Thermodynamics of accelerating AdS_4 black holes from the covariant phase space

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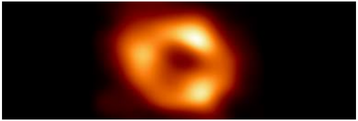
Introduction and motivation

Desire to understand the thermodynamics of black holes:
[Image: Event Horizon Telescope]



- Crucial observations: entropy $\propto$ horizon area **[Bekenstein '72, '73]**, black holes radiate at finite T **[Hawking '74]**

$$S_{\mathsf{BH}} = \frac{A}{4G}$$

- First law for stationary, asymptotically flat black holes **[Bardeen-Carter-Hawking '73]**:

$$\delta M = T\delta S_{\mathsf{BH}} + \Omega_H \delta J + \Phi_e \delta Q_e$$

Our goal in this talk is to understand the first law for accelerating, asymptotically locally AdS_4 black holes

Theory and solutions

- Minimal $d = 4, \mathcal{N} = 2$ gauged supergravity: $(\Lambda = -3, F = dA)$

$$S_{\text{bulk}} = S_{\text{EM}} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left(R + 6 - F^2\right)$$

- Progenitive accelerating solution is the C-Metric **[Kinnersley-Walker '70], [Plebanski-Demianski '76], [Griffiths-Podolsky '05], [Podolsky-Vratny '22]**

$$ds^2 = \frac{1}{H^2} \left\{ -\frac{Q}{r^2} \frac{1}{\kappa^2} dt^2 + \frac{r^2}{Q} dr^2 + \frac{r^2}{P} d\theta^2 + P r^2 K^2 \sin^2\theta d\varphi^2 \right\},$$

where

$$H(r,\theta) = 1 - \alpha r \cos\theta, \quad P(\theta) = 1 - 2\alpha m \cos\theta + \alpha^2 (e^2 + g^2) \cos^2\theta,$$
$$Q(r) = (r^2 - 2mr + e^2 + g^2)(1 - \alpha^2 r^2) + r^4.$$

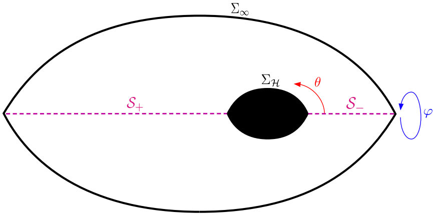
- 6 parameters $(m, e, g, \alpha, K, \kappa)$, note the time scaling $\kappa > 0$
- 2 Killing vectors: $\partial_t, \partial_\varphi \implies$ stationary + axisymmetric
- We focus on the non-rotating case

Accelerating spindles

How is this solution accelerating? Look at metric as $\theta \approx \theta_\pm = \{\pi, 0\}$

$$ds^2_{\theta,\varphi} \simeq \left[\frac{r^2}{PH^2}\right]_{\theta=\theta_\pm} \left[d\theta^2 + (KP_\pm)^2(\theta - \theta_\pm)^2 d\varphi^2\right]$$

$P_+ \neq P_-$ when $\alpha m \neq 0 \implies$ cosmic strings $\{\mathcal{S}_-, \mathcal{S}_+\}$



- Strings have tensions:

$$\mu_\pm = \frac{1}{4G}\left[1 + P_\pm K\right]$$

- $\mu_\pm$ "accelerate" the black hole
- Assumption: Slow acceleration

- If $K = (n_+P_+)^{-1} = (n_-P_-)^{-1}$ with gcd$(n_+, n_-) = 1$ then $\Sigma_{\theta\phi} \cong \mathbb{WCP}^1_{[n_-,n_+]}$, more commonly known as a spindle
- Smooth uplift to $d = 11$ supergravity **[Ferrero et al. '20]**

Covariant phase space

[Wald '93, Iyer-Wald '94] Diffeomorphism covariant theory with d-form Lagrangian, $\mathbf{L}[\psi]$. A variation of $\mathbf{L}$ satisfies

$$\delta\mathbf{L}[\psi] = \delta\psi\,\mathbf{E}[\psi] + \mathsf{d}\boldsymbol{\Theta}[\psi;\delta\psi]$$

- $\boldsymbol{\Theta}$ is the $(d-1)$-form (pre)symplectic potential
$$\boldsymbol{\omega}[\psi;\delta_1\psi,\delta_2\psi] = \delta_2\boldsymbol{\Theta}[\psi;\delta_1\psi] - \delta_1\boldsymbol{\Theta}[\psi;\delta_2\psi]$$
- $\boldsymbol{\omega}$ is the (pre)symplectic current and $d\boldsymbol{\omega} \overset{\mathbf{E}=\delta\mathbf{E}=0}{=} 0$

Consider the case when $\delta_2 = \delta_\xi = \mathcal{L}_\xi$

$$\mathbf{J}[\xi] = \boldsymbol{\Theta}[\psi;\mathcal{L}_\xi\psi] - i_\xi\mathbf{L}$$

- $\mathbf{J}$ is the Noether current and $d\mathbf{J} \overset{\mathbf{E}=\delta\mathbf{E}=0}{=} 0 \implies \mathbf{J} = d\mathbf{Q}[\xi]$ where $\mathbf{Q}[\xi]$ is the Noether charge

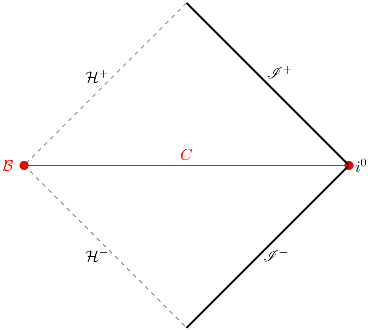
The Hamiltonian charge associated to ξ, H_ξ, on a (1-end) hypersurface C

$$\delta H_\xi[\psi] = \Omega_C[\psi;\delta\psi,\mathcal{L}_\xi\psi] = \int_C \boldsymbol{\omega}[\psi;\delta\psi,\mathcal{L}_\xi\psi] = \int_{\partial C_\infty} \delta\mathbf{Q}[\xi] - i_\xi\boldsymbol{\Theta}$$

Black hole entropy = Noether charge

Initial application by **[Wald '93]** in asymptotically flat space times

- Consider $\xi = \partial_t$ s.t. $\xi|_{\mathcal{B}} = 0$



$$0 = \int_C \boldsymbol{\omega}[\psi; \delta\psi, \mathcal{L}_\xi \psi] = \delta M - \int_{\mathcal{B}} \delta \mathbf{Q}[\xi]$$

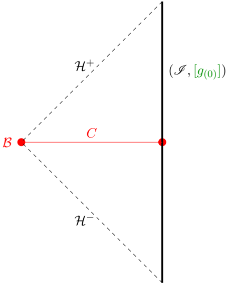
First law! $T\delta S_{\mathrm{BH}} = \int_{\mathcal{B}} \delta \mathbf{Q}[\xi]$ i.e. "black hole entropy is Noether charge" in all diff. covariant theories of gravity

Covariant phase space for AdS

- Asymptotically locally AdS: **[Papadimitriou-Skenderis '05]**
- First law:

$$\delta M = T\delta S + \Phi_e \delta Q_e$$

- Dirichlet boundary conditions: $\delta[g_{(0)}] = 0 \implies \delta S_{\text{ren}} = 0$, $(S_{\text{ren}} = S_{\text{bulk}} + S_{\text{GHY}} + S_{\text{ct}})$



Idea: **[Kim-Kim-Lee-AP '23]** extend these techniques to accelerating solution in order to derive their conserved charges and first law of thermodynamics

- Gain insight into acceleration (α, K) and time scaling κ parameters
- Describe the role of cosmic strings $\mathcal{S}_\pm$ covariantly and geometrically
- Application to spindles **[Ferrero et al. '20, Cassani et al. '21]**

Conformal boundary geometry

Need conformal class $[g_{(0)}]$, i.e. use **[Fefferman-Graham '85]** expansion:

$$ds^2 = \frac{1}{\rho^2}\left(d\rho^2 + \sum_{n=0}^{n=\infty} \rho^n g_{(n)ij} dx^i dx^j\right)$$

$$ds^2_{(0)} = -\frac{\tilde{P}}{\kappa^2} dt^2 + \frac{1}{P\tilde{P}} d\theta^2 + P K^2 \sin^2\theta d\varphi^2, \quad \tilde{P} = 1 - \alpha^2 P \sin^2\theta$$

- Boundary Cotton tensor

$$C^{ij}_{(0)} = \varepsilon^{ikl}_{(0)} \nabla^{(0)}_k \left(R^{(0)j}{}_l - \frac{1}{4}\delta^j_l R^{(0)}\right),$$

- To analyse Dirichlet BCs:

$$\delta\left(\sqrt{-g_{(0)}} C_{(0)}{}^i{}_j\right) = 0 \iff \delta[g_{(0)}] = 0$$

- **[Kim-Kim-Lee-AP '23]** Solution: $\delta m = \delta\alpha = \delta K = \delta e = \delta g = \delta\kappa = 0$
- Trivial! Dirichlet BCs cannot be imposed!

Variational problem and boundary conditions

More general BCs needed: **[Compère-Marolf '08]**

- General variational problem

$$\delta S^g_{\text{ren}} \approx \frac{3}{32\pi G}\int_{\mathscr{I}} d^3x\,\sqrt{-g_{(0)}}\,g^{ij}_{(3)}\delta g_{(0)ij} = 0$$

recall **[de Haro-Skenderis-Solodukhin '00]**: $g_{(3)ij} = 16\pi G\langle T_{ij}\rangle/3,\ \langle T^i_i\rangle = 0$

- Solving the more general problem results in the following constraint **[Kim-Kim-Lee-AP '23]**:

$$-2K\alpha\kappa\Xi\delta\alpha + 2K(\alpha^2\Xi - 1)\delta\kappa + \kappa(2\alpha^2\Xi - 1)\delta K = 0,$$

where

$$\Xi = 1 + \alpha^2(e^2 + g^2).$$

- Previous forms of first law **[Appels-Gregory-Kubiznak '16, '17], [Gregory '17], [Anabalon et al. '18, '18], [Cassani et al. '21]** considered ill-posed variations

Mass charge

- Holographic mass **[Hollands-Ishibashi-Marolf '05]**, **[Papadimitriou-Skenderis '05]**

$$\mathcal{M}_{\text{hol}} = \int_{\Sigma_\infty} d^2x \sqrt{-g_{(0)}} T^t_t = \frac{Km(1-\alpha^2\Xi)}{\kappa G}$$

- Unlike **[Papadimitriou-Skenderis '05]**, this does NOT appear to be equal to the Wald Hamiltonian

$$\delta H_{\partial_t} = \int_{\Sigma_\infty} \delta \mathbf{Q}[\partial_t] - i_{\partial_t}\mathbf{\Theta} = \mathcal{O}(1/\rho) \neq \delta\mathcal{M}_{\text{hol}}$$

- Cosmic strings pierce $\mathscr{I} \implies \partial\Sigma_\infty = S^1_+ \sqcup S^1_-$
- Need a corner improvement **[Compère-Fiorucci-Ruzziconi '20]**

$$\delta H^{\text{ren}}_{\partial_t} = \int_{\Sigma_\infty} \delta \mathbf{Q}[\partial_t] - i_{\partial_t}\mathbf{\Theta} + d i_{\partial_t}\mathbf{\Theta}_{\text{ct}} = \delta\mathcal{M}_{\text{hol}},$$

$\mathbf{\Theta}_{\text{ct}}[h_{ij};\delta h_{ij}]$ is symplectic potential for $\mathbf{L}_{\text{ct}}$.

First law

Idea of derivation:

$$0 = \int_C \boldsymbol{\omega}[\psi; \delta\psi, \mathcal{L}_{\partial_t}\psi] = \int_{\Sigma_\infty} \boldsymbol{k}_{\partial_t} - \int_{\Sigma_\mathcal{H}} \boldsymbol{k}_{\partial_t} + \int_{\mathcal{S}_-} \boldsymbol{k}_{\partial_t} - \int_{\mathcal{S}_+} \boldsymbol{k}_{\partial_t}$$

where

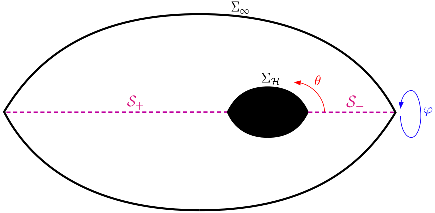
$$\boldsymbol{\omega}[\psi; \delta\psi, \mathcal{L}_{\partial_t}\psi] = d\boldsymbol{k}_\xi, \quad \boldsymbol{k}_\xi = \delta\mathbf{Q}[\xi] - i_\xi\mathbf{\Theta}[\psi; \delta\psi] - di_\xi\mathbf{\Theta}_{\text{ct}}[h_{ij}; \delta h_{ij}]$$

First law: $T\delta S = \delta\mathcal{M} - \Phi_e\delta Q_e - \Phi_m\delta Q_m + \lambda_-\delta\mu_- + \lambda_+\delta\mu_+$

- String integrals define thermodynamic lengths:

$$\lambda_\pm = \frac{r_+}{\kappa(1 \pm \alpha r_+)} \mp \frac{\alpha}{\kappa}P_\pm$$

- Correction to $\lambda_\pm$ from **[Cassani et al. '21]**, due to well-posedness



Spindles and supersymmetry

Constant (t, r) surfaces with the topology of a fixed spindle $\cong \mathbb{WCP}^1_{[n_-,n_+]}$

$$\frac{1}{n_\pm} = 1 - 4G\mu_\pm, \qquad \delta n_\pm = 0 \iff \delta\mu_\pm = 0$$

Spindle first law: $T\delta S = \delta\mathcal{M} - \Phi_e \delta Q_e - \Phi_m \delta Q_m$

We also want to apply the supersymmetry conditions **[Klemm-Nozawa '13]**:

$$g = \alpha m, \qquad g^2 = \Xi(\Xi - 1)$$

and extremality condition **[Ferrero et al. '20]**

$$e = 0, \qquad \text{(when supersymmetric)}$$

- Problem: SUSY solutions are NOT slowly accelerating black holes
- (Partial) solution: Close-to-SUSY, close-to-extremal: $g = \alpha m$, $e = 0$

Close-to-SUSY and close-to-extremal spindle: $T\delta S = \delta\mathcal{M}$

Summary and future directions

Summary:

- R: Application of the covariant phase space in order to construct the conserved charges of slowly accelerating black holes in AdS_4
- R: First law of thermodynamics - covariant (and corrected) form of the terms due to the cosmic string singularities
- R: Application to close-to-supersymmetric and close-to-extremal spindles

Future directions:

- Q: Angular momentum for spinning spindle? Need suitable non-rotating frame at $\mathscr{I}$ - **[Papadimitriou-Skenderis '05], [Compère-Fiorucci-Ruzziconi '19]**
- Q: Rapidly accelerating solutions? i.e. those with acceleration horizons - Would allow for application to supersymmetric and extremal solutions
- Q: Addition of other charges? e.g. NUT parameter **[Godazgar-Guisset '22]**
- Q: All in $d = 4$ - can we use similar corner renormalisations to define charges for spacetimes with cosmic p-brane singularities etc?