Center for Quantum SpaceTime

Workshop on Cosmology and
Quantum Space Time (CQUeST 2023)
: 임채호 교수님 추모 학회

Jeonju, 2023-07-31 ~ 2023-08-04

Maxwell field in the geometry of a charged rotating object

Wonwoo Lee
(CQUeST, Sogang U)

based on

H.-C. Kim, B.-H. Lee, WL, Y. Lee, PRD101, 064067 (2020), e-Print: 1912.09709 [gr-qc]

H.-C. Kim, B.-H. Lee, WL, Y. Lee , e-Print: 2112.04131 [gr-qc], 17th Italian-Korean Symposium on Relativistic Astrophysics

H.-C. Kim, B.-H. Lee, WL, ( ) .... Charged rotating wormhole (charge without charge)

Research Directions in Quantum Field Theory and String Theory

'Research Directions in Quantum Field Theory and String Theory'

Venue : R1029, Sogang University (26th) & Best Western Premier Seoul Garden Hotel (27th)

Date : 26(Wed) ~ 27(Thu) February, 2020

Home Page link : http://www.apctp.org/plan.php/RDQFTST2020

The application deadline :

Notice: Due to the COVID-19 pandemic, the workshop has been canceled!

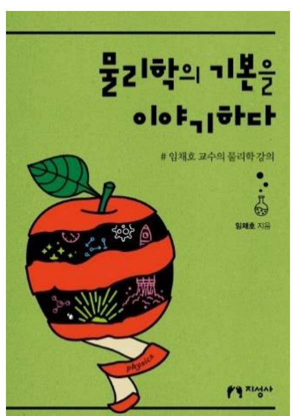
By the way, we plan to have dinner together on Wednesday, February 26.

Group Photo

임채호 교수 퇴임 기념 Mini Workshop
Research Directions in Quantum Field Theory and String Theory
일시/장소 | 2020년 2월 26일 / 서강대학교
주관 및 후원 | 서강대학교, CQUeST, 한국연구재단, APCTP

We have shared the memory.

우주와 원자시대



The plan of this talk

1. Motivations

2. Spherically symmetric static object

3. Charged rotating object by employing the Newman-Janis algorithm

4. Summary and discussions

1. Motivations

Observations of astrophysical objects show that most of them are rotating.

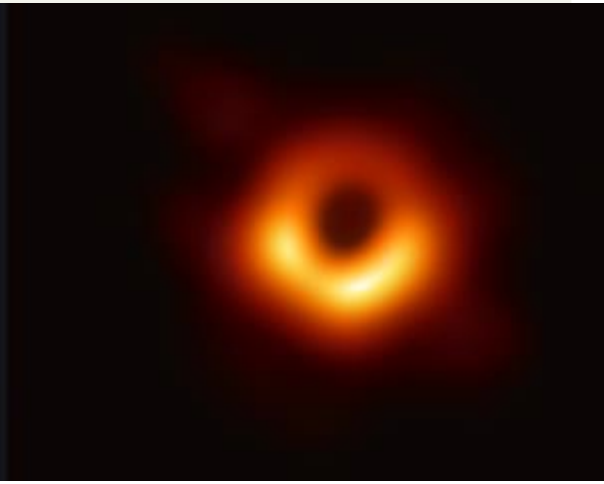
It is believed that the gravitational collapse of a super-massive star forms a rotating black hole eventually.

The energy extraction mechanism from black holes is a promising candidate describing astrophysical events such as active galactic nuclei, gamma-ray bursts, and ultra-high-energy cosmic rays.

⟹ For this reason, the spacetime geometry describing these rotating objects has attracted the attention of researchers for decades.

In addition to this aspect, observationally, the black hole has recently gained the most attention among astrophysical objects, thanks to the observational reports on the shadow of a black hole by the Event Horizon Telescope.

The recent detections of the gravitational waves coming from binary black hole collisions have also opened a new horizon on the studies of astrophysical phenomena and the gravitational theory itself.



Furthermore, actual astrophysical black holes reside in the background of matters or fields.

⟹ Therefore, we need to find a way of describing a realistic black hole that coexists with a matter field!

If a charged rotating black hole has an additional matter field, can it be more efficient when extracting energy out of the rotating black hole?

H.-C. Kim, B.-H. Lee, WL, Y. Lee, PRD101, 064067 (2020), e-Print: 1912.09709 [gr-qc] ⟹ extracted energy = rotating energy + contribution from charge + contribution from additional matter

⟹ Energetics of a rotating black hole is one of the big issues, so B.-H. Lee, S. Lee, WL, and Y.-H. Qi are collaborating on that issue.

Charge without charge

ANNALS OF PHYSICS: 2, 525–603 (1957)

Classical Physics as Geometry

Gravitation, Electromagnetism, Unquantized Charge, and Mass as Properties of Curved Empty Space*

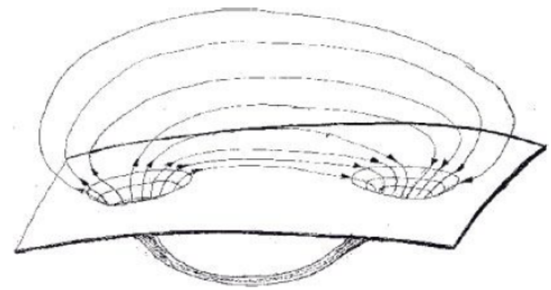
CHARLES W. MISNER† AND JOHN A. WHEELER‡

Lorentz Institute, University of Leiden, Leiden, Netherlands, and Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

If classical physics be regarded as comprising gravitation, source free electromagnetism, unquantized charge, and unquantized mass of concentrations of electromagnetic field energy (geons), then classical physics can be described

GEONS, BLACK HOLES, AND QUANTUM FOAM

W. W. NORTON & COMPANY



The idea of "charge without charge": Electric field lines that seem to begin at one place and end at another may be connected, thanks to a wormhole in "multiply connected" space.

(Drawing by John Wheeler.)

Charged rotating wormhole?

S.-W. Kim and H. Lee, PRD 63, 064014 (2001) [arXiv:gr-qc/0102077].
H. C. Kim and Y. Lee, JCAP 09, 001 (2019) [arXiv:1905.10050 [gr-qc]].

2. Spherically symmetric static object

We consider the action

$$I = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi}(R - F_{\mu\nu}F^{\mu\nu}) + \mathcal{L}_{\rm am}\Big] + I_{\rm b},$$

(G = 1 for simplicity)

where $\mathcal{L}_{\rm am}$ **describes effective anisotropic matter fields.**

We obtain (1) the Einstein equation

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu}\,,$$

(2) The source-free Maxwell equations are given by

$$\nabla_\mu F^{\mu\nu} = \frac{1}{\sqrt{-g}}[\partial_\mu(\sqrt{-g}F^{\mu\nu})] = 0\,.$$

The additional (fluid) matter does not have an independent equation of motion.

We will show the procedure using only metric functions to make it applicable to the general case and later show the specific case.

The static spherically symmetric charged object(black hole and wormhole) solution is given by

$$ds^2 = -f(r)dt^2 + \frac{1}{g(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\psi^2)\,,$$

where we consider the f(r) and g(r) metric functions are different from each other.

비공개

Let me show you the famous static spherically symmetric Reissner-Nordström black hole solution with f(r)=g(r).

One can consider the action

$$I = \int_{\mathcal{M}} \sqrt{-g} d^4x \left[\frac{1}{16\pi} (R - F_{\mu\nu} F^{\mu\nu}) + \mathcal{L}_m \right] + I_b$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$\nabla_\mu F^{\mu\nu} = \frac{1}{\sqrt{-g}} [\partial_\mu (\sqrt{-g} F^{\mu\nu})] = 0$$

In static spherically symmetric spacetime, the stress-energy tensor takes the form

$$T_\mu^\nu = g^{\nu\beta} T_{\mu\beta} = \frac{1}{4\pi} \left[F_{\mu\alpha} F^{\nu\alpha} - \frac{1}{4} \delta_\mu^\nu F_{\alpha\beta} F^{\alpha\beta} \right]$$

$$= \mathrm{diag}(-1, -1, +1, +1) \frac{Q^2}{8\pi r^4} = \mathrm{diag}(-\varepsilon, p_r, p_\theta, p_\phi)$$

$$F_{tr} = \partial_t A_r(r) - \partial_r A_t = \Phi'(r) = -E_r, \qquad (E^r = \frac{Q}{r^2})$$

$$A_\mu = \left(-\frac{Q}{r}, 0, 0, 0 \right)$$

$$p_r(r) = -\varepsilon(r)$$

$$p_\theta(r) = p_\phi(r) = \varepsilon(r)$$

The black hole solution is

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega_2^2$$

3. Charged rotating object

How can we obtain the solution describing the rotating black hole?

1) **Solve the Ernst equation** [Ernst, PR 167, 1175 (1968)]

2) **Employ the Newman-Janis algorithm**

[Newman & Janis, JMP 6, 915 (1965)]

3) ....

One should obtain solutions of the Maxwell field and the additional matter in the geometry of the charged rotating object.

It is not easy. Anyway, you'll see a new world you've never get the experience before.

To get Rotating object

One can take three steps!

Consider null coordinate of a static spherically object

⬇

Employ Newman-Janis algorithm

⬇

Then one can obtain the null coordinate of the charged rotating object.

A simple diagram of Newman-Janis algorithm

Schwarzschild (1916) → **Kerr (1963)**

Newman and Janis, JMP 6, 915 (1965)

Reissner – Nordström (1916, 1918) → **Kerr-Newman (1965)**

Newman, Chinnapared, Exton, Prakash and Torrence, JMP 6, 918 (1965)

Black hole with anisotropic matter

I. Cho and H. C. Kim, Chin. Phys. C 43, no. 2, 025101 (2019)
M. Visser, Class.Quant.Grav. 37 (2020) 4, 045001

Black hole with am (2019)

Rotating BH with am (2020)

‘(Charged) rotating black holes with an anisotropic matter field’
H.-C. Kim, B.-H. Lee, WL, Y. Lee, PRD101, 064067 (2020), e-Print: 1912.09709 [gr-qc]
H.-C. Kim, B.-H. Lee, WL, Y. Lee , e-Print: 2112.04131 [gr-qc], 17th Italian-Korean Symposium on Relativistic Astrophysics

비공개

비공개

비공개

For the black hole, using $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$, the non-vanishing components of the Einstein tensor are obtained and given by

$$
\begin{aligned}
G_{tt} &= \frac{2[r^4 - 2r^3 b + a^2 r^2 - a^4 \sin^2\theta\cos^2\theta]b'}{\rho^6} - \frac{ra^2\sin^2\theta b''}{\rho^4}, \\
G_{rr} &= -\frac{2r^2 b'}{\triangle\ \rho^2}, \qquad G_{\theta\theta} = -\frac{2a^2\cos^2\theta b'}{\rho^2} - rb'', \\
G_{t\phi} &= \frac{2a\sin^2\theta[(r^2+a^2)(a^2\cos^2\theta - r^2) + 2r^3 b]b'}{\rho^6} + \frac{ra\sin^2\theta(r^2+a^2)b''}{\rho^4} \\
G_{\phi\phi} &= -\frac{a^2\sin^2\theta[(r^2+a^2)(a^2+(2r^2+a^2)\cos 2\theta) + 4r^3\sin^2\theta b]b'}{\rho^6} \\
&- \frac{r\sin^2\theta(r^2+a^2)^2 b''}{\rho^4},
\end{aligned}
$$

where a prime denotes differentiation with respect to r.

$$
\begin{aligned}
2b &= 2M - Q^2 r^{-1} + Kr^{1-2w}, \quad 2b' = Q^2 r^{-2} + (1-2w)Kr^{-2w} \\
2b'' &= -2Q^2 r^{-3} - 2(1-2w)wKr^{-2w-1}.
\end{aligned}
$$

Let us consider physical quantities in an orthonormal frame, $(e_{\hat{t}}, e_{\hat{r}}, e_{\hat{\theta}}, e_{\hat{\phi}})$, introduced by Carter(1968), in which the stress-energy tensor for the anisotropic matter field is diagonal,

$$e^{\mu}_{\hat{t}} = \frac{(r^2+a^2,0,0,a)}{\rho\sqrt{\Delta}}, \qquad e^{\mu}_{\hat{r}} = \frac{\sqrt{\Delta}(0,1,0,0)}{\rho}$$
$$e^{\mu}_{\hat{\theta}} = \frac{(0,0,1,0)}{\rho}, \qquad e^{\mu}_{\hat{\phi}} = -\frac{(a\sin^2\theta,0,0,1)}{\rho\sin\theta}$$

The components of the energy-momentum tensor are expressed in terms of $G_{\mu\nu}$ as

$$8\pi\varepsilon = e^{\mu}_{\hat{t}} e^{\nu}_{\hat{t}} G_{\mu\nu}, \quad 8\pi p_{\hat{r}} = e^{\mu}_{\hat{r}} e^{\nu}_{\hat{r}} G_{\mu\nu}, \; 8\pi p_{\hat{\theta}} = e^{\mu}_{\hat{\theta}} e^{\nu}_{\hat{\theta}} G_{\mu\nu}, \; 8\pi p_{\hat{\phi}} = e^{\mu}_{\hat{\phi}} e^{\nu}_{\hat{\phi}} G_{\mu\nu}.$$

We obtained with ($r_o^{2w} = (1-2w)K$)

$$\varepsilon = \varepsilon_e + \varepsilon_{am} = \frac{Q^2}{8\pi\rho^4} + \frac{r_o^{2w} r^{2(1-w)}}{8\pi\rho^4}, \quad p_{\hat{r}} = -\varepsilon,$$
$$p_{\hat{\theta}} = p_{\hat{\phi}} = \frac{Q^2}{8\pi\rho^4} + [\rho^2 w - a^2\cos^2\theta]\frac{\varepsilon_{am}}{r^2},$$

For Kerr-Newman one

$$\varepsilon = \frac{Q^2}{8\pi\rho^4}, \quad p_{\hat{r}} = (-\varepsilon) = -\frac{Q^2}{8\pi\rho^4},$$
$$p_{\hat{\theta}} = (p_{\hat{\phi}}) = \frac{Q^2}{8\pi\rho^4}$$

K=0

비공개

Maxwell tensor

For the non-rotating charged black hole, the gauge field has only the component $A_0 = -Q/r$. While for the rotating charged black hole, how can we obtain the gauge field (or Maxwell field)?

One could consider this as a general relativistic version of the problem for finding the potential of a rotating charge.

When the electric charge rotates, the A_φ component is non-vanishing. Thus, $A_\mu = (A_0, 0, 0, A_\varphi)$.

Three complex invariants (Maxwell Newman-Penrose scalar) can be defined by the electromagnetic field tensor as follows:

$$\Phi_0 \equiv F_{\mu\nu} l^\mu m^\nu = \frac{1}{4}\mathscr{F}_{\mu\nu} V^{\mu\nu},$$

$$\Phi_1 \equiv \frac{1}{2} F_{\mu\nu}(l^\mu n^\nu + \bar{m}^\mu m^\nu) = -\frac{1}{8}\mathscr{F}_{\mu\nu} W^{\mu\nu},$$

$$\Phi_2 \equiv F_{\mu\nu} \bar{m}^\mu n^\nu = \frac{1}{4}\mathscr{F}_{\mu\nu} U^{\mu\nu},$$

where the six real components of $F_{\mu\nu}$ are replaced by the three complex Φ's and

$$V_{\mu\nu} = l_\mu m_\nu - m_\mu l_\nu,\ U_{\mu\nu} = -n_\mu \bar{m}_\nu + \bar{m}_\mu n_\nu,\ W_{\mu\nu} = -l_\mu n_\nu + n_\mu l_\nu + m_\mu \bar{m}_\nu - \bar{m}_\mu m_\nu.$$

The Maxwell tensor could be recovered from three complex invariants as

$$F_{\mu\nu} = \Phi_0 U_{\mu\nu} + \Phi_1 W_{\mu\nu} + \Phi_2 V_{\mu\nu} + (\bar{\Phi}_0 \bar{U}_{\mu\nu} + \bar{\Phi}_1 \bar{W}_{\mu\nu} + \bar{\Phi}_2 \bar{V}_{\mu\nu}).$$

비공개

비공개

In the asymptotic rest frame with r≫a for charged rotating black hole with additional fluid matter, the electric field through $F^{\hat{a}\hat{b}} = e^{\hat{a}}_{\mu} e^{\hat{b}}_{\nu} F^{\mu\nu}$ **takes the form**

$$E^{\hat{r}} = \frac{Q}{r^2} + \mathscr{O}\left(\frac{1}{r^3}\right), \quad E^{\hat{\theta}} = \mathscr{O}\left(\frac{1}{r^4}\right)$$

while the magnetic field takes the form

$$B^{\hat{r}} = -2\frac{Qa}{r^3}\cos\theta + \mathscr{O}\left(\frac{1}{r^4}\right), \quad B^{\hat{\theta}} = -\frac{Qa}{r^3}\sin\theta + \mathscr{O}\left(\frac{1}{r^4}\right)$$

This is a dipole magnetic field and $\mathscr{M}$ **= Qa corresponds to the magnetic moment of the black hole.**

H.-C. Kim, B.-H. Lee, WL, Y. Lee , e-Print: 2112.04131 [gr-qc], 17th Italian-Korean Symposium on Relativistic Astrophysics

비공개

4. Summary and discussions

We have presented a family of new charged rotating object solutions to Einstein's equations and Maxwell equations with an anisotropic matter field. The rotating geometry was obtained from the known static solution by employing the Newman-Janis algorithm.

To get the solutions, I modified the function Phi_1, which was introduced by Janis & Newman.

Thank you for your attention!