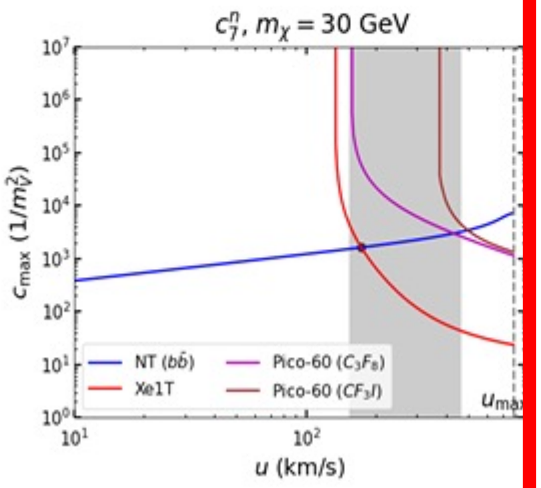
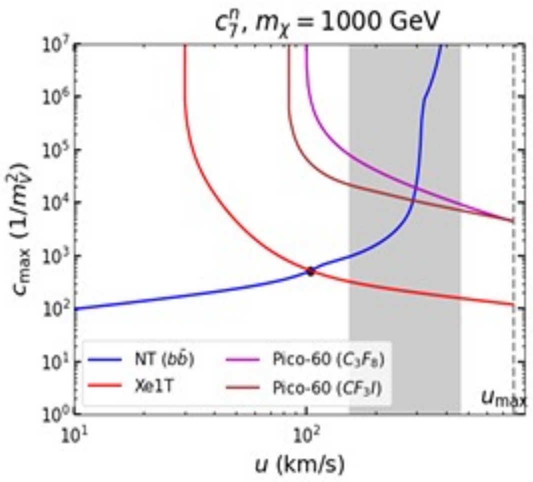
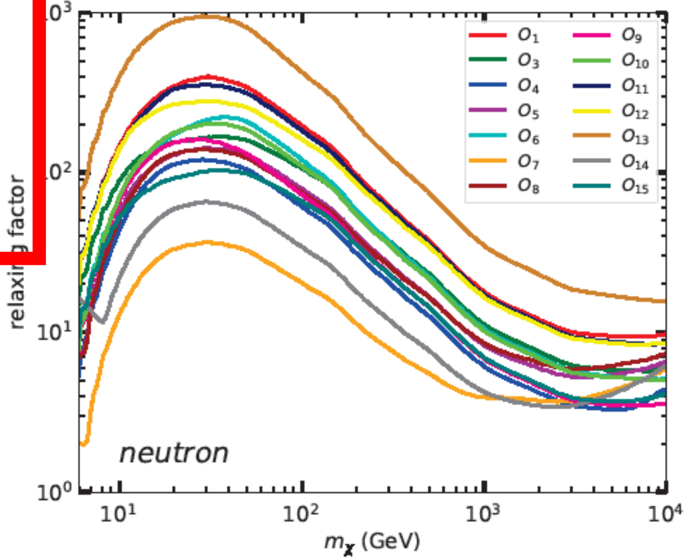
Halo independent approach









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$$r_f^2 \simeq 2c_*^2 \left\langle \frac{1}{(c^{\exp})^2_{\max}} \right\rangle_{\text{bulk}},$$

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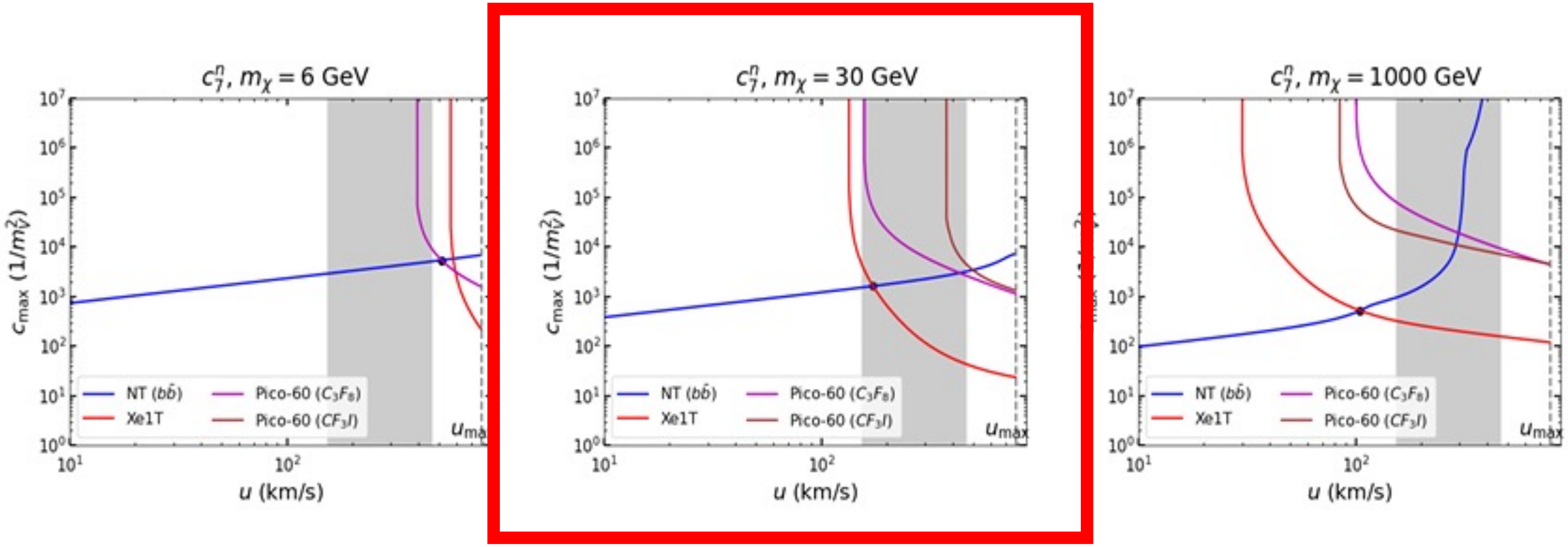
Halo independent approach



- small or large mass range
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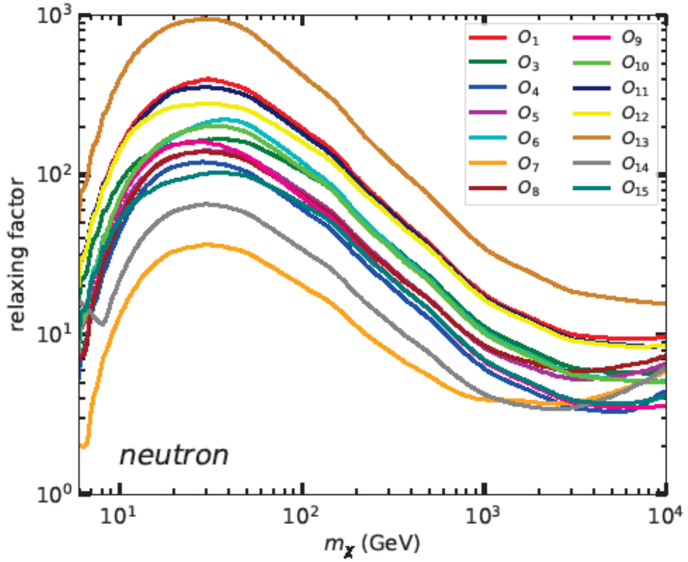

$$r_f^2 \simeq 2c_*^2 \left\langle \frac{1}{(c^{\mathrm{exp}})^2_{\mathrm{max}}} \right\rangle_{\mathrm{bulk}},$$

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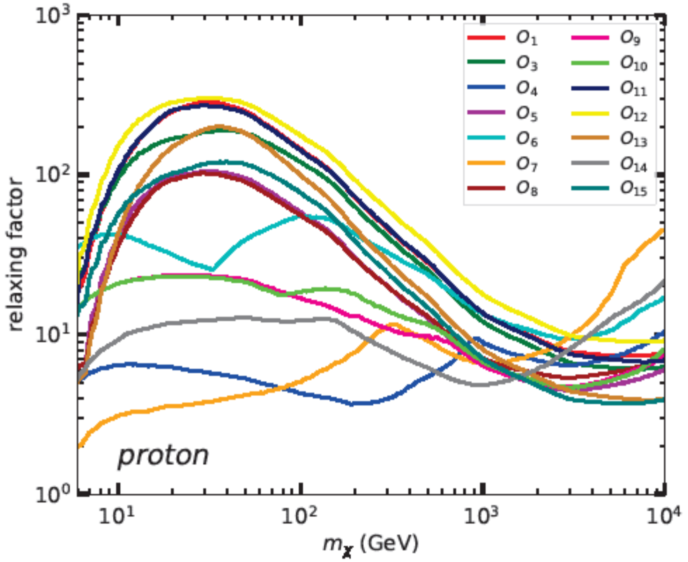
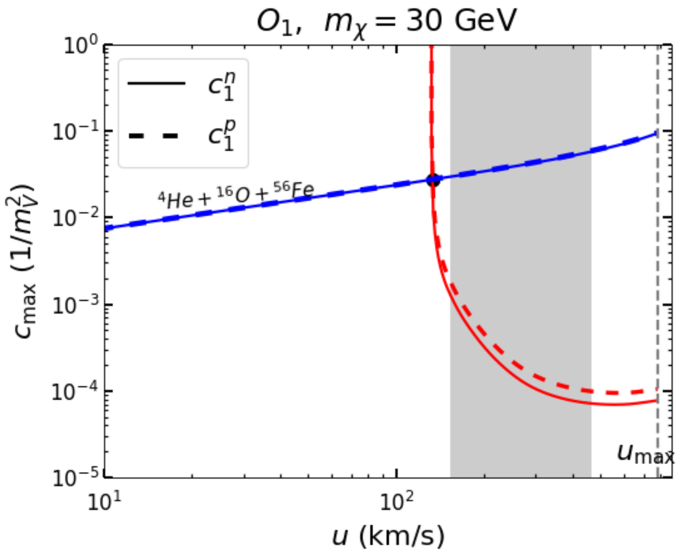
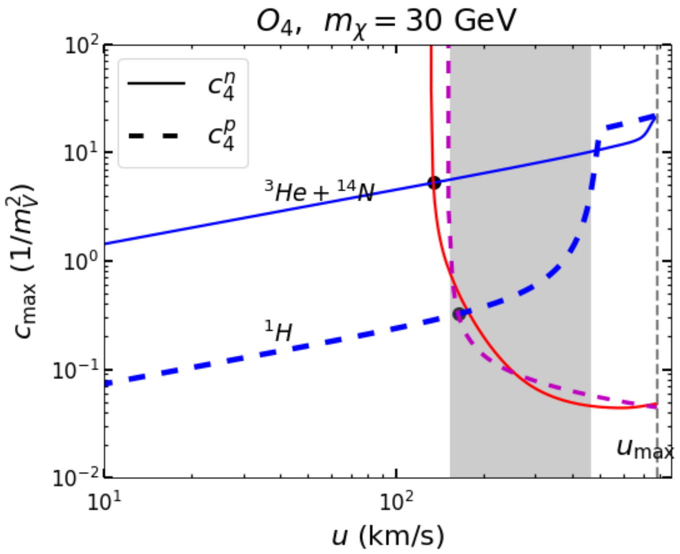
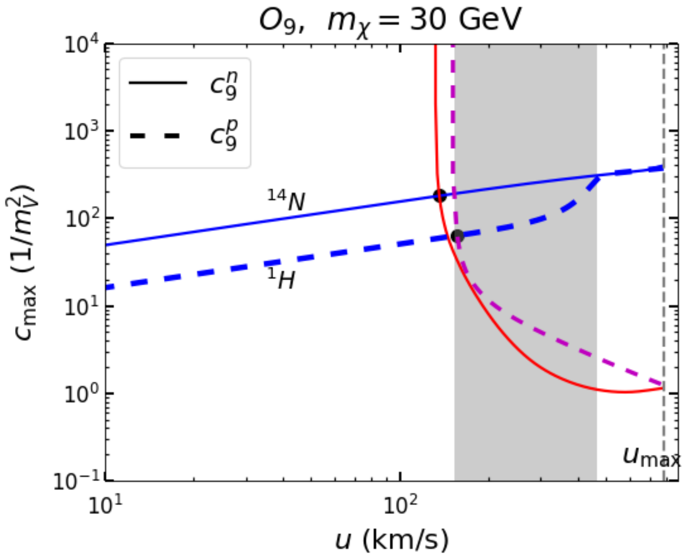
Halo independent approach

- small relaxing factors
  - $O_{4,7}$: SD with no q suppression
  - $O_{9,10,14}$: SD with q^2 suppression
  - O_6 : SD with q^4 suppression









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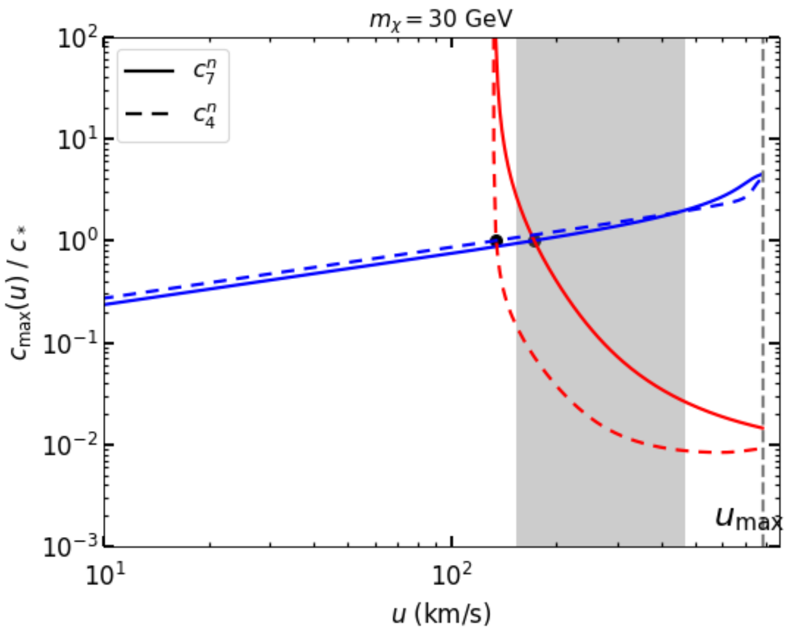
Halo independent approach

- Velocity dependent operators($O_{7,14}$)
  - small relaxing factor
  - velocity dependence → weaker limit



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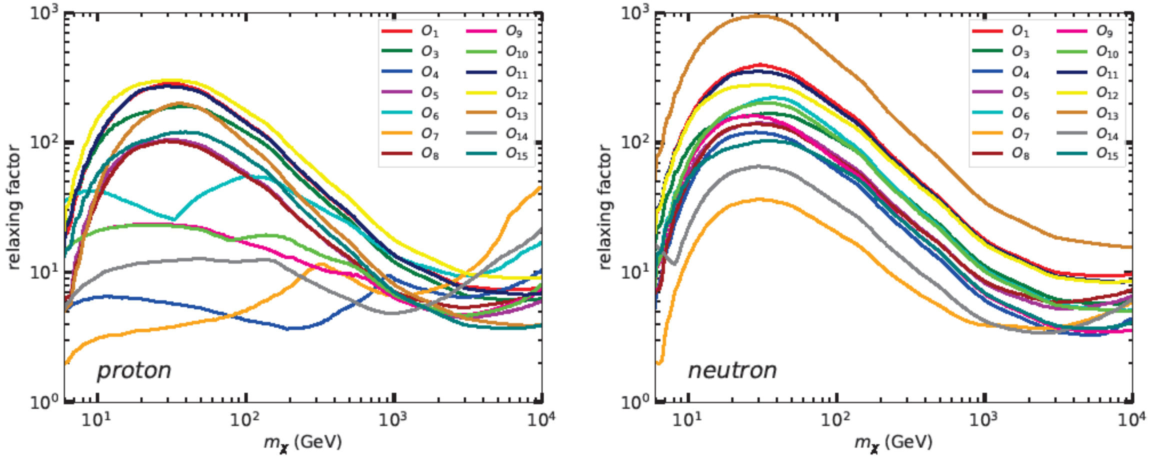

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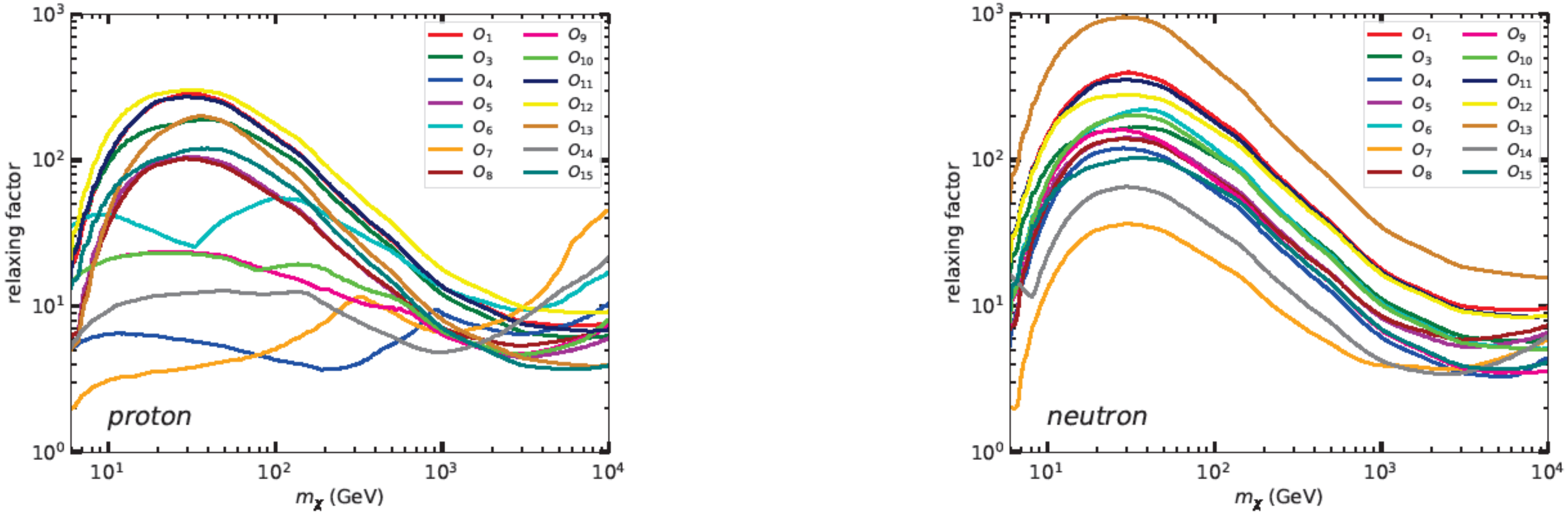
Halo independent approach

- High relaxing factor:
  the halo-independent method can weaken the bound

$$r_f^2 = \frac{2c_*^2}{\left(c_{SHM}^{exp}\right)^2} = 2c_*^2 \int_0^{u_{max}} du \frac{f_M(u)}{(c^{exp})^2_{max}(u)} = 2c_*^2 < \frac{1}{(c^{exp})^2_{max}} > \cong 2c_*^2 < \frac{1}{(c^{exp})^2_{max}} >_{bulk}$$



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Summary

- Halo independent method can be applied to any speed distribution

- Combining results of direct detection experiments and capture in the Sun may provide halo-independent bounds on each couplings

- In most cases the relaxation of halo independent bounds is moderate in low and high m_χ

- More moderate values of the relaxation is obtained with c_{SD}^{p}

- High relaxing factor: halo independent method weaken the bounds
  → sensitive on speed distribution