WIMPs in Dilatonic Einstein-Gauss-Bonnet Cosmology

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Modified gravity

- The difficulty to fit General Relativity (GR) with other fundamental interactions may imply that the theory of GR is incomplete
(GR is an effective theory valid below the cut-off scale $M_{PL} \sim 10^{19}$ GeV)

- Discovery of Gravitational Waves (GWs) and direct measurements of merger events of compact binaries open up a new era of precision tests of gravity

- Such tests can complement constraints from Cosmology, etc.

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- Among them, Horndeski's theory is the most general scalar-tensor theory
[e.o.m 2nd-order in 4 d spacetime $\Rightarrow$ no ghost modes]
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- At the level of e.o.m, the simplest example of Horndeski's theory containing higher-curvature terms is the dilatonic Einstein-Gauss-Bonnet (dEGB) theory

Possible probes of the dEGB scenario

- We have no direct probe of the Universe expansion rate, composition or reheating temperature before Big Bang Nucleosynthesis (BBN)
- However, an understanding of the present Universe cannot avoid the inclusion of Inflation, Dark Matter (DM), Baryon asymmetry, etc.
- All such events that take place before BBN can be used to shed light on physics beyond Standard GR
- On the other hand, GW data from Black Hole (BH) or Neutron Star (NS) binary mergers in the late Universe can also constrain such scenarios
- dEGB theory has been extensively studied in many of such realizations

Possible probes of the dEGB scenario: WIMPs

- Cold Dark Matter (CDM): provides $\sim$25% of the energy density of the present Universe
- Standard Model (SM) of particle physics cannot explain CDM
- Weakly Interacting Massive Particles (WIMPs): one of the most popular candidates for CDM; [mass in GeV – TeV scale]

  Decoupled from thermal bath in the early Universe before BBN
- We study the thermal decoupling of WIMP DM in the early Universe under modified dEGB Cosmology and use the WIMP DM search results to probe the dEGB scenario
- Constraints on dEGB from WIMP DM indirect searches are nicely complementary to late-time constraints from compact binary mergers

- dEGB action:

$$S = \int_{\mathcal{M}} \sqrt{-g}\; d^4x \left[\frac{R}{2\kappa} - \frac{1}{2}\nabla_\mu \phi \nabla^\mu \phi - V(\phi) + f(\phi) R^2_{\rm GB} + \mathcal{L}_m^{\rm rad} \right]$$

$\kappa \equiv 8\pi G = 1/M_{PL}^2$; $\quad g = \det(g_{\mu\nu})$;
$R \equiv$ scalar curvature of the spacetime $\mathcal{M}$ (3+1 d)

ϕ : scalar field (dilaton field); $\quad V(\phi)$: scalar field potential

$R^2_{\rm GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ (Gauss-Bonnet term)

$f(\phi)$: describes the coupling between ϕ and the Gauss-Bonnet term

$\mathcal{L}_m^{\rm rad}$: interactions of radiation and matter fields

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  $\mathcal{L}_m^{\rm rad}$: interactions of radiation and matter fields

- If $f(\phi)$ is constant, the Gauss-Bonnet term (in 3+1 d) reduces to a surface term and does not contribute to the e.o.m

- $f(\phi)$ in principle can be arbitrary
  – an exponential or a power law form is frequently adopted
  – the two forms can be connected by field redefinition

- We adopt: $f(\phi) = \alpha e^{\gamma\phi}$ $\qquad$ [α and γ have both signs]

$$S = \int_{\mathcal{M}} \sqrt{-g}\; d^4x \left[\frac{R}{2\kappa} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi) + \; f(\phi)R^2_{\rm GB} + \mathcal{L}^{\rm rad}_m \right]$$

$$(R^2_{\rm GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})$$

Equations of motion:

(1) $\Box\phi - V' + f'R^2_{\rm GB} = 0$ $\quad\quad$ $\Box = \nabla_\mu\nabla^\mu\,;\; V' = \partial V/\partial\phi\,;\; f' = \partial f/\partial\phi$

(2) $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa\left(T^{\{\phi+{\rm GB}\}}_{\mu\nu} + T^{\rm rad}_{\mu\nu}\right) \equiv \kappa T^{\rm tot}_{\mu\nu}$

(Additional terms are moved to the r.h.s to get the familiar form of the Einstein Equation)

Energy-momentum tensor for radiation: $T^{\rm rad}_{\mu\nu} = -2\frac{\delta\mathcal{L}^{\rm rad}_m}{\delta g^{\mu\nu}} + \mathcal{L}^{\rm rad}_m g_{\mu\nu}$

$T^{\{\phi+{\rm GB}\}}_{\mu\nu} = T^{\phi}_{\mu\nu} + T^{\rm GB}_{\mu\nu}$ $\quad$ (for notation purpose)

$$T^{\phi}_{\mu\nu} = \nabla_\mu\phi\nabla_\nu\phi - \left(\frac{1}{2}\nabla_\rho\phi\nabla^\rho\phi + V\right)g_{\mu\nu}$$

$$\begin{aligned} T^{\rm GB}_{\mu\nu} \;\; = \;\; & 4\left[R\nabla_\mu\nabla_\nu f(\phi) - g_{\mu\nu}R\Box f(\phi)\right] - 8\left[R_\nu{}^\rho\nabla_\rho\nabla_\mu f(\phi) + R_\mu{}^\rho\nabla_\rho\nabla_\nu f(\phi)\right. \\ & \left. - R_{\mu\nu}\Box f(\phi) - g_{\mu\nu}R^{\rho\sigma}\nabla_\rho\nabla_\sigma f(\phi) + R_{\mu\rho\nu\sigma}\nabla^\rho\nabla^\sigma f(\phi)\right] \end{aligned}$$

$$S = \int_{\mathcal{M}} \sqrt{-g}\; d^4x \left[\frac{R}{2\kappa} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi) + \ f(\phi)R^2_{\rm GB} + \mathcal{L}_m^{\rm rad} \right]$$

We consider the spatially flat FLRW metric:

$$ds^2 = -dt^2 + a^2(t)\,\delta_{ij}\,dx^i dx^j$$

$\Rightarrow$ Equations of motion depend only on time (t)

Energy density: $\rho_I = -T_I{}^0{}_0$, Pressure: $p_I\,\delta^i{}_j = T_I{}^i{}_j$, $[I \equiv \{\phi + {\rm GB}\}, {\rm rad}]$

Cosmology in dEGB theory

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Friedmann equations:

$$H^2 = \frac{\kappa}{3}\left(\rho_{\{\phi+{\rm GB}\}} + \rho_{\rm rad}\right) \equiv \frac{\kappa}{3}\rho_{\rm tot}$$

$$\dot{H} = -\frac{\kappa}{2}\left[\left(\rho_{\{\phi+{\rm GB}\}} + p_{\{\phi+{\rm GB}\}}\right) + \left(\rho_{\rm rad} + p_{\rm rad}\right)\right] \equiv -\frac{\kappa}{2}(\rho_{\rm tot} + p_{\rm tot})$$

$$\ddot{\phi} + 3H\dot{\phi} + V' - f' R^2_{\rm GB} = 0 \qquad \text{where} \quad R^2_{\rm GB} = 24H^2(\dot{H} + H^2)$$

"dot" $\Rightarrow d/dt$, "prime" $\Rightarrow d/d\phi$

Expansion rate of the Universe: $H = \dot{a}/a$

$\rho_{\rm rad} \sim g_* T^4$, $[\Rightarrow H_{\rm rad} \sim \sqrt{g_*}\, T^2]$ ($T \equiv$ temperature of the Universe)
$p_{\rm rad} = \frac{1}{3}\rho_{\rm rad}$

Cosmology in dEGB theory

$$\rho_{\{\phi+\mathrm{GB}\}} \equiv \rho_\phi + \rho_{\mathrm{GB}}, \quad p_{\{\phi+\mathrm{GB}\}} \equiv p_\phi + p_{\mathrm{GB}} \qquad \text{(for notation purposes)}$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \qquad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$\rho_{\mathrm{GB}} = -24\dot{f}H^3 = -24f'\dot{\phi}H^3$$

$$p_{\mathrm{GB}} = 8\left(f''\dot{\phi}^2 + f'\ddot{\phi}\right)H^2 + 16f'\dot{\phi}H(\dot{H}+H^2) = 8\frac{d(f'\dot{\phi}H^2)}{dt} - \frac{2}{3}\rho_{\mathrm{GB}}$$

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$$H^2 = \frac{\kappa}{3}\left(\frac{1}{2}\dot{\phi}^2 + V - 24f'\dot{\phi}H^3 + \rho_{\mathrm{rad}}\right)$$

$$\dot{H} = -\frac{\kappa}{2}\left(\dot{\phi}^2 + 8\frac{d(f'\dot{\phi}H^2)}{dt} - 8f'\dot{\phi}H^3 + \rho_{\mathrm{rad}} + p_{\mathrm{rad}}\right)$$

$$\ddot{\phi} + 3H\dot{\phi} + V' - 24f'H^2(\dot{H}+H^2) = 0$$

Friedmann Equations

- We assume $V(\phi) = 0$ to reduce the parameter space
  (To avoid early accelerated expansion before matter-radiation equivalence $V(\phi)$ should be zero or close to zero at Big Bang Nucleosynthesis (BBN))
- Coupling $f(\phi) = \alpha e^{\gamma\phi}$ [α and γ have both signs]
- Dynamics is controlled by derivative(s) of $f(\phi)$
- $f'(\phi) = 0$ (α and/or $\gamma = 0$)
  $\Rightarrow$ No dEGB ; only kination [$\dot{\phi} \sim a^{-3}$ or $\rho_\phi(= \frac{1}{2}\dot{\phi}^2) \sim a^{-6}$]
- Unit convention: [$\kappa = 8\pi G = 1 = c$] $\Rightarrow \alpha$ (in km^2), γ (dimension less)

Solutions of Friedmann equations in dEGB theory

- We convert $\text{time}\,(t) \rightarrow \text{temperature}\,(T)$ and solve the cosmological equations in terms of T

- The relation between t and T is obtained using the conservation of entropy (sa^3 = constant) :

$$\frac{dT}{dt} = -\frac{H\,T}{(1 + \frac{1}{3} d\ln g_{*s}/d\ln T)}, \qquad s \sim g_{*s} T^3$$

$[sa^3 = \text{constant} \quad \Rightarrow a \sim T^{-1}]$

- BBN is the earliest process in Cosmology that provides a successful confirmation of Standard Cosmology
  $\Rightarrow$ any departure from Standard Cosmology for $T \leq T_{\rm BBN} \simeq 1$ MeV is strongly constrained

- We solve the Friedmann equations by setting initial conditions at $T = T_{\rm BBN} = 1$ MeV and evolving the variables towards higher T and see how cosmology is modified at the early Universe

Solutions of Friedmann equations in dEGB theory

$$H^2 = \frac{\kappa}{3}\left(\frac{1}{2}\dot{\phi}^2 - 24f'\dot{\phi}H^3 + \rho_{\rm rad}\right)$$
$$\dot{H} = -\frac{\kappa}{2}\left(\dot{\phi}^2 + 8\frac{d(f'\dot{\phi}H^2)}{dt} - 8f'\dot{\phi}H^3 + \rho_{\rm rad} + p_{\rm rad}\right)$$
$$\ddot{\phi} + 3H\dot{\phi} - 24f'H^2(\dot{H} + H^2) = 0$$

V = 0

$f(\phi) = \alpha e^{\gamma\phi}$

Parameter space:
α, γ, $\phi_{\rm BBN}$ and $\dot{\phi}_{\rm BBN}$

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- Contribution of $\rho_\phi(T_{\rm BBN}) = \frac{1}{2}\dot{\phi}^2_{\rm BBN}$ to $\rho_{\rm tot}$ at BBN is constrained by the effective number of neutrino flavors $N_{eff} \leq 2.99 \pm 0.17$

  $\Rightarrow \rho_\phi(T_{\rm BBN}) \lesssim 3 \times 10^{-2}\rho_{\rm BBN}$, [$\rho_{\rm BBN} \equiv$ radiation energy density at BBN]

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- ϕ (hence $\phi_{\rm BBN}$) appears in Friedmann equations only through $f(\phi)$ $(= \alpha e^{\gamma\phi})$

  Define $\tilde{\alpha} = \alpha\, e^{\gamma\phi_{\rm BBN}}$; $[\Rightarrow f(\phi)_{\rm BBN} = \tilde{\alpha}$, $f'(\phi)_{\rm BBN} = \tilde{\alpha}\gamma$, $f''(\phi)_{\rm BBN} = \tilde{\alpha}\gamma^2]$

  $\tilde{\alpha}$ invariant under $\phi'_{\rm BBN} = \phi_{\rm BBN} + \phi_0$, $\;$ with $\;\alpha' = \alpha\, e^{-\gamma\phi_0}$, $\;\gamma' = \gamma$

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$$H^2 = \frac{\kappa}{3}\left(\frac{1}{2}\dot\phi^2 - 24 f'\dot\phi H^3 + \rho_{\rm rad}\right)$$

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- We show our results in terms of $\tilde\alpha$ and adopting $\phi_{\rm BBN} = 0$
  [The results are independent of $\phi_{\rm BBN}$]

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- $H_{\rm BBN}$ is obtained from $\phi_{\rm BBN}$ & $\dot{\phi}_{\rm BBN}$ by solving the 1st Eqn. (cubic in $H_{\rm BBN}$)

- Using $\dot{\phi}_{\rm BBN}$, $\phi_{\rm BBN}$, $H_{\rm BBN}$ the Friedmann equations are solved to obtain the solutions ϕ, $\dot{\phi}$, H, at $T > T_{\rm BBN}$

Numerical solutions of Friedmann equations

$\rho_\phi(T_{\rm BBN}) = 0$ $\Rightarrow$ Standard Cosmology in the absence of the GB term (i.e. when $\tilde{\alpha}$ and/or $\gamma = 0$)

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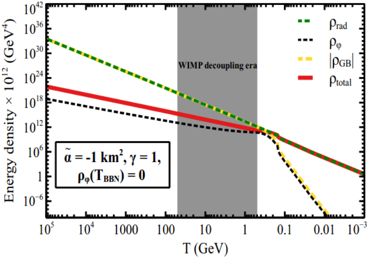
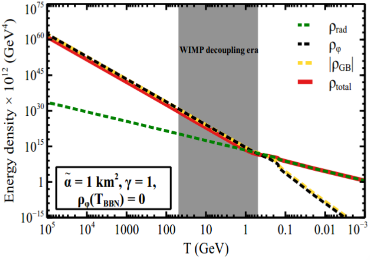
Energy density vs. temperature:

$\tilde{\alpha} = -1\ \mathrm{km}^2, \quad \gamma = 1$



$\tilde{\alpha} = 1\ \mathrm{km}^2, \quad \gamma = 1$



$[H^2 = \frac{\kappa}{3}\rho_{\rm tot}]$

(time $\rightarrow$)

The GB term plays an important role on the scalar field dynamics
$\Rightarrow$ reduces (enhances) the expansion rate H compared to the Standard one

N.B.: only $\rho_{\rm tot}$ and $\rho_{\rm rad}$ represent physical energy densities
while ρ_ϕ and $\rho_{\rm GB}$ are shown for illustrative purposes
($\rho_{\rm GB}$ can be negative and is plotted in absolute value)

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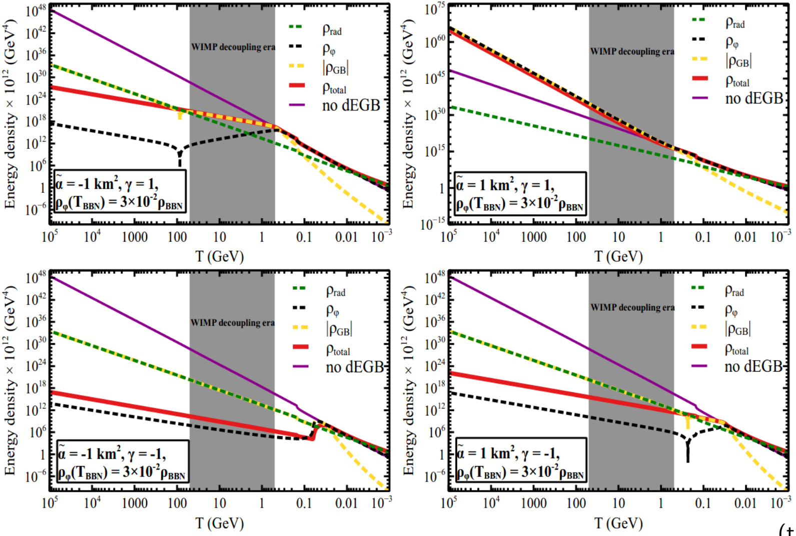
Numerical solutions of Friedmann equations

$\rho_\phi(T_{\rm BBN}) = 3\times10^{-2}\rho_{\rm BBN}$ (max.), $\tilde\alpha = \pm1\ {\rm km}^2$, $\gamma = \pm1$ $\quad [H^2 = \frac{\kappa}{3}\rho_{\rm tot}]$



no dEGB
$\Rightarrow f' = 0$
$(\tilde\alpha\gamma = 0)$

$\Rightarrow$kination
$(\rho \sim T^6)$

(time $\rightarrow$)

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Numerical solutions of Friedmann equations

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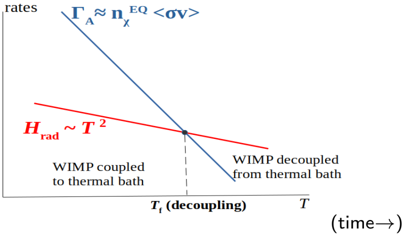
  slows down (speeds up) the evolution of the scalar field

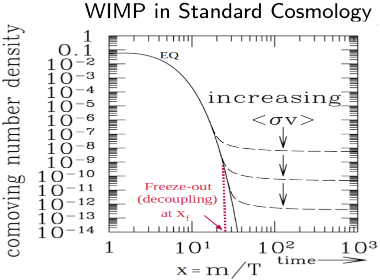
  reduces (enhances) the expansion rate H

- H is larger (smaller) at high T when ρ_ϕ $(= \frac{1}{2}\dot{\phi}^2)$ evolves with T faster (slower) than kination ($\rho_\phi \sim T^6$)

Physics of WIMP DM in dEGB modified cosmology

WIMP thermal freeze-out and relic density





$\Gamma_{\rm A}(= n_\chi \langle \sigma v \rangle)$: Rate of WIMP annihilation to SM particles

$H(T)$: Expansion rate of the Universe

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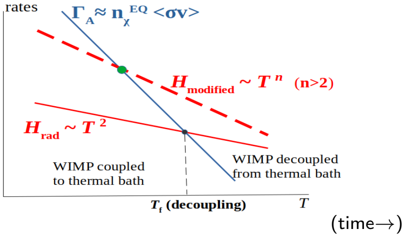

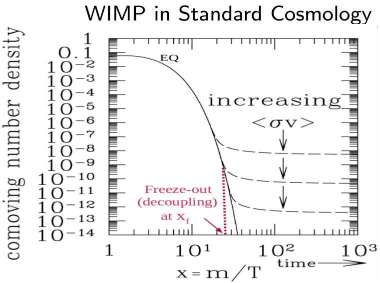

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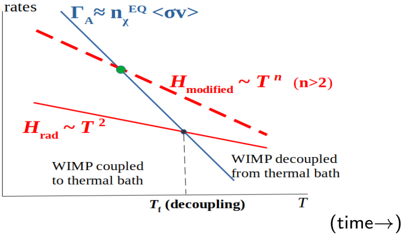
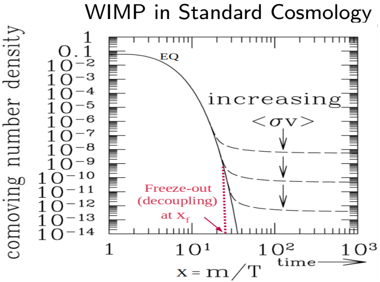


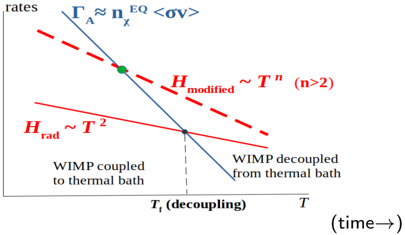


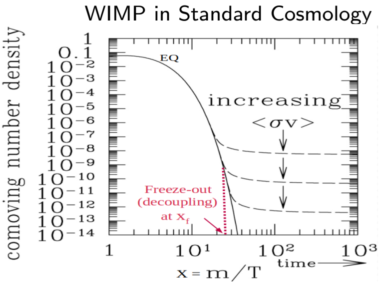
$\Gamma_{\rm A} (= n_\chi \langle \sigma v \rangle)$: Rate of WIMP annihilation to SM particles
$H(T)$: Expansion rate of the Universe

Evolution of WIMP comoving number density ($Y_\chi = n_\chi / s$):

$$\frac{dY_\chi}{dx} = -\frac{\beta\, s}{H(x)\, x} \langle \sigma\, v \rangle \left(Y_\chi^2 - (Y_\chi^{\rm eq})^2 \right), \qquad x = m_\chi / T$$

$m_\chi \equiv$ WIMP mass ; $s \sim g_{*s} T^3$ (entropy density) ; $\beta = (1 + \frac{1}{3} d \ln g_{*s} / d \ln T)$
Equilibrium comoving density: $Y_\chi^{\rm eq} \sim x^{3/2} \exp(-x)$





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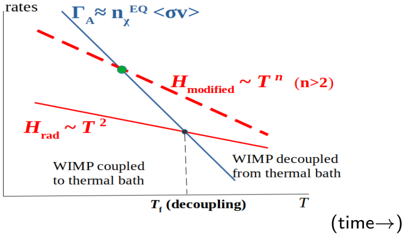
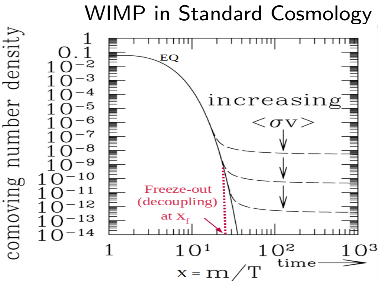
WIMP thermal freeze-out and relic density





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Observation: $\Omega_\chi h^2 \big|_{\rm obs} = 0.12$

WIMP thermal freeze-out and relic density

In Standard Cosmology, $\langle \sigma v \rangle_f \simeq 3 \times 10^{-26}\mathrm{cm}^3\mathrm{s}^{-1}$ gives $\Omega_\chi h^2 = 0.12$

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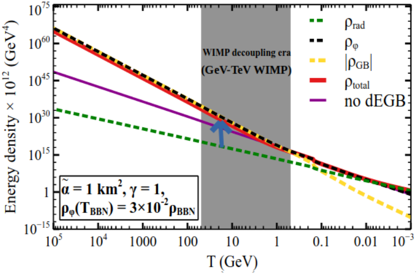
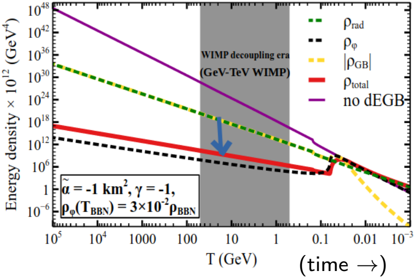
$[H^2 = \frac{\kappa}{3}\rho_{\rm tot}]$



In dEGB cosmology H gets modified during WIMP freeze-out

$\Rightarrow$ affects $\Omega_\chi h^2$

$\Rightarrow$ $\langle\sigma v\rangle_f$ different from $\langle\sigma v\rangle_f^{\rm standard}$ so that $\Omega_\chi h^2 = 0.12$



(time $\rightarrow$)

Enhancement factor for the expansion rate

"Enhancement of the expansion rate": $A(T) \equiv H(T)/H_{\rm rad}(T)$

Standard Cosmology $\Rightarrow A = 1$

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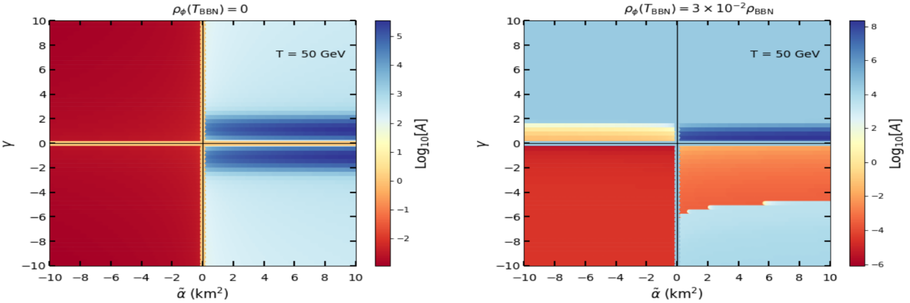
@ $T = 50$ GeV (decoupling temperature of TeV WIMP)



A. Biswas, AK, B-H. Lee, H. Lee, W. Lee, S. Scopel, L. V-Sevilla, L. Yin, [2303.05813]

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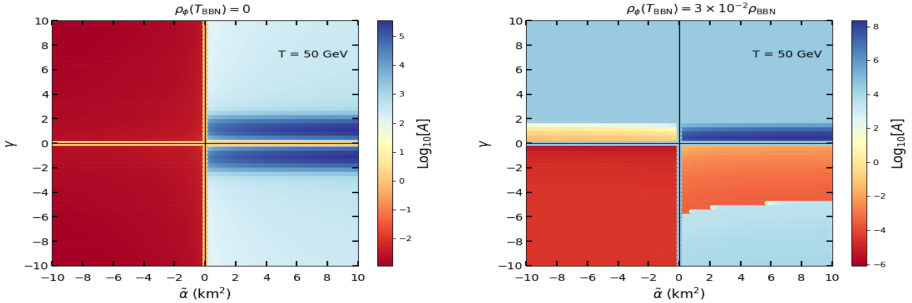

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In order to get $\Omega_\chi h^2 = 0.12$:

$A > 1 \Rightarrow \langle \sigma v \rangle_f > \langle \sigma v \rangle_f^{\rm standard}$ $\qquad$ $A < 1 \Rightarrow \langle \sigma v \rangle_f < \langle \sigma v \rangle_f^{\rm standard}$

WIMP annihilation cross-section $\langle \sigma v \rangle$

- In general $\langle \sigma v \rangle$ can be a function of T

  Expansion of $\langle \sigma v \rangle$ in powers of $v^2/c^2 \ll 1$:

$$\langle \sigma v \rangle \simeq a + b \left(\frac{T}{m_\chi} \right) \qquad [a \rightarrow \text{s-wave}\,; \quad b \rightarrow \text{p-wave}]$$

- We assume s-wave annihilation

  $\Rightarrow \langle \sigma v \rangle$ independent of T

  $\Rightarrow \langle \sigma v \rangle_f = \langle \sigma v \rangle_0$ (today)

Constraints on WIMP annihilation from Indirect Detection searches

- **Indirect Detection:** searches for γ-ray/ν's/e^+/$\bar{p}$ signals produced by WIMP annihilations in the late Universe (e.g., in the Galaxy, in local dwarf galaxies)

  $\chi\chi \to b\bar{b}$, $\tau^+\tau^-, W^+W^-, ... \Rightarrow \gamma$-rays, $e^\pm$, $p(\bar{p})$, $\nu(\bar{\nu})$

  Experiments: Fermi-LAT (γ-rays from dwarf galaxies, Galactic Center) ; AMS (e^+, $\bar{p}$ in cosmic-rays) ; CMB measurements

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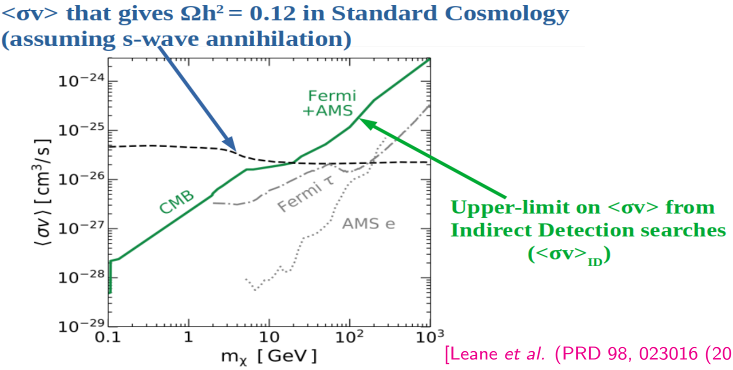
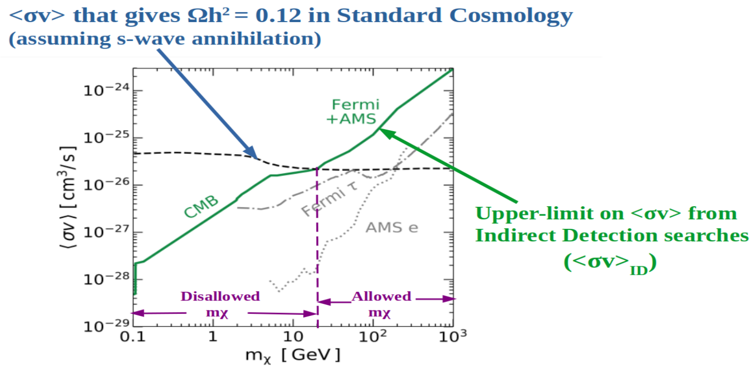


[Leane *et al.* (PRD 98, 023016 (2018))]

- The Indirect Detection upper-limit is obtained combining all possible SM annihilation channels and different existing experimental observations

Constraints on WIMP annihilation from Indirect Detection searches

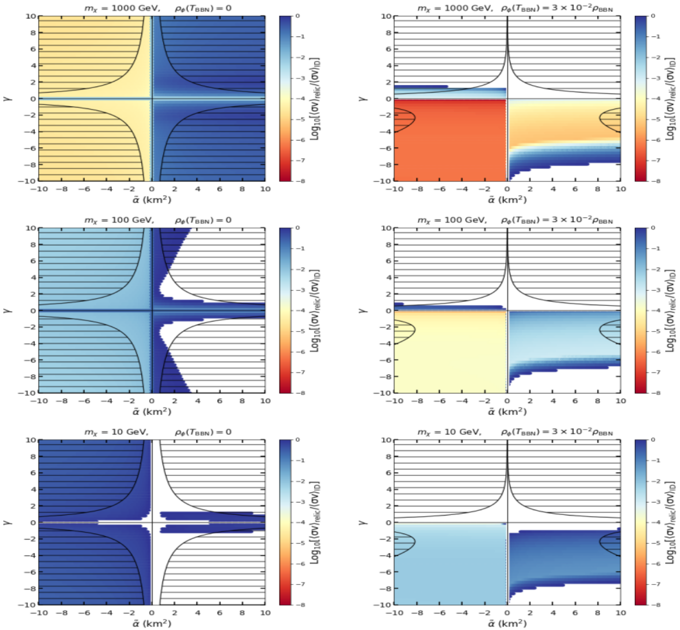


$\langle \sigma v \rangle_{\rm relic}$: gives $\Omega_\chi h^2 = 0.12$
$\langle \sigma v \rangle_{\rm ID}$: upper-limit on $\langle \sigma v \rangle$ from Indirect Detection

$\langle \sigma v \rangle_{\rm relic} / \langle \sigma v \rangle_{\rm ID} > 1 \Rightarrow$ Disallowed WIMPs
$\langle \sigma v \rangle_{\rm relic} / \langle \sigma v \rangle_{\rm ID} \leq 1 \Rightarrow$ Allowed WIMPs

- In Standard Cosmology $m_\chi \lesssim 20$ GeV is disallowed by existing searches
- In modified cosmology $\langle \sigma v \rangle_{\rm relic}$ is different

dEGB parameter space favoured/disfavoured by WIMP searches



left-column:
$\rho_\phi(T_{\mathrm{BBN}}) = 0$

right-column:
$\rho_\phi(T_{\mathrm{BBN}}) = 3 \times 10^{-2} \rho_{\mathrm{BBN}}$
(max.)

Colored regions $\Rightarrow$
$\langle \sigma v \rangle_{\mathrm{relic}} / \langle \sigma v \rangle_{\mathrm{ID}} \leq 1$
(favoured)

White regions $\Rightarrow$
$\langle \sigma v \rangle_{\mathrm{relic}} / \langle \sigma v \rangle_{\mathrm{ID}} > 1$
(disfavoured)

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Constraints on dEGB from Black Hole (BH) & Neutron Star (NS) mergers

- Near a BH or a NS the density of ϕ field in the dEGB scenario is distorted compared to its background value

  $\Rightarrow$ leads to a local departure from standard GR that can modify the Gravitational Wave (GW) signal from BH and NS binary mergers

  $\Rightarrow$ GW data (taken by LIGO-Virgo) from BH and NS binary mergers can constrain dEGB

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$$S = \int_{\mathcal{M}} \sqrt{-g}\; d^4x \left[\frac{R}{2\kappa} - \frac{1}{2}\nabla_\mu \phi \nabla^\mu \phi + \; f(\phi) R^2_{\rm GB} + \mathcal{L}^{\rm rad}_m \right]$$

$$|f'(\phi_{\rm Late})| \lesssim \sqrt{8\pi}\,(1.18)^2\,{\rm km}^2$$

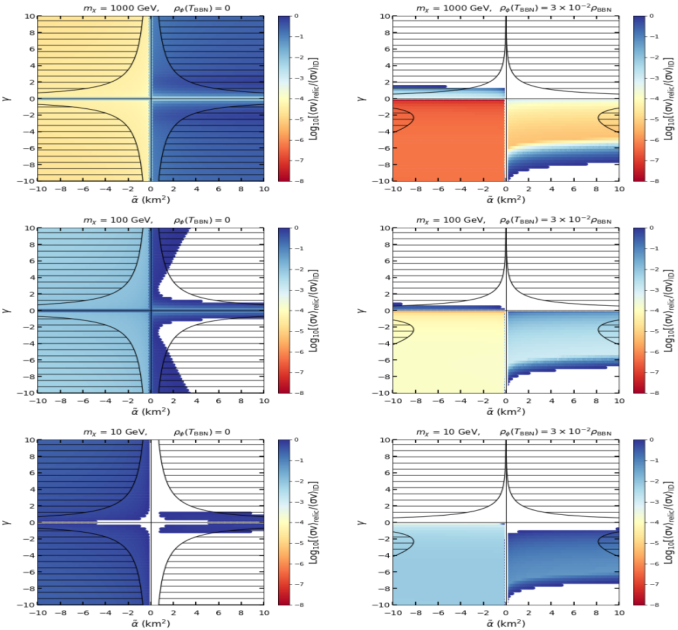
[Lyu *et al.* (PRD 105, 064001 (2022))]

$$f(\phi) = \alpha e^{\gamma\phi}$$

- In our notation:

$$|\tilde{\alpha}\gamma e^{\gamma(\phi_{\rm Late} - \phi_{\rm BBN})}| = |\tilde{\alpha}\gamma e^{\gamma \frac{\dot{\phi}_{\rm BBN}}{H_{\rm BBN}}}| \leq \sqrt{8\pi}\,(1.18)^2\,{\rm km}^2 \qquad \tilde{\alpha} = \alpha e^{\gamma\phi_{\rm BBN}}$$

Complementarity between GW and WIMP search constraints



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Colored regions $\Rightarrow$ $\langle\sigma v\rangle_{\mathrm{relic}}/\langle\sigma v\rangle_{\mathrm{ID}} \leq 1$ (favoured by WIMP)

White regions $\Rightarrow$ $\langle\sigma v\rangle_{\mathrm{relic}}/\langle\sigma v\rangle_{\mathrm{ID}} > 1$ (disfavoured by WIMP)

Hatched regions $\Rightarrow$ GW exclusion

A. Biswas, AK, B-H. Lee, H. Lee, W. Lee, S. Scopel, L. V-Sevilla, L. Yin, [2303.05813]

Summary

- We study cosmologies in a dilatonic Einstein-Gauss-Bonnet (dEGB) scenario [GB term is non-minimally coupled to a scalar field with vanishing potential]

- Standard Cosmology is modified irrespective of the initial conditions on ϕ and $\dot{\phi}$ (even with $\phi_{\rm ini}$ and $\dot{\phi}_{\rm ini} = 0$), if the coupling $f(\phi)$ is not constant

- In dEGB cosmology, WIMP annihilation cross-section $\langle \sigma v \rangle$ required to predict correct relic density is modified compared to its standard value

  $\Rightarrow$ $\langle \sigma v \rangle$ can be larger than the upper-limit obtained from Indirect Detection

  $\Rightarrow$ dEGB parameter space can be favoured/disfavoured by WIMP searches

- WIMP mass $m_\chi \lesssim 20$ GeV is inconsistent with Standard Cosmology

  In dEGB scenario, $m_\chi \lesssim 20$ GeV can be accommodated

- WIMP search constraints on the dEGB parameter space are nicely complementary to late-time constraints from compact binary mergers

  $\Rightarrow$ It could be interesting to use other early Cosmology processes to probe the dEGB scenario

Thank You

Backup slides

Continuity equations

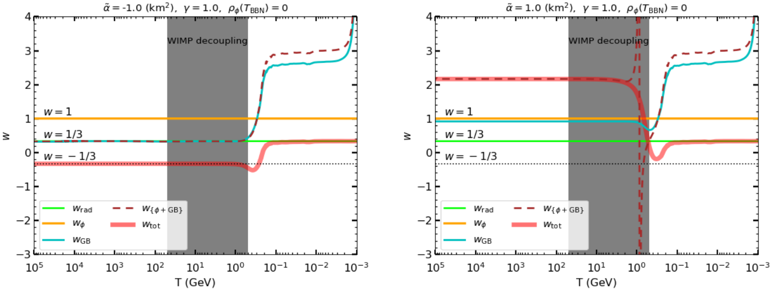
Continuity equation of energy-momentum tensor:

$$\dot{\rho}_{\rm tot} + 3H(\rho_{\rm tot} + p_{\rm tot}) = 0$$

$$\dot{\rho}_{\rm rad} + 3H(\rho_{\rm rad} + p_{\rm rad}) = 0$$

$$\dot{\rho}_{\{\phi+{\rm GB}\}} + 3H\left(\rho_{\{\phi+{\rm GB}\}} + p_{\{\phi+{\rm GB}\}}\right) = 0$$

$\rho_\phi(T_{\rm BBN}) = 0, \quad \tilde{\alpha} = \pm 1 \text{ km}^2, \gamma = 1$



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Equation of states

$\rho_\phi(T_{\rm BBN}) = 3 \times 10^{-2} \rho_{\rm BBN}$ (max.), $\tilde{\alpha} = \pm 1\ {\rm km}^2$, $\gamma = \pm 1$