QRF at CQUeST Workshop 2023

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Physical Applications of Quantum Reference Frame (QRF)

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Based on the collaboration with Jaewon Shin (GIST)
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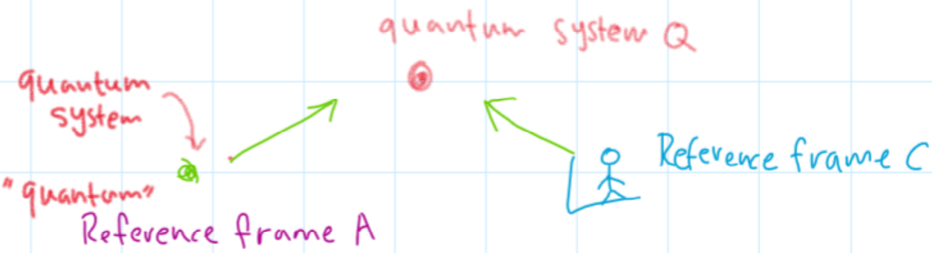
Consider two reference frames A & C.



One (or both) of the reference frames is a "quantum system",
it is called a quantum reference frame (QRF)

Oddly enough, this quantum nature of reference frame
is also contagious to other reference frames
through superpositions and entanglements

Reference frame transformations $A \rightleftarrows C$

- Translations
- Galilean (or Lorentz) boosts
- Accelerations (gravity via the equivalence principle)

Why is quantum reference frame important?

출처: <https://chat.openai.com/>

Chat GPT gives us a nice answer. ^_^

1.
A quantum reference frame is important because it provides a consistent way to describe and measure quantum states and phenomena. In classical physics, a reference frame is a coordinate system that is used to describe the position and motion of objects.
In quantum physics, however, the concept of a reference frame is more complex because it involves the interaction between the observer and the observed system.

2.

In quantum mechanics, the measurement process is essential for understanding the properties of quantum systems.
The outcome of a measurement is dependent on the reference frame from which it is observed.
This means that different observers may obtain different results when measuring the same quantum system.

3.
A quantum reference frame provides a way to reconcile these different results by establishing a common set of coordinates that all observers can agree on.
It allows for the consistent measurement of quantum states and phenomena, which is essential for the development of quantum technologies such as quantum computing and quantum cryptography.

4.
Moreover, a quantum reference frame is also important for the study of foundational issues in quantum mechanics such as the measurement problem and the nature of entanglement.
It provides a framework for exploring these fundamental questions and developing new insights into the nature of the quantum world.

A Background Independent Algebra in Quantum Gravity

Edward Witten

Lecture at Strings 2023

Abstract: To define an algebra of observables in quantum gravity in a way that is universal and does not depend on a background spacetime, one can consider the observables along the worldline of an observer, rather than the observables in a region of spacetime.

A third problem concerns the question of why we want to define an algebra in the first place – what is this algebra supposed to mean? In ordinary quantum mechanics, an observer is external to the system and we are quite free to make what assumptions we want about the capability of the observer. In quantum field theory without gravity, we can imagine an observer who can probe a system at will but only in a specified region $\mathcal{U} \subset M$, and that is the context in which it makes sense to consider the algebra $\mathcal{A}_{\mathcal{U}}$. In gravity, at least in a closed universe or in a typical cosmological model, there is no one who can probe the system from outside so an algebra only has operational meaning if it is the algebra of operators accessible to some observer.

A Parodox and its Resolution Illustrate Principles of de Sitter Holography.

Leonard Susskind

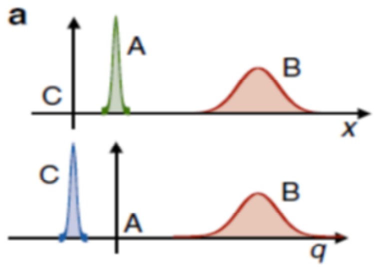
Stanford Institute for Theoretical Physics and Department of Physics,
Stanford University, Stanford, CA 94305-4060, USA

and Google,
Mountain View, CA

Abstract

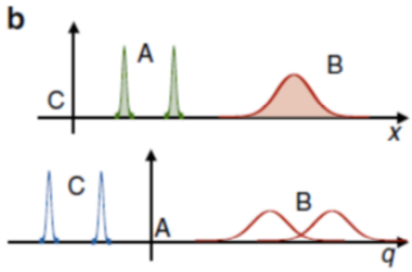
Semiclassical gravity and the holographic description of the static patch of de Sitter space appear to disagree about properties of correlation functions. Certain holographic correlation functions are necessarily real whereas their semiclassical counterparts have both real and imaginary parts. The resolution of this apparent contradiction involves the fact that time-reversal is a gauge symmetry in de Sitter space—a point made by Harlow and Ooguri— and the need for an observer (or quantum reference frame) as advocated by Chandrasekaran, Longo, Penington, and Witten.

Intuitive Exposition of QRF



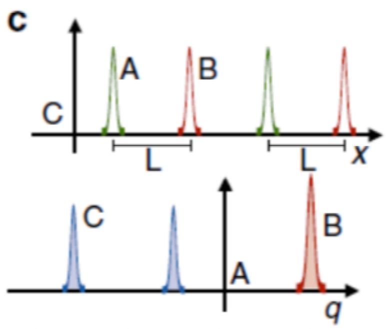
In **a** A's state is well localised from the point of view of C.
In A's reference frame, B has the same state as seen from C,
but translated, and C is well-localised.
This case corresponds to the translation of a classical reference frame.

Some examples of transformed states according to the map in Eq. (2) are given in Fig. 3. In particular, we see in Fig. 3a that when the new reference frame A is very sharp in position basis and the initial state in C's reference frame is $|\psi\rangle_{AB} = |x_0\rangle_A|\psi\rangle_B$, from A's point of view the state of B is translated by x_0, and the state of C is also sharp. The state in the new reference frame would then be $|\psi\rangle_{BC} = \int dq_B \psi(q_B + x_0)|q_B\rangle_B|-x_0\rangle_C$. This corresponds to the translation of a classical reference frame by an amount x_0, since transformation $\hat{S}_x$ applied to the well-localised state of A takes the form of the standard translation operator $\hat{S}_x|x_0\rangle_A = \hat{\mathcal{P}}_{AC} e^{\frac{i}{\hbar}x_0\hat{p}_B}|x_0\rangle_A$. (Up to the parity-swap operator that specifies the relative position of the two reference frames, which is usually ignored in the standard framework.)

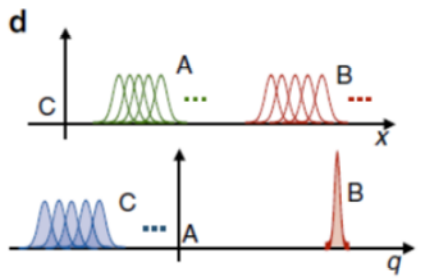


In **b** A and B are in a product state,
and A is in a superposition of two sharp-position states that do not overlap.
From A's point of view,
B and C are entangled,
but the relative distance between the states is unchanged.

In Fig. 3b we illustrate the case in which the state of A is a superposition of two sharp states, i.e. $|\phi\rangle_A = \frac{1}{\sqrt{2}}(|x_1\rangle_A + |x_2\rangle_A)$. In general, if C describes the joint state of A and B as a product state $|\phi\rangle_A|\psi\rangle_B$, the state in the reference frame of A is entangled and is obtained as the convolution product of the two, $\hat{S}_x|\phi\rangle_A|\psi\rangle_B = \int dq_B dq_C \phi(-q_C)\psi(q_B - q_C)|q_B\rangle_B|q_C\rangle_C$. Analogously, if the states of A and B are entangled in the initial reference frame, this property might not hold after changing to the reference frame of A. Examples of this situation are given in Fig. 3c, d. In particular, in Fig. 3c we consider an entangled state of A and B in position basis, where there is a perfect correlation between A and B, i.e. $|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|x_1\rangle_A|x_1 + L\rangle_B + |x_2\rangle_A|x_2 + L\rangle_B)$. From the point of view of A, the state of B and C is in a product state. In particular, B appears localised at the position $q_B = L$, while the state of C is in the superposition state $\frac{1}{\sqrt{2}}(|-x_1\rangle_C + |-x_2\rangle_C)$. Similarly, if A and B are entangled in the EPR state $|\psi\rangle_{AB} = \int dx|x\rangle_A|x + X\rangle_B$ as in Fig. 3d, A sees B localised at position $q_B = X$, while C is spread over the whole space.



In **c** A and B are entangled and perfectly correlated,
i.e. the relative distance between them is always L.
In A's reference frame B is in a well-defined position
and C is in a superposition of positions.



a superposition of positions. Finally, in **d** A and B are entangled in an EPR state from C's point of view, i.e. $|\psi\rangle_{AB} = \int dx |x\rangle_A |x+X\rangle_B$. Changing to A, B appears in a fixed position, while C is spread over the whole space

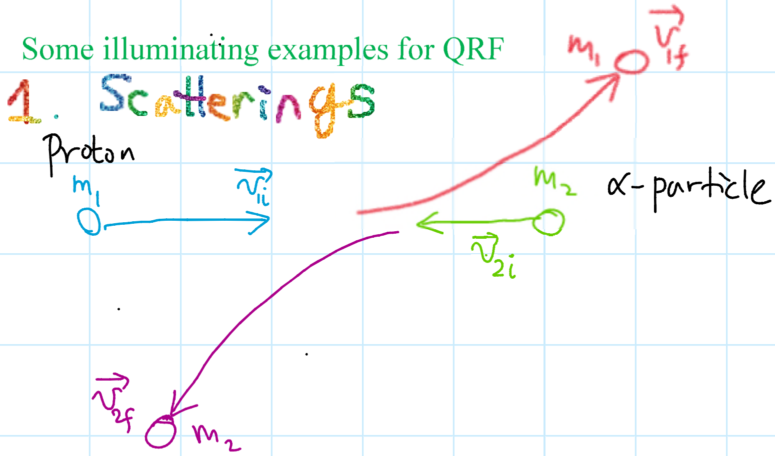
From this example, we see that the notion of entanglement and superposition are observer-dependent feature!

I think the spin entanglement also shares this property, i.e. may be reprented by the superposition of spin states in spin doublet space through QRF transformations (spin rotations $\vec{S}\cdot\hat{n}$)

No result yet. So prove this claim.

Some illuminating examples for QRF

1. Scatterings



Scattering can be most easily described in the center of mass frame

$$\text{Lab frame } (\vec{r}_1, \vec{r}_2) \longmapsto \text{CM frame } (\vec{R}, \vec{r})$$

where $\vec{R} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$, $\vec{r} = \vec{r}_1 - \vec{r}_2$

In some scattering events which are not prepared by an experimenter, the center of mass momentum may be not known exactly.

The superposition of all possible CM momenta is necessary to describle the scattering event, because (proton + α-particle) is a quantum system.

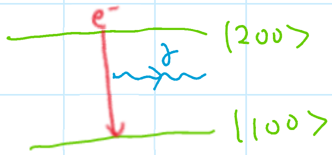
In CM frame, scattering particles are often entangled in momentum space and spin space.

Spin entanglement: $\pi^0 \to 2\gamma$, $\eta \to \mu^+ + \mu^-$

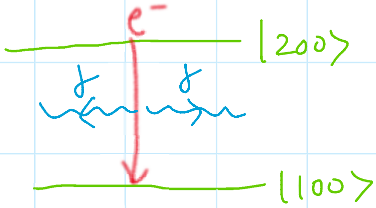
Charge entanglement: $K^0 \to \pi^+ + \pi^-$

2. Two photon decay in hydrogen atom

H-atom in $|200\rangle$ state



dipole transition
is forbidden
due to the angular momentum conservation



Two photon decay
is allowed (Breit & Teller, 1940)
because

$$\left[(S=1)\otimes(S=1)\right]_S = \overset{S=2}{3} \oplus \overset{S=0}{1}$$

EPR state

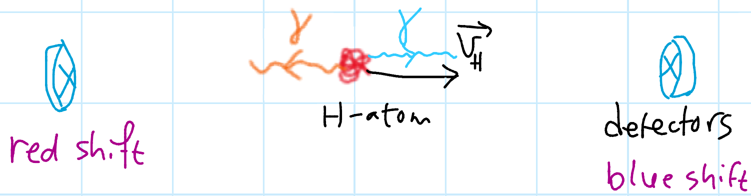
$$|S\rangle = \frac{1}{\sqrt{2}}\left(|R,+\vec{k}\rangle|R,-\vec{k}\rangle + |L,\vec{k}\rangle|L,-\vec{k}\rangle\right)$$

parity-even

Now consider a moving hydrogen atom which decays to the ground state by emitting two photons

Reference frames: $\begin{cases}\text{CM frame of H-atom} = H \\ \text{Pair of detectors} = C\end{cases}$

Quantum system: electron + $2\gamma = Q$



Spin entanglement

$\begin{cases}\text{polarization} \\ \text{Doppler shift}\end{cases}$

Momentum entanglement

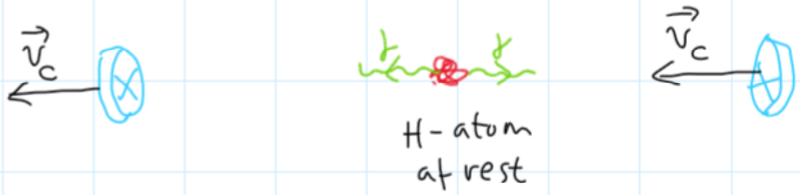
Suppose that we don't know the momentum state of H-atom (CM-fram) exactly:

Superpose all possible momentum states $|\vec{P}_H = m_H\vec{V}_H\rangle$ of H-atom.

$\Rightarrow$ Superposition of Doppler shifts

Quantum Reference Frame Transformation:

$$H \longrightarrow C$$

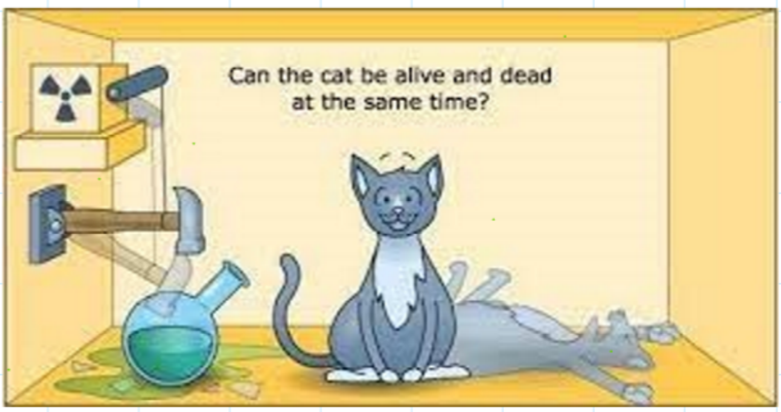


We need to superpose the momentum states $|\vec{P}_c = m_c \vec{v}_c\rangle$ of detectors to reproduce the superposition of Doppler shifts in the H-frame.

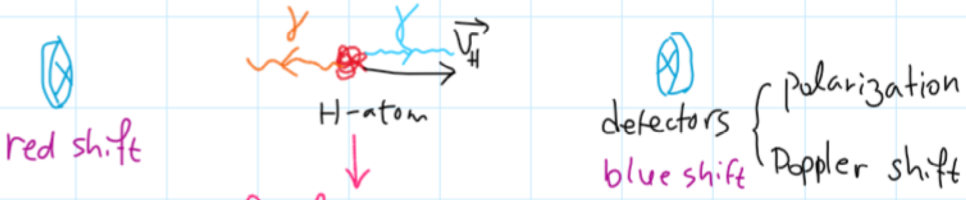
Oddly enough, this quantum nature of reference frame is also contagious to other reference frames through superpositions and entanglements

This kind of strange entanglement of micro and macro worlds also appears in the famous "Schrödinger's cat"

Reincarnation of Schrödinger's cat!



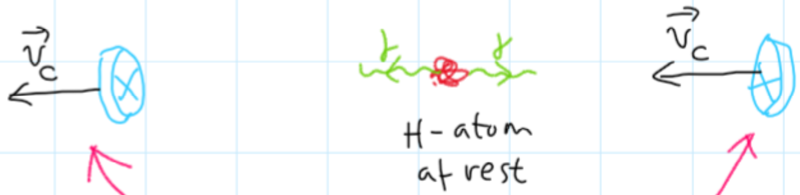
Acceleration and weak equivalence principle



Uniformly accelerating (superposed)

Quantum Reference Frame Transformation:

$$H \longrightarrow C$$



uniformly accelerating
(superposed)

Equivalence principle:

Accelerating reference frame

$\simeq$ Quantum (H+Q) system
in a gravitational field

$\boxed{\text{Superposition of acceleration} \simeq \text{superposition of gravitational fields}}$

The equivalence principle still hold even in quantum world (gravity couples with quantum matters)

Unlike the claim of Roger Penrose!

To resolve this puzzle, the concept of QRFs plays an important role.

Challenging questions?

1. Find a QRF transformation for tunneling

2. Is it possible to extend QRFs to internal symmetries (e.g., gauge transformation)?

   Maybe YES. (Rotations ⟷ magnetic fields)

3. Address the famous paradox in the context of QRFs:

   Electromagnetic radiation by accelerating electric charges
   $\simeq$ EM radiation in a uniform gravitational field

This paradox is very interesting.
mind-invoking puzzle.

In memory of Professor Chaiho Rim

Before leaving CQUeST in February 2020, I had been working with Prof. Chaiho Rim on the mirror symmetry of the moduli space of Calabi-Yau manifolds (specifically with boundaries).

Unfortunately, I left CQUeST without completing this research and promised Prof. Chaiho Rim that I would complete this unfinished work as soon as I had time.

Sadly, he is no longer with us.
Nevertheless, I hope to finish this incomplete study sooner or later, in memory of him.

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3.3. Computing the antiholomorphic involution $M_{\mu\nu}$

We want to compute the antiholomorphic involution $M_{\mu\nu}$. For this computation, we use its connection with the transition matrix T.

Since both $\omega^{\pm}_{\alpha\mu}(\phi)$ and $\sigma^{\pm}_{\alpha\mu}(\phi)$ are bases of periods defined as integrals over the cycles in the same space $H_5(\mathbb{C}^5, \mathrm{Re}\, W_0(x) = \pm\infty)$, they are connected by some constant matrix $(T^{\pm})^{\nu}_{\mu}$, which is independent of α:

$$\omega^{\pm}_{\alpha\mu}(\phi) = (T^{\pm})^{\nu}_{\mu}\, \sigma^{\pm}_{\alpha\nu}(\phi)$$

where the periods are defined by

$$\omega^{\pm}_{\alpha\mu}(\phi) = \int_{Q^{\pm}_{\mu}} e_{\alpha}(x)\, e^{\mp W(x,\phi)}\, d^5x$$

$$Q^{\pm}_{\mu} = (T^{\pm})^{\nu}_{\mu}\, \Gamma^{\pm}_{\nu}$$

$$\sigma^{\pm}_{\alpha\mu}(\phi) = \int_{\Gamma^{\pm}_{\mu}} e_{\alpha}(x)\, e^{\mp W(x,\phi)}\, d^5x$$

It follows from $e_0(x) = 1$ that $\omega^{\pm}_{0\mu}(\phi) = \omega^{\pm}_{\mu}(\phi)$ and $\sigma^{\pm}_{0\mu}(\phi) = \sigma^{\pm}_{\mu}(\phi)$.

Due to our choice of the cycles $\Gamma^{\pm}_{\mu}$, we also have $\sigma^{\pm}_{\alpha\mu}(\phi=0) = \delta_{\alpha\mu}$, and $\omega^{\pm}_{\alpha\mu}(\phi=0) = (T^{\pm})^{\alpha}_{\mu}$.

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To find the matrix T, it suffices to take a few first terms of the expansion over ϕ of the periods $\omega^{\pm}_{\mu}(\phi)$ and $\sigma^{\pm}_{\mu}(\phi)$ and to use the expansion

$$\omega^{\pm}_{\mu}(\phi) = (T^{\pm})^{\nu}_{\mu}\, \sigma^{\pm}_{\nu}(\phi)$$

Here we use another strategy. We first choose a real basis of cycles.
Especially, we choose Lefschetz thimbles $L^{\pm}_{\mu}$ as the basis of such cycles.

The matrix T can then be found as the transition matrix that connects the cycles $\Gamma^{\pm}_{\mu}$ and Lefschetz thimbles $L^{\pm}_{\mu}$:

$$\Gamma^{\pm}_{\mu} = (T^{-1})_{\mu\nu}\, L^{\pm}_{\nu}.$$

Using the normalization of cycles $\Gamma^{\pm}_{\mu}$,

$$\delta^{\mu}_{\nu} = \int_{\Gamma^{\pm}_{\mu}} e_{\nu}(x)\, e^{\mp W_0(x)}\, d^5x = \int_{(T^{-1})_{\mu\lambda} L^{\pm}_{\lambda}} e_{\nu}(x)\, e^{\mp W_0(x)}\, d^5x$$

$$= (T^{-1})_{\mu\lambda} \underbrace{\int_{L^{\pm}_{\lambda}} e_{\nu}(x)\, e^{\mp W_0(x)}\, d^5x}_{= T_{\lambda\nu}}$$

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Thus we get

$$T_{\mu\nu} = \int_{L^{\pm}_{\mu}} e_{\nu}(x)\, e^{\mp W_0(x)}\, d^5x,$$

After computing this integral, we obtain the matrix M from the formula

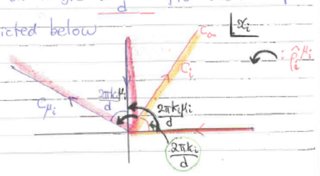
$$M = T^{-1}\, \overline{T}.$$

The Lefschetz thimbles $L^{\pm}_{\mu}$ are products of one-dimensional cycles C_{μ_i}

$$L^{+}_{\mu} = \prod_{i=1}^{5} C_{\mu_i}$$

and $C_{\mu_i} = \hat{\rho}_i^{\mu_i} \cdot C_i$ with $\rho_i = e^{\frac{2\pi i k_i}{d}}$.

This definition of the one-dimensional cycle C_{μ_i} means that this cycle is the path in the x_i plane obtained by the operation $\hat{\rho}_i^{\mu_i}$ of rotating counterclockwise through an angle $\frac{2\pi k_i \mu_i}{d}$ from the basic path C_i depicted below



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By construction, $L^{\pm}_{\mu}$ are the steepest descent/acsent cycles for $\mathrm{Re}\, W_0$. We now compute $T_{\alpha\mu}$ explicitly,

$$T_{\alpha\mu} = \int_{L^{+}_{\alpha}} e_{\mu}(x)\, e^{-W_0(x)}\, d^5x$$

$$L^{+}_{\alpha} = \prod_{i=1}^{5} C_{\alpha_i} = \prod_{i=1}^{5} \hat{\rho}_i^{\alpha_i} \cdot C_i$$

$$e_{\mu}(x) = \prod_{i=1}^{5} x_i^{\mu_i}, \quad W_0(x) = \sum_{i=1}^{5} x_i^{\frac{d}{k_i}}$$

Therefore we get the transition matrix $T_{\alpha\mu}$ by

$$T_{\alpha\mu} = \prod_{i=1}^{5} \int_{C_{\alpha_i} = \hat{\rho}_i^{\alpha_i} \cdot C_i} x_i^{\mu_i}\, e^{-x_i^{\frac{d}{k_i}}}\, dx_i \qquad x_i \to e^{2\pi i k_i \alpha_i / d}\, x_i$$

$$= \prod_{i=1}^{5} e^{2\pi i \alpha_i k_i (\mu_i + 1)/d} \int_{C_i} x_i^{\mu_i}\, e^{-x_i^{\frac{d}{k_i}}}\, dx_i$$

$$* \int_{C_i} x_i^{\mu_i}\, e^{-x_i^{\frac{d}{k_i}}}\, dx_i = \left(\int_{\infty}^{0} + \int_{0}^{C_\infty} \right) x_i^{\mu_i}\, e^{-x_i^{\frac{d}{k_i}}}\, dx_i$$

$$= \int_{0}^{C_\infty} x_i^{\mu_i}\, e^{-x_i^{\frac{d}{k_i}}}\, dx_i - \int_{0}^{\infty} x_i^{\mu_i}\, e^{-x_i^{\frac{d}{k_i}}}\, dx_i$$

$$x_i = e^{\frac{2\pi i k_i}{d}}\, \xi_i, \quad 0 \leq \xi_i < \infty.$$