2022 12 28 Cosmology Ch 12-1

2022년 12월 25일 일요일 오후 11:44

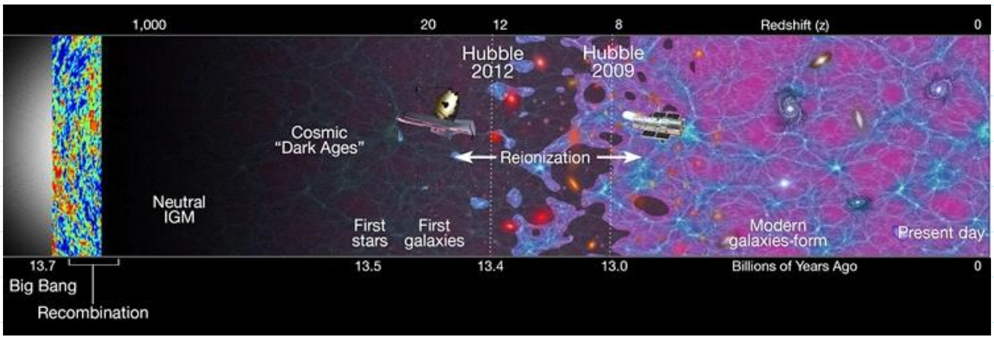
12.1 What i s cosmology? - The universe in the large
Cosmology is the study of the universe as a whole: its history, evolution, composition, dynamics.

The primary aim - the large-scale structure of the universe from the Big Bang,
providing the clues to the small-scale structure : galaxies, stars, planets, people.

Fundamental questions of physics related to Cosmology:
what are the laws of physics at the very highest possible energies,

←			
10^{19}GeV	TeV GeV	MeV	eV
Quantum Gravity?	Particle Physics	Nuclear	Atom,Molecule, Solid
String Theory?	Standard Model		-chemistry, Biology-

Evolution of the Universe



t= 0	1sec	3min.	50,000 yr	370,000 yrs		~800Myrs	10^{10} yrs	1.38×10^{10} yrs
T=∞	10^{10}K	10^9 K	8,700K	3,000K	~60K	~30K	3.8K	2.73K
z= ∞			3,400	1,100	20	10	0.66	0
	ν-decpl	BBN	$\rho_{\mathrm{rad}} = \rho_{\mathrm{Matt}}$	Recombination	1st galaxies	re-ionization	$\rho_{\mathrm{Matt}} = \rho_{\Lambda}$	

← Radiation Dominated →|← Matter Dominated →|← Λ Dominate
opaque →|← transparent; CMB →|

quarks,	baryons(p,n)	Nucleus	
leptons	leptons(e,ν)	(p,n,**He**,)	atoms(H, He)
gauge bosons		e,ν	ν
	γ	γ	γ

Q: how did the Big Bang happen, what came before the Big Bang,
We know only from $t\sim10^{-43}$s after Big Bang
how did the building blocks of matter (electrons, protons, neutrons, atoms) get made?
Particle Physics, Nucleosynthesis, (Supernova)
Ultimately, the origin of every system and structure in the natural world,
Inflation, Dark Energy, Dark Matter, ΛCDM model

Gravity theory to the universe on large scales : Newtonian vs Eistein General Relativity
M/R≡ $R_{HorM}/R \ll 1$: Newtonian theory is an adequate approximation
M/R ≳ 1 : General Relativity
(Note : [GM/c^2]=[length], ex) $R_{HM_\odot}$ =$2GM_\odot$ = 2.953 km

Note 1) M/R ≳ 1
i) if R becomes small faster than M
Ex) compact or collapsed objects: neutron stars and black holes

ii) if the system's mass M increases faster than its radius R
Ex) cosmology

Note 2) Let space is filled with matter of roughly the same density ρ everywhere,
Let
$\frac{G}{c^2}\rho \equiv \frac{1}{r_\rho{}^2}$ ($\frac{G}{c^2}$=0.742x10^{-27}m/kg =
then
$$M/R \sim \rho R^3/R \sim \left(\frac{R}{r_\rho}\right)^2$$ GR becomes important if $R \gtrsim r_\rho$

Ex) Water (~Sun)
$\rho = 1$ g/cm^3=10^3kg/m^3 =0.742x10^{-24}/m^2 $r_\rho \sim 10^{12}$m ~10^9km
Mean density of the universe
$\rho = 10^{-26}$kg/m^3 =0.742x10^{-53}/m^2 $r_\rho \sim 10^{26}$m ~10^{23}km~3x10^9pc
(1= $\frac{M}{R}$ = $\frac{4\pi}{3}\rho R^3/R$~6 Gpc < size of the observable universe. Need GR for cosmology!)

Ex) Sun $\frac{M_\odot}{R_\odot} \sim \frac{km}{10^6 km} \sim 10^{-6} \sim \left(\frac{R_\odot}{r_\rho}\right)^2$ No need for GR
Galaxy $\frac{M_{Gal}}{R_{Gal}} \sim \frac{10^{11} M_\odot}{15\ kpc} \sim \frac{10^{11} M_\odot}{1.5\times10^4\times10^7 R_\odot} \sim 10^{-6}$ No need for GR
(This applies to the galaxy as a whole: small regions, including the very center, may be dominated by black holes or other relativistic objects.)
Cluster $\frac{M_{cl}}{R_{cl}} \sim \frac{10^3 M_{Gal}}{Mpc} \sim \frac{10^3 M_{Gal}}{10\ R_{Gal}} \sim 10^{-4}$ No need for GR

For larger scales than galaxy clusters, we enter the domain of cosmology & need GR.

In the cosmological picture, galaxies and even clusters are very small-scale structures,
mere atoms in the larger universe.

Cosmological Principle : On this large scale, the universe is observed to be
1) homogeneous, to have
roughly the same density of galaxies, and roughly the same types of galaxies, everywhere.
2) Isotropic

The Friedmann-Robertson-Walker metric from the GR
no boundaries, no edges

Newtonian gravity could not consistently make such models,
$\nabla^2\Phi = 4\pi G\rho$
is ambiguous if there is no outer edge on which to set a boundary condition for the equation.

So only with Einstein could cosmology work.

the converse question: if we live in a universe whose overall structure is GR,
- how is it that we can study our local region of the universe without reference to cosmology?
- How can we apply general relativity to the study of neutron stars and black holes as if they were embedded in an empty asymptotically flat spacetime, when actually they exist in a highly relativistic cosmology?
- How can astronomers study individual stars, geologists individual planets, biologists individual cells – all without reference to GR?

The answer, of course, is that
- in GR spacetime is locally flat:
  - as long as your experiment is confined to the local region, the large-scale geometry is not needed.

- This separation of local and global is not possible in Newtonian gravity,
  - where even the local gravitational field within a large uniform-density system depends on the boundary conditions far away, on the shape of the distant "edge" of the universe.

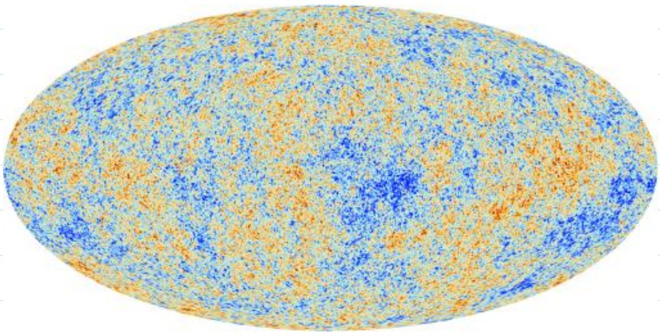
So GR not only explains cosmology, it explains why can study other sciences without GR!

The cosmological arena

- In recent years, with the increasing power of ground- & space-based observatories, cosmology has become a precision science, with some of the most fundamental questions.

- Cosmological Principle (when averaged over large distance scales, say, 1Gpc)
  (Homogeniety) universe is homogeneous , expanding at the same rate everywhere.
  (Isotropy) The universe is isotropic: it looks the same, on average, in every direction we look.

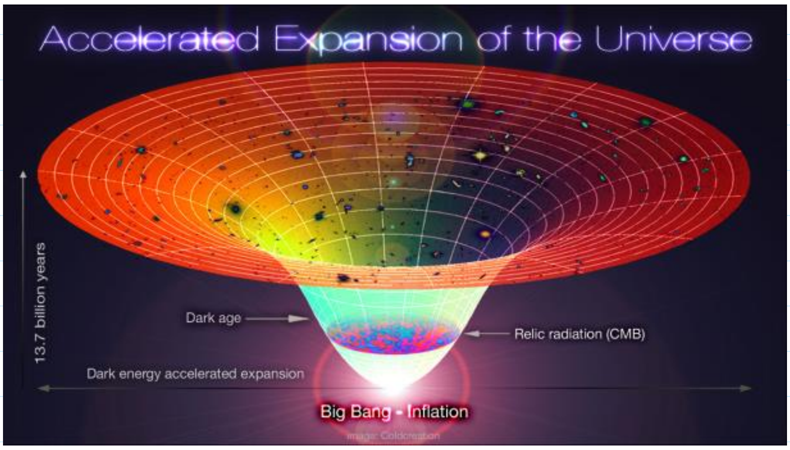
- CMB (Cosmic Microwave Background)
  The universe is filled with a black-body thermal radiation, with a temperature of 2.725K.
  However, small temperature fluctuations of 3×10^{-5} K should exist :
  due to the Large Scale Structures



The Cosmic Microwave Background as seen from the Planck satellite.
Credit: ESA https://www.esa.int/ESA_Multimedia/Images/2013/03/Planck_CMB
출처: <https://en.wikipedia.org/wiki/Cosmic_microwave_background#/media/File:Planck_satellite_cmb.jpg>

- Expanding Universe
  - The expansion means that the universe has a finite age, or
  - at least that it has expanded in a finite time from a state of very high density.
  - The thermal radiation was initially much hotter than today, and has cooled as it expanded.
  - The expansion resolves the oldest of all cosmological conundrums, Olbers' Paradox.
    The sky is dark at night because we do not receive light from all stars in our infinite homogeneous universe, but only from stars that are close enough for light to have traveled to us during the age of the universe.
  - Recent accelerated expansion.

- Evolution of the Universe
  - Q: how the universe evolved to its present state and what it was like much earlier.
  - how the first stars formed, why they group into galaxies, why galaxies form clusters:
  - where did the density irregularities come from that have led to the enormously varied structure of the universe on scales smaller than 200 Mpc?
  - how the elements formed, what the universe was like when it was too dense and too hot to have normal nuclei, and what the very hot early universe can tell us about the laws of physics at energies higher than we can explore with particle accelerators.
  - Does the observed homogeneity and isotropy of the universe have a physical explanation?



Lambda-CDM, accelerated expansion of the universe. The time-line in this schematic diagram extends from the Big Bang/inflation era 13.7 Byr ago to the present cosmological time.

출처: <https://en.wikipedia.org/wiki/Lambda-CDM_model>

- The homogeneity problem can be solved by **inflation**
- During which the extremely early universe expanded exponentially rapidly,
- this would as a bonus help to explain the density fluctuations that led to the observed galaxies and clusters.
- **Dark Matter** : it appears that most of the matter in the universe is in an unknown form,
- **the dark energy**: Even more strangely, the universe seems to be pervaded by a relativistic energy density that carries negative pressure and which is driving the expansion faster and faster (accelerated expansion);

- Modern cosmology answers are becoming more precise and more definite at a rapid pace.

Λ –CDM Model (Lambda - Cold Dark MatterModel)

Lambda: Cosmological Constant, Dark Energy; Cold: Nonrelativistic

12.2 Cosmological kinematics : observing the expanding universe

Homogeneity and isotropy of the universe : the remarkable observed large-scale uniformity.

- We see, on scales much larger than 200 Mpc, not only a uniform average density but uniformity in other properties: types of galaxies, their clustering densities, their chemical composition and stellar composition.
- when we look very far away we are also looking back in time, and see a younger universe.
- But the evolution we see is again the same in all directions, even when we look at parts of the early universe that are very far from one another.
- We therefore conclude that, on the large scale, the universe is homogeneous.
- What is more, on scales much larger than 10 Mpc the universe seems to be isotropic about every point: we see no consistently defined special direction.
  - A universe could be homogeneous but anisotropic, if, for instance, it had a large-scale magnetic field which pointed in one direction everywhere and whose magnitude was the same everywhere.
  - On the other hand, an inhomogeneous universe could not be isotropic about every point, since most – if not all – places in the universe would see a sky that is ‘lumpy’ in one direction and not in another.

A third feature of the observable universe is **the uniformity of its expansion**:

- galaxies, on average, seem to be receding from us at a speed which is proportional to their distance from us. This recessional velocity is called the Hubble flow.
- This kind of expansion is easily visualized in the ‘balloon’ model (see Fig. 12.1).
  Any point will see all other points receding at a rate proportional to their distance.

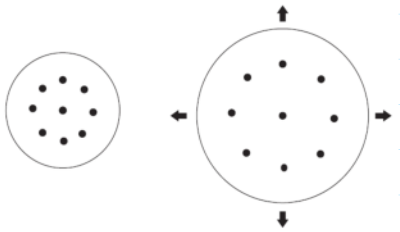


Figure 12.1 As the figure is magnified, all relative distances increase at a rate proportional to their magnitudes.

- This proportionality preserves the **homogeneity** of the distribution of dots with time.
- Our location in the universe is not special, even though we appear to see everything else receding away from us. We are no more at the 'center' of the cosmological expansion than any other point is.
- The Hubble flow is compatible with the Copernican Principle, the idea that the universe does not revolve around (or expand away from) our particular location.
- The universe would be homogeneous and **anisotropic if** every point saw a recessional velocity larger in, say, the x direction than in the y direction. The ellipsoid balloon would have to expand faster along its longest axis than along the others to keep its shape .
- Our universe does not have any measurable velocity anisotropy.

Hubble parameter, Hubble time

- the relation btwn recessional velocity & distance with a single constant of proportionality H:

  v = Hd (12.1)

  H : Hubble's parameter.

  Its present value is called *Hubble's constant*, H_0.

  $H_0 = 100h\ \text{km s}^{-1}\text{Mpc}^{-1} = h(9.777752\text{Gyr})^{-1}$

  $= h(3.08566 \times 10^{17}\text{s})^{-1}$

  $\frac{H_0}{c} = h \times (0.9257 \times 10^{26}\text{m})^{-1}$

  $1\ \text{Mpc} = 3.1 \times 10^{22}\ \text{m},\ h \approx 0.7$

  Hubble time t_H

  $t_H = H_0^{-1} = h^{-1} \times 3.08566 \times 10^{17}\text{s}$

  $= h^{-1} \times 9.777752\text{Gyr} \approx 13.968\text{Gyr}$

The age of the universe will not exactly be this, since in the past the expansion speed varied, but this gives the order of magnitude.

We may object that the above discussion ignores the relativity of simultaneity.
If the universe is changing in time – expanding –

- then it may be possible to find some definition of time such that hypersurfaces of constant time are homogeneous and isotropic,
- but this would not be true for other choices of a time coordinate.
- Moreover, Eq. (12.1) cannot be exact since, for d > 1.3×10^{26} m = 4200 Mpc, the velocity exceeds the velocity of light!

This objection is right on both counts.

- Our discussion was a local one (applicable for recessional velocities $\ll 1$)
- and took the point of view of a particular observer, ourselves.

- Fortunately, the cosmological expansion is slow, so that over distances of 1000 Mpc, large enough to study the average properties of the homogeneous universe, the velocities are essentially nonrelativistic.
- Moreover, the average random velocities of galaxies relative to their near neighbors is typically less than 100 km s—1, which is certainly nonrelativistic, and is much smaller than the systematic expansion speed over cosmological distances.
- Therefore, **the correct relativistic description of the expanding universe** is that, in our neighborhood, there exists a ***preferred choice of time***, whose hypersurfaces are **homogeneous and isotropic**, and with respect to which **Eq. (12.1) is valid in the local inertial frame of any observer who is at rest with respect to these hypersurfaces at any location**.
- The existence of a preferred cosmological reference frame may at first seem startling: did we not introduce special relativity as a way to get away from special reference frames?
- There is no contradiction: the laws of physics themselves are invariant under a change of observer. But there is only one universe, and its physical make-up defines a convenient reference frame.

- would it be silly for us to develop the theory of cosmology in a frame that does not take advantage of the simplicity afforded by the large-scale homogeneity.
- From now on we will, therefore, work in the cosmological reference frame, with its preferred definition of time.

Models of the universe: the cosmological principle

- two different inaccessible regions of the universe.
  - The first inaccessible region is the region which is so distant that no information (traveling on a null geodesic) could reach us from it no matter how early this information began traveling. This region is everything that is **outside our past light-cone**. Such a region usually **exists if the universe has a finite age**, as ours does (see Fig. 12.2). This 'unknown' region is unimportant in one respect: what happens there has no effect on the interior of our past light cone, so how we incorporate it into our model universe has no effect on the way the model describes our observable history. On the other hand, our past light cone is a kind of horizon, which is called **the particle horizon**: as time passes, more and more of the previously unknown region enters the interior of our past light cone and becomes observable. So the unknown regions across the particle horizon can have a real influence on our future.
  - In this sense, cosmology is a retrospective science: it reliably helps us understand only our past.
  - It must be acknowledged, however, that if information began coming in tomorrow that yesterday's 'unknown' region was in fact very different from the observed universe, say

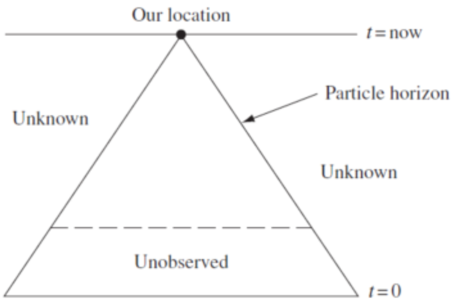


Figure 12.2 Schematic spacetime diagram showing the past history of the Universe, back to t = 0. The 'unknown' regions have not had time to send us information; the 'unobserved' regions are obscured by intervening matter.

highly inhomogeneous, then we would be posed difficult physical and philosophical questions regarding the apparently special nature of our history until this moment.

  - we usually assume that the unknown regions are very like what we observe, and in particular are homogeneous and isotropic. Consider, in Fig. 12.2, two hypothetical observers within our own past light cone, but at such an early time in the evolution of the universe that their own past light-cones are disjoint. Then they are outside each other's particle horizon. But we can see that the physical conditions near each of them are very similar: we can confirm that if they apply the principle that regions outside their particle horizons are similar to regions inside, then they would be right!
  - This modern version of the Copernican Principle is called the Cosmological Principle, or more informally the Assumption of Mediocrity, the ordinary-ness of our own location in the universe.
  - It is, mathematically, an extremely powerful (i.e. restrictive) assumption. We shall adopt it, but we should bear in mind that predictions about the future depend strongly on the assumption of mediocrity.
- The second inaccessible region is that part of the interior of our past light cone which our instruments cannot get information about. This includes galaxies so distant that they are too dim to be seen; processes that give off radiation – like gravitational waves – which we have just been able to detect; and events that are masked from view, such as those which emitted electromagnetic radiation before the epoch of decoupling (see below) when the universe ceased to be an ionized plasma and became transparent to electromagnetic waves. The limit of decoupling is sometimes called our optical horizon since no light reaches us from beyond it (from earlier times). But gravitational waves do propagate freely before this, so eventually we will begin to make observations across this 'horizon': the optical horizon is not a fundamental limit in the way the particle horizon is.

Cosmological metrics

- The metric tensor that represents a cosmological model must incorporate the observed homogeneity and isotropy. We shall therefore adopt the following idealizations about the universe:

- (i) spacetime can be sliced into hypersurfaces of constant time which are perfectly homogeneous and isotropic; and
- (ii) the mean rest frame of the galaxies agrees with this definition of simultaneity.

○ Let us adopt comoving coordinates:
each galaxy is idealized as having no random velocity, and we give each galaxy a fixed set of coordinates $\{x^i$, i = 1, 2, 3}.
We choose our time coordinate t to be proper time for each galaxy. The expansion of the universe – the change of proper distance between galaxies – is represented by time-dependent metric coefficients.
Thus, if at one moment, t_0, the hypersurface of constant time has the line element

$$dl^2(t_0) = h_{ij}(t_0)dx^i dx^j \qquad (12.2)$$

then the expansion of the hypersurface can be represented by

$$dl^2(t_1) = f(t_1, t_0)\, h_{ij}(t_0)dx^i dx^j = h_{ij}(t_1)dx^i dx^j \qquad (12.3)$$

This form guarantees that all the h_{ij}s increase at the same rate; otherwise the expansion would be anisotropic (see Exer. 4, § 12.6).
In general, then, Eq. (12.2) can be written

$$dl^2(t) = R^2(t)h_{ij}dx^i dx^j \ , (12.4)$$

where
R is an overall scale factor which equals one at t_0,
h_{ij} is a constant metric equal to that of the hypersurface at t0.

First we extend the constant-time hypersurface line element to a line element for the full spacetime.
In general, it would be

$$ds^2(t) = -dt^2 + g_{0i}dtdx^i + R^2(t)h_{ij}dx^i dx^j \ , (12.5)$$

where $g_{00} = -1$, because t is proper time along a line dx^i = 0.
However, if the definition of simultaneity given by t = const. is to agree with that given by the local Lorentz frame attached to a galaxy (idealization (ii) above), then
$\mathbb{e}_0$ must be orthogonal to $\mathbb{e}_i$ in our comoving coordinates.
This means that

$g_{0i} = \mathbb{e}_0 \cdot \mathbb{e}_i$ must vanish,

and we get

$$ds^2(t) = -dt^2 + R^2(t)h_{ij}dx^i dx^j \ , (12.6)$$

○ What form can h_{ij} take?
Since it is isotropic, it must be spherically symmetric about the origin of the coordinates, which can of course be chosen to be located at any point we like.
a spherically symmetric metric always has the line element (last part of Eq. (10.5))

$$dl^2(t) = e^{2\Lambda(r)}dr^2 + r^2 d\Omega^2 \ . (12.7)$$

This form of the metric implies only isotropy about one point.
We want a stronger condition, namely that the metric is homogeneous.
A necessary condition for this is certainly that the Ricci scalar curvature of the three-dimensional metric, $R^i{}_i$, must have the same value at every point: every scalar must be independent of position at a fixed time. We will show below, remarkably, that this is sufficient as well, but for now we just treat it as the next constraint we place on the metric in Eq. (12.7). We can calculate $R^i{}_i$ as in § 6.9.
Alternatively, we can use Eqs. (10.15)–(10.17) of our discussion of spherically symmetric spacetimes in Ch. 10, realizing that G_{ij} for the line element, Eq. (12.7), above is obtainable from G_{ij} for the line element, Eq. (10.7), of a spherical star by setting Φ to zero. We get

$$G_{rr} = -\frac{1}{r^2}e^{2\Lambda}\left(1 - e^{-2\Lambda}\right),$$
$$G_{\theta\theta} = -re^{-2\Lambda}\Lambda', \qquad (12.8)$$
$$G_{\phi\phi} = \sin^2\theta \ G_{\theta\theta}.$$

we simply require that the trace G of the threedimensional Einstein tensor be a constant. (In fact, this trace is just −1/2 of the Ricci scalar.)
The trace is

$$G \ = G_{ij}g^{ij}$$
$$= -\frac{1}{r^2}e^{2\Lambda}\left(1 - e^{-2\Lambda}\right)e^{-2\Lambda} - 2re^{-2\Lambda}\Lambda' r^{-2}$$

$$= -\frac{1}{r^2} + \frac{1}{r^2} e^{-2\Lambda}(1 - 2r\Lambda')$$
$$= -\frac{1}{r^2}\left[1 - (re^{-2\Lambda})'\right]. \ (12.9)$$

Demanding homogeneity means setting G to some constant κ:

$$\kappa = -\frac{1}{r^2}\left[1 - (re^{-2\Lambda})'\right]$$

This is easily integrated to give

$$g_{rr} = e^{2\Lambda} = \frac{1}{1 + \frac{1}{3}\kappa r^2 - \frac{A}{r}}$$

where A is a constant of integration. As in the case of spherical stars, we must demand local flatness at r = 0 (compare with § 10.5): $g_{rr}(r = 0) = 1$. This implies A = 0. Defining the more conventional curvature constant k = −κ/3 gives

$$g_{rr} = \frac{1}{1 - kr^2}$$
$$dl^2 = \frac{dr^2}{1-kr^2} + r^2 d\Omega^2. \qquad (12.12)$$

We have not yet proved that this space is isotropic about every point; all we have shown is that Eq. (12.12) is the unique space which satisfies the necessary condition that this curvature scalar be homogeneous.

Thus, if a space that is isotropic and homogeneous exists at all, it must have the metric, Eq. (12.12), for at least some k.

In fact, the converse is true: the metric of Eq. (12.12) is homogeneous and isotropic for any value of k. We will demonstrate this explicitly for positive, negative, and zero k separately in the next paragraph. General proofs not depending on the sign of k can be found in, for example, Weinberg (1972) or Schutz (1980b).

- We conclude that the full cosmological spacetime has the metric

$$\boxed{ds^2(t) = -dt^2 + R^2(t)\left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2\right]. \qquad (12.13)}$$

This is called the *Robertson–Walker metric*.

The conformal time

$$d\tau = \frac{dt}{R(t)} \quad \text{or} \quad \tau = \int_0^t \frac{dt'}{R(t')}$$

This can be rewritten (as will be shown below) as

$$ds^2(t) = R^2(t)\left[-d\tau^2 + \left\{\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right\}\right]$$

or

$$ds^2(t) = -dt^2 + R^2(t)\left[d\chi^2 + \begin{Bmatrix} \sin^2\chi \\ \chi^2 \\ \sinh^2\chi \end{Bmatrix} d\Omega^2\right] \qquad \begin{cases} S_{+1}(\chi) = r = \sin\chi \\ S_0(\chi) = r = \chi \\ S_{-1}(\chi) = r = \sinh\chi \end{cases}$$
$$= -dt^2 + R^2(t)\left[d\chi^2 + {S_k}^2(\chi)d\Omega^2\right]$$

or

$$ds^2(t) = R^2(t)\left[-d\tau^2 + d\chi^2 + {S_k}^2(\chi)d\Omega^2\right]$$

Note) we can, without loss of generality, scale the coordinate r in such a way as to make k take one of the three values +1, 0,−1.

To see this, consider for definiteness k = −3. Then re-define $\tilde{r} = \sqrt{3}r$ and $\tilde{R} = 1/\sqrt{3}R$, and the line element becomes

$$ds^2(t) = -dt^2 + \tilde{R}^2(t)\left[\frac{d\tilde{r}^2}{1-k\tilde{r}^2} + \tilde{r}^2 d\Omega^2\right]. \quad (12.14)$$

What we cannot do with this rescaling is change the sign of k. Therefore there are only three spatial hypersurfaces we need consider: k = (−1, 0, 1).

Three types of space

1) k = 0 ⇝ $\mathbb{R}^3$ flat 3-dim Euclidean space

Then, at any moment t_0, the line element of the hypersurface (setting dt = 0) is

$$dl^2(t_0) = R^2(t_0)\left[dr^2 + r^2 d\Omega^2\right] = d(r')^2 + (r')^2 d\Omega^2, \ (12.15)$$

with $r' = R(t_0)r$.

This is clearly the metric of flat Euclidean space. This is the *flat* Robertson–Walker universe. That it is homogeneous and isotropic is obvious.

2) k = +1 ⇝ $\mathbb{S}^3$ 3-dim sphere

Let us define a new coordinate χ(r) such that

$$d\chi^2 = \frac{dr^2}{1-r^2} \quad (12.16)$$

and χ = 0 where r = 0. This integrates to

r = sin χ, (12.17)

so that the line element for the space $t = t_0$ is

$$dl^2(t_0) = R^2(t_0)\left[d\chi^2 + \sin^2\chi\,(d\theta^2 + \sin^2\theta\, d\phi^2)\right]. \quad (12.18)$$

this is the metric of a three-sphere of radius $R(t_0)$.

the balloon analogy of cosmological expansion is for $\mathbb{S}^2$.

It is clearly homogeneous and isotropic: no matter where we stand on the three-sphere, it looks the same in all directions. the fourth spatial dimension – the radial direction to the center of the three-sphere – has no physical meaning: all our measurements are confined to our three-space.

3) k = −1 ⇝ $\mathbb{H}^3$ 3-dim hyperbolic space

An analogous coordinate transformation gives the line element

$$dl^2(t_0) = R^2(t_0)\,(d\chi^2 + \sinh^2\chi\, d\Omega^2). \quad (12.19)$$

This is called the *hyperbolic*, or *open*, Robertson–Walker model.

As the proper radial coordinate χ increases away from the origin, the circumferences of spheres increase as sinh χ, more rapidly with proper radius than in flat space.

this hypersurface is not realizable as a 3-dim hypersurface in a 4 or higher-dim Euclidean space.

The space is called 'open' because, unlike for k = +1, circumferences of spheres increase monotonically with χ: there is no natural end to the space.

In fact, this geometry is the geometry embedded in Minkowski spacetime. Specifically, of events that all have the same timelike interval from the origin. Since this hypersurface has the same interval from the origin in any Lorentz frame, this hypersurface is indeed homogeneous and isotropic.

Distance :

Cosmological redshift as a distance measure

- The distance of an object at a cosmological distance in an expanding universe is a little ambiguous, due to the long time it takes light to travel from the object to us.
- Its separation from our location when it emitted the light that we receive today may have been much less than its separation at present, i.e. on the present hypersurface of constant time.

1) Comoving proper distance ($dt = 0$) from the origin to the object at comoving coord r.

$$d_{comoving} = \int_0^r \frac{dr}{\sqrt{1-kr^2}} = \begin{cases} \sin^{-1} r \\ r \\ \sinh^{-1} r \end{cases}$$

Physical distance

$$d_{phs} = a(t)\, d_{comoving}$$

The redshift z

- Instead, a different measurement of separation is in commonly use:
  the redshift z of the spectrum of the light emitted by the object, let us say a galaxy.

- To compute the redshift in our cosmological models, let us assume that the galaxy has a fixed coordinate position on some hypersurface
  at the cosmological time t at which it emits the light
  we eventually receive at time t_0.
- Recall our discussion of conserved quantities in § 7.4:
  if the metric is independent of a coordinate, then the associated covariant component of momentum is constant along a geodesic.
  In the cosmological case, the homogeneity of the hypersurfaces ensures that the covariant components of the spatial momentum of the photon emitted by our galaxy are constant along its trajectory.
  Suppose that we place ourselves at the origin of the cosmological coordinate system, so that light travels along a radial line θ = const., ϕ = const. to us. In each of the cosmologies the line-element restricted to the trajectory has the form

$$0 = -dt^2 + R^2(t)d\chi^2. \quad (12.20)$$

It follows that the relevant conserved quantity for the photon is p_χ.

- Now, the cosmological time coordinate t is proper time, so the energy as measured by a local observer at rest in the cosmology anywhere along the trajectory is $-p^0$.
  We argue in Exer. 9, § 12.6, that conservation of p_χ implies that p^0 is inversely proportional to R(t).

Redshift

Suppose wave crest emitted at time t_e and observed at t_0 then,

First wave crest :

$$c\int_{t_e}^{t_0} \frac{dt}{R(t)} = \int_0^\chi d\chi = \chi$$

Next wave crest

$$c\int_{t_e+\frac{\lambda_e}{c}}^{t_0+\frac{\lambda_0}{c}} \frac{dt}{R(t)} = \int_0^\chi d\chi = \chi$$

Hence,

$$c\int_{t_e}^{t_e+\frac{\lambda_e}{c}} \frac{dt}{R(t)} = c\int_{t_0}^{t_0+\frac{\lambda_0}{c}} \frac{dt}{R(t)}$$

$$\frac{\lambda_e}{R(t_e)} = \frac{\lambda_0}{R(t_0)}$$

- It follows that the wavelength as measured locally (in proper distance units) is proportional to R(t), and hence that the redshift z of a photon emitted at time t and observed by us at time t_0 is given by
  $$1 + z = R(t_0)/R(t_e). \quad (12.21)$$
- Note) z is a measure of t_e, $\quad 0 \le z < \infty$
- Often interpret z using relativistic Doppler shift formula
  $$1+z = \sqrt{\frac{c+v}{c-v}}$$
- this is just the cosmological part of any overall redshift: if the source or observer is moving relative to the cosmological rest frame, then a further factor of $1 + z_{\text{motion}}$ multiplied into the RHS of Eq. (12.24).

- Define the Hubble expansion parameter H(t) as :
  $$H(t) = \frac{\dot{R}(t)}{R(t)}. \quad (12.22)$$
  Ex) If $R(t) = At^\alpha$, then $H(t) = \frac{\alpha}{t}$
  Note) At the present time t_0 the proper distance of the galaxy at the comoving coordinate χ from the origin (in the constant-time hypersurface) is :
  $$d_0 = R(t_0)\chi. \quad (12.23)$$
- Claim : the Hubble parameter $H(t)$ is the instantaneous relative rate of expansion :
  Proof) Differentiate (12.23), $v = \dot{d}_0 = \dot{R}(t_0)\chi = \dot{R}(t_0)\frac{d_0}{R_0} = \left(\frac{\dot{R}}{R}\right)_0 d_0 = H_0 d_0$ hence,
  $$v = \left(\frac{\dot{R}}{R}\right)_0 d_0 = H_0 d_0 \quad (12.24)$$
  where H_0 is the present value of the Hubble parameter. By comparison with Eq. (12.1), we see that this is just the present value of the Hubble parameter $\left(\frac{\dot{R}}{R}\right)$.
- We show in Exer. 10, § 12.6 that this velocity is just v = z, provided the galaxy is not far away.
  In our cosmological neighborhood, therefore, the cosmological redshift is a true Doppler shift.
  Moreover, the redshift is proportional to proper distance in our neighborhood, with the Hubble constant as the constant of proportionality.

- Redshift dependance of various distance measures.
  Integrating Eq. (12.22) $H(t) = \frac{\dot{R}(t)}{R(t)}$, the scale factor of the Universe $R(t)$ is given by
  $$R(t) = R_0 \exp\left[\int_{t_0}^t H(t')dt'\right]. \quad (12.25)$$
  Ex) If $H(t) = \frac{\alpha}{t}$. then,
  $$R(t) = R_0 \exp\left[\int_{t_0}^t H(t')dt'\right] = R_0 \exp\left[\int_{t_0}^t \frac{\alpha}{t'}dt'\right] = R_0 e^{\alpha \ln\frac{t}{t_0}} = R_0\left(\frac{t}{t_0}\right)^\alpha$$

(Take $R_0 = At_0{}^{\alpha}$)

$= At^{\alpha}$

Ex) If $H(t) = H_0 = const,$ then, $R(t) = R_0 \exp H_0(t - t_0)$

The Taylor expansion

- $R(t)$ in terms of $(t - t_0)$

$$R(t) = R_0\left[1 + H_0(t-t_0) + \frac{1}{2}\frac{\ddot{R}_0}{R_0}(t-t_0)^2 + \cdots\right]$$
$$= R_0\left[1 + H_0(t-t_0) - \frac{1}{2}q_0 H_0{}^2(t-t_0)^2 + \cdots\right]$$
$$= R_0\left[1 + H_0(t-t_0) + \frac{1}{2}(H_0{}^2 + \dot{H}_0)(t-t_0)^2 + \cdots\right], \quad (12.26)$$

Or,

$$a(t) = \underbrace{a(t_0)}_{1} + \underbrace{\dot{a}(t_0)}_{=\frac{\dot{a}_0}{a_0}a_0 = H_0}(t-t_0) + \frac{1}{2}\underbrace{\ddot{a}(t_0)}_{=-q_0 H_0{}^2}(t-t_0)^2 + \cdots$$

∴

$$a(t) = 1 + H_0(t-t_0) - \frac{1}{2}q_0 H_0{}^2(t-t_0)^2 + \cdots$$

Note) $\dot{H}_0 = \frac{\ddot{R}_0}{R_0} - H_0{}^2$

$$-q_0 \equiv \frac{R_0\ddot{R}_0}{\dot{R}_0{}^2} = \frac{\ddot{R}_0}{R_0 H_0{}^2} = (1 + \frac{\dot{H}_0}{H_0{}^2}). \text{ deceleration parameter} \quad (12.27)$$

Note) $\dot{H}_0$ can be converted to q_0 and vice-versa.

Note) $q_0 \approx -0.5$ acceleration

Ex) If $R(t) = At^{\alpha}$, then $H(t) = \frac{\alpha}{t}, \dot{H}_0 = -\frac{\alpha}{t_0{}^2}, \; q_0 = -\left(1 - \frac{\frac{\alpha}{t_0{}^2}}{\left(\frac{\alpha}{t_0}\right)^2}\right) = -(1 - \frac{1}{\alpha})$

- the universe is accelerating, so the minus sign in the definition and the name 'deceleration parameter' reflecting the assumption that gravity would be slowing down the expansion has gone out of fashion.

- $z(t)$, $R(t)$ vs $(t - t_0)$

Combining Eq. (12.25) with Eq. (12.21) we get

$$1 + z(t) = \frac{R_0}{R(t)} = \exp\left[-\int_{t_0}^{t} H(t')dt'\right] = \exp\left[\int_{t}^{t_0} H(t')dt'\right]. \quad (12.28)$$

Ex) If $H(t) = \frac{\alpha}{t}$. then, $R(t) = At^{\alpha}$, $1+z(t) = \left(\frac{t_0}{t}\right)^{\alpha} = t_0{}^{\alpha}t^{-\alpha}$

The Taylor expansion (Let $f(t) \equiv \exp\left[\int_t^{t_0} H(t')dt'\right]$)

Then $f(t_0) = 1, \dot{f}(t) = \exp\left[\int_t^{t_0} H(t')dt'\right](-H(t)),\; \ddot{f}(t) = \exp\left[\int_t^{t_0} H(t')dt'\right](H^2(t) - \dot{H}(t))$

We get,

$$z(t) = \frac{1}{a(t)} - 1 = H_0(t_0 - t) + \frac{1}{2}(H_0{}^2 - \dot{H}_0)(t_0 - t)^2 + \cdots. \quad (12.29)$$

This is not directly useful yet, since we have no independent information about the time t at which a galaxy emitted its light.

- Look-back time $t_0 - t(z)$ in terms of the redshift z

Invert the series Eq. (12.29) to give an expansion for the look-back time to an event with redshift z:

$$t_0 - t(z) = H_0{}^{-1}\left[z - \frac{1}{2}\left(1 - \frac{\dot{H}_0}{H_0}{}^2\right)z^2 + \cdots\right]. \quad (12.30)$$

- $H(z)$ in terms of z

From the simple expansion

$$H(t) = H_0 + \dot{H}_0(t - t_0) + \cdots$$

substitute in the first term of the previous equation and get an expansion for H as a function of z:

$$H(z) = H_0\left(1 - \frac{\dot{H}_0}{H_0{}^2}z + \cdots\right). \quad (12.31)$$

Note) Eq. (12.28) can also be inverted to give the exact and very simple relation

$$H(t) = -\frac{\dot{z}}{1+z}. \quad (12.32)$$

(Newtonian description)

- We imagine a spherical region uniformly filled with galaxies, starting at some time with radially outward velocities that are proportional to the distance from the center of the sphere.

- If we are not near the edge then we can show that the expansion is homogeneous and isotropic about every point.
- The galaxies just fly away from one another, and the Hubble constant is the scale for the initial velocity: it is the radial velocity per unit distance away from the origin.
- The problem with this Newtonian model is not that it cannot describe the local state of the universe, it is that, with gravitational forces that propagate instantaneously, the dynamics of any bit of the universe depends on the structure of this cloud of galaxies arbitrarily far away.
- Only in a relativistic theory of gravity can we make sense of the dynamical evolution of the universe.

- When light is redshifted, it loses energy. Where does this energy go?
  - The fully relativistic answer is that it just goes away: since the metric depends on time, there is no conservation law for energy along a geodesic.
  - Interestingly, in the Newtonian picture of the universe just described, the redshift is just caused by the different velocities of the diverging galaxies relative to one another. As the photon moves outward in the expanding cloud, it finds itself passing galaxies that are moving faster and faster relative to the center. It is not surprising that they measure the energy of the photon to be smaller and smaller as it moves outwards.
- Comoving distance χ in terms of the look-back time $(t_0 - t)$ or the redshift z

$$R_0\chi(t) = R_0\int_0^{\chi(t)} d\chi = c\int_t^{t_0}\frac{dt}{a(t)},\quad a(t) = 1 + H_0(t-t_0) - \frac{1}{2}q_0{H_0}^2(t-t_0)^2 + \cdots$$

$$= c\int_t^{t_0}[1 - H_0(t_0 - t) + \cdots]dt$$

$$= c(t-t_0)[1 + H_0(t-t_0) + \cdots]$$

(Using Eq.12-30 $t_0 - t(z) = {H_0}^{-1}\left[z - \frac{1}{2}\left(1 - \frac{\dot{H}_0}{H_0}^2\right)z^2 + \cdots\right]$)

$$= c{H_0}^{-1}\left[z - \frac{1}{2}(1+q_0)z^2 + \cdots\right]$$

Particle Horizon - The Size of the Observable Universe

In co-moving coordinates, the greatest distance $r_{\max}$ that we can see is the distance that light has travelled since the Big Bang. Light path : $ds^2(t) = R^2(t)\left[-d\tau^2 + d\chi^2 + {S_k}^2(\chi)d\Omega^2\right]$ ⇝ $0 = -d\tau^2 + d\chi^2$

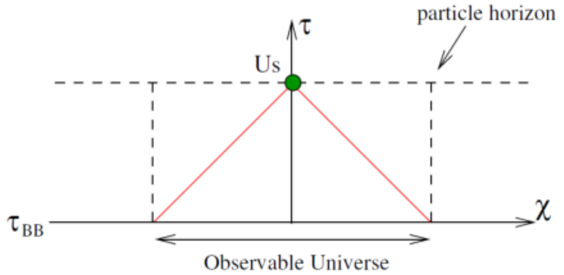
Then,

$$c\tau = c\int_0^{\tau}d\tau' = c\int_{t_{BB}}^{t}\frac{dt}{R(t)} = \int_0^{r_{\max}(t)}\frac{dr}{\sqrt{1-kr^2}} = \int_0^{\chi_{\max}(t)}d\chi = \chi_{\max}(t)$$

The particle horizon $d_H(t)$, the maximum size of the observable universe is then given by
The corresponding physical distance

$$d_H(t) = a(t)\chi_{\max}(t) = a(t)\int_0^{r_{\max}(t)}\frac{dr}{\sqrt{1-kr^2}} = ca(t)\int_{t_{BB}}^{t}\frac{dt'}{a(t')} = a(t)c\tau$$

Note that this size $\neq c(t - t_{BB})$= the naive distance that light has travelled since the Big Bang.
If the integral does not converge at t_{BB}, in which case the maximum distance $r_{\max}(t)$ would be infinite.
Nothing outside the particle horizon can influence us today.



Ex) $a(t) = \left(\frac{t}{t_0}\right)^n \quad (0 \le n < 1)$

$$\rightsquigarrow \int_0^t \frac{dt'}{a(t')} = (t_0)^n\int_0^t\frac{dt'}{t'^n} = \frac{t}{1-n}\left(\frac{t_0}{t}\right)^n = \frac{t}{1-n}\frac{1}{a(t)}$$

$$d_H(t) = ca(t)\int_{t_{BB}}^{t}\frac{d}{a(t')} = \frac{1}{1-n}ct$$

The (cosmological) Event Horizon

The particle horizon tells us that there are parts of the universe that we cannot presently see.
Can, as time progresses, all of spacetime comes into view? In fact, this need not be the case.
Examples

1) the collapsing universe in the future
there is a second time $t_{BC} > t_0$ where $a(t_{BC}) = 0$. This is referred to as the Big Crunch.
The limit on how far we can communicate before the universe comes to an end is given by

$$c\int_t^{t_{BC}} \frac{dt'}{a(t')} = \int_0^{r_{\max}(t)} \frac{dr}{\sqrt{1-kr^2}} = \int_0^{\chi_{\max}(t)} d\chi = \chi_{\max}(t)$$

2) Even if the universe continues to expand and the FRWmetric holds for $t \to \infty$, then there could still be a maximum distance that we can influence.
The relevant equation is now

$$c\int_t^{\infty} \frac{dt'}{a(t')} = \int_0^{r_{\max}(t)} \frac{dr}{\sqrt{1-kr^2}} = \int_0^{\chi_{\max}(t)} d\chi = \chi_{\max}(t)$$

The maximum co-moving distance $r_{\max}(t)$ is finite provided that the left-hand side converges.
Ex) $a(t) \sim e^{Ht}$ as $t \to \infty$. This seems to be the most likely fate of our universe.

Note) Let $a(t) \sim t^n \sim t^{\frac{2}{3(1+w)}}$
Then,

$\dot{a}(t) \sim n t^{n-1}$,

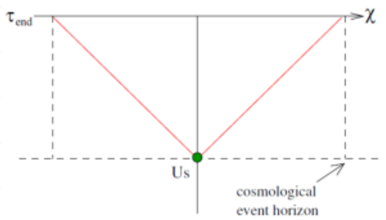
$\ddot{a}(t) \sim n(n-1)t^{n-2}$

w	$w < -1$	$\pm\infty$	1	$1/3$	0	$-1/3$	$-2/3$	-1
n	$n < 0$	0	$1/3$	$1/2$	$2/3$	1	2	∞
$a(t)$	↘	↤↦		↗				
$\dot{a}(t)$	$-$↘	↤↦		$+$↘		↤↦	$+$↗	
$\ddot{a}(t)$	$+$	↤↦		$-$		↤↦	$+$	

And,

$$\int_t^{\infty} \frac{dt'}{a(t')} \sim t^{1-n}$$

As Schroedinger described, it is quite possible that two friends could move apart from each other, only to find that they've travelled too far and can never return.



In this context, the distance $r_{\max}(t)$ is called the (co-moving) **cosmological event horizon**.
We cannot influece to the events beyond the event horizon.

the analogy with the black hole.
Regions beyond the cosmological horizon are beyond our reach; if we choose to sit still, we will never see them and never communicate with them.
Important distinctions- In contrast to the event horizon of a black hole, the concept of cosmological event horizon depends on the choice of observer.

Cosmography: measures of distance in the universe

- Cosmography refers to the description of the expansion of the universe and its history.
- In cosmography we do not yet apply the Einstein equations, we simply use distance measures and the evolution of the Hubble parameter.
  By analogy with Eq. (12.1), we would like to replace t in Eq. (12.29) with distance.

○ But what measure of distance is suitable over vast cosmological separations?

- **comoving coordinate distance** : it would be unmeasurable.
  1) Comoving proper distance ($dt = 0$) from the origin to the object at comoving coord r.

$$d_{comoving} = \int_0^r \frac{dr}{\sqrt{1-kr^2}}$$
$$= \begin{cases} \sin^{-1} r \\ r \\ \sinh^{-1} r \end{cases}$$

- **The physical (proper) distance**

$$d_{phs} = a(t)\, d_{comoving}$$

is not good either

  - The proper distance between the events of emission and reception of the light is zero, since light travels on null lines.
  - The proper distance between the emitting galaxy and us at the present time is also unmeasurable: in principle, the galaxy may not even exist now, perhaps because of a collision with another galaxy.

○ Distances to nearby galaxies are almost always inferred **from luminosity measurements**.

- Consider an object at rest, and near enough. The relation for flux F, luminosity L & the distance d is

$$L = 4\pi d^2 F \quad \text{or} \quad F = \frac{L}{A}, \ A = 4\pi d^2 \quad (12.33)$$

- If L is known, then a measurement of F leads to the distance d= $(L/4\pi F)^{1/2}$.
- Astronomers have used brightness measurements to build up a carefully calibrated cosmological distance ladder to measure the scale of the universe.
- For each step on this ladder they identify what is called **a standard candle**, which is a class of objects whose absolute luminosity L is known (say from a theory of their nature or from reliably calibrated distances to nearby examples of this object).
- The distance ladder starts at the nearest stars, the distances to which can be measured by parallax (independently of luminosity), and continues all the way to very distant high-redshift galaxies.

I. **Define the luminosity distance d_L** to any object, no matter how distant, by inverting Eq. (12.33):

$$d_L = \left(\frac{L}{4\pi F}\right)^{1/2}. \quad \left(\text{Note } A = 4\pi d_L^2 = \frac{L}{F} \text{ or } F = \frac{L}{A} = \frac{L}{4\pi d_L^2}\right) \text{ at rest} \quad (12.34)$$

- In FWR metric

$$ds^2(t) = -dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] = -dt^2 + R^2(t)\left[d\chi^2 + S_k^2(\chi) d\Omega^2\right]$$

the proper area of the sphere

$$A = R^2(t) r^2(t) \oiint d\Omega^2 = 4\pi R^2(t) r^2(t) = 4\pi R^2(t) S_k^2 \equiv 4\pi d_L^2$$

or $d_L = R(t) r(t) = R(t) S_k(t)$

- if the intrinsic luminosity L is known or can be inferred, then a measurement of its brightness F determines the luminosity distance.
- The luminosity distance is the proper distance of the object in a Euclidean space if it were at rest.

○ **The luminosity distance d_L in an expanding cosmology** ($\neq$ the proper distance to the object)

$$d_L = \left(\frac{L}{4\pi F}\right)^{1/2} = R_0 r(1+z)$$

(Derivation)

- Consider an object emitting with luminosity L at a time t_e.
- Suppose only photons of frequency ν_e at time t_e.
  (This frequency drops out in the end, so our result is perfectly general.)
- We place the object at the origin of the coordinate system, and suppose that we sit at coordinate position r in this system, as given in Eq. (12.13).

- # of photons the object emits in a small time interval δt_e (in a spherically symmetric manner).

$$N = L\delta t_e / h\nu_e \ . \quad (12.35)$$

- In other words, the luminosity L (or the emitted energy /time $= E_e/\delta t_e$) in terms of the # of photons N

$$L = \frac{E_e}{\delta t_e} = \frac{N h\nu_e}{\delta t_e} \quad (E_e = N h\nu_e)$$

- the proper area of the sphere when the photons reach our coordinate distance

$$A = R_0^2 r^2 \oiint d\Omega^2 = 4\pi R_0^2 r^2. \ (12.36)$$

- Now, the photons have been redshifted by the amount (1 + z) = R_0/R(t_e) to frequency ν_0:
  hν_0 = hν_e/(1 + z). (12.37)
- Moreover, they arrive spread out over a time δt_0, which is also stretched by the redshift:
  δt_0 = δt_e(1 + z). (12.38)
- The total energy E_0 received during δt_0 (4π angle) $E_0 = Nh\nu_0 = Nh\nu_e/(1+z) = \frac{E_e}{1+z}$

  Or, the Power received by whole area A $P_0 = \frac{E_0}{\delta t_0} = \frac{E_e/(1+z)}{\delta t_e(1+z)} = \frac{L}{(1+z)^2}$
- The energy flux at the observation time t_0 is thus Nhν_0/(Aδt_0), from which it follows that

$$F = \frac{P_0}{A} = \frac{L}{A(1+z)^2} = \frac{L}{4\pi R_0{}^2 r^2 (1+z)^2}$$

$$\rightsquigarrow d_L = \left(\frac{L}{4\pi F}\right)^{1/2} = R_0 r\,(1+z) = R(t_e) r (1+z)^2$$

  Or

$$F = E_0/(A\delta t_0) = Nh\nu_0/(A\delta t_0) = \frac{Nh\nu_e}{1+z}/(A\delta t_e(1+z))$$

$$= \frac{L}{A(1+z)^2} = \frac{L}{4\pi R_0{}^2 r^2 (1+z)^2}. \quad (12.39)$$

  The luminosity distance from Eq. (12.34) & (12.39)

$$d_L = \left(\frac{L}{4\pi F}\right)^{1/2} = R_0 r(1+z) = R(t_e) r(1+z)^2. \quad (12.40) \text{ QED}$$

○ **the comoving coordinate r as a function of the redshift z**

▪ the photon trajectory (use Eq. (12.13) : ds^2 = 0 (a photon world line) and $d\Omega^2$ = 0 (photon traveling on a radial line).
  leads to the differential equation

$$\frac{dr}{(1-kr^2)^{1/2}} = -\frac{dt}{R(t)} = \frac{dz}{R_0 H(z)}, \quad (12.41)$$

  where the last step follows from differentiating Eq. (12.21) $1+z = R(t_0)/R(t)$.
- for small r and z the curvature parameter k will come into the solution only at second order. Ignoring k and working only to first order beyond the Euclidean relations, we get

$$d_L = R_0 r(1+z) = \left(\frac{cz}{H_0}\right)\left[1 + \left(1 + \frac{1}{2}\frac{\dot{H}_0}{H_0{}^2}\right)z\right] + \cdots$$

$$= \left(\frac{cz}{H_0}\right)\left[1 + \frac{1}{2}(1 - q_0)z\right] + \cdots. \quad (12.42) \text{ (k=0)}$$

- If we can measure the luminosity distances d_L and redshifts z of a number of objects, then we can in principle measure H_0 & $\dot{H}_0$.
- Measurements of this kind led to the discovery of the accelerating expansion of the universe (below).

II. **the angular diameter distance**.
- in a Euclidean space the angular size θ of an object at a distance d can be inferred if we know the proper diameter D of the object transverse to the line of sight,
  θ = D/d.
- This leads to the definition of
  the **angular diameter distance d_A** :
  d_A = D/θ. (12.43)

  The dependence of d_A on redshift z is

$$d_A = R_e r = (1+z)^{-2} d_L$$

$$= R_0 r(1+z)^{-1}, \quad (12.44)$$

  where R_e is the scale factor of the universe when the photon was emitted. The analogous expression to Eq. (12.42) is

$$d_A = R_0 r/(1+z) = \left(\frac{z}{H_0}\right)\left[1 + \left(-1 + \frac{1}{2}\frac{\dot{H}_0}{H_0{}^2}\right)z\right] + \cdots. \quad (12.45)$$

▪ There are situations where we have an estimate of the comoving diameter D of an emitter.
  the temperature irregularities in maps of the cosmic microwave background radiation have a length scale that is determined by the physics of the early universe.
▪ Observing the objects with very high redshifts.
  ▪ Some galaxies and quasars are known at redshifts greater than z = 6.
  ▪ The CMB originated at redshift z ~ 1100, and is our best tool for understanding the Big Bang.
  ▪ Even so, the universe was already some 300 000 years old at that redshift. Gravitational wave detectors may detect random radiation from the Big Bang itself, originating when the universe was only a fraction of a second old.
- in the extension of the nonrelativistic formula v = Hd into relativistic language, we were forced to re-think the meaning of all the terms in the equations and to go back to the quantities we can directly measure. :

The universe is accelerating!

- the expansion of the universe is not slowing down, but rather speeding up.
- This was done by essentially making a plot of the luminosity distance against redshift, but where luminosities are given in magnitudes. This is called the magnitude–redshift diagram, and we derive its low-z expansion in Exer. 13, § 12.6.
- Two teams of astronomers, called the High-Z Supernova Search Team (Riess et al., 1998) and the Supernova Cosmology Project (Perlmutter et al., 1999), respectively, used supernova explosions of Type Ia as standard candles out to redshifts of order 1. The data from the High-Z Team are shown in Fig. 12.3. See Filippenko (2008) for a full discussion.
- The top diagram shows the flux (magnitude) measurement for each of the supernovae in the sample, along with error bars. The trend seems to curve upwards, meaning that at high redshifts the supernovae are dimmer than expected. This would happen if the universe were speeding up, because the supernovae would simply be further away than expected.
- Three possible fits are shown, and the best one has a large positive cosmological constant, which we shall see below is the simplest way, within Einstein's equations, that we can accommodate acceleration.
- The lower diagram shows the same data but plotting only the residuals from the fit to a flat universe. This shows more clearly how the data favor the curve for the accelerating universe.
- These studies were the first strong evidence for acceleration, but by now there are several lines of investigation that lead to the same conclusion.
- Gravity is universally attractive. If the energy density of the universe exerts attractive gravity, the expansion should be slowing down. Instead it speeds up.

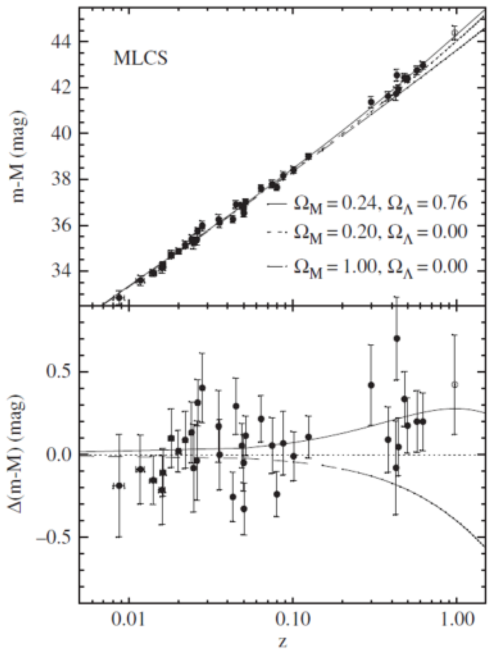
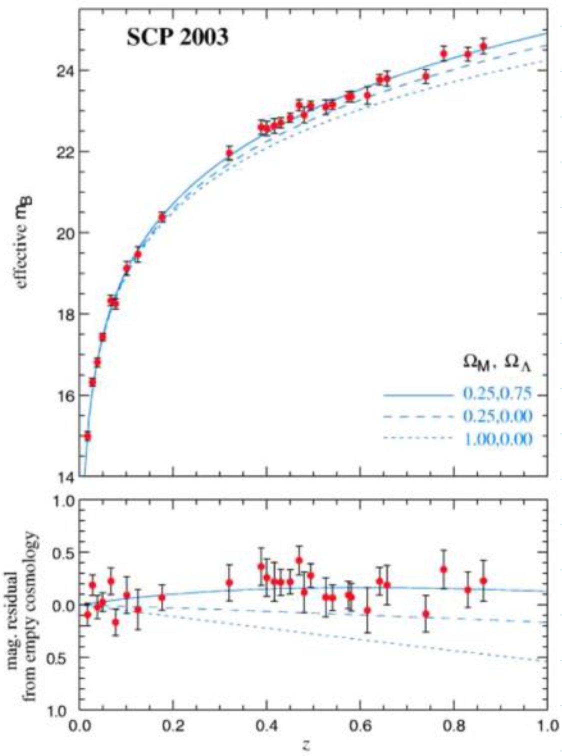


Figure 12.3 The trend of luminosity versus redshift for Type Ia supernovae is fit best with an accelerating universe. The lower part of this curve determines H0, the upper part demonstrates acceleration. (High -Z Supernova Search Team: Riess, et al, 1998.)

- What can be the cause of this repulsion? ⟶ Dark Energy!



12.3 Cosmological dynamics : understanding the expanding universe

We will study Einstein's equations, with relatively simple perfect-fluid physics and with the cosmological constant that seems to be implied by the expansion.

Dynamics of Robertson–Walker universes: Big Bang and dark energy

- **(Metric)**
  - A homogeneous & isotropic universe is described by the Robertson–Walker metrics given by Eq. (12.13).

$$ds^2(t) = -dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]. \quad (12.13)$$ R(t) : the scale factor

  - Depending on just one function, R(t), which will be determined by the Einstein's equations.

- **(the stress-energy tensor)**
  - We idealize the universe as filled with a homogeneous perfect fluid.
  - The fluid must be at rest in the preferred cosmological frame, for otherwise its velocity would allow us to distinguish one spatial direction from another: the universe would not be isotropic.
  - Therefore, the stress-energy tensor will take the form of Eq. (4.36) in the cosmological rest frame.

$$T^{\alpha\beta} = (\rho c^2 + p)U^\alpha U^\beta + p\,\eta^{\alpha\beta} \quad \text{at rest} \quad \mathbb{U}_0 = (1, \vec{0}) \text{ gives } T_0{}^{\alpha\beta} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

  - Because of homogeneity, all fluid properties depend only on time:
    ρ = ρ(t), p = p(t), etc.

the equation of motion for matter

$$T^{\mu\nu}{}_{;\nu} = 0$$

which follows from the Bianchi identities of Einstein's field equations.

Because of isotropy, only the time component μ = 0 is nonzero, giving (see Exer. 14, § 12.6)

$$\frac{d}{dt}(\rho R^3) = -p\frac{d}{dt}(R^3). \quad (12.46)$$

Note) the LHS = the rate of change of its total energy in a volume,
the RHS = the work it does as it expands (−p d V).

Eq. of state

$$p = w\rho c^2$$

Or,

$$\dot{\rho}c^2 + 3H(\rho c^2 + p) = 0 \;(\rightsquigarrow\; \dot{\rho} + 3(1+w)\rho = 0)$$

Equation of state

$p = w\rho$

w :	-1	$-1/3$	0	$+\frac{1}{3}$
phantom	cos const	curvature	nonrel	relativistic

Two simple cases, a matter-dominated cosmology and a radiation dominated cosmology.

In a matter-dominated era,
The main energy density of the cosmological fluid is in cold nonrelativistic matter particles, which have random velocities that are small and which therefore behave like dust: p = 0.
So we have

Matter-dom : $\frac{d}{dt}(\rho R^3) = 0. \rightsquigarrow \quad \rho \sim R^{-3}$ (12.47)

In a radiation-dominated era,
the principal energy density of the cosmological fluid is in radiation or hot, highly relativistic particles, which have an equation of state p = 1/3 ρ (Exer. 22, § 4.10). Then we get

Radiation-dom : $\frac{d}{dt}(\rho R^3) = -\frac{1}{3}\rho\frac{d}{dt}(R^3),$ (12.48)

or

Radiation-dom :$\frac{d}{dt}(\rho R^4) = 0. \rightsquigarrow \quad \rho \sim R^{-4}$ (12.49)

In general,

$$\frac{d}{dt}(\rho R^3) = -w\rho\frac{d}{dt}(R^3) \rightsquigarrow \frac{d\rho}{\rho} + (1+w)\frac{dR^3}{R^3} = 0 \rightsquigarrow \rho R^{3(1+w)} = const$$
$$\rightsquigarrow \rho \sim R^{-3(1+w)}$$

Or,

$$\dot{\rho} + 3(1+w)H\rho = 0 \rightsquigarrow \frac{\dot{\rho}}{\rho} + 3(1+w)\frac{\dot{R}}{R} = 0 \quad \rightsquigarrow \rho R^{3(1+w)} = const$$

w :		-1	$-1/3$	0	$+\frac{1}{3}$
$a(t) \sim t^{\frac{2}{3(1+w)}}$	$t^{-\#}$	$t^{\infty} \sim e^{Ht}$	t^1	$t^{2/3}$	$t^{1/2}$
$\rho \sim R^{-3(1+w)}$	$R^{+\#}$	R^{-0}	R^{-2}	R^{-3}	R^{-4}
	phantom	cos const Dark Energy	curvature	nonrel Matter	relativistic Radiation

The Einstein equations $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^2}T_{\mu\nu}/c^2$

- Isotropy will guarantee that
  G_{tj} = 0 for all j,
  and also that
  $G_{jk} \propto g_{jk}$.
- only two components are independent,
  G_{tt} and (say) G_{rr}.
- But the Bianchi identity will provide a relationship between them, which we have already used in deriving the matter equation in the previous paragraph. (The same happened for the spherical star.)
  Therefore we only need compute one component of the Einstein tensor (see Exer. 16, § 12.6):
  $G_{tt} = 3(\dot{R}/R)^2 + 3k/R^2.$ (12.50)
  Therefore, besides Eqs. (12.47) or (12.49), we have only one further equation,
  the Einstein equation with cosmological constant Λ
  $G_{tt} + \Lambda g_{tt} = \frac{8\pi G}{c^2}T_{tt}/c^2.$ (12.51)
  The cosmological constant can be given the notation
  $T_{\Lambda}^{\alpha\beta}/c^2 = -(\frac{\Lambda c^2}{8\pi G})g^{\alpha\beta}$. (12.52)
  the energy density and pressure of the cosmological constant 'fluid' are
  $\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}, \quad p_\Lambda = -\rho_\Lambda c^2$. (12.53)

the dark energy ρ_Λ ,
- is expected to be positive, $\Lambda \geq 0$. while its associated dark pressure p_Λ has the opposite sign.
- an energy that is not associated with any known matter field.
- As the universe expands, the dark energy density (and dark pressure) remain constant.

In these terms the tt-component of the Einstein equations can be written

$$\boxed{\frac{1}{2}\dot{R}^2 = -\frac{1}{2}k + \frac{4\pi}{3}R^2(\rho_{\text{matt}} + \rho_\Lambda),} \quad (12.54)$$

where ρ_{matt} for the energy density of the matter (including radiation).

Note) the observed **acceleration of our universe**.
- It appears (see below) that k = 0, or at least that the k-term is negligible.
- the term $R^2\rho_{\text{matt}}$ decreases as R increases, while the term $R^2\rho_\Lambda$ increases rather strongly.
- At present, $\rho_\Lambda > \rho_{\text{matt}}$, and the result is that $\dot{R}$ increases as R increases.
- This trend must continue now forever, provided the acceleration is truly propelled by a cosmological constant, and not by some physical field that will go away later.

How, physically, can a positive dark energy density drive the universe into accelerated expansion? Is not positive energy gravitationally attractive, so would it not act to slow down the expansion?

- To answer this it is helpful to look at the **spatial part of Einstein's equations**.
- it follows from the two basic equations Eq. (12.46) and the time-derivative of Eq. (12.54).
  the combination of these two equations implies the following
  simple 'equation of motion' for the scale factor:
  $\frac{\ddot{R}}{R} = -\frac{4\pi}{3}(\rho + 3p)$, (12.55)
  where ρ & p are the total energy density & pressure, including both the normal matter & the dark energy.
- The acceleration is produced, not by the energy density alone, but by ρ + 3p.
  We have met this combination before, in Exer. 20, § 8.6, where we showed that the source of the far-away Newtonian field is ρ + 3p(the active gravitational mass), not just ρ.
  In the cosmological context, the same combination generates the cosmic acceleration.
- the negative pressure associated with the cosmological constant can make this sum negative, driving the universe faster and faster.
- Einstein's gravity with a cosmological constant has a kind of in-built anti-gravity!

- Notice that a negative pressure is not by any means unphysical.
- Negative stress is called tension, and in a stretched rubber band, for example, the component of the stress tensor along the band is negative.
- Interestingly, our analogy using a balloon to represent the expanding universe also introduces a negative pressure, the tension in the stretched rubber.
  What is remarkable about the dark energy is that its tension is so large, and it is isotropic.
  See Exer. 18, § 12.6 for a further discussion of the tension in this 'fluid'.

The dynamics of R

For Eq. (12.54),

$\frac{1}{2}\dot{R}^2 = -\frac{1}{2}k + \frac{4\pi}{3}R^2(\rho_m + \rho_\Lambda)$, (12.54)

the LHS looks like a 'kinetic energy' and the RHS contains a constant (−k/2) that plays the role of the 'total energy' and a potential term proportional to $R^2(\rho_m + \rho_\Lambda)$.

The dynamics of R will be constrained by this energy equation.

Or,

$$\frac{1}{2}H^2(t) = -\frac{kc^2}{2R^2} + \frac{4\pi G}{3}(\rho_m + \rho_\Lambda) = \frac{4\pi G}{3}(\rho_m + \rho_k + \rho_\Lambda)$$

$$H^2(t) = -\frac{kc^2}{R^2} + \frac{\Lambda c^2}{3} + \frac{8\pi G}{3}\rho_m = -\frac{kc^2}{R^2} + \frac{8\pi G}{3}(\rho_m + \rho_\Lambda) = \frac{8\pi G}{3}(\rho_m + \rho_k + \rho_\Lambda)$$

where

H = $\dot{R}$/R the Hubble parameter

$\rho_k = -\frac{3k}{8\pi R^2} = \frac{3}{8\pi G}(-\frac{kG}{R^2})$ $\boldsymbol{\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}}$

- In the far distant future : (Let $\rho_\Lambda \geq 0$)
  - (k = −1)
    if ρ_Λ > 0, Λ-dominating era; if ρ_Λ = 0, k-dominant era
    $\rho_\Lambda + \rho_k$ > 0 an expanding hyperbolic universe will never stop expanding.
  - (k = 0) For the flat universe ,
  - if ρ_Λ > 0, an expanding universe will also never stop ;
  - if ρ_Λ = 0, then matter dominating era!
    it could asymptotically slow down to a zero expansion rate as R approaches infinity, since the matter density will decrease at least as fast as R^{-3}.
  - (k = 1)
    An expanding closed universe
  - If ρ_Λ = 0, matter-dominat era (expansion)
    reach a maximum expansion radius and then re-collapse, eventually reaches another singularity, called the Big Crunch!
  - But if ρ_Λ > 0, then the ultimate fate of an expanding closed universe depends on the balance of ρ_m and ρ_Λ.
- a Big Bang(the scale factor R =0) at a finite time in the past?
  - Eq. (12.54) shows that, as R gets smaller, the matter term gets more and more important compared to the curvature term−k/2.
  - Therefore, $\dot{R} \neq 0$ at any time in the past since our universe is expanding now.
  - The existence of a Big Bang, i.e. whether we reach R = 0 at a finite time in the past,
  - depends only on the behavior of the matter;
  - the curvature term is not important,
  - and all three kinds of universes have qualitatively similar histories.

- **a radiation-dominated universe** with Λ = 0 for simplicity.
  We write ρ = BR^{-4} for some constant B. Neglecting k in Eq. (12.54) gives
  $$\dot{R}^2 = \frac{8}{3}\pi B R^{-2},$$
  or
  $$\frac{dR}{dt} = \left(\frac{8}{3}\pi B\right)^{1/2} R^{-1}. \quad (12.56)$$
  This has the solution

$R^2 = \left(\frac{32}{3}\pi B\right)^{\frac{1}{2}}(t-T)$, (12.57)

where T is a constant of integration.

So, indeed, R = 0 was achieved at a finite time in the past,

and we adjust our zero of time so that R = 0 at t = 0, i.e., T = 0, hence,

an expansion rate where

R(t) $\propto t^{1/2}$.

- **for a matter-dominated cosmology** with ρ_Λ = 0,
  we find (see Exer. 19,§ 12.6)
  R(t) $\propto t^{2/3}$.

- If $\rho_\Lambda > 0$, there is no qualitative change in the conclusion,
  since the term involving ρ_Λ simply increases the value of $\dot{R}$ at any value of R, and this brings the time where R = 0 closer to the present epoch.
- If ρ_m>0 and if $\Lambda \geq 0$, then Einstein's equations make the Big Bang inevitable:
  the universe began with R = 0 at a finite time in the past.
  the cosmological singularity: the curvature tensor is singular, tidal forces become infinitely large, and Einstein's equations do not allow us to continue the solution to earlier times. Within the Einstein framework we cannot ask questions about what came before the Big Bang: time simply began there.
- How certain, then, is our conclusion that the universe began with a Big Bang?
- First, we must ask if isotropy and homogeneity were crucial; the answer is no.
  The 'singularity theorems' of Penrose and Hawking (see Hawking and Ellis 1973) have shown that our universe certainly had a singularity in its past, regardless of how asymmetric it may have been. But the theorems predict only the existence of the singularity: the nature of the singularity is unknown, except that it has the property that at least one particle in the present universe must have originated in it. Nevertheless, the evidence is strong indeed that we all originated in it.
- The singularity theorems of necessity assume (1) something about the nature of Tμν, and (2) that Einstein's equations (without cosmological constant) are valid at all R.
- The assumption about the positivity of the energy density of matter can be challenged if we allow quantum effects. Fluctuations can create negative energy for short times.
- In principle, therefore, our conclusions are not reliable if we are within one
  Planck time t_{Pl}= G$M_{\text{Pl}}/c^2 \sim 10^{-43}$s of the Big Bang!
  (Recall the definition of the Planck mass in Eq. (11.111).)
  This is the domain of quantum gravity, governing the universe before the Big Bang.
- Philosophically satisfying as this might be, it has little practical relevance to the universe we see today. We might not be able to start our universe model evolving from t = 0, but we can certainly start it from, say, t = 100t_{Pl} within the Einstein framework.
  The primary uncertainties about understanding the physical cosmology that we see around us are, as we will discuss below, to be found in the physics of the early universe, not in the time
  immediately around the Big Bang.

- So far we have restricted our attention to the case of a positive cosmological constant.
  Cosmologies with negative cosmological constant left to the exercises.
- Einstein introduced the cosmological constant in order to allow his equations to have a static solution, $\dot{R}$ = 0.
  He did not know about the Hubble flow at the time, and he followed the standard assumption of his day that the universe was static.
  Even in the framework of Newtonian gravity, this would have presented problems.
  We have to do more than just set $\dot{R}$ = 0 in Eq. (12.54); we have to guarantee that the solution is an equilibrium one, that the dynamics won't change $\dot{R}$, i.e. that the universe is at a minimum or maximum of the 'potential' we discussed earlier.
- We show in Exer. 20,§ 12.6 that the static solution requires
  $\rho_\Lambda = \frac{1}{2}\rho_0$.
  For Einstein's static solution, the dark energy density has to be exactly half of the matter energy density. We shall see below that in our universe the measured value of the dark energy density is about twice that of the matter energy density, so we are near to but not exactly at Einstein's static solution.

In general,

$$H^2(t) = \frac{8\pi}{3}\rho_w \sim R^{-3(1+w)} \rightsquigarrow \dot{R} \sim R^{-(1+3w)/2} \rightsquigarrow dR R^{(1+3w)/2} \sim dt \rightsquigarrow R^{\frac{3(1+w)}{2}} \sim t$$

$$\rightsquigarrow R \sim t^{\frac{2}{3(1+w)}}$$

Critical density and the parameters of our universe

If we divide Eq. (12.54) by $\frac{4\pi}{3}R^2$, we obtain :

$$\frac{3H^2}{8\pi G} = -\frac{3k}{8\pi G R^2} + \rho_m + \rho_\Lambda = \rho_k + \rho_m + \rho_\Lambda \text{ (12.58)}$$

or,

$$H^2 = \frac{8\pi G}{3}(\rho_{\text{Matt}} + \rho_{\text{k}} + \rho_{\Lambda}) \quad \text{(12.58-a)}$$

Define the critical density (at present)

$$\rho_{\text{c}} = \rho_{H_0} = \frac{3}{8\pi G}{H_0}^2 = \rho^0_{\text{Matt}} + \rho^0_k + \rho^0_{\Lambda}. \quad \text{(12.59)}$$

and

$$\Omega_i(t) = \rho_i(t)\,/\rho_{\text{c}}$$

Then, (by dividing both side with ${H_0}^2 = \frac{8\pi G}{3}(\rho^0_{\text{Matt}} + \rho^0_k + \rho^0_{\Lambda}) = \frac{8\pi G}{3}\rho_{\text{c}}$

$$H^2(t) = {H_0}^2 \frac{1}{\rho_{\text{c}}}(\rho_{\text{Matt}}(t) + \rho_{\text{k}}(t) + \rho_{\Lambda}(t))$$

$$= {H_0}^2(\Omega_{\text{k}}(t) + \Omega_{\text{m}}(t) + \Omega_{\Lambda}(t))$$

$$= {H_0}^2\left(\Omega^0_{\text{rad}}a^{-4}(t) + \Omega^0_{\text{M}}a^{-3}(t) + \Omega^0_k a^{-2}(t) + \Omega^0_{\Lambda}a^{-3(1+w)}(t)\right) \quad w = -1 \text{ for } \Lambda\text{CDM}$$

where the Hubble parameter H = $\dot{R}$/R.

This equation becomes

$$\rho_{\text{H}} = \rho_{\text{k}} + \rho_{\text{m}} + \rho_{\Lambda}$$

where energy densities are defined as

$$\rho_{\text{H}} = 3H^2/8\pi\text{G}, \text{ and } \rho_{\text{k}} = -3\text{k}/8\pi GR^2 \qquad \rho_{\Lambda} = \frac{\Lambda}{8\pi G}.$$

- The Hubble energy density ρ_{H} is a threshold, the critical energy density ρ_{c} for the universe:

$$\rho_{\text{c}} = \rho_{\text{H}} = \frac{3}{8\pi G}{H_0}^2. \quad \text{(12.59)}$$

- If $\rho_{\text{m}} + \rho_{\Lambda} < \rho_{\text{c}}$, then $\rho_{\text{k}}>0$ i.e., k<0, and the universe has hyperbolic hypersurfaces.
- Conversely, if $\rho_{\text{m}} + \rho_{\Lambda} > \rho_{\text{c}}$, then, k>0, and the universe will be the closed model.
- we can divide the earlier energy-density equation, evaluated at the present time, by ρ_{c} to get

$$1 = \Omega_{\text{k}} + \Omega_{\text{m}} + \Omega_{\Lambda}. \quad \text{(12.60)}$$

where $\Omega_{\text{i}} = \rho_{\text{i}}/\rho_{\text{c}}$.

- The data from supernovae, the cosmic microwave background, and studies of the evolution of galaxy clusters (below) all suggest that our universe at present has

$$\Omega_{\Lambda} = 0.7,\ \Omega_{\text{m}} = 0.3,\ \Omega_{\text{k}} = 0 \text{ at present. (12.61)}$$

we live in a flat universe, dominated by a positive cosmological constant.

- What size do these numbers have? H_0 has the value

$$H_0 = 100h \text{ km s}^{-1}\text{Mpc}^{-1} \text{ with } h = 0.71. \quad \text{(12.62)}$$

Using this, the critical energy density is

$$\rho_{\text{c}} = 1.87847(23) \times 10^{-26}h^2\text{kgm}^{-3} = 9.5 \times 10^{-27}\text{kgm}^{-3}.$$

The density of baryonic matter (normal matter made of protons, neutrons, and electrons, including stars and galaxies,etc.) has Ω_{b} = 0.04.

- The remaining is the dark matter Ω_{d}, which is non-baryonic, does not emit light, and can be studied only indirectly, through its gravitational effects.

$$\Omega_{\text{m}} = \Omega_{\text{b}} + \Omega_{\text{d}},\quad \Omega_{\text{b}} = 0.04,\quad \Omega_{\text{d}} = 0.26. \quad \text{(12.63)}$$

- The variety of possible cosmological evolutions and the data are captured in the diagram in Fig. 12.4. Questions: where in physics does this dark energy come from? at present there is simply no good theory for it.

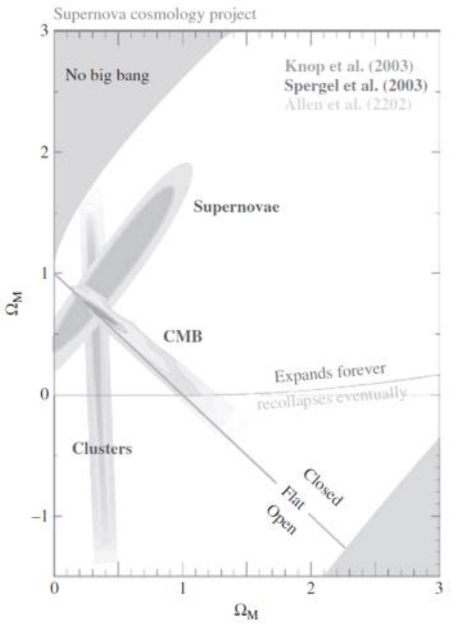


Figure 12.4 In the Ω_{m} v. Ω_{Λ} plane one sees the variety of possible cosmological models, their histories and futures. The constraints from studies of supernovae (Knop et al. 2003), the cosmic microwave background radiation (Spergel et al. 2003), and galaxy clustering (Allen et al. 2002) are consistent with one another and all overlap in a small region of parameter space centered on Ω_{Λ} = 0.7 and Ω_{m} = 0.3. This means that Ω_{k} = 0 to within the errors. Figure courtesy the Supernova Cosmology Project.

- From the point of view of general relativity, one of the most intriguing ways of studying the dark energy is with the LISA gravitational wave detector. As mentioned in § 9.5, LISA observed coalescences of black holes at high redshifts and measure their distances. To do cosmography, we have to combine these luminosity distance measures with redshifts. A gravitational-wave measurement would be a very desirable complement to other studies of the dark energy, because it needs no calibration: it would be independent of the assumptions of the cosmic distance ladder. It would therefore be an important check on the systematic errors of other methods.

z : 3,400 0.6 0

t= 0	1sec	3min.	50,000 yr	370,000 yrs		~800Myrs	10^{10} yrs	1.38x10^{10} yrs
T=∞	10^{10}K	10^9 K	8,700K	3,000K	~60K	~30K	3.8K	2.73K
z= ∞			3,400	1,100	20	10	0.66	0
	ν-decpl	BBN	$\rho_{\text{rad}} = \rho_{\text{Matt}}$	Recombination	1st galaxies	re-ionization	$\rho_{\text{Matt}} = \rho_\Lambda$	

⟵ Radiation Dominated ⟶ | ⟵ Matter Dominated ⟶ | ⟵ Λ Dominate

opaque ⟶ | ⟵ transparent; CMB ⟶ |

quarks, baryons(p,n) Nucleus
leptons leptons(e,ν) (p,n,**He**,) atoms(H, He)
gauge bosons e,ν ν
γ γ γ

6-parameter Lambda-CDM model

The first Friedmann equation gives the expansion rate in terms of the matter+radiation density ρ, the curvature k, and the cosmological constant Λ,

$$H^2(t) = -\frac{kc^2}{R_0^2 a^2} + \frac{8\pi G}{3}\rho_{\text{m}} + \frac{\Lambda c^2}{3} = \frac{8\pi G}{3}(\rho_{\text{Matt}} + \rho_{\text{k}} + \rho_\Lambda)$$

where

$$\rho_{\text{k}} = -\frac{3kc^2}{8\pi G R_0^2 a^2} = \frac{3}{8\pi G}\left(-\frac{kc^2}{R^2}\right) \qquad \rho_\Lambda = \frac{\Lambda c^2}{8\pi G}$$

A critical density ρ_{crit} is the present-day density, which gives zero curvature $k = 0$, assuming the cosmological constant Λ is zero, regardless of its actual value.

Substituting these conditions to the Friedmann equation gives

$$\rho_{\text{crit}} = \frac{3{H_0}^2}{8\pi G}\left(= \rho^0_{\text{Matt}} + \rho^0_k + \rho^0_\Lambda\right) \qquad \left[\frac{3{H_0}^2 c^2}{8\pi G}\right] = \left[\frac{energy}{vol}\right]$$

=1.87847(23) × $10^{-26} h^2$kgm^{-3} = 0.95 × 10^{-26}kgm^{-3}

=mass of # 5.678 H-atom m^{-3}

Cf) Mass of the Hydrogen atom m_{H}=1.673 × 10^{-27}kg

- If the cosmological constant were actually zero,

the critical density would also mark the dividing line between eventual recollapse of the universe to a Big Crunch, or unlimited expansion.

- For the Lambda-CDM model with a positive cosmological constant (as observed),

the universe is predicted to expand forever regardless of whether the total density is slightly above or below the critical density;

- though other outcomes are possible in extended models where the dark energy is not constant but actually time-dependent.

the Friedmann equation can be conveniently rewritten in terms of the various density parameters as

$$H^2(a) \equiv \left(\frac{\dot a}{a}\right)^2 = {H_0}^2 \frac{1}{\rho_{\text{c}}}\left(\rho_{\text{Matt}}(t) + \rho_{\text{k}}(t) + \rho_\Lambda(t)\right)$$

$$= {H_0}^2\left(\Omega_{\text{k}}(t) + \Omega_{\text{m}}(t) + \Omega_\Lambda(t)\right)$$

$$H(a) = H_0\sqrt{\Omega^0_{\text{rad}} a^{-4}(t) + \Omega^0_{\text{M}}(= \Omega^0_{CDM} + \Omega^0_b) a^{-3}(t) + \Omega^0_k a^{-2}(t) + \Omega^0_{\text{DE}} a^{-3(1+w)}(t)}$$

where w is the equation of state parameter of dark energy, and assuming negligible neutrino mass (significant neutrino mass requires a more complex equation).

The various Ω^0 parameters add up to 1 by construction.

This is integrated to give $a(t)$ and also observable distance-redshift relations for any chosen values of the cosmological parameters, which can then be compared with observations such as supernovae and baryon acoustic oscillations.

In the minimal 6-parameter Λ-CDM model, it is assumed that curvature $\Omega^0_k = 0$ and $w = -1$, so this simplifies to

$$H(a) = H_0\sqrt{\Omega^0_{\text{rad}} a^{-4}(t) + \Omega^0_{\text{M}}(= \Omega^0_{CDM} + \Omega^0_b) a^{-3}(t) + \Omega^0_\Lambda}$$

Observations show that the radiation density is very small today, $\Omega^0_{\text{rad}} \sim 10^{-4}$;

if this term is neglected the above

$$H(a) = H_0 \sqrt{\Omega_M^0 a^{-3}(t) + \Omega_\Lambda^0}$$

has an analytic solution[13]

$$a(t) = \left(\frac{\Omega_M^0}{\Omega_\Lambda^0}\right)^{1/3} \sinh^{2/3}(t/t_\Lambda)$$

where

$$t_\Lambda \equiv \frac{2}{3H_0\sqrt{\Omega_\Lambda^0}};$$

this is fairly accurate for $a > 0.01$ or $t > 10$ million years.

Solving for $a(t) = 1$ gives the present age of the universe t_0 in terms of the other parameters.

the transition from decelerating to accelerating expansion (the second derivative occurred when

$$a = \left(\frac{\Omega_M^0}{2\Omega_\Lambda^0}\right)^{1/3}$$

which evaluates to $a \sim 0.6$ or $z \sim 0.66$ for the best-fit parameters estimated from the *Planck* spacecraft.

출처: <https://en.wikipedia.org/wiki/Lambda-CDM_model>

Note) Let $a(t) \sim t^n \sim t^{\frac{2}{3(1+w)}}$

Then,

$\dot{a}(t) \sim n t^{n-1}$,

$\ddot{a}(t) \sim n(n-1) t^{n-2}$

w	$w < -1$	$\pm\infty$	1	1/3	0	−1/3	−2/3	−1
n	$n < 0$	0	1/3	1/2	2/3	1	2	∞
$a(t)$	↘	⟻⟼		↗				
$\dot{a}(t)$	−↘	⟻⟼		+↘		⟻⟼	+↗	
$\ddot{a}(t)$	+	⟻⟼		−		⟻⟼	+	

And,

$$\int_t^\infty \frac{dt'}{a(t')} \sim t^{1-n}$$

Note) Let $a(t) \sim t^n \sim t^{\frac{2}{3(1+w)}}$

Then,

$\dot{a}(t) \sim n t^{n-1}$,

$\ddot{a}(t) \sim n(n-1) t^{n-2}$

w	$-\infty$	−1	$-\frac{2}{3}$	−1/3	0	1/3	1	∞
3(1+w)	$-\infty$	0	1	2	3	4	6	∞
n	−0	$-\infty/+\infty$		1	2/3	1/2	1/3	0
$\rho(t) \sim a^{-3(1+w)}$	$a^{+\infty}$	const		a^{-2}	a^{-3}	a^{-4}	a^{-6}	$a^{-\infty}$
$a(t)$	↘	⟻⟼	↗					
$\dot{a}(t)$	−↘	⟻⟼	+↘		⟻⟼	+↗		
$\ddot{a}(t)$	−	⟻⟼	−	⟻⟼	+			

Standard Model of Particle Physics

Interactions :	**gauge boson**	**Mass**	**Gauge Group**	**Quantum Field Theory**
Strong,	gluon g	0	SU(3) ("color"charge)	Quantum Chromodynamics(QCD)
electromagnetic,	photon γ	0	U(1) ("charge")	Quantum Electrodynamics(QED)
weak	W^+, W^-,	$m_W = 80.39$ GeV/c^2	SU(2)("weak"charge)	(cf : Electroweka theory)
	Z	$m_Z = 91.19$ GeV/c^2		
gravitational	graviton $g_{\mu\nu}$	0	General Linear Group	(Quantum General Relativity?)

Note : # gauge bosons for SU(N) $= N^2 - 1$

Matter

Quarks$(q_I, \bar{q}_I, I = (D, c, f)(D = u, d; c, f = 1,2,3)$ D:doublet, c:color, f:family

+Leptons $(l_I, \bar{l}_I, I = (D, f)(D = \nu, e; f = 1,2,3)$

	Mass lepton	Mass-quarks
Spin 1/2 Fermions,		
3-family,	$m_e = 0.511$ MeV/c^2	$m_{u,d} \sim$ few MeV/c^2
doublets (under weak interaction)	$m_\mu = 105$ MeV/c^2	$m_s = 96$ MeV/c^2
	$m_\tau = 1.78$ GeV/c^2	$m_c = 1.28$ GeV/c^2
Antimatter : same mass & spin, opposite charge		
		$m_b = 4.2$ GeV/c^2
		$m_t = 173$ GeV/c^2

Gauge bosons+Higgs boson

Spin 1 gauge boson, (spin 0 Higgs boson) $m_H = 125$ GeV/c^2

$(\bar{g}_A = g_A, A = 1, \ldots, 8 = 3^2 - 1$

$\bar{\gamma} = \gamma$, $\bar{Z} = Z$, (real) $\bar{W}^\pm = W^\mp$ (complex)

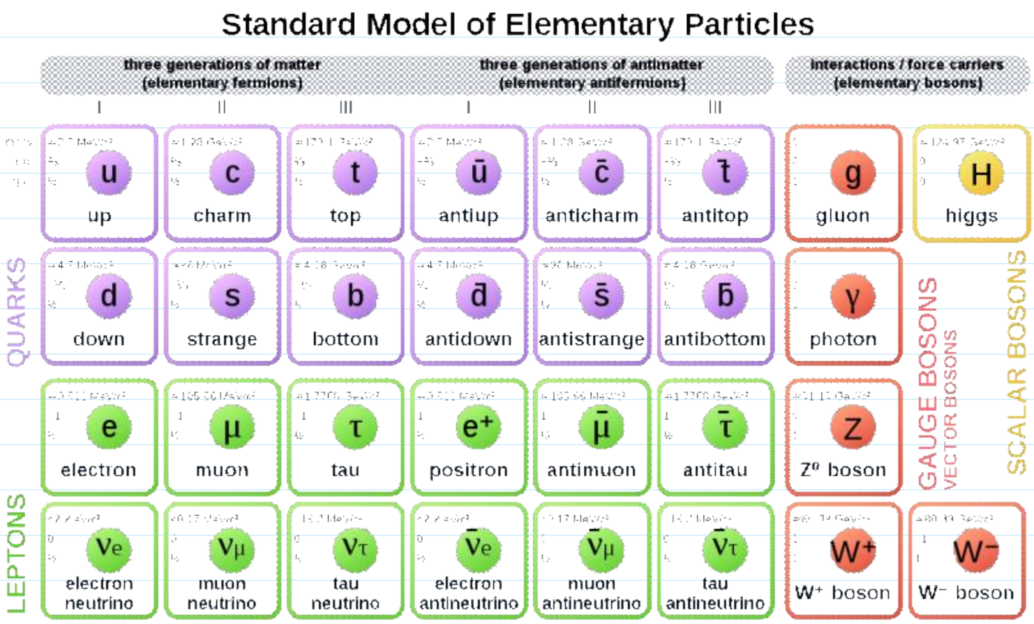
$\bar{H} = H$ real)

The Standard Model elementary particles can be summarized as follows:

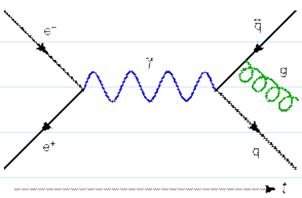
			Elementary particles			
	Elementary fermions Half-integer spin Obey the Fermi–Dirac statistics				**Elementary bosons** Integer spin Bose–Einstein statistics	
Quarks and antiquarks Spin = 1/2 color charge strong, E&W		**Leptons & antileptons** Spin = 1/2 No color charge Electroweak interactions (No strong interaction)		**Gauge bosons** Spin = 1 Force carriers		**Scalar bosons** Spin = 0
3 generations 1. Up (u), Down (d) 2. Charm (c), Strange (s) 3. Top (t), Bottom (b)		**Three generations** 1. Electron (e−),[†] Electr neutrino(νe) 2. Muon (μ−), Muon neutrino(νμ) 3. Tau (τ−), Tau neutrino(ντ)		**Four kinds** 1. Photon(γ) (el-mag inter) 2. W&Z(W±, Z) (weak inter) 3. #8 gluons (g) (strong inter)		**Unique** Higgs boson (H0)

Notes: [†] An anti-electron (e+) is conventionally called a "positron".

출처: <https://en.wikipedia.org/wiki/Standard_Model>

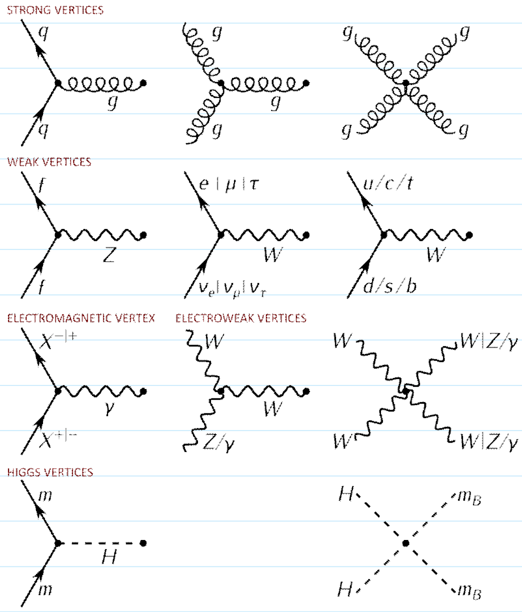


Interaction



In this Feynman diagram, an electron (**e**) and a positron (**e**⁺) annihilate, producing a photon (**γ**, represented by the blue sine wave) that becomes a quark–antiquark pair (quark **q**, antiquark **q**□), after which the antiquark radiates a gluon (**g**, represented by the green helix)

출처: <https://en.wikipedia.org/wiki/Feynman_diagram>



Interactions in the Standard Model.

All Feynman diagrams in the model are built from combinations of these vertices.

q is any quark, g is a gluon, X is any charged particle, γ is a photon, f is any fermion,

m is any particle with mass (with the possible exception of the neutrinos),

mB is any boson with mass.

In diagrams with multiple particle labels separated by / one particle label is chosen.
In diagrams with particle labels separated by | the labels must be chosen in the same order.
For example, in the four boson electroweak case the valid diagrams are WWWW, WWZZ, WWγγ, WWZγ. The conjugate of each listed vertex (reversing the direction of arrows) is also allowed.

출처: <https://en.wikipedia.org/wiki/Standard_Model>

Stability of leptons :only the lightest leptons ($e^{\pm}, \nu_i$) are stable

$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}$ $\begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}$ $\begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}$ neutrino : oscillation (stable)

$\mu^- \longrightarrow e^- + \bar{\nu}_e + \nu_\mu$ mean life time $\lesssim 10^{-6}$sec

$\tau^- \longrightarrow \mu^- + \bar{\nu}_\mu + \nu_\tau$

Overview : Thermal History of the Universe

Overview : Thermal History of the Universe

What	When (t)	When (z)	When (T)	When (E)
Neutrino Decoupling	1 second	6x10^9	10^{10}K	1 MeV
Nucleosynthesis (BBN)	3 minutes	4x10^8	10^9 K	0.1MeV
Matter-Radiation Equality	50,000 years	3,400	8,700 K	~ 0.8 eV
Recombination	~370,000 years	1,100	3,000 K	~0.3eV
Matter-Λ Equality	10^{10} years	0.4	3.8 K	~3.2x$10^{-4}eV$
Today	1.4x10^{10} years	0	2.73 K	~2.3x$10^{-4}eV$

t= 0 1sec 3min. 50,000 yr 370,000 yrs ~800Myrs 10^{10} yrs 1.4x10^{10} yrs

T=∞ 10^{10}K 10^9 K 8,700K 3,000K ~60K ~30K 3.8K 2.73K

z= ∞ 3,400 1,100 20 10 0.4 0

Energy 200MeV 1MeV 0.1MeV ~ 0.8 eV ~0.3eV ~3.2x$10^{-4}eV$ ~2.3x$10^{-4}eV$

QCD Tr ν-decpl BBN $\rho_{\rm rad} = \rho_{\rm Matt}$ Recombination 1st galaxies re-ionization $\rho_{\rm Matt} = \rho_\Lambda$

⟵ Radiation Dominated ⟶ | ⟵ Matter Dominated ⟶ | ⟵ Λ Dominated

opaque ⟶ | ⟵ transparent; CMB ⟶ |

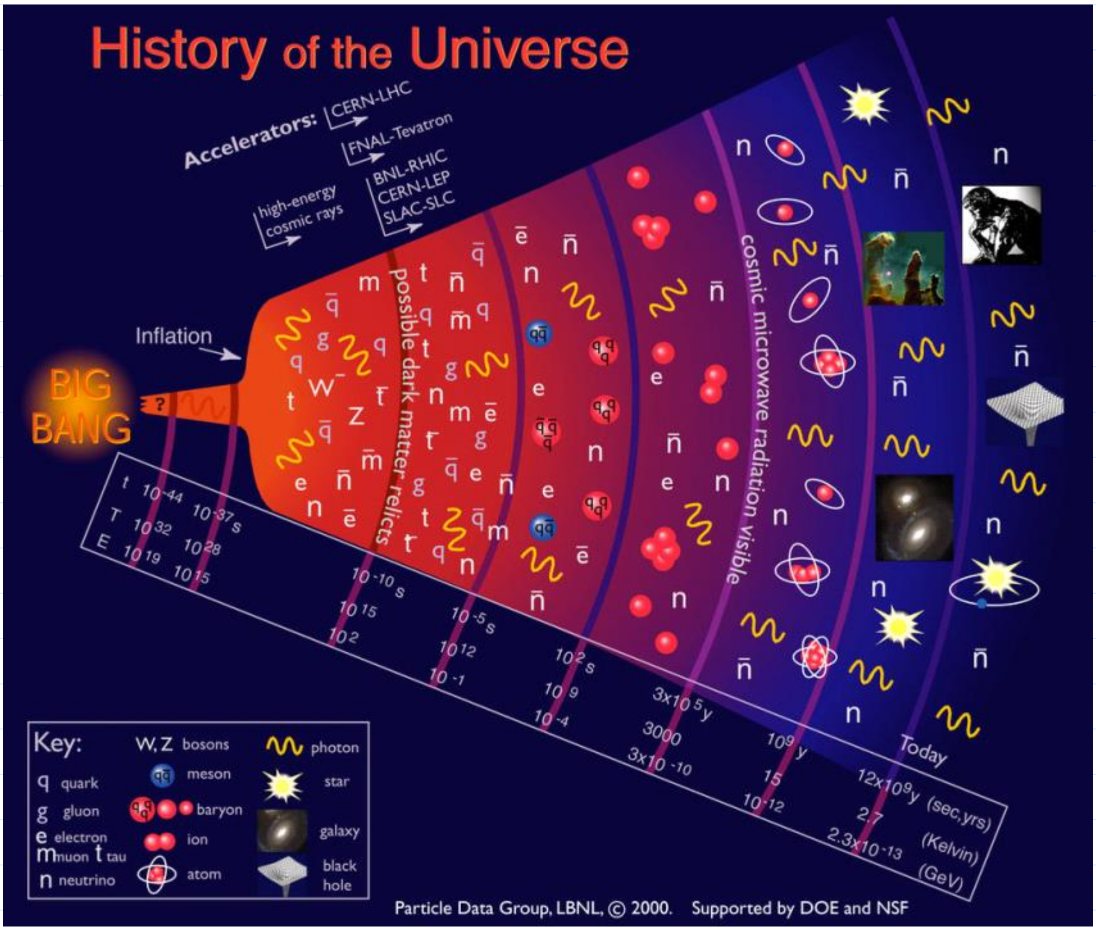
quarks, baryons(p,n) Nucleus

leptons leptons(e,ν) **(p,He,)** atoms(H, He)

gauge bosons e,ν ν

(gluons, γ, ...) γ γ γ

Energy



Time t vs Temperature T in radiation dominated era

$$\left(\frac{t}{1\text{ second}}\right) \approx \frac{2.4}{\sqrt{g_*}}\left(\frac{1\text{MeV}}{k_B T}\right)^2$$

Note : $a \sim t^{1/2}$ and $a \sim \frac{1}{T}$

Ex) $k_B T \approx 1$ MeV at $t = 1$ sec

Energy density (Einstein Eq. $H^2(a) \equiv \left(\frac{\dot{a}}{a}\right)^2 = {H_0}^2 \frac{1}{\rho_c}\left(\rho_{\text{Matt}}(t) + \rho_{\text{k}}(t) + \rho_{\Lambda}(t)\right)$

1) Nonrelativistic, classical

Number density

$$n = g\, \underbrace{e^{\beta(\mu - mc^2)}}_{\mu^{\text{Nonrel}}} \left(\frac{mk_B T}{2\pi\,\hbar^2}\right)^{3/2}$$

Energy density

$$\rho_{\text{Nonrel,cl}}(t) = n(mc^2 + \frac{3}{2}k_B T)$$

2) Relativistic

Number density

$$n = \frac{g\zeta(3)}{\pi^2 \hbar^3 c^3}\left(k_B T\right)^3 \begin{cases} 1 & \text{Boson} \\ \frac{3}{4} & \text{Fermion} \end{cases}$$

Energy density

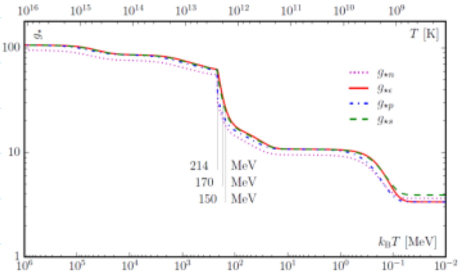
$$\rho_{\text{Rad}} = \frac{g\pi^2}{30(\hbar c)^3}\left(k_B T\right)^4 \begin{cases} 1 & \text{Boson} \\ \frac{7}{8} & \text{Fermion} \end{cases}$$

In the cosmology,

$\rho_{\text{Rad}}(t) = g_*(t)\frac{\pi^2}{30(\hbar c)^3}(k_B T(t))^4$,

$g_*(t) = \sum_{\text{Boson}} g_i + \frac{7}{8}\sum_{\text{Fermion}} g_i$

# of relativisitc degree of freedom



Note : # polarization for spin s =2s+1, in general,
- ex) spin 1, $m \neq 0$, # pol =3
- spin 1, $m = 0$, # pol =2
- spin 1/2, $m \neq 0$, # pol =2
- left-handed ν # pol =1

Ex) $E \gtrsim 100$ GeV

Boson : (#pol g_i)x $\#i$		g_i x $\#i$	Fermion		
W,Z	$g_i = 3$, $\#i = \#(\pm,0) = 3$	3 x 3=9	quark $q_I, \bar{q}_I$		g_{q_I}x $\#I$ x$(q,\bar{q})$
γ	$g_\gamma = 2$	2 x 1 =2	$g_{q_I} = 2$		
g_I	$g_{g_I} = 2$ $\#I$=8=$3^2 - 1$	2 x 8=16	$\#I = (D,c,f)$ =2x3x3=18		2 x 18x2=72
Higgs	$g_H = 1$	1 x 1 = 1	leptons :		g_i x $\#i$ x$(l,\bar{l})$
	total	28	$g_{e_f} = 2$	$\#e_f$ =3	2 x 3 x2=12
			g_{ν_f} =1	$\#\nu_f$ =3	1 x 3 x2 = 6
				total	90

$\boldsymbol{g_*} = 28 + \frac{7}{8}$x90=**106.75**

1) **QCD Phase Transition** (at T~200 MeV)

Quark Confinement-Deconfinement Phase Transition **cf) Ice-water, water-vapor phase transition**

Phase	Deconfinement		Confinement
t	$t_{BB} = 0$	$t \approx 10^{-5}$ s	Confinement
E, T	$k_B T, E = \infty$ $(g_* \simeq 80)$	E, T~200 MeV	$(g_* \simeq 10)$
Particles	Quanrk-Gluon plasma $q_i, \bar{q}_i, gluons,$		Hadrons(Baryons, Mesons) Baryons (qqq) $p, n, \Lambda, \Delta, \Xi, \Sigma$ etc. Antibaryon $(\bar{q}\bar{q}\bar{q})$ $\bar{p}$ etc. Mesons $(q\bar{q})$ π, ρ, η, etc.
Particles Relativistic	quarks, antiquark leptons $\nu_i, \bar{\nu}_i$ $e^\pm$, $g, \gamma,$		$\nu_i, \bar{\nu}_i$ $e^\pm$ $\gamma,$
Nonrelativisitc	$(\mu^\pm, \tau^\pm), (W^\pm, Z, H)$		$p, n,$

*Stability : Only the lightest baryon (proton) is stable.

Ex) Heavy Baryons($\Lambda, \Delta, \Xi, \Sigma$ etc.)$\longrightarrow p, n$ $(+\nu's)$: life time $\lesssim 10^{-6}$sec

$n \longrightarrow p + e^- + \bar{\nu}_e$ mean life time $\simeq$ 10 min.

Ex) $E >\approx 1$ MeV

Relativistic species (No hadrons, neither μ, τ)

Boson :

γ $g_\gamma = 2$

Fermions : (+antiparticles)			g_{l_I}x $\#I$ x$(l,\bar{l})$
ν_e, ν_μ, ν_τ	$\bar{\nu}_i$	$g_{\nu_f} = 1$, $\#\nu_f$=3	1 x 3 x2= 6
e^-	e^+	$g_e = 2$, e : 1	2 x2= 4
			g_{Fermions}=10

$\boldsymbol{g_*} = g_{\text{Boson}} + \frac{7}{8}xg_{\text{Ferm}} = 2 + \frac{7}{8}$x10 = **10.75**

2) The e^-e^+- annihilation (around $k_BT \simeq 2mc^2 \simeq 1$MeV or $t \simeq 1$sec)

t $t_{BB} = 0$ 1sec

$k_BT = \infty$ T_{before} 1MeV T_{after}

$\#e^- \simeq \#e^+$ small $\#e^- = \#p = \eta\ \#\gamma = 10^{-9}\#\gamma$

$\simeq \#\gamma \sim (k_BT)^3$ $\#e^+ \simeq 0$

Particles

Relativistic $\nu_i, \bar{\nu}_i, e^-, e^+$ $\nu_i, \bar{\nu}_i, e^-$

$\gamma,$ $\gamma,$

Nonrelativisitc $p, n,$ $p, n,$

Note 1) **The CP-symmetry violation**

Slight eccess of the "particles"e^- to the "antiparticles" e^+ before annihilation

$\frac{\#e^-}{\#e^+} = 1 + \eta \simeq 1 + 10^{-9}$

2) **Temperature increase due to the annihilation**

$T_{\text{after}} = \left(\frac{11}{4}\right)^{1/3} T_{\text{before}}$

Ex) $E \lesssim 1$ MeV

Relativistic species (No hadrons, neither e, μ, τ)

Boson :

γ $g_\gamma = 2$

Fermions : (+antiparticles) g_{l_l}x $\#I$ x$(l, \bar{l})$

ν_e, ν_μ, ν_τ $g_{\nu_f} = 1$, $\#\nu_f$=3 1 x 3 x2= 6

g_{Fermions}=6

$\boldsymbol{g}_* = g_{\text{Boson}} + \frac{7}{8}xg_{\text{Ferm}}$

$= 2 + \frac{7}{8}$x6= 7.25 (Assuming $T_\gamma = T_\nu$)

Note : Actually, $\boldsymbol{g}_* = \mathbf{3.4}$ since $T_\gamma > T_\nu$ (due to $e^- + e^+ \longrightarrow 2\,\gamma$ increasing T_γ) annihilation

1) Neutrino decoupling (around $k_BT \simeq 0.8$MeV or $t \simeq 2$sec)

t $t_{BB} = 0$ 2 sec

$k_BT = \infty$ 0.8 MeV

	$\Gamma \gg H$	$\Gamma \simeq H$	$\Gamma < H$
weak inter :	thermal equil	ν-decoupling, n freeze-out	out of equil
Relativistic	γ, e^-, e^+ $\nu_i, \bar{\nu}_i,$		γ, e^- $\nu_i, \bar{\nu}_i,$
Nonrelativisitc	$p, n,$		$p, n,$
	$\frac{n_n}{n_p} \simeq e^{-\beta\Delta mc^2}$		ν-decoupled

* $m_nc^2 = 939.6$ MeV
$m_pc^2 = 938.3$ MeV
$\Delta mc^2 = 1.3$MeV

Interaction rate

$\Gamma = n\sigma v$ ($n \sim T^3, \sigma v \sim G_FT^2$)

$\sim T^5$

Hubble parameter

$H(t) = \frac{\dot{a}(t)}{a(t)} \sim \frac{1}{t} \sim \frac{1}{a^2} \sim T^2$

Note 2) $\left(\frac{n_n}{n_p}\right)_{dec} \simeq e^{-\frac{\Delta mc^2}{k_B T_{dec}}} = e^{-\frac{1.3\text{MeV}}{0.8\text{MeV}}} \approx \frac{1}{5}$

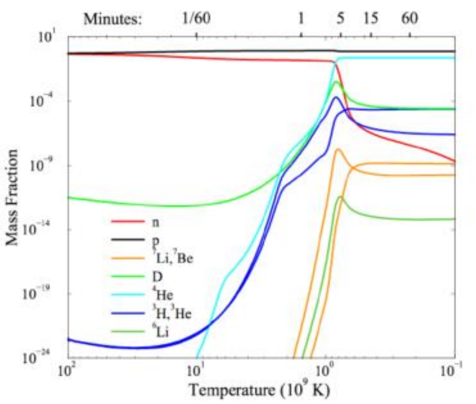
Note 3) After decoupling ($t > t_{dec} \simeq 2$sec),

neutron density decreases only by β-decay ($n \longrightarrow p + e^- + \bar{\nu}_e$, $\tau_n \simeq$880 sec)

$$n_n(t) \simeq \frac{1}{5} n_p(t_{dec}) e^{-\frac{t}{\tau_n}}$$

1) **Nucleosynthesis** (around t~min)

		Nucleosynthesis	
t	$t_{BB} = 0$	360sec	→
$k_B T$	$= \infty$	$T_D \simeq$0.06MeV D-formation	
Relativistic	$\nu_i, \bar{\nu}_i$ (decoupled) e^- γ	$\nu_i, \bar{\nu}_i$ (decoupled) $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$ e^- γ	
Nonrelativisitc	$p, n,$	$\underbrace{p}_{0.75}, \underbrace{{}^4\text{He}}_{0.25}, \underbrace{{}^2\text{D}}_{10^{-5}}, \underbrace{{}^3\text{He}}_{10^{-5}}, \underbrace{{}^3\text{H}}_{10^{-5}}, \underbrace{\text{Li}}_{10^{-9}}, \underbrace{\text{Be}}_{10^{-9}}$	



- Neutron n

$$n_n(t) \simeq \frac{1}{5} n_p(t_{dec}) e^{-\frac{t}{\tau_n}}$$

- Deuterium D ($= (n,p)$ bounds, $E^D_{\text{bind}} \simeq 2.2$MeV) when

$$\frac{n_D}{n_p} \simeq \eta \left(\frac{k_B T}{m_p c^2}\right)^{3/2} e^{\frac{E^D_{\text{bind}}}{k_B T}} \approx 1 \text{ when } k_B T_D \lesssim 0.06\text{MeV} \text{ or } t_D \approx 360\text{sec}$$

- Heavier elements (He, Li, etc.) forms after $t_D \approx 360$sec ($T < T_D$)

All neutrons at $t = t_D \approx 360$sec,

$$\frac{n_n}{n_p}(t_D) \simeq \frac{1}{5} e^{-\frac{360}{800}} \simeq 0.13$$

are bound to ^{4}He ($E^{{}^4\text{He}}_{\text{bind}} \simeq 28$MeV)

$$\frac{n_{\text{He}}}{n_{\text{H}}} = \frac{n_n/2}{n_p - n_n} \simeq 0.07 \quad (\text{\# fraction} \simeq 0.07, \text{mass fraction} \simeq 0.28)$$

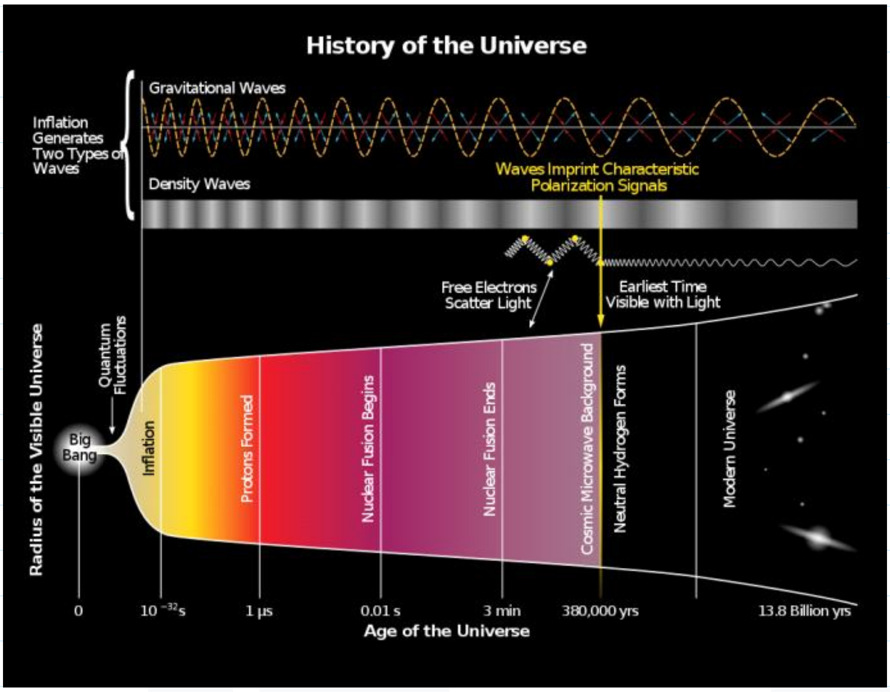
Helium occupies 25% of the baryonic mass. (Remaining 75% Hydrogen)

cf) Mass fraction of

Deuterium D $\sim 10^{-5}$ ($E^D_{\text{bind}} \simeq 2.2$MeV)

Helium-3 ^{3}He$\sim 10^{-5}$ ($E^{{}^3\text{He}}_{\text{bind}} \simeq 7.7$MeV)

Note : Dark Matter decoupling around $k_B T \simeq 10^2$GeV DM decoupling



History of the Universe – gravitational waves are hypothesized to arise from cosmic inflation, a faster-than-light expansion just after the Big Bang

출처: <https://en.wikipedia.org/wiki/Physical_cosmology>

t	$t_{BB}=0$	DM decoupling	QCD phs tr 10^{-5} s	e^-e^+- annih 1sec	ν-decpl 2sec	Nucleosynthesis 360sec
k_BT	$=\infty$	T_{DM} 10^2GeV	T_{QCD} 200MeV	$T_{e^-e^+}$ 1MeV	$T_{\nu\text{-dcpl}}$ 0.8MeV	T_D 0.06MeV Deuterium formation

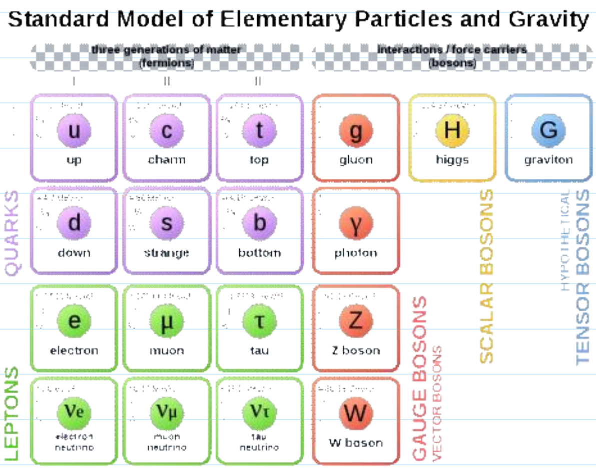
Relativistic: quarks, $q_i, \bar{q}_i$, leptons $l_i, \bar{l}_i$, $g, \gamma, W^\pm, Z, H$ | $\nu_i, \bar{\nu}_i$, e^-, e^+, γ, | $\nu_i, \bar{\nu}_i$, e^-, γ, | $\nu_i, \bar{\nu}_i$, e^-, γ, | $\nu_i, \bar{\nu}_i$, e^-, γ,

Nonrelativisitc: (DM) | $p, n,$ | $p, n,$ | $p, n,$ | $p,$ He^{++},D$^+$

$T_\gamma = T_\nu = T_{DM}$ (γ, ν,DM all equil)

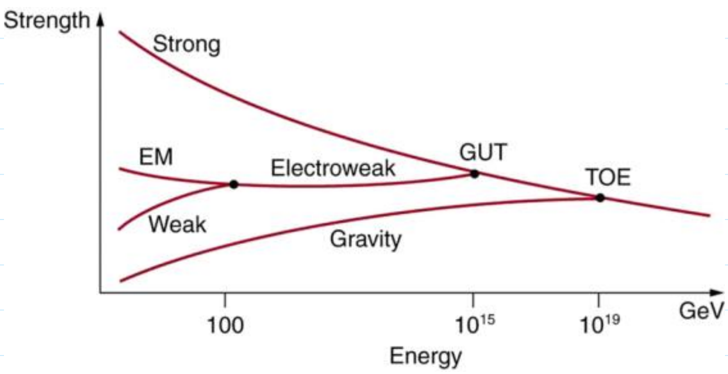
$T_\gamma = T_\nu \neq T_{DM}$ $\qquad$ $T_\gamma = T_\nu \neq T_{DM}$ (γ, ν : at equil, DM : out of equil (decouped))

$T_\gamma \neq T_\nu \neq T_{DM}$ (ν : out of equil (ν & DM decoupled))



The Standard Model of elementary particles + hypothetical Graviton

출처: <https://en.wikipedia.org/wiki/Unified_field_theory>



	Inflation		EW Phase Tr.	QCD phs tr	e^-e^+- annih/ ν-decpl
$t \quad t_{BB}=0$	10^{-36} s		10^{-12} s	10^{-6} s	1sec
	T_{Infl}		T_{EW}	T_{QCD}	$T_{e^-e^+}$
$k_BT \quad =\infty$	10^{15}GeV		10^2GeV	200MeV	1MeV
	Inflation epoch	EW epoch	Quark epoch	Hadron epoch	

Relativistic	quarks, $q_i, \bar{q}_i$, leptons $l_i, \bar{l}_i$ $g, \gamma, W^\pm, Z, H$	quarks, $q_i, \bar{q}_i$, leptons $l_i, \bar{l}_i$ $g, \gamma,$	$\nu_i, \bar{\nu}_i$, e^-, e^+ $\gamma,$	$\nu_i, \bar{\nu}_i$, e^- $\gamma,$
Nonrelativisitc			$p, n,$	$p, n,$

1. the **electroweak epoch** :
   160GeV $\approx T_{EW} \lesssim T \lesssim T_{GUT} \approx 10^{15}$GeV $\qquad 10^{-36}$sec $\approx t_{GUT} \lesssim t \lesssim t_{EW} \approx 10^{-12}$sec

- the strong force separated from the electroweak interaction,
- electromagnetism & the weak interaction remain merged into a single electroweak interaction
- the critical temperature for electroweak symmetry breaking T_{EW} ≈159.5±1.5 GeV .
- the electroweak epoch may be at the start of the inflationary epoch, t~10^{-36} sec
  or at 10^{-32} sec the inflaton field filling the universe with a dense, hot quark–gluon plasma.

- large numbers of W and Z bosons and Higgs bosons in this phase.
- when $t \approx t_{EW} \approx 10^{-12}$sec, W and Z bosons ceased to be created at observable rates. The remaining W and Z bosons decayed quickly, and the weak interaction became a short-range force in the following quark epoch.

- The electroweak epoch ended with an electroweak phase transition.
- If first order, this could source a gravitational wave background.[6][7]
  The electroweak phase transition is also a potential source of baryogenesis, provided the Sakharov conditions are satisfied.
- In the minimal Standard Model, the transition during the electroweak epoch was not a first- or a second-order phase transition but a continuous crossover, preventing any baryogenesis, or the production of an observable gravitational wave background.
- many extensions to the Standard Model including supersymmetry and the two-Higgs-doublet model have a first-order electroweak phase transition (but require additional CP violation).

See also

- Chronology of the universe

출처: <https://en.wikipedia.org/wiki/Electroweak_epoch>

Baryogenesis

(Redirected from Electroweak baryogenesis)

baryogenesis :

- the physical process to produce baryonic asymmetry,
  i.e. the imbalance of matter (baryons) and antimatter (antibaryons) in the observed universe.
- One of the outstanding problems in modern physics is the predominance of matter over antimatter in the universe.
- A number of theoretical mechanisms are proposed to account for this discrepancy, namely identifying conditions that favour symmetry breaking and the creation of normal matter (as opposed to antimatter).
- This imbalance has to be exceptionally small, on the order of 1 in every 1630000000 (~2×10^{-9}[9]) particles.

Two main Baryogenesis theories are
1) electroweak baryogenesis (standard model), which would occur during the electroweak epoch,
2) and the GUT baryogenesis, which would occur during or shortly after the grand unification epoch.

Baryogenesis is followed by primordial nucleosynthesis, when atomic nuclei began to form.

Background
- Once the universe expanded and cooled to a critical temperature of approximately 2×10^{12} K, quarks combined into normal matter and antimatter and proceeded to annihilate up to the small initial asymmetry of about one part in five billion, leaving the matter around us.

GUT Baryogenesis under Sakharov conditions[edit]

In 1967, Andrei Sakharov proposed[8] a set of three necessary conditions that a baryon-generating interaction must satisfy to produce matter and antimatter at different rates.
The three necessary "Sakharov conditions" are:
- Baryon number violation.
- C-symmetry and CP-symmetry violation.
- Interactions out of thermal equilibrium.

- Baryon # violation is a necessary condition to produce an excess of baryons over anti-baryons.
- But C-symmetry violation is also needed so that the interactions which produce more baryons than anti-baryons will not be counterbalanced by interactions which produce more anti-baryons than baryons.
- CP-symmetry violation is similarly required because otherwise equal numbers of left-handed baryons and right-handed anti-baryons would be produced, as well as equal numbers of left-handed anti-baryons and right-handed baryons.
- Finally, the interactions must be out of thermal equilibrium, since otherwise CPT symmetry would assure compensation between processes increasing and decreasing the baryon number.[11]

- Currently, there is no experimental evidence of particle interactions where the conservation of baryon number is broken perturbatively: this would appear to suggest that all observed particle reactions have equal baryon number before and after.

- However, the Standard Model is known to violate the conservation of baryon number only non-perturbatively: a global U(1) anomaly.

- To account for baryon violation in baryogenesis, such events (including proton decay) can occur in Grand Unification Theories (GUTs) and supersymmetric (SUSY) models via hypothetical massive bosons such as the X boson.
- The second condition – violation of CP-symmetry – was discovered in 1964 (direct CP-violation, that is violation of CP-symmetry in a decay process, was discovered later, in 1999).[13]
  Due to CPT symmetry, violation of CP-symmetry demands violation of time inversion symmetry, or T-symmetry.
- In the out-of-equilibrium decay scenario,[14] the last condition states that the rate of a reaction which generates baryon-asymmetry must be less than the rate of expansion of the universe. In this situation the particles and their corresponding antiparticles do not achieve thermal equilibrium due to rapid expansion decreasing the occurrence of pair-annihilation.

Baryogenesis within the Standard Model[edit]
- The Standard Model can incorporate baryogenesis, though the amount of net baryons (and leptons) thus created may not be sufficient to account for the present baryon asymmetry.

- Baryogenesis within the Standard Model requires the electroweak symmetry breaking to be a first-order phase transition, since otherwise sphalerons wipe off any baryon asymmetry that happened up to the phase transition.
- Beyond this, the remaining amount of baryon non-conserving interactions is negligible.
- The phase transition domain wall breaks the P-symmetry spontaneously, allowing for CP-symmetry violating interactions to break C-symmetry on both its sides.
- Quarks tend to accumulate on the broken phase side of the domain wall, while anti-quarks tend to accumulate on its unbroken phase side.
- Due to CP-symmetry violating electroweak interactions, some amplitudes involving quarks are not equal to the corresponding amplitudes involving anti-quarks, but rather have opposite phase (see CKM matrix and Kaon);
- since time reversal takes an amplitude to its complex conjugate, CPT-symmetry is conserved in this entire process.

- Though some of their amplitudes have opposite phases, both quarks and anti-quarks have positive energy, and hence acquire the same phase as they move in space-time. This phase also depends on their mass, which is identical but depends both on flavor and on the Higgs VEV which changes along the domain wall.[16]
- Thus certain sums of amplitudes for quarks have different absolute values compared to those of anti-quarks.
- In all, quarks and anti-quarks may have different reflection and transmission probabilities through the domain wall, and it turns out that more quarks coming from the unbroken phase are transmitted compared to anti-quarks.

- Thus there is a net baryonic flux through the domain wall. Due to sphaleron transitions, which are abundant in the unbroken phase, the net anti-baryonic content of the unbroken phase is wiped off as anti-baryons are transformed into leptons.[17]
- However, sphalerons are rare enough in the broken phase as not to wipe off the excess of baryons there.
- In total, there is net creation of baryons (as well as leptons).
- In this scenario, non-perturbative electroweak interactions (i.e. the sphaleron) are responsible for the B-violation, the perturbative electroweak Lagrangian is responsible for the CP-violation, and the domain wall is responsible for the lack of thermal equilibrium and the P-violation; together with the CP-violation it also creates a C-violation in each of its sides.

Matter content in the universe[edit]

See also: Baryon asymmetry

- The central question to Baryogenesis is what causes the preference for matter over antimatter in the universe, as well as the magnitude of this asymmetry. An important quantifier is the *asymmetry parameter*, given by

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma}$$

where n_B and $n_{\bar{B}}$ refer to the number density of baryons and antibaryons respectively and n_γ is the number density of cosmic background radiation photons.[19]
- According to the Big Bang model, matter decoupled from the cosmic background radiation (CBR) at a temperature of roughly 3000 kelvin, corresponding to an average kinetic energy of 3000 K / (10.08×10^3 K/eV) = 0.3 eV. After the decoupling, the *total* number of CBR photons remains constant. Therefore, due to space-time expansion, the photon density decreases. The photon density at equilibrium temperature T per cubic centimeter, is given by

$$n_\gamma \approx 20.3 \left(\frac{T}{1K}\right)^3 cm^{-3}$$

,

with k_B as the Boltzmann constant, $\hbar$ as the Planck constant divided by 2π and c as the speed of light in vacuum, and $\zeta(3)$ as Apéry's constant.[19]
- At the current CBR photon temperature of 2.725 K, this corresponds to a photon density n_γ of around 411 CBR photons per cubic centimeter.
- Therefore, the asymmetry parameter η, as defined above, is *not* the "best" parameter. Instead, the preferred asymmetry parameter uses the entropy density s,

$$\eta_s = \frac{n_B - n_{\bar{B}}}{s}$$

because the entropy density of the universe remained reasonably constant throughout most of its evolution. The entropy density is

with p and ρ as the pressure and density from the energy density tensor $T_{\mu\nu}$, and g_* as the effective number of degrees of freedom for "massless" particles at temperature T (in so far as $mc^2 \ll k_B T$ holds),

,

for bosons and fermions with g_i and g_j degrees of freedom at temperatures T_i and T_j respectively. At the present epoch, s = 7.04 n_γ.[19]

출처: <https://en.wikipedia.org/wiki/Baryogenesis#Electroweak_baryogenesis>

2. the **Quark epoch**

- In this period the fundamental interactions of gravitation, electromagnetism, the strong interaction and the weak interaction had taken their present forms,

but the temperature of the universe was still too high to allow quarks to bind together to form hadrons.
- The quark epoch began approximately 10^{-12} sec after the Big Bang, when the preceding electroweak epoch ended as the electroweak interaction separated into the weak interaction and electromagnetism.
- During the quark epoch the universe was filled with a dense, hot quark–gluon plasma, containing quarks, leptons and their antiparticles. Collisions between particles were too energetic to allow quarks to combine into mesons or baryons.
- The quark epoch ended when the universe was about 10^{-6} seconds old, when the average energy of particle interactions had fallen below the binding energy of hadrons.

The following period, when quarks became confined within hadrons, is known as the hadron epoch.

출처: <https://en.wikipedia.org/wiki/Quark_epoch>