Hairy Black Holes by Spontaneous Symmetry Breaking[1] [2]

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“CQUeST Workshop 2023"

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[1] 2205.00907, Phys.Rev.D 106 (2022) 8, 084024

[2] 2305.19814

Test General Relativity

Detection of gravitational waves

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- General relativity alone struggles to explain the presence of dark matter, dark energy, and inflationary expansion.
- To improve general relativity, many alternative theories of gravity have been proposed.
- In this talk, I will consider Einstein-Scalar-Gauss-Bonnet Theory.

Einstein-Scalar-Gauss-Bonnet Theory

Let us consider ESGB theory

$$S = \int \mathrm{d}^4\mathrm{x}\sqrt{-\mathrm{g}}\left[\frac{\mathrm{R}}{2\kappa^2} - \frac{1}{2}\nabla_\alpha\varphi\nabla^\alpha\varphi + \mathrm{f}(\varphi)\mathcal{G}\right], \quad (1)$$

$$\mathcal{G} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \quad (2)$$

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- yields 2^{nd} order differential equation (topological in $4-$dim.) : Hondenski theory
- **evasion of no-hair theorem**[3] [4] [5]

If $f(\varphi_\infty) = 0$ or $\varphi_1 = 0$, the no-hair theorem is evaded

$$\text{when} \qquad f(\varphi) > 0$$

If $f(\varphi_\infty) \neq 0$ and $\varphi_1 \neq 0$, the no-hair theorem fails. Solutions might exist

$$\text{when} \qquad f(\varphi) > 0 \;\; \text{and} \;\; f(\varphi) < 0$$

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Hairy Black Holes for in ESGB

- Our metric ansatz

$$ds^2 = -A(r)dt^2 + \frac{1}{B(r)}dr^2 + r^2 d\Omega_2,$$

- Boundary conditions

$$\begin{aligned} &\text{Near horizon}: \quad A(r) \sim A_h \epsilon, \qquad B(r) \sim B_h \epsilon, \qquad \varphi(r) \sim \varphi_h + \varphi_{h,1}\epsilon \\ &\text{Near infinity}: \quad A(r) \sim 1, \qquad B(r) \sim 1, \qquad \varphi(r) \sim \varphi_\infty \end{aligned}$$

where

$$\varphi'(r_h) = \varphi_{h,1} = -\frac{r_h}{4\dot{f}_h}\left(1 \mp \sqrt{1 - \frac{96}{r_h^4}\dot{f}_h^2}\right), \qquad B_h = \frac{2}{r_h}\left(1 \pm \sqrt{1 - \frac{96}{r_h^4}\dot{f}_h^2}\right)^{-1}$$

- To avoid $\varphi''(r_h)$ being divergent the inside of the root should not be zero, namely

$$\dot{f}_h^2 < \frac{r_h^4}{96}.$$

Hairy Black Holes for $f = \alpha e^{\gamma\varphi}$ in ESGB

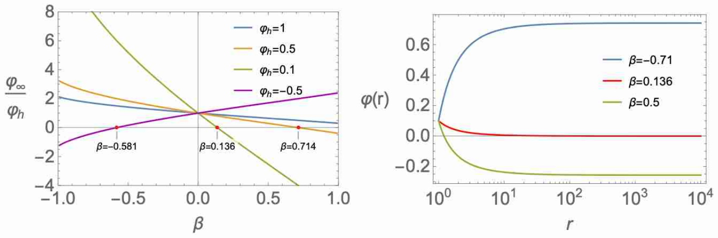

FIG. 2. For $f = \alpha e^{\gamma\varphi}$ (left) φ_∞/φ_h vs β and (right) $\varphi(r)$ for different values of β fixing $\varphi_h = 0.1$

Hairy Black Holes for $f = \alpha\varphi^2$ in ESGB

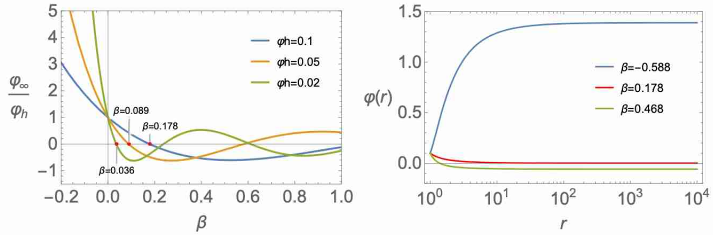

FIG. 4. For $f = \alpha\varphi^2$, (left) φ_∞/φ_h vs β and (right) $\varphi(r)$ for different values of β fixing $\varphi_h = 0.1$

Formation of Hairy Black Holes[6] [7] [8]

“How hairy black holes acquire their hair from non-hairy ones?"

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Formation of Hairy Black Holes[6] [7] [8]

“How hairy black holes acquire their hair from non-hairy ones?"

Here, we attempt to construct a generating mechanism for hairy black holes associated with the symmetry of the theory.

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Spontaneous Symmetry Breaking (SSB)

"The underlying theory has a symmetry while the underlying vacuum state does not share the same symmetry with the theory."

- Global symmetry : $\varphi(r) \to \varphi(r)e^{i\chi}$ for global $U(1)$

$$\mathcal{L} = \nabla^\mu \varphi^* \nabla_\mu \varphi - V(\varphi), \qquad V(\varphi) = -\mu^2 \varphi^* \varphi + \lambda(\varphi^* \varphi)^2$$

- Gauge symmetry : $\varphi(r) \to \varphi(r)e^{i\chi(r)}$ for local $U(1)$

$$\mathcal{L} = D^\mu \varphi^* D_\mu \varphi - V(\varphi) - \frac{1}{4}F^2, \qquad V(\varphi) = -\mu^2 \varphi^* \varphi + \lambda(\varphi^* \varphi)^2$$

: Ginzburg–Landau theory, superconductivity

- Higgs mechanism in standard model : responsible for giving mass to elementary particles in standard model.

Hairy Black Holes by SSB in ESGB with global $U(1)$

- We are interested in the situation that

  “Scalar fields are about to grow from non-hairy black holes. Finally the non-hairy ones evolve to hairy black holes."

ESGB theory with global $U(1)$: $\alpha < 0$

The following Lagrangian respects the global $U(1)$ symmetry : $\varphi(r) \rightarrow e^{i\chi}\varphi(r)$

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \nabla_\alpha\varphi^*\nabla^\alpha\varphi + f(\varphi)\mathcal{G}\right], \tag{3}$$

$$\mathcal{L}_\varphi = -\nabla_\alpha\varphi^*\nabla^\alpha\varphi + f(\varphi)\mathcal{G} = T - V, \qquad V = -f(\varphi)\mathcal{G}, \tag{4}$$

$$f(\varphi) = \alpha\,\varphi^*(r)\varphi(r) - \lambda\left(\varphi^*(r)\varphi(r)\right)^2, \qquad (\lambda > 0) \tag{5}$$

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- “$V = -f(\varphi)\mathcal{G}$" as an “interacting potential" :
  - effective near the black hole horizon
  - not effective at infinity ($V \to 0$ as $r \to \infty$, since $\mathcal{G} \to 0$ as $r \to \infty$)

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- physical fields are the excitation above the vacuum

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- In the presence of symmetry, the conserved current is defined as

$$\partial_\alpha J^\alpha = 0, \qquad J_\alpha = i\,g\,(\varphi^*\partial_\alpha\varphi - \varphi\partial_\alpha\varphi^*).$$

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- The flux for a timelike hypersurface near the horizon is given by

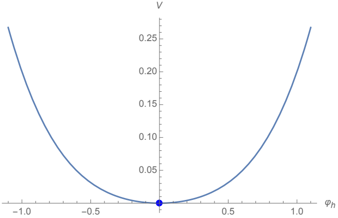
$$\int_\Sigma J_\alpha n^\alpha\sqrt{-h}\,\mathrm{d}^3\mathrm{y} = \int_\Sigma \left[\mathrm{g}(\varphi_2\partial_\mathrm{r}\varphi_1 - \varphi_1\partial_\mathrm{r}\varphi_2)\right]\left[\sqrt{\mathrm{A(r)B(r)}}\,\mathrm{r}^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi\,\mathrm{dt}\right] = 0$$

where

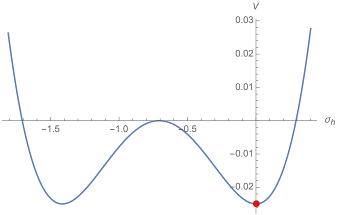
$$\varphi(r) = \frac{1}{\sqrt{2}}\left(\varphi_1(r) + i\,\varphi_2(r)\right)$$

ESGB theory with global $U(1)$

"When $\alpha < 0$, vacuum is symmetric under global $U(1)$"



"When $\alpha > 0$, vacuum is changed and not symmetric under global $U(1)$"



EsGB theory with global $U(1)$: $\alpha > 0$

When $\alpha > 0$: V forms degenerate vacuums and the stable minima are described by

$$\langle\varphi\rangle = v\, e^{i\beta}, \qquad v = \sqrt{\frac{\alpha}{2\,\lambda}} \tag{6}$$

We expand a field around a ground state v by reparameterizing it as follows

$$\varphi(r) = \left(v + \frac{\sigma(r)}{\sqrt{2}}\right) e^{i\theta(r)}. \tag{7}$$

the new Lagrangian is written as

$$\mathcal{L}_\varphi = -\frac{1}{2}\nabla_\alpha\sigma(r)\nabla^\alpha\sigma(r) - \left(v + \frac{\sigma(r)}{\sqrt{2}}\right)^2 \nabla_\alpha\theta(r)\nabla^\alpha\theta(r) + f(\sigma)\,\mathcal{G} \tag{8}$$

where

$$f(\sigma) = -\alpha\,\sigma(r)^2 - \sqrt{\alpha\,\lambda}\,\sigma(r)^3 - \frac{\lambda}{4}\,\sigma(r)^4. \tag{9}$$

ESGB theory with global $U(1)$

Field $\theta(r)$ is decoupled from the system, and the solution for $\theta'(r)$ reads

$$\theta'(r) = \frac{c_2}{4r^2\sqrt{A(r)B(r)}}\left(v + \frac{\sigma(r)}{\sqrt{2}}\right)^{-2}$$

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The flux for a timelike hypersurface

$$\int_\Sigma J_\alpha n^\alpha \sqrt{-h}\, \mathrm{d}^3\mathrm{y} = \int_\Sigma \left[-2\mathrm{g}\left(\mathrm{v} + \frac{\sigma(\mathrm{r})}{\sqrt{2}}\right)^2 \theta'(\mathrm{r})\right]\left[\sqrt{\mathrm{A(r)B(r)}}\,\mathrm{r}^2 \sin\theta \mathrm{d}\theta \mathrm{d}\phi \mathrm{dt}\right] = -8\pi \mathrm{g c_2}$$

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“The hairy black holes in this theory can only possess trivial Goldstone bosons hair."

symmetric and *symmetry-broken phase*

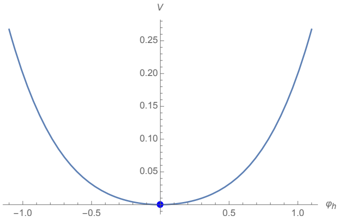

Figure: *symmetric* (left) and *symmetry-broken phase* (right)

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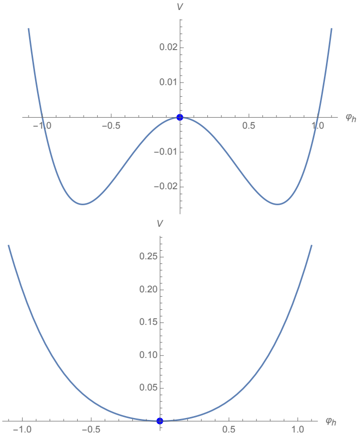

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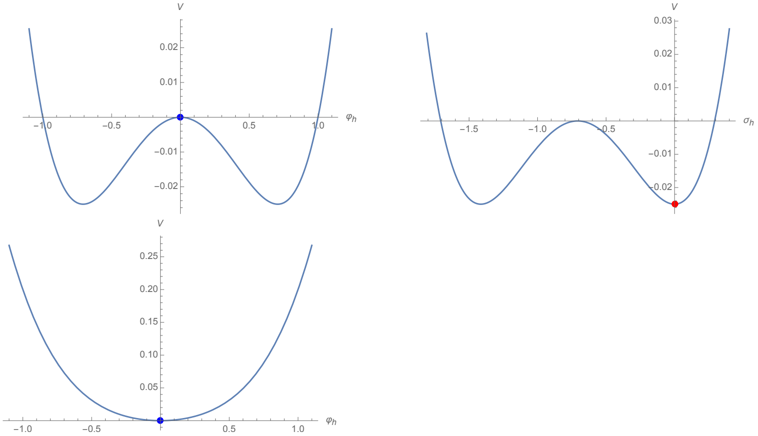


Figure: *symmetric* (left) and *symmetry-broken phase* (right)

scalar field perturbation in EsGB : instability

The linearized equation then becomes

$$\Big(\nabla_\alpha \nabla^\alpha + f_{\varphi^*\varphi}\,\mathcal{G}\Big)\delta\varphi(r) = 0, \qquad m_{\rm eff}^2 = -\,f_{\varphi^*\varphi}\,\mathcal{G}$$

$$\delta\varphi(t,r,\theta,\phi) = \sum_{l,m} \frac{\Phi(r)Y_{lm}(\theta,\phi)}{r} e^{-i\omega t}$$

the perturbation equation is written as

$$\Phi''(r_*) - (V_{\rm eff} - \omega^2)\Phi(r_*) = 0, \qquad {\rm dr}_* = \frac{1}{\sqrt{\rm AB}}{\rm dr},$$

$$V_{\rm eff}(r) = \frac{l(l+1)A}{r^2} + \frac{1}{2\,r}\Big(A'B + AB'\Big) - f_{\varphi^*\varphi}\,A\,\mathcal{G},$$

The system becomes unstable if the following condition is met

$$\int_{r_h}^{\infty} dr \frac{1}{\sqrt{AB}} V_{\rm eff}(r) < 0.$$

Instability for Schwarzschild black hole

$$\mathrm{ds}^2 = -\mathrm{A(r)dt}^2 + \frac{1}{\mathrm{B(r)}}\mathrm{dr}^2 + \mathrm{r}^2(\mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\phi^2), \qquad \mathrm{A} = \mathrm{B} = 1 - \frac{2\mathrm{M}}{\mathrm{r}} \tag{10}$$

Instability check yields

$$\alpha > \frac{5}{6}\big(2\,l(l+1)+1\big)M^2 = \alpha_{\mathrm{Sch.}} \tag{11}$$

When $M = \frac{1}{2}$ and $l = 0$,

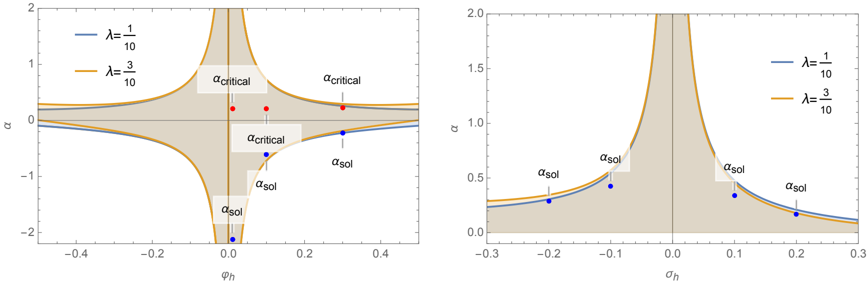
$$\alpha_{\mathrm{Sch.}} = \frac{5}{24} \approx 0.2083 \tag{12}$$

“Schwarzschild black holes are unstable if $\alpha > \alpha_{\mathrm{Sch.}}$"

Phase space

The regularity condition yields

$$1 - 96(\dot{f}_h)^2 > 0$$



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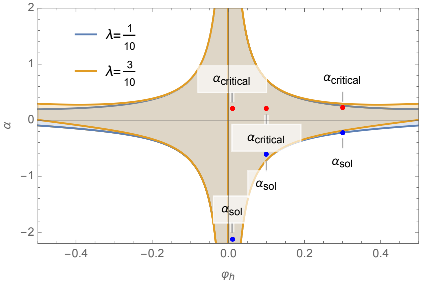


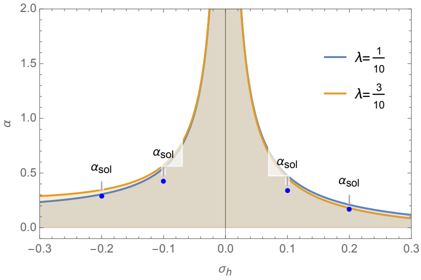
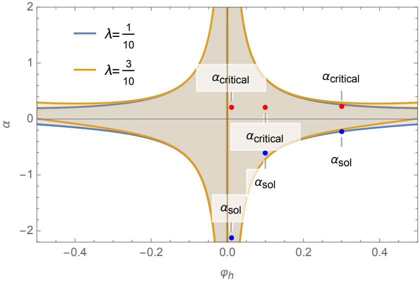


Figure: Phase space for symmetric phase (left) and symmetry-broken phase (right)

$$\text{“}\,\alpha_{\text{critical}} \approx \alpha_{\text{Sch.}}\,\text{”}$$

Hairy black holes in *symmetric phase*



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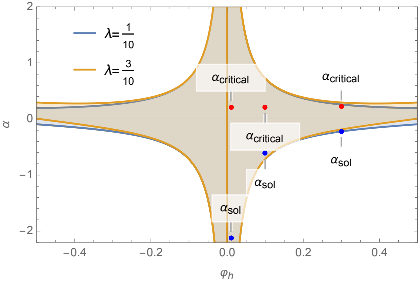

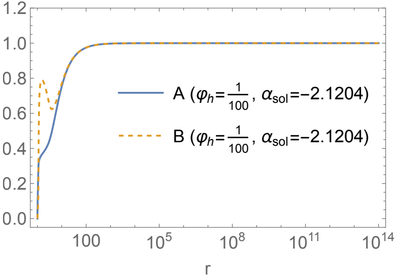

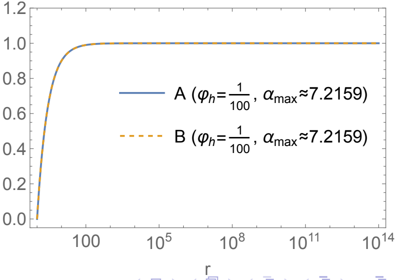

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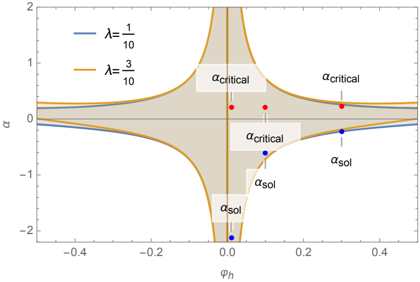

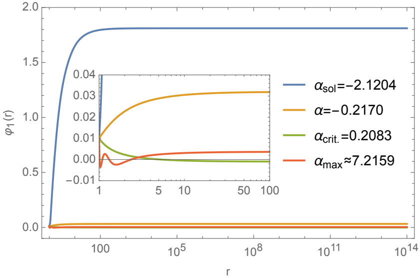

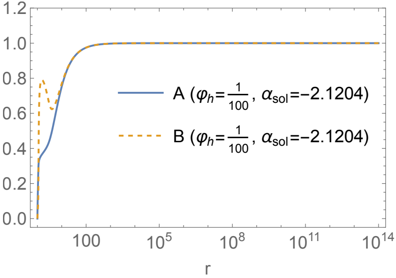

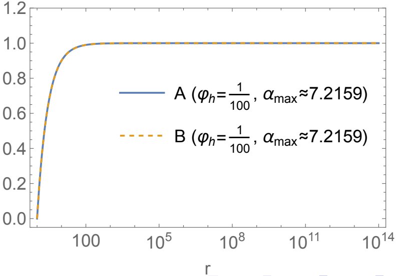

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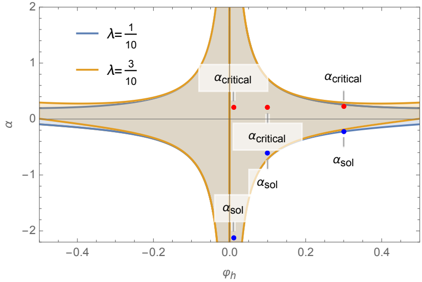
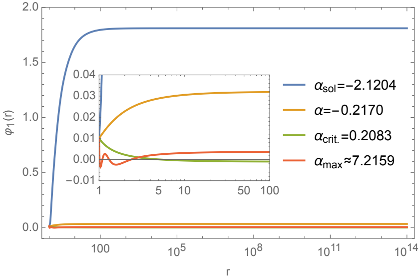
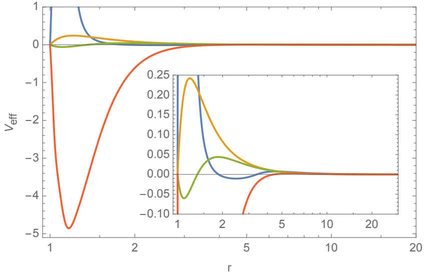
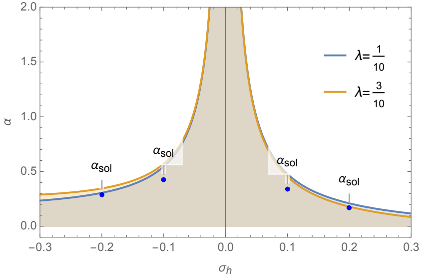






Hairy black holes in *symmetry broken phase*



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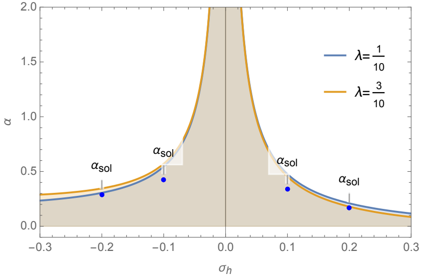

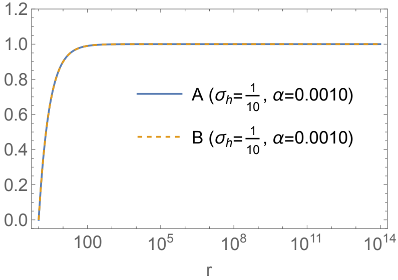

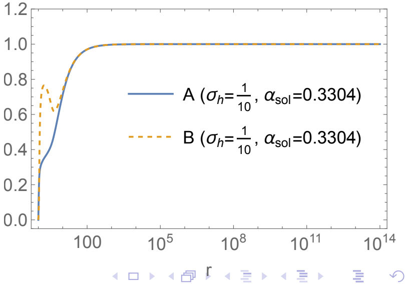

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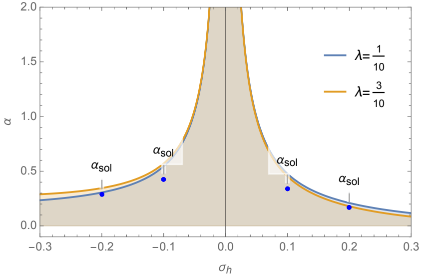

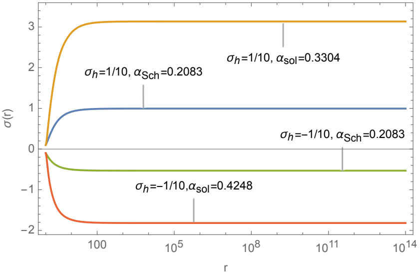

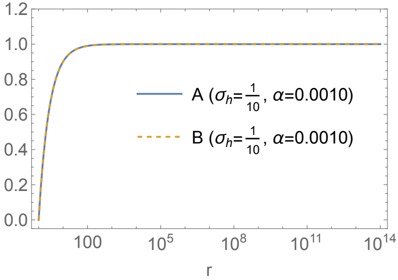

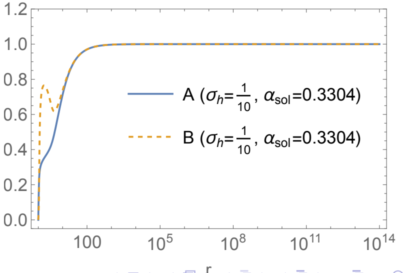

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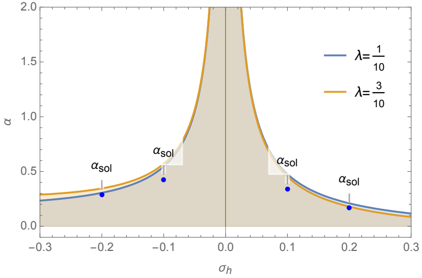

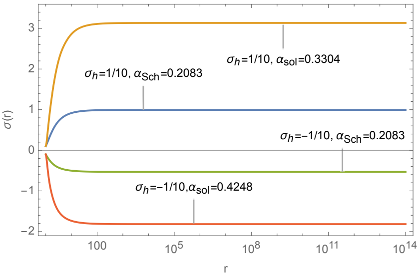

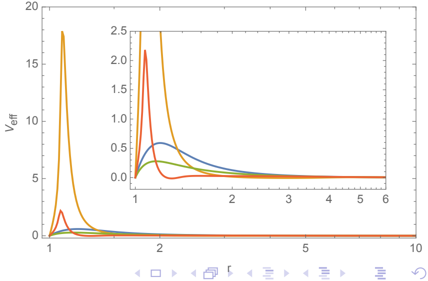

SSB in ESGB theory with local $U(1)$

This action is invariant under local $U(1)$ symmetry : $\varphi \to \varphi\, e^{i\chi(r)}$

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2\,\kappa^2}R - \frac{1}{4}F^2 - D_\alpha\varphi^* D^\alpha\varphi + f(\varphi^*,\varphi)\,\mathcal{G}\right], \tag{13}$$

where $F = dP$ and $D_\alpha = \nabla_\alpha - iqP_\alpha$.

- When $q = 0$,

$$A(r) \sim 1 + \frac{A_1}{r} + \frac{P_1^2}{4\,r^2} - \frac{A_1\varphi_1^2}{12\,r^3} + \cdots$$
$$B(r) \sim 1 + \frac{A_1}{r} + \frac{P_1^2 + 2\,\varphi_1^2}{4\,r^2} - \frac{A_1\varphi_1^2}{4\,r^3} + \cdots$$
$$P(r) \sim P_\infty + \frac{P_1}{r} - \frac{P_1\,\varphi_1^2}{12\,r^3} + \cdots$$
$$\varphi(r) \sim \varphi_\infty + \frac{\varphi_1}{r} - \frac{A_1\,\varphi_1}{2\,r^2} - \frac{\varphi_1\left(-4\,A_1^2 + P_1^2 + \varphi_1^2\right)}{12\,r^3} + \cdots$$

- When $q \neq 0$, the asymptotic expansions of the gauge field and scalar fields yield $P_\infty = P_1 = 0$ or $\varphi_\infty = \varphi_1 = 0$.
- This may imply that either the gauge field or the scalar field falls off faster than $1/r^n$ at infinity, or that there are no electrically-charged scalar hairy black hole solutions.

Hairy Black Holes in EsGB theory with local $U(1)$

Numerical solutions for $q = 0$ case

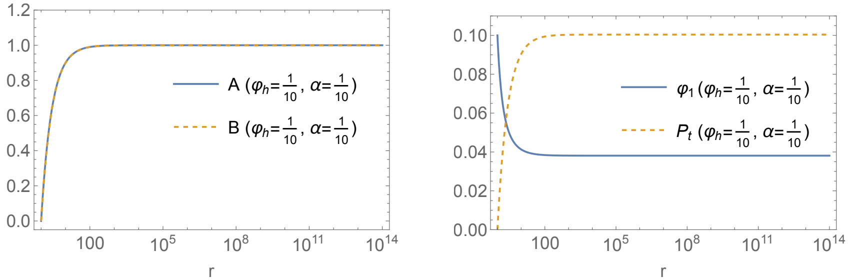

Figure: Hairy black hole solutions when $q = 0$

* We were not able to find hairy black hole solutions with charged scalar hairs ($q \neq 0$).

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- In the symmetry-broken phase, the hairy black hole solutions are all stable against the scalar field perturbation.
- Goldstone bosons are decoupled from other equations and only trivial solutions are accepted.
- Thus, we expect that the Schwarzschild black holes in the unstable range of α $(\alpha > \alpha_{\rm Sch.})$ would evolve into the hairy black holes in the symmetry-broken phase.
- Spontaneous symmetry breaking associated with local U(1) cannot be realized in this theory.

Thank you!