2022 12 27 Basics of Cosmology

2021년 11월 20일 토요일 오후 11:30

Observed Properties of the Universe

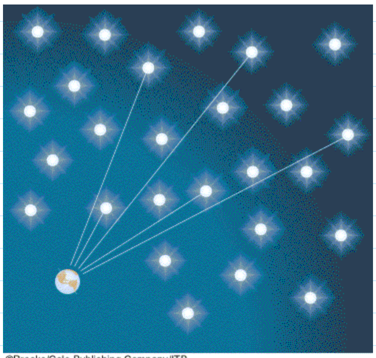
Cosmology

- The word Universe refers to all existing regions of space.
- Cosmology is the study of the properties of the Universe.
- Cosmology includes the study of how the Universe's properties change with time.

Observed Properties of the Universe

1. The Sky is Dark at Night (Olbers' Paradox)

- Early astronomers Edmund Halley and Heinrich Olbers (1758 - 1840) wondered why the sky is dark at night.
- They wondered, if the sky is filled with an infinite number of stars, the sky shouldn't be dark.
- This statement is known as Olbers' paradox.
  (Although the brightness of a star is inversely proportional to the square of the distance, the number of stars counted increases as the square of the distance.)



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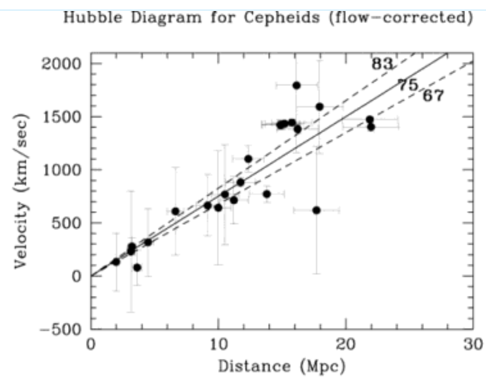
- Edgar Allen Poe (1809 - 1849) came up with the following solution to Olbers' paradox:
  Since light has a finite speed, if the Universe is not infinitely old, then light from the most distant sources has not had enough time to travel to us.
- Poe's solution suggests that the Universe had an early period when no stars (or galaxies) existed, and that they only formed some time ago. In other words the properties of the Universe are not constant in time.

2. Hubble's Redshift – Distance Relation.

Out to distances of 4000 Mpc (or a redshift of z=1) every galaxy whose distance can be measured obeys Hubble's redshift relation: The further away a galaxy is away from us, the larger its redshift is.



Evidence supporting the view that a galaxy's redshift (z) is proportional to its distance away from us for other galaxies:

1. Galaxies outside of our Local Group are all redshifted (no observations of blueshifted galaxies beyond a Mpc).
2. Over the last 80 years, as more powerful telescopes have been built, these telescopes have been used to find the distance to much further galaxies and these galaxies have all obeyed Hubble's redshift law. Therefore: Hubble's redshift law is not just a "local" effect.
3. Whenever we observe gravitationally lensed galaxies, the lensed arc-shaped image of the background galaxy is always at a larger redshift then the foreground galaxy (or cluster of galaxies) which causes the lensing.
4. The quasars with smaller redshifts ($z < 1.0$) have been easily resolved into the shape of a galaxy, while the larger redshift quasars (at $z \sim 4$) are difficult to resolve. This suggests that the high redshift quasars are further away and thus are more difficult to resolve.

The graph reveals a linear relation between galaxy velocity (*v*) and its distance (*d*)

$v = H_0 \times d$.

The expansion rate at the present time,

$H_\circ \approx 70$ km/s/Mpc$\equiv 100h$ km/s/Mpc, $h = 0.7$

$H_\circ$ =Hubble "constant" (=Hubble parameter at present ($H(t=t_{present})$))

The inverse of the Hubble Constant is the Hubble Time,

$t_H = d_H/v = 1/H_0 = 3/h \times 10^{17}$ s $= 1/h \times 10^{10}$ yr$\approx$ 14 Gyr

the time since a linear cosmic expansion has begun

(extrapolating a linear Hubble Law back to time t = 0);

it is thus related to the age of the Universe from the Big-Bang to today.

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$d = H_0^{-1} v = zc H_0^{-1}$ z= z= z Ex)

$$\frac{c}{H_0} = \frac{3\times10^5 \text{km/s}}{100h \text{ km/s/Mpc}} = \frac{3{,}000\ c}{ch/\text{Mpc}} = \frac{3\times10^3\text{Mpc}}{h} = \frac{10^4\text{Mly}}{h}$$

$1/H_0 =$ $(3.26/z_0 h)$Mpc yr/pc $=(0.978\text{x}10^4/h)$ Myr$=(0.978/h)$ 10 Gyr

$c/H_0 = (0.978\text{x}10^4/h)$ Mly$=(9.78/h)$ Gly

$$d = z\frac{10^4\text{Mly}}{h}$$

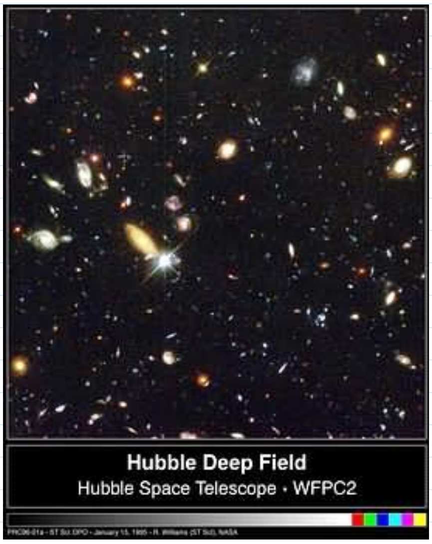
z=0.1 ⇝ d=1,000h^{-1} Mly, z=0.32 ⇝ d= h^{-1} Gpc

z=0.01⇝ d=100h^{-1} Mly, z=0.032 ⇝ d=100h^{-1} Mpc,

z=0.001⇝ d=10h^{-1} Mly, z=0.0032 ⇝ d=10h^{-1} Mpc,

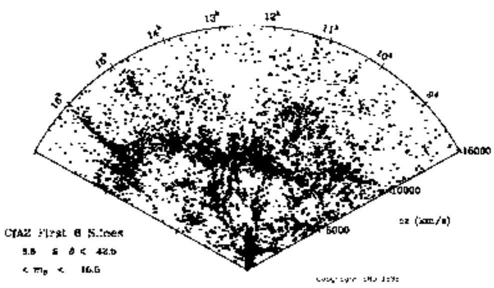
3. Evolution of Galaxy Properties

- Properties of galaxies with large redshifts tend to be different than those with small redshifts.
- For instance,
  - quasars only appear at larger redshifts.
  - Galaxies in the Hubble Deep and Ultra-Deep Fields are much smaller than galaxies today.
- This suggests that the properties of galaxies have evolved with time and as an extension, the properties of the universe have changed with time.
- This suggests that t**he universe is not static or unchanging**.



4. Uniformity of the Universe at Large Scales

- **Large scale structures :** Maps of the universe (Cfa, Los-Campanos, 2dF, SDSS and other redshift surveys) at distance scales of **a few 100 Mpc** show **large scale structures** (superclusters (The Great Wall) and large voids with no galaxies).
- **Smooth at larger scale** : Maps of the universe **at larger scales** (out to z = 0.3) show that the distribution of galaxies is **smoother** when one **averages over 1000 Mpc**.



- **The universe is isotropic** : At the largest distance scales that we can measure out to, there is **no direction** in the sky with more galaxies than the other directions. : the distribution of galaxies at the largest scales is the same in all directions.
- This isotropy could be due to the Milky Way being at the "centre" of the universe. However most astronomers adopt the Copernican philosophy which states that we are not at a special location in the universe.
- If we are not at a special location, then observers living in another galaxy cluster will measure a similar sky map.
- **The universe is homogeneous**. This leads to the assumption that the universe is **homogeneous** in space.
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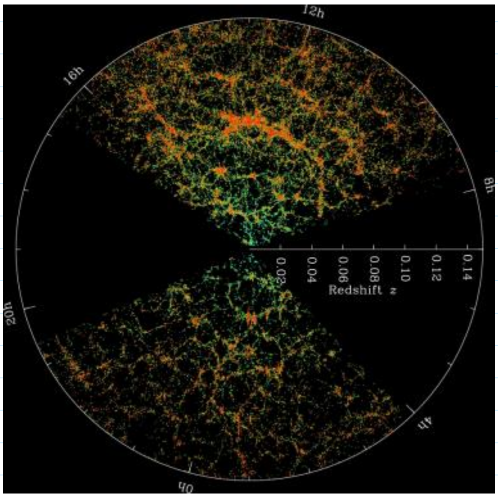
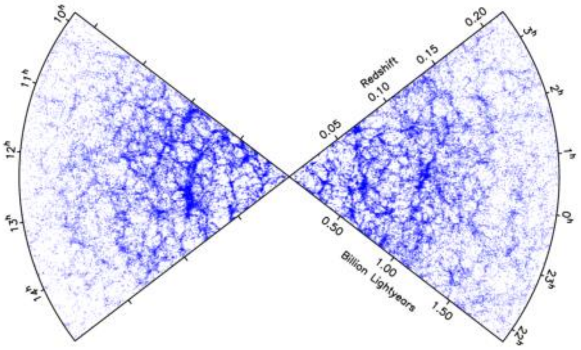


The SDSS's map of the Universe. Each dot is a galaxy; the color is the g-r color of that galaxy .

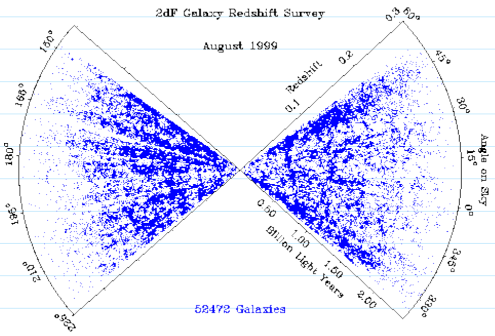
출처: <https://www.sdss.org/science/orangepie/>

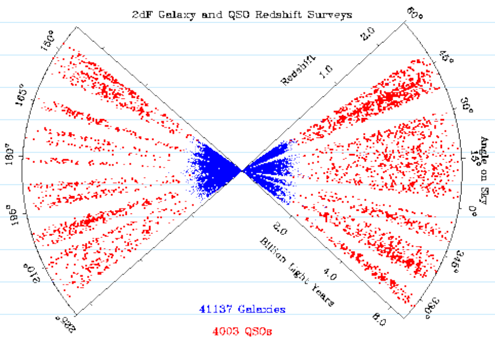


Slice image (blue on white)

출처: <https://www.roe.ac.uk/~jap/2df/2df_main.html>

The 2dF Galaxy Redshift Survey (2dfgrs.net)





Redshift cone diagram showing the distribution of objects from the 2dF galaxy and QSO redshift surveys as of August 1999. The top panel shows the distribution of the galaxies alone, and the bottom panel shows the distributions of both galaxies and QSOs

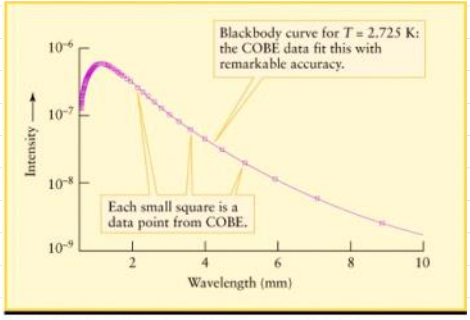
출처: <https://www.atnf.csiro.au/pasa/17_3/colless/paper/node2.html>

5. Cosmic Microwave Background Radiation

- When we say that the sky looks dark at night, this is

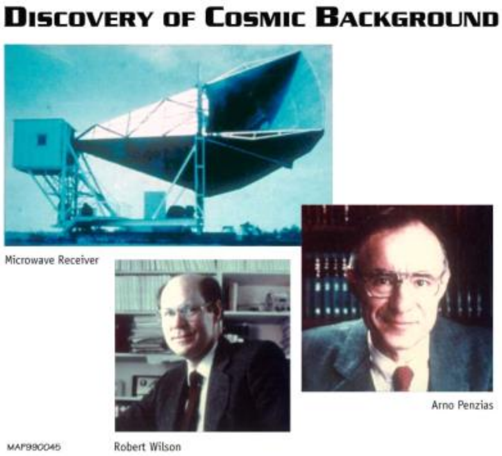
only because our eyes are sensitive to only a tiny part of the electromagnetic spectrum.

- If we look at any part of the sky with a radio telescope, **we see a glow of radiation coming from all directions.**
- **The spectrum of the radiation** from the sky can be measured and **corresponds to blackbody radiation of a temperature of 2.73 K** (Nobel Prize, 2006).
- (In order to see this spectrum we have to look at a part of the sky where the Milky Way and other nearby stars are absent.)
- This spectrum peaks a radio wavelength near 1mm which is sometimes called microwave.
- This radiation from the sky is called the Cosmic Microwave Background Radiation (CMBR).



(b) The spectrum of the cosmic microwave background

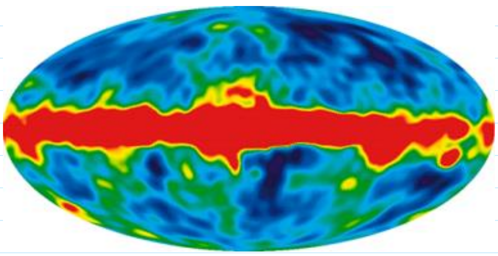
- Hot emission ~5K was predicted by theorists as a leftover of the hot early stage of the Universe evolution.
- This 2.73 K emission was discovered by Robert Wilson and Arno Penzias in the early 1960's.
- They were testing a very sensitive microwave receiver which was to be used to communicate with a satellite.
- They were discouraged by a constant hiss which they continued to detect whenever they pointed the receiver at apparently empty parts of the sky.



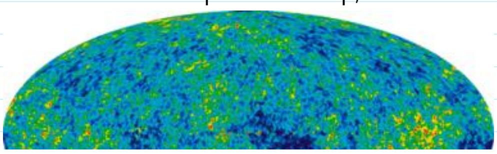
- They soon learned that the "Big Bang" model of the universe predicts that there was an early phase of the universe's life when it was filled with radiation.
- The early predictions were that the radiation should have a temperature less than 10 K.
- Penzias and Wilson were awarded the Nobel Prize in physics for their discovery.

Detailed CMBR Maps reveal fluctuations of temperature

- Modern measurements of the CMBR show that the temperature is very smooth, that it is almost constant in all directions.
- This smoothness lends support for the claim that **the universe is isotropic (and homogeneous) at large distance scales** .
- However, small temperature fluctuations should exist: some areas of the sky are hotter or cooler than 2.73

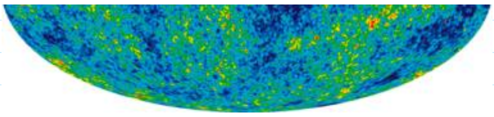


COBE temperature map, 1998

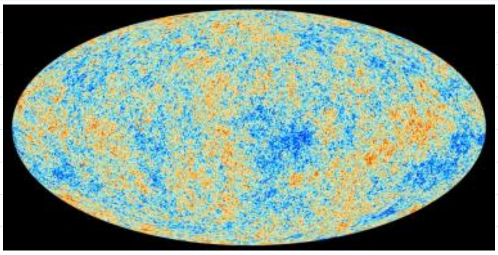


K by 3 x 10^{-5} K

- Such fluctuations at the level 3 x 10^{-5} K were discovered by the COBE satellite in 1992. (Nobel prize 2006)
- Now they are measured very accurately, for example by WMAP and recently launched Planck satellites.
- The Milky Way galaxy lies along the horizontal axis, and its light has been subtracted out of WMAP picture.
- The red spots are hotter than average.
- The blue spots are cooler than average.
- Cooler regions correspond to regions of the early universe which were denser than average.



WMAP temperature map, 2008



detected with the greatest precision yet by the Planck mission. Planck is a European Space Agency mission, with significant participation from NASA.

출처: <https://planck.ipac.caltech.edu/image/planck13-001a>

6. Abundance of Elements

- The abundance of elements seen in the oldest (Pop II) stars is approximately 75% Hydrogen (by mass) and 25% Helium with only a tiny fraction of a percentage of other elements.
- Hydrogen and Helium are two of the simplest elements.
- The fact that the oldest stars are composed of practically only Hydrogen and Helium, suggests that at the time the first stars were formed only Hydrogen and Helium existed.
- A theory of the Universe should explain why only Hydrogen and Helium are seen and their

출처: ·

TABLE 24.2 Redshift, Distance, and Look-Back Time

Redshift	V/C	Present Distance (Mpc)	Present Distance (10^6 light-years)	Look-Back Time (millions of years)
0.000	0.000	0	0	0
0.010	0.010	42	137	137
0.025	0.025	105	343	338
0.050	0.049	209	682	665
0.100	0.095	413	1350	1290
0.200	0.180	809	2640	2410
0.250	0.220	999	3260	2920
0.500	0.385	1880	6140	5020
0.750	0.508	2650	8640	6570
1.000	0.600	3320	10,800	7730
1.500	0.724	4400	14,400	9320
2.000	0.800	5250	17,100	10,300
3.000	0.882	6460	21,100	11,500
4.000	0.923	7310	23,800	12,100
5.000	0.946	7940	25,900	12,500
6.000	0.960	8420	27,500	12,700
10.000	0.984	9660	31,500	13,200
50.000	0.999	12,300	40,100	13,600
100.000	1.000	12,900	42,200	13,700
∞	1.000	14,600	47,500	13,700

The Expanding Universe

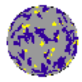
Concept of the expanding space

Balloon example

- Surface of the baloon represents two-dimensional (2D) space (balloon itself is three-dimensional (3D) object)
- Our Universe **may have been** 3D surface of 4D baloon

- Say, in this 2D space live flat 2D galaxies
- Expansion of the Universe is similar to the stretching of the **balloon surface**
- Its area increases (volume for us), the distances between every pairs of galaxies increases.
- We speak about "space expansion" but object that kept together by forces, are not stretched. Galaxies are not stretched, they kept to the same size by gravity. So do groups and clusters of galaxies, but larger structures are being stretched.

Cosmological redshift



- Proper interpretation of the **redshift of light from distant galaxies** is not Doppler effect, but **stretching of the photons as they propagate through the space.**
- As Universe expands, local velocity of photons is always **c**
- **Local velocity** of galaxies never exceeds **c** either. Even for distant ones, distance to which is increasing faster than c ! Actually, if only expansion is present, galaxies are at rest relative to the local space.
- This is because the position of distance galaxy at exactly the same moment as observation is unobservable in any experiments.

Uniform Expansion

- Consider an overly-simplified one-dimensional model of uniform expansion.
- In this model, we have a strip of 7 dots, each one representing a galaxy.
- The galaxies are spaced equal distance from each other.
- In the first diagram, we have a snap-shot of the "universe" taken at some early time called t_1.
- For example, make the distance between galaxies at t1 to be 1 m.

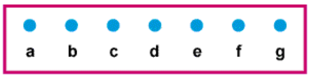


Photo of the Universe at time = t_1.

- If we now make the one-dimensional universe expand uniformly, each galaxy will move further apart from each other, keeping the distance (at any time) between galaxies constant.
- at a later time t_2, the Universe looks similar, but the distance between galaxies is larger.
- For example, make $t_2 = t_1 + 1$ s.
- At t_2 make all the distances between galaxies to increase twice.
- So betweeen the neighbouring galaxies the distance is now 2m.
- Again, the distance between galaxies changes, but the actual size of the galaxy stays constant. The galaxies are treated as point particles.

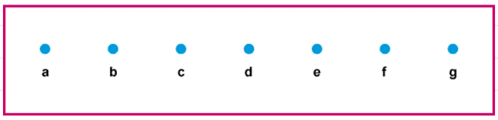
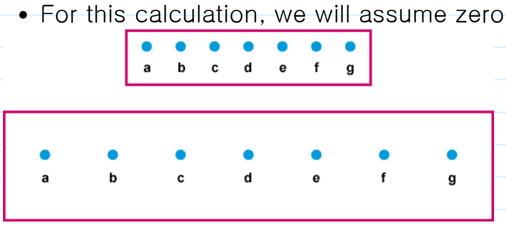


Photo of the Universe at time = t_2.

What does an Astronomer living in Galaxy d Measure?

- An astronomer will calculate the relative velocities and distances to other galaxies.
- For this calculation, we will assume zero acceleration between t_1 and t_2.



Galaxy c

- Define $\mathbf{d_c}$ distance between galaxies c and d at t_2.
- Define $\mathbf{v_c}$ relative velocity between galaxies c and d.
- At time t_2, $\mathbf{d_c} = \mathbf{2\ m}$ (to the left of d)
- The relative velocity of galaxy c with respect to galaxy d is:

Galaxy e

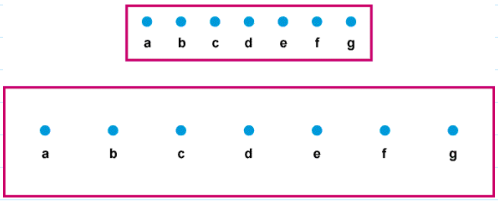
- A similar calculation shows that galaxy e (to the right of d) has:
- $\mathbf{d_e}$ = 2 m
- $\mathbf{v_e}$ = + 1m/s

v_c = (change in distance)/(change in time)

= (2m - 1m)/1s

= (1m)/(1s) (moving to the left)

- (At time t_1 the galaxies are separated by 1m and at t_2 they are separated by 2m, so the change in distance is 1m over a time period of 1s.)

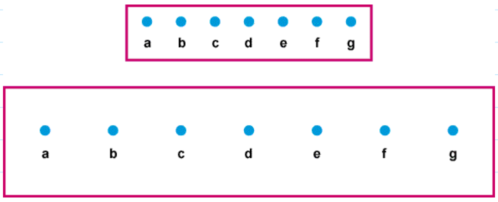


Galaxy b

- At t_2, the displacement and velocity relative to d for galaxy b are:
- $\mathbf{d_b}$ = 4 m (to the left)
- $\mathbf{v_b}$ = (4m - 2m)/1s = 2 m/s (to the left)

Galaxy f

- At t_2, the displacement and velocity relative to d for galaxy f are:
- $\mathbf{d_f}$ = 4 m (to the right)
- $\mathbf{v_f}$ = (4m - 2m)/1s = 2 m/s (to the right)



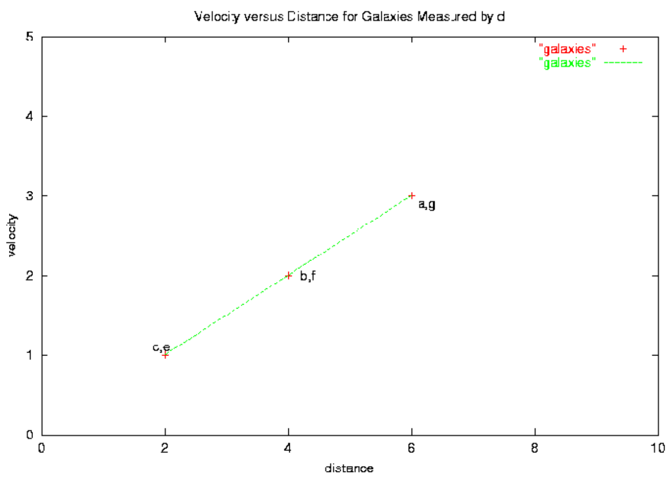
Galaxy a

- At t_2, the displacement and velocity relative to d for galaxy a are:
- $\mathbf{d_a}$ = 6 m (to the left)
- $\mathbf{v_a}$ = (6m - 3m)/1s = 3 m/s (to the left)

Galaxy g

- At t_2, the displacement and velocity relative to d for galaxy g are:
- $\mathbf{d_g}$ = 6 m (to the right)
- $\mathbf{v_g}$ = (6m - 3m)/1s = 3 m/s (to the right)

The astronomer living in galaxy d can now plot velocity versus distance on a graph, as shown below.



- The data points from measurements of galaxies a,b,c,e,f,g all fall on a straight line.
- The slope of this line is
  slope = rise/run = (3m/s − 1m/s)/(6m − 2m) = 2/(4s) = 0.5/s
- This is analogous to Hubble's law with a value of Hubble's constant H_0 = 0.5/s
- The Hubble time is $t_H = 1/H_0$ = 2 s.

Observations made by astronomers in other galaxies

- The astronomer in galaxy **d** observes that all galaxies are moving away from galaxy d and that the velocity increases linearly with distance.
- Another astronomer in another galaxy (say galaxy **c**) can also make similar measurements.
- The astronomer at galaxy **c** will also find that all galaxies are moving away from galaxy **c** and that the slope of the graph of velocity versus distance is 0.5/s, exactly the same as for the astronomer in galaxy **d** .
- However, one important difference: This model has two galaxies to the left of galaxy c and four galaxies to the right of galaxy c.
- The astronomer in galaxy c will see that more galaxies are in one direction, and thus can tell that galaxy d is at a special location.
- If we wish to adopt the **Copernican principle**, that we are not in a special location in the universe, we should add another complication to this simple one-dimensional model. Two ways to change the model are:
  1. The universe should be infinite in size, so that an infinite number of galaxies exist. This means that there is no edge to the universe and that every astronomer in every galaxy sees that there are an infinite number of galaxies to the right and to the left. This type of model is called **open**.
  2. The universe has a finite number of galaxies, but they exist on a closed loop, so that galaxy a is next to galaxy g. In this universe it is possible to travel in one direction for and end up at the starting point. This type of model is called **closed**.
- Both open and closed models have the property that there is no special galaxy. All astronomers making cosmological measurements from different galaxies agree on the properties of the universe.
- Balloon example (in 3D) is exactly the **closed** model of the Universe.
- An infinite 3D plane is an example of an **open** model of the Universe (but there are others).

The Big Bang Model of the Universe

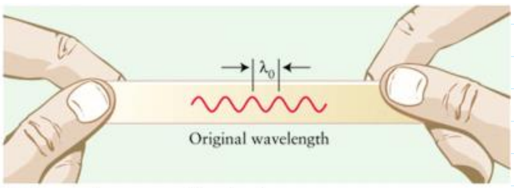
- The Big Bang model of our Universe is similar to these simple models, but it is three-dimensional.
- During the uniform expansion distances in all directions change by the same amount.
- The Big Bang refers to the extrapolation down to t=0, when all galaxies are separated by zero distance.
- The time t=0 corresponds to infinite density and is not properly explained by the equations. It is called a singularity.
- The singularity occurs everywhere in space at t=0.
- There is no special "place" where the Big Bang happened.
- The physics of the universe at time t>0 is well defined.
- The basic Big Bang model can not explain why the Big Bang happened or "what happened" before t=0.
- The Big Bang theory is actually a family of models, some of which are open and some are closed.
- There is also a special open model called the critical model.
- One of the goals of the science of cosmology is to discover whether our Universe is open, closed or critical.

Cosmological redshift and expansion of the Universe

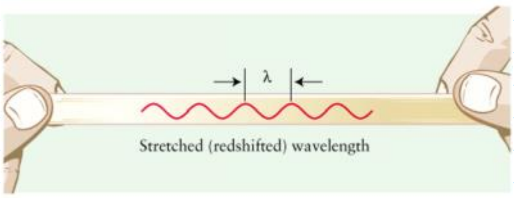
- We understood that cosmological redshift

  $z = (\lambda_0 - \lambda)/\lambda$

  is the result of stretching the space.
- Thus, the redshift can be immediately related to how much the Universe has expanded between the photon emission and observation.



(a) A wave drawn on a rubber band ...



(b) ... increases in wavelength as the rubber band is stretched.

Caution: &lambda and &lambda$_0$ in Figure and formulas are exchanged

Let us describe the expansion of the Universe with the help of scale factor **a(t)** which gives and average distance between galaxies. Then $a(t_0)/a(t)$ gives the amount of expansion between time **t** and **t_o**.

Then, this is the amount by which the wavelength of the photon was stretched, thus

$z = \lambda_0/\lambda - 1 = a(t_0)/a(t) - 1$

- We usually write

$$1 + z = a(t_0)/a(t) = \lambda_0/\lambda$$

- Redshift is convenient way to measure time. Saying something happened at redshift z means when the Universe was compressed 1+z times, independently on how exactly the expansion was proceeding.
- At the present moment z=0, and z increases for more and more earlier times. (z→ ∞ at Big Bang)

출처: <https://sites.ualberta.ca/~pogosyan/teaching/ASTRO_122/lect30/lecture30.html>

$$z=\frac{\delta\lambda}{\lambda} = \frac{v}{c} = \frac{H_0 d}{c} = \frac{1}{c}100h\frac{\text{km}}{\text{s Mpc}}\, d = \frac{1}{c}100h\frac{\text{km}}{\text{s Mpc}} d = \frac{1\text{s}}{3\times10^5\text{km}}10^2 h\frac{\text{km}}{\text{s Mpc}} d = \frac{h}{3\times10^3} d_{\text{Mpc}}$$
$$= \frac{h}{3.26\times3\times10^3} d_{\text{Mly}} = \frac{h}{0.978}10^{-4} d_{\text{Mly}} = \frac{h}{0.978}10^{-1} d_{\text{Gly}}$$

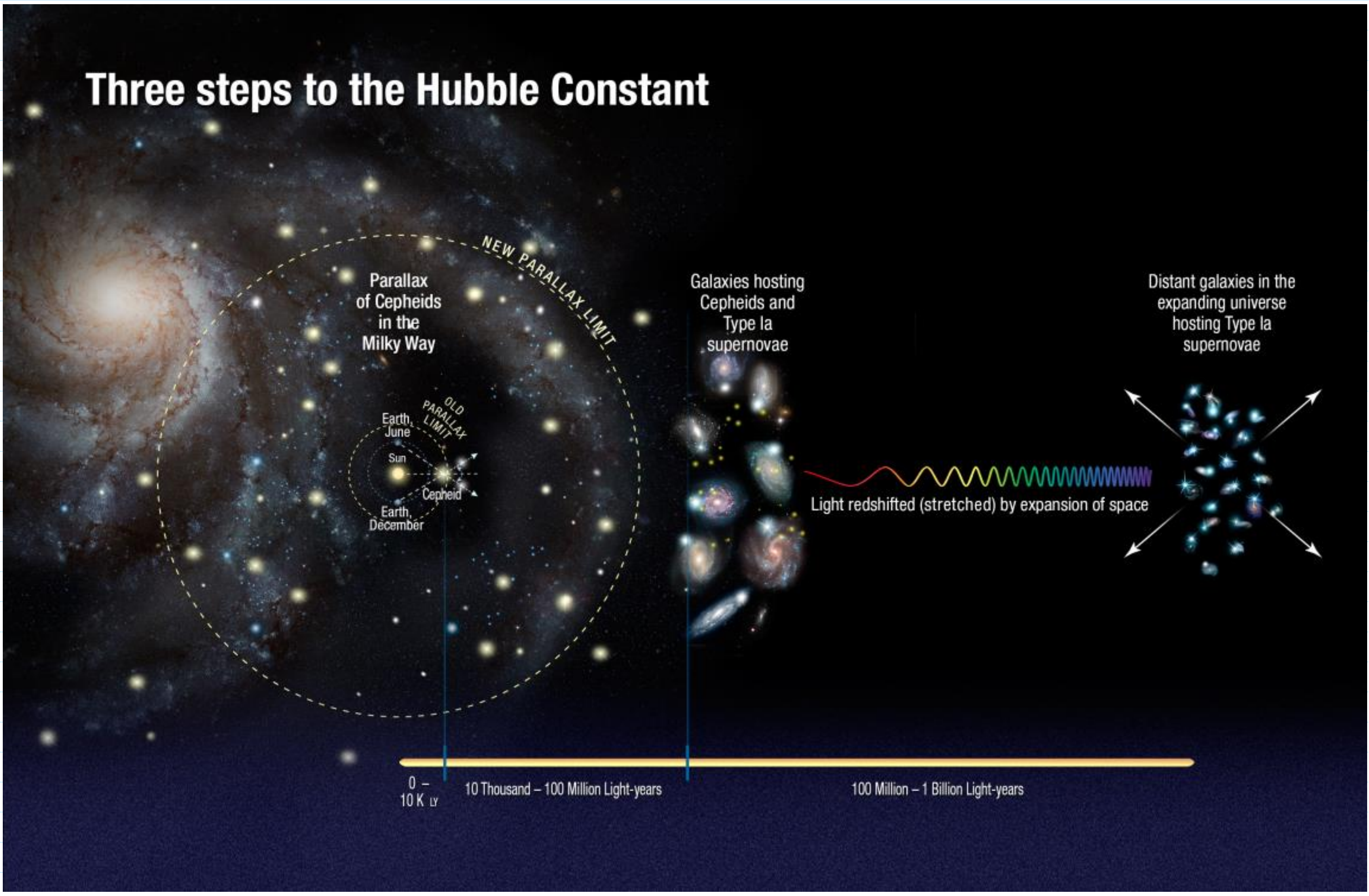
$$d_{\text{Mly}} = 10^4 z\, 0.978\, h^{-1} \qquad d_{\text{Mpc}} = 3\times10^3 \text{z}\, h^{-1}$$

Ex) z=0.1 d=1 h^{-1}Gly
z=0.01 d=100 h^{-1}Mly
z=0.001 d=10 h^{-1}Mly

How to measure H_0?

Three Steps to Measuring the Hubble Constant

출처: <https://www.nasa.gov/image-feature/goddard/2016/three-steps-to-measuring-the-hubble-constant/>



The Expanding Universe

- expansion rate (Hubble constant)

  $H_\circ = 100h$ km/s/Mpc, $\quad h = 0.7$

  $= h/3 \times 10^{-17}\ \mathrm{s}^{-1} = h\ 10^{-10}\ \mathrm{yr}^{-1}$

- the *Robertson–Walker metric* : homogeneous & isotropic solution of the Einstein Eq.

$$ds^2(t) = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \qquad k = +1, 0, -1 \text{ (curvature)}$$

Einstein Equation for the scale factor

$$\frac{1}{2}\dot{a}^2 = -\frac{1}{2}k + \frac{4}{3}\pi a^2(\rho_{\text{matter}} + \rho_\Lambda)$$

or

$$\frac{3H^2}{8\pi} = -\frac{3k}{8\pi R^2} + \rho_{\text{matter}} + \rho_\Lambda \qquad H(t) = \frac{\dot{a}}{a} \quad \textit{Hubble parameter}$$

Or

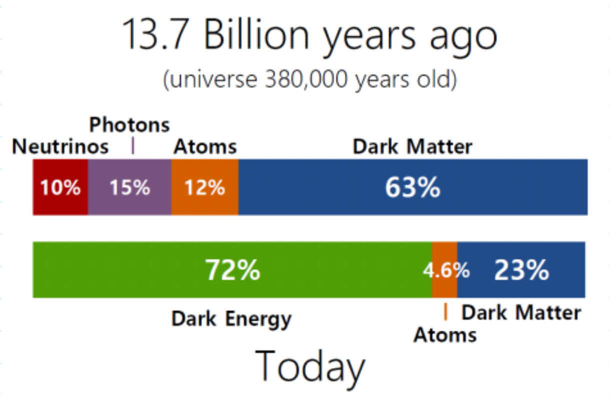
$$H^2 = \frac{8\pi}{3}(\rho_{\mathrm{k}} + \rho_{\text{matter}} + \rho_\Lambda), \quad \rho_{\mathrm{c}} = \rho_{H_0} = \frac{3}{8\pi}{H_0}^2, \quad \rho_{\mathrm{k}} = -\frac{3k}{8\pi R^2}$$

$$1 = \Omega_k^{(0)} + \Omega_{matt}^{(0)} + \Omega_\Lambda^{(0)} \qquad = \Omega_{\mathrm{k}} + \Omega \quad \Omega_i^{(0)} = \rho_i^0/\rho_{\mathrm{c}}$$

Another form of the equation (acceleration)

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3p)$$

For ordinary matter or energy, $\ddot{a} < 0$ i.e., decceleration.



Comparison of the contents of the universe today to 380,000 years after the Big Bang as measured with 5 year WMAP data (from 2008). (Due to rounding errors, the sum of these numbers is not 100%). This reflects the 2008 limits of WMAP's ability to define dark matter and dark energy.

출처: <https://en.wikipedia.org/wiki/Universe>

Matter content of the Universe

Matter (nonrelativistic)

usual matter, **Baryons** (consisting of quarks, ex)proton, neutron) & Leptons(ex) electron) ; forming atoms & molecules. Can be **luminous**

Dark Matter : nonbaryonic, "should" exist but, we do not know yet what it is

Radiation (relativistic matter)

Photons (highly relativistic matters behaves as the "radiation")

Dark Energy

(Curvature)

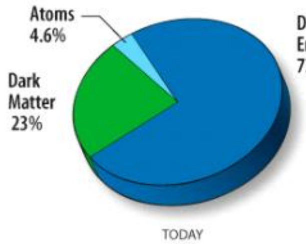
Matter : Equation of state $\quad \boldsymbol{p = w\rho}$

radiation (rel. Matt.)	Nonrel. Matt.	Curvature	Dark energy
$w = +1/3\ (p = \frac{1}{3}\rho)$	$0\ (p = 0)$	$-1/3$	$-1\ (p = -\rho)$

What do we have in our Universe at the present time ? (they are function of time)

- We can see luminous matter . We can count stars in our galaxy, then count galaxies to get an estimate. We get density of luminous matter ρ_b ("b" stands conventionally for "baryons" – protons and neutrons)
  ρ_b = 4.2 x 10^{-28} kg/m^3
- Presence of the dark matter reveals that there is other gravitating matter that we do not see. Current estimates for the total matter density ρ_m
  ρ_m = 2.4 x 10^{-27} kg/m^3 (≈ mass of 1.5 protons per cubic meter) ($m_{\text{proton}} = 1.6726 \times 10^{-27}$kg)
  i.e we see 17% of the total matter is the luminous matter
- There is Cosmic Microwave Background radiation. How much energy is in it ? This can be determined very accurately. We express is as mass density of radiation ρ_{rad} (mass=E/c^2)
  ρ_{rad} = 4 σ T^4/c^3= 4.6 x 10 $^{-31}$ kg/m^3 (cf: $m_{\text{electron}} = 0.91 \times 10^{-30}$kg)

Is is a lot or a little ?



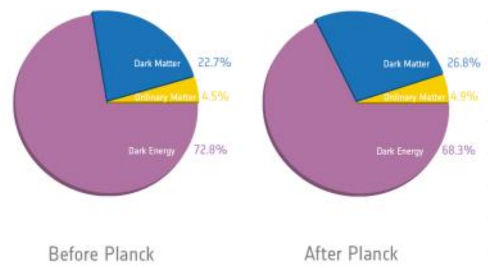
- Need to compare with something cosmological.
- There is a **critical density ρ_c** that is associated with the expansion rate (Hubble constant)

H∘ = $100h$ km/s/Mpc = h/3× 10^{-17} s^{-1}, h=0.7

$\boldsymbol{\rho_c = 3H_0^2/(8\pi G) = 1\text{x}10^{-26}\text{kg/m}^3}$

(G=6.67430x10^{-11}m^{3}kg^{-1}s^{-2})

Check : $\rho_c = \frac{3}{8\pi}\left(\frac{h}{3}\right)^2 \times 10^{-34} \, \frac{1}{6.6743} \times 10^{11}\text{m}^{-3}\text{kg}^1$
$= 2\text{h}^2 \times 10^{-26}\text{kg m}^{-3}$



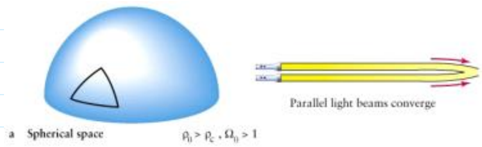
- So we have the following ratios to the critical density (at present)
- $\Omega_{\text{rad}} = \rho_{\text{rad}}/\rho_c$=4.6x10^{-5}, $\Omega_b = \rho_b/\rho_c$=0.042, $\Omega_m = \rho_m/\rho_c$= 0.24
- We also define total mass density ratio

$\Omega_0 = \Sigma\ \Omega_i = \Omega_{\text{matt}} + \Omega_\Lambda = \Omega_{\text{rad}} + \Omega_b(\textit{ordinary matter}) + \Omega_m(\textit{Dark Matter}) + \Omega_\Lambda$ **(sum of all contributions)** Einstein Eq. 1= $\Omega_k + \Omega_{\text{matt}} + \Omega_\Lambda, = \Omega_k + \Omega_0$

Matter content and Spatial Geometry

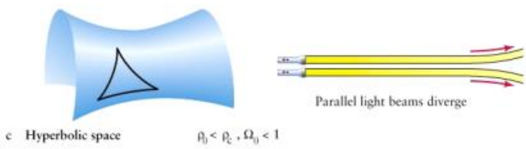
- There are also three types of geometries the Universe could have.
- The three types of geometries are related to Ω_0
- One of the goals of observational cosmology is to determine the geometry of the universe.

▪**If $\Omega_0 > 1$**, then k=+1, positive curvature
-Finite yet unbounded spherical Universe then
-Parallel light rays converge
the universe has a **closed geometry**.
Ex) If the universe were two-dimensional, its geometry would be like the surface of a sphere.



$$ds^2 = \frac{dr^2}{1-r^2} + r^2 d\phi^2 = d\theta^2 + \sin^2\theta \, d\phi^2$$

▪**If $\Omega_0 < 1$**, then k=-1, negative curvature
-Infinite, hyperbolic Universe
-Parallel light rays diverge
the universe is **open**.
Ex) If it were two-dimensional would have the geometry of a hyperboloid, which is saddle-shaped.



$$ds^2 = \frac{dr^2}{1+r^2} + r^2 d\phi^2 = d\chi^2 + \sinh^2\chi \, d\phi^2$$

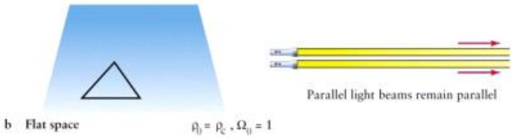
▪**If $\Omega_0 = 1$**, then k=0, Critical (Flat) curvature

-Infinite, flat Universe

-Parallel light rays stay parallel

the universe is **flat**, the critical solution.

Ex) The geometry of the critical solution is flat, so that distances between points can be calculated using the Pythagorean Theorem.



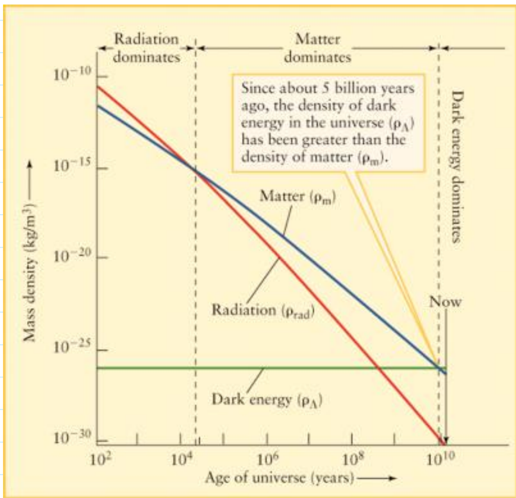
$$ds^2 = dr^2 + r^2 d\phi^2 = d\theta^2 + \sin^2\theta \, d\phi^2$$

So what about our Universe ?

- One should not think that density causes the space to adopt a geometry. Remember critical density depends on expansion rate. Sop it is more like if the Universe has a given geometry and a matter content, it will expand with the rate to reflect it.
- So we have $\Omega_{\mathrm{rad}}+\Omega_{\mathrm{m}}$=0.24. Does it mean our space is open with hyperbolic geometry ? Only if we are sure that we accounted for all possible kinds of matter and that Einstein equations are correct.
- However, one can measure the curvature of the space independently, and more accurately that accounting for different types of matter.
- You need a standard rod (something with known physical size) at a known distance . Distances better be large, cosmological !
- Then measuring its angular diameter and comparing with small angle formula, one can find the geometry of space. Small angle formula is valid only if space is flat !
- We shall discuss how CMB can provide such standard rod. The conclusion is
  Ω_0 = 1.00–1.02
  i.e our space is flat (k=0)!
- So where is the 75% of density ? There must be some other invisible matter (or modification to Einstein equations). It is known under the name of cosmological constant , Λ –term, dark energy …

Evolution of density throughout the time

- As Universe expands, the density of matter changes
- We say "the expansion of the universe is dominated by Λ term", or "universe is radiation dominated", which specifies what component has the largest contribution to the total density
- However, the density of different types of matter changes differently as Universe expands
  - Ordinary matter. Its density changes because the volume of the Universe changes, while no new matter is created. ρ_{m} ~ 1/V ~ 1/a^3 where "a" is the distance scale between galaxies. Therefore with redshift
    $\rho_{\mathrm{m}}(t)/\rho_{\mathrm{m}}(t_0)= a(t_0)^3/a^3 = (1+z)^3$
  - Radiation. Best to think about photons. Its **number density** changes ~ 1/V as well, whoever each photon wavelength is redshifted, so energy of individiual photon also changes as E ~ (1+z) ~ 1/a. Therefore the **energy density** (expressed as mass density) of radiation evolves as
    ρ_{rad} ~ $1/a^4$ or
    $\rho_{\mathrm{rad}}/\rho_{\mathrm{rad}}(t_0)= a(t_0)^3/a^3 \times a(t_0)/a = (1+z)^4$



Summary

ρ_{rad} ~ $1/a^4$ m=0 or E≫m

ρ_{m} ~ 1/V ~ $1/a^3$ nonrelativistic

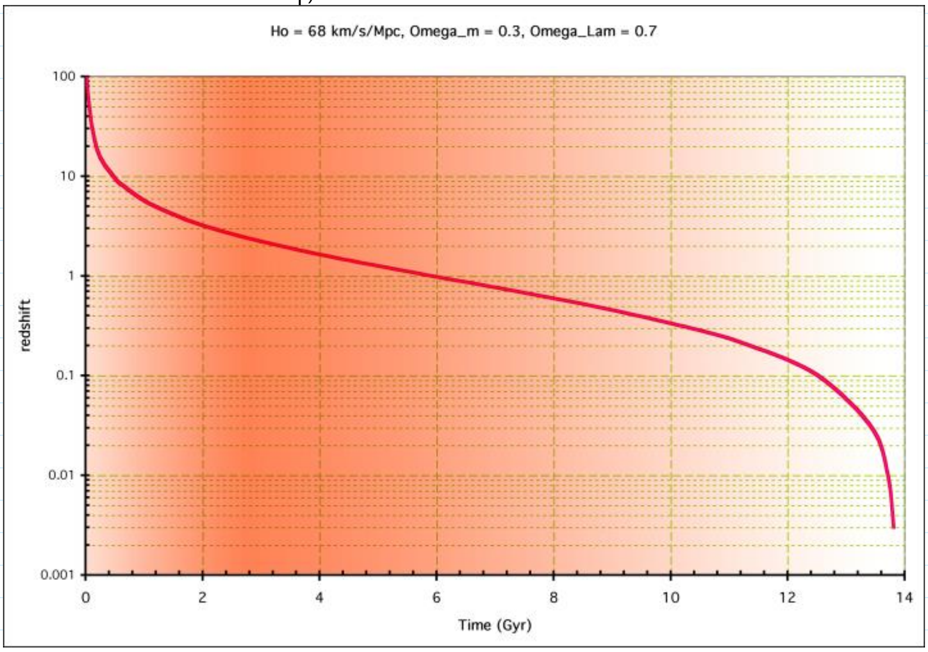
ρ_{k} ~ $1/a^2$ curvature term

ρ_Λ ~ $1/a^0$ dark energy

- Dark energy - its **density** does not change at all, that's why it is called cosmological constant !
  ρ_Λ = const
- Therefore in the past the balance between the different types of matter was different.
  The Universe passes through
  radiation-dominated → **matter**-dominated → cosmological constant(**dark energy**)-dominated stages
- At the present time, according to the accepted model, the Universe is dark energy dominated

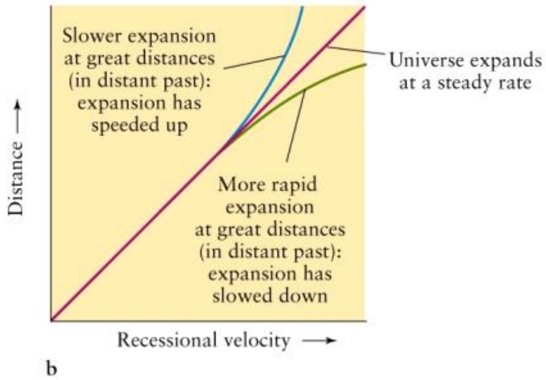
Redshift as look back time – a reminder

- In cosmology the light signals were emitted in the past and are observed now.
- Earlier the light signal was emitted, more it is redshifted by the time it reaches us.
- Thus, the redshift **z** of the source tells how far & **how old** in the past the light was emitted.
- Higher 'z' – further in the past the source is (and also – further away).
- z=0 is the present moment (the light is emitted right here, and took no time to travel, so photons did not change their wavelength at all).
- z=infinity at the start of Big Bang, when we consider t=0
- Relation between redshift and time depends on how exactly the expansion of the Universe proceeded. (Dynamics, need to solve the Einstein Eq.)



Changes in the Expansion Rate

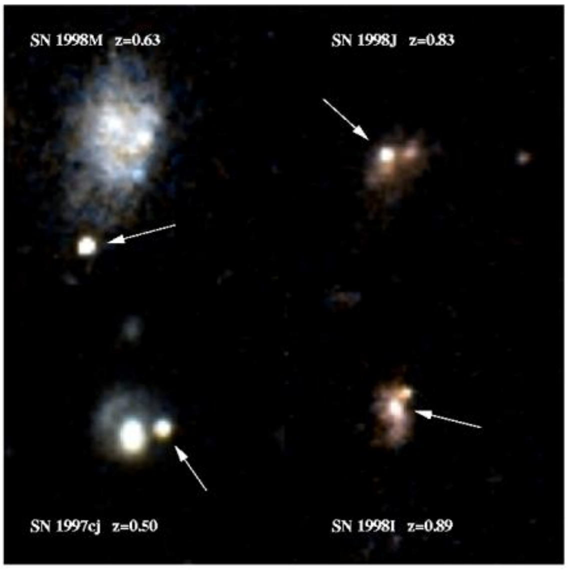
- In our simplistic model we assumed that there is no acceleration.
- However, there **must** be an acceleration, since all of the galaxies will feel the gravitational attraction of each other.
- Consider a model involving only two galaxies.
- Suppose that at an initial time the two galaxies are moving away from each other.
- Their gravitational attraction provides a force which tries to pull them back together.



- The gravitational attraction is a deceleration which slows down the expansion.
- We then expect that the same galaxy was moving at a faster speed in the past and that it's speed in the future will be slower.
- When we observe galaxies at large distances from us, we are seeing them as they were in the past.
- Since these galaxies were moving faster in the past than they are today, our graph of velocity versus distance should not be exactly a straight line.

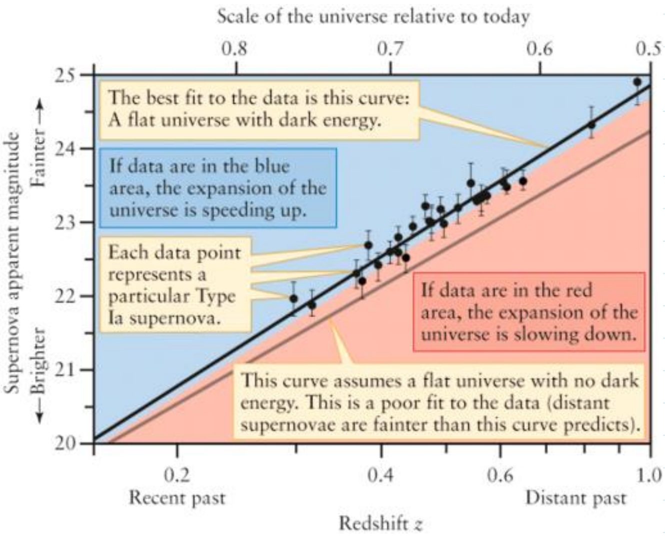
Measurements of the Universe's Acceleration

- Recently astronomers have observed Type Ia supernovae in order to use them as a standard candle to find the distances to the galaxies where the supernovae occur.



Evidence for Acceleration from Type Ia Supernovae

- The astronomers plotted (M-m) versus redshift (z), which is equivalent to plotting distance versus velocity.
- The data favours a universe whose expansion is accelerating so that the expansion speed increases with time!
- opposite to what was expected!
- The expansion changes depends on the matter content and type of the Universe,(If $\rho + 3p > 0$, then decceleration)
- The **deceleration** parameter from Einstein equations is **q(t)** = $\Omega_{\mathrm{rad}}(t)$ **+ 1/2** $\Omega_{\mathrm{m}}(t)$ - $\Omega_{\Lambda}(t)$
- when **q > 0,** expansion decelerates, while if **q < 0,** it accelerates



$$\rho + 3p = \left(\rho_{\mathrm{rad}} + 3 \times \frac{1}{3}\rho_{\mathrm{rad}}\right) + (\rho_{\mathrm{m}} + 0) + (\rho_{\Lambda} - 3\rho_{\Lambda})$$
$$\approx \rho_{\mathrm{m}} - 2\rho_{\Lambda}$$
$$\propto \frac{1}{2}\,\Omega_{\mathrm{m}} - \Omega_{\Lambda}$$

- Neglecting Ω_{rad}, we conclude that Universe was decelerating while $\Omega_{\mathrm{m}} > 2\,\Omega_{\Lambda}$ and accelerating afterwards.
  Note: all Ω's are functions of time. For example $\Omega_{\mathrm{m}}(t) = \rho_{\mathrm{m}}(t)/\rho_{\mathrm{crit}}(t)$, etc. Whether Universe accelerates or not does not depend on the time dependence of critical density.
- Note that ordinary matter always causes deceleration. Acceleration requires an unusual type of energy whose density does not decrease with expansion! This is very strong argument for existence of "dark energy"

Einstein's Theory of General Relativity

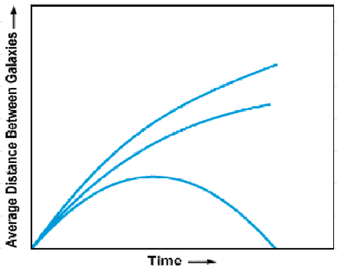
- Einstein's theory of general relativity is the description of gravity which keeps the speed of light as the ultimate speed limit for all information, including gravity itself.
- Einstein solved his equations for a simple model of the universe which has a uniform distribution of galaxies.
- He found that if he placed all the galaxies at rest at constant separation from each other, they would all collapse to a point due to their mutual gravitational attraction.
- If all the galaxies were given an initial velocity so that they were all moving away from each other, it was possible to find solutions which did not collapse, however, no static solution was possible. This was proven by Friedman.

The Cosmological Constant

- Einstein was very unhappy about the expanding universe solutions to his equations.
- Einstein derived the equations before Hubble and Humason discovered the redshift-distance relation which can imply that the universe expands.
- Einstein thought that the universe should be static and unchanging.
- He realised that the only way to get a static universe was to throw an extra constant into his equations.
- This extra constant is called the Cosmological Constant.
- The cosmological constant has the effect of providing an outwards force sort of like "anti-gravity".
- If the "anti-gravity" effect is chosen large enough to cancel the gravitational attraction of the galaxies, the universe can be static.
- Later when Hubble's law was discovered Einstein discarded the constant.
- Today, we introduce the constant back, however the value of the constant must be much larger than Einstein thought it should be.
- The preferred view is that cosmological constant is an contribution of an unusual form of energy (dubbed "dark").
- If a cosmological constant exists, it has the effect of accelerating the expansion of the universe so that galaxies speed up in the future instead of slowing down.
- If we can measure both the density of matter and the cosmological constant, we can determine the fate of the universe.
- The data from Type Ia supernovae favours the existence of a cosmological constant and a low matter density.
- Their data suggests that the universe will expand forever.
- The physical reason for the existence of the cosmological constant is still unknown.
- An important area of present-day research in theoretical astrophysics is on physical methods for creating a cosmological constant.

Future of the Universe

- There are three types of solutions to Einstein's cosmological equations.
- They can be understood by analogy with a universe with only two galaxies with equal masses M which are thrown in opposite directions from each other.

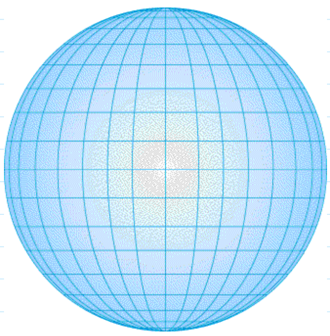


a. If the two galaxies are given a large enough initial speed (away from each other) that is larger than the escape velocity, the galaxies continue to move away from each other forever. This type of solution is called **unbounded**.
b. If the initial speed is lower than the escape velocity, the galaxies move out to some maximum separation and then return to each other so that they come closer to each other. A universe such as this ends in a **Big Crunch**. This type of universe is called **bounded**.
c. If the initial speed is exactly the escape velocity, then the galaxies continue to move apart from each other for all time. This is a special solution called the **marginally bounded**.

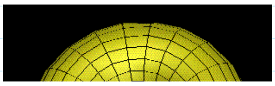
- The distance between galaxies for the three types of solutions are shown.
- If there was no "cosmological constant", open and flat Universes would expand forever, while closed Universe will end in big crunch. Geometry would have determined the future fate.
- However, once Universe become "cosmological constant dominated" it will expand with acceleration forever.
- Unless "dark energy" decay somehow in another form of matter.

Spatial Topology

- If the geometry of the universe is closed, Einstein's equations do not tell you what the topology is.
- Topology tells you how points on a surface are connected.
- For 2-dimensional surfaces we can classify the types of topology by the number of holes.
- The simplest topology is a sphere which has no holes.



- The next simplest topology is a torus, which is the surface of a donut.
- While it may seem unlikely, the universe could have a complicated torus-like topology.

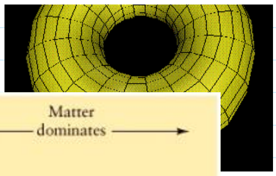


• If so, we would expect patterns in the sky to repeat in regular ways.

• A different way to view a torus is as a square where opposite sides have been glued together.

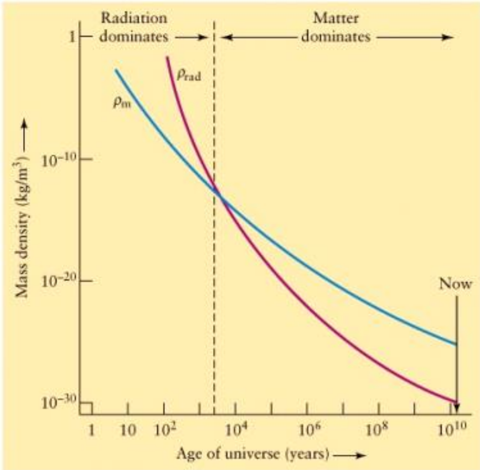
• A neat way to visualize a torus can be found through some simple games at http://www.geometrygames.org/TorusGames/html/Maze.html

출처: <https://sites.ualberta.ca/~pogosyan/teaching/ASTRO_122/lect30a/lecture30a.html>



The Cosmic Microwave Background Radiation

Temperature and Density of the Universe



- The basic premise of the Big Bang theory of the Universe is that the Universe used to be **hotter and denser** than it is today.
- The present density is estimated to be 10^{-27} kg per cubic meter.
- The matter density is inversely proportional to the cube of the typical distance between galaxies.

  $\rho_m(t) = \rho_m(t_0)\,(1+z(t))^3$
- For example: About 6.5 billion years ago, the distances between the Local Group of galaxies and the Coma Cluster of galaxies was about half of the present distance.
- The volume of the sphere containing the Local group and the Coma Cluster was 1/8 the present volume.
- This means that the density of matter was 8 times larger at this early time.

Overview : Thermal History of the Universe

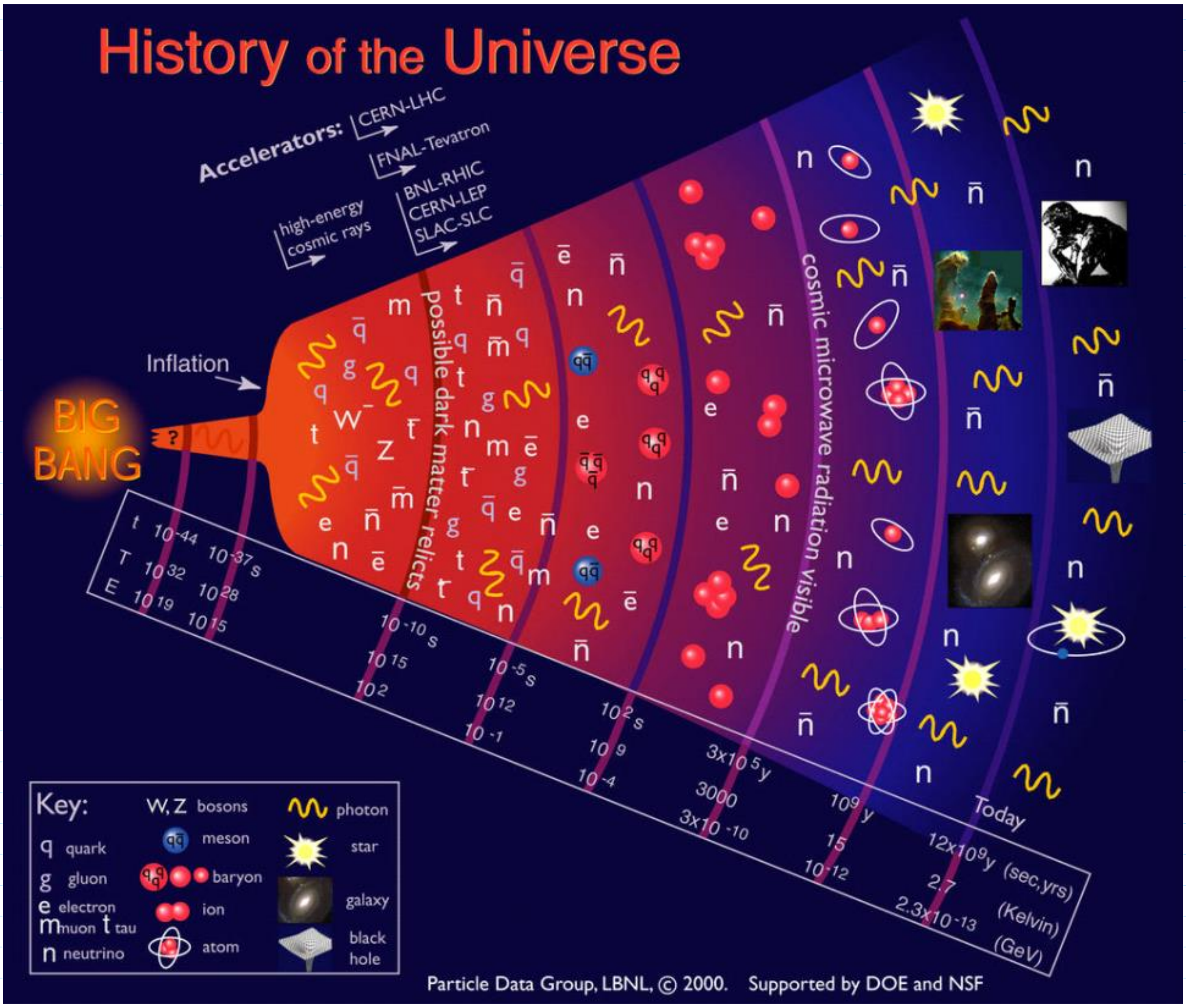
What	When (t)	When (z)	When (T)	When (E)
Neutrino Decoupling	1 second	$6\text{x}10^9$	10^{10}K	1 MeV
Nucleosynthesis (BBN)	3 minutes	$4\text{x}10^8$	10^9 K	0.1MeV
Matter-Radiation Equality	50,000 years	3,400	8,700 K	~ 0.8 eV
Recombination	~370,000 years	1,100	3,000 K	~0.3eV
Matter-Λ Equality	10^{10} years	0.4	3.8 K	~$3.2\text{x}10^{-4}eV$
Today	$1.4\text{x}10^{10}$ years	0	2.73 K	~$2.3\text{x}10^{-4}eV$

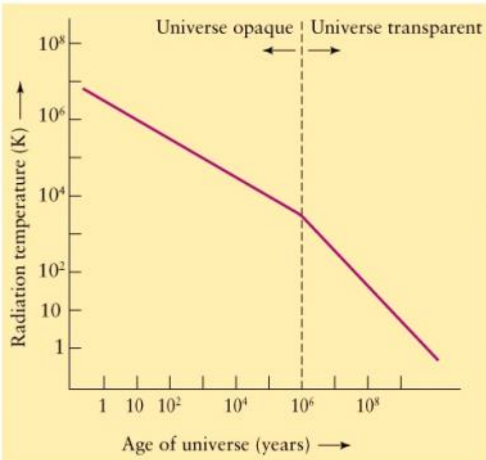
t= 0	1sec	3min.	50,000 yr	370,000 yrs	10^{10} yrs	$1.4\text{x}10^{10}$ yrs
T=∞	10^{10}K	10^9 K	8,700K	3,000K	3.8K	2.73K
z= ∞			3,400	1,100	0.4	0
	ν-decpl	BBN	$\rho_{rad} = \rho_{Matt}$	Recombination	$\rho_{Matt} = \rho_\Lambda$	

⟵ Radiation Dominated ⟶ | ⟵ Matter Dominated ⟶ | ⟵ Λ Dominated

opaque ⟶ | ⟵ transparent; CMB ⟶ |

quarks, leptons, gauge bosons	baryons(p,n), leptons(e,ν)	Nucleus **(p,n,He,)**	
		e,ν	atoms(H, He)
			ν
	γ	γ	γ





Temperature of the Universe

- The present temperature of the Universe is 2.725 K. (Based on measurements of the CMBR.)
- The temperature of the Universe in the Big Bang theory is inversely proportional to the typical distance between galaxy clusters.

  $T = T(t_0)\ (1+z)$

  or

  $T(t) \sim 1/a(t)$

- For example:
  - About 6.5 billion years ago, the distances between the Local Group of galaxies and the Coma Cluster of galaxies was about half of the present distance.
  - Astronomers living 6.5 billion years ago would have measured the Cosmic Background radiation to be 6K.

Problem: At what redshift did the Universe changed from radiation to matter dominated ? What was the temperature then ?
Solution

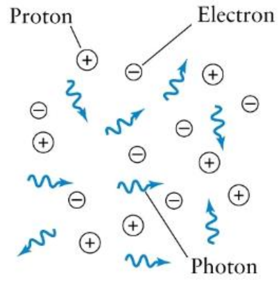
- Very early he Universe was radiation-dominated $\rho_{\mathrm{rad}}(t)$ **>** $\rho_{\mathrm{m}}(t)$.

- Transition took place when $\rho_{\mathrm{rad}}(t) = \rho_{\mathrm{m}}(t)$. i.e, when
  $\rho_{\mathrm{rad}}(t_0)\ (1+z)^4 = \rho_{\mathrm{m}}(t_0)\ (1+z)^3$
- Therefore, the redshift of transition is given by the **present day densities** of matter and radiation as
  $1+z = \rho_{\mathrm{m}}(t_0)\ /\rho_{\mathrm{rad}}(t_0) \sim 5000$
- (Actually 1.68 times less, because besides Cosmic Microwave Background Radiation there are relativistic Cosmic neutrinos, which constitute 68% of the amount of CMB and behave as radiation)
- The temperature of the Cosmic background Radiation changes at this redshift is
  $T = T(t_0)\ (1+z) \approx 2.725$ K x 5000 = 13600 K

Can we compute the time of this or other events for which we now the redshift ?

- This is complicated, need to solve Einstein equations. This is done in this calculator, for example.

The Opaque Period of the Universe

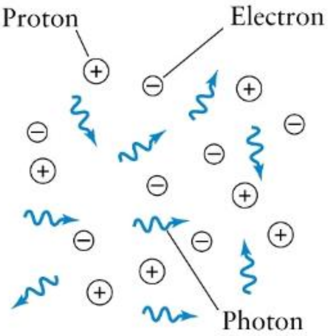


Q: What would be seen in the radiation dominated period of the Universe?

A: Not much.

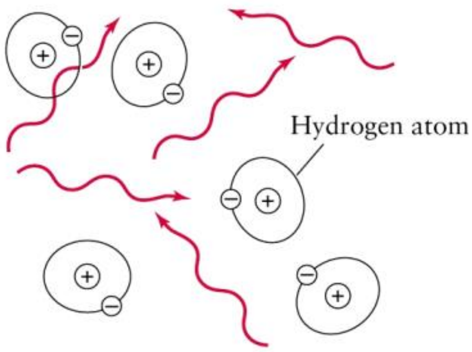
- At high temperatures above a few thousand Kelvin neutral atoms tend to become ionized and the Universe will be a plasma
- .Light has a great deal of trouble traveling through a plasma.
  For example, inside the Sun (which is a plasma) it takes a photon about one million years to travel the radius of the Sun.
- In the early universe, photons are scattered so much through collisions with charged particles that the universe would appear to be opaque.
- You would only be able to see "out" a small distance and all that you would see is light with a blackbody spectrum corresponding to the temperature of the Universe.

The Era of Recombination



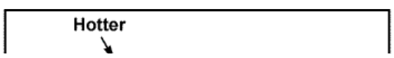
a) Before the recombination

- Today, the Universe is transparent, since photons from quasars a few billion light-years can reach our telescopes.
- How can the Universe become transparent?
- Get rid of the plasma: When a plasma is cool enough the protons and electrons will combine to form neutral Hydrogen which doesn't hinder photons. This occurs at a temperature near 3000 K.
- Question: If recombination occured at T=3000 K, at what redshift did it happen ?
- Answer
  $1+z = T\ /\ T(t_0) = 3000$ K $/2.725$ K ≈ 1100
- The time when the temperature is 3000 K is around 370,000 years after the Big Bang (use calculator) and is called the time of **"recombination"**.
- The reaction p + e ⟶ H is called recombination, although in the early universe the protons and electrons had never been joined, so it should really be called "combination".
- The time of recombination is the last time that photons collide with matter (until the photons reach our eyes).
- When the photons hit our eyes, we call them the **Cosmic Background Radiation**.
- This time is also called the **time of last scattering**.

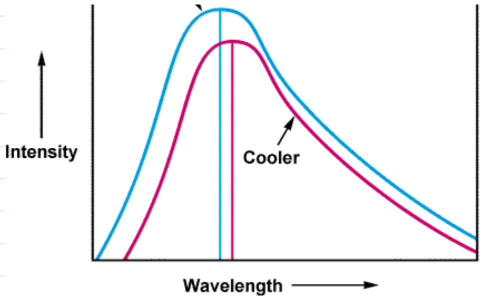


b After recombination

Redshift of the Cosmic Background Radiation

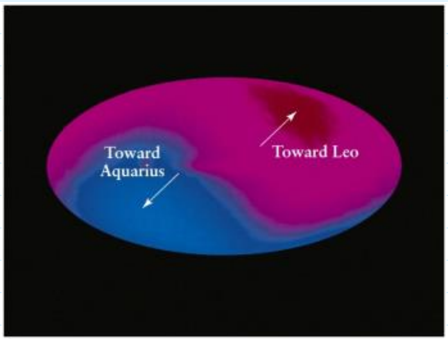


- As the photons travel, the universe expands causing the photons' wavelength to expand (redshift) so the photons correspond to a blackbody with a cooler temperature.
- The photons originated in the Big Bang have a wavelength of about 1mm today, which corresponds to a temperature of 3 K : the **Cosmic Microwave Background (Radiation)** (CMB(R)).



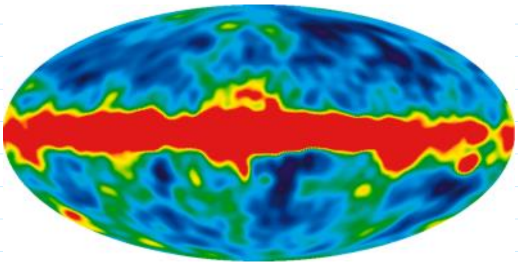
The Microwave Sky (from the COBE Satellite)

- The average Temperature of the sky is T = 2.725 K
- The Colour scale is backwards:
  - Red = Temp hotter by 0.0033 K than average.
  - Blue = Temp cooler by 0.0033 K than average.
- This indicates that the Earth is moving in the direction towards the constellation Leo at v = 371 km/s.
- Dipole in the temperature distribution!



Snapshots of the Universe at the Time of Last Scattering (T-variation vs density variation)

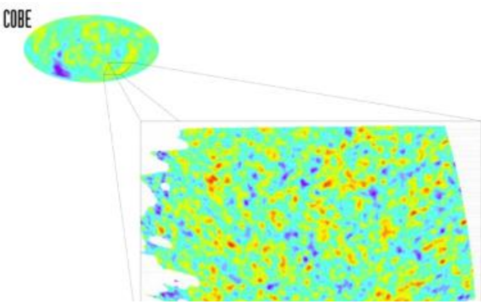
- The 3 K photons last collided with matter at the time of last scattering.
- These photons which we detect then show us how matter was distributed at the time of last scattering, when the Universe was only a few hundred thousand years old.



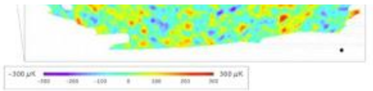
- Regions which were a little bit denser than average have light which is gravitationally redshifted and corresponds to a slightly lower blackbody temperature. These regions later will collapse to form galaxy clusters.
- Regions which were a little bit less dense than average have slightly higher than average temperatures and evolve to form voids.
- The Cobe satellite (mid 1990's) mapped the whole sky and showed the temperature (and hence density) variations in the CMB.
- Typical angular size of regions are about 7 degrees in the sky.
- Red = hotter than average by 30 microKelvin.
- Blue = cooler than average by 30 microKelvin.

Boomerang

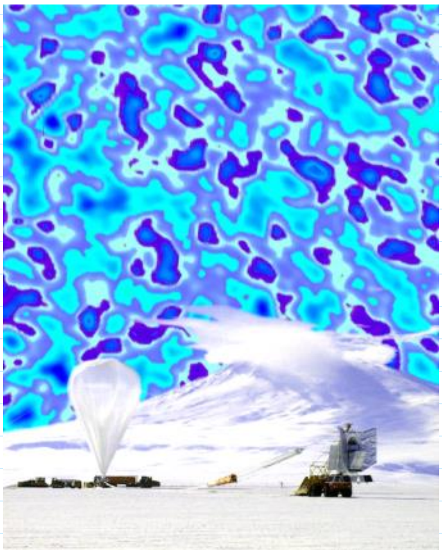
- The Boomerang experiment (1999) mapped a smaller part of the sky than Cobe, but at much greater resolution.
- The typical anglar size of constant density regions is about 1 degree.
- Red = Hotter than average by 300 microKelvin.



- Blue = Cooler than average by 300 microKelvin.

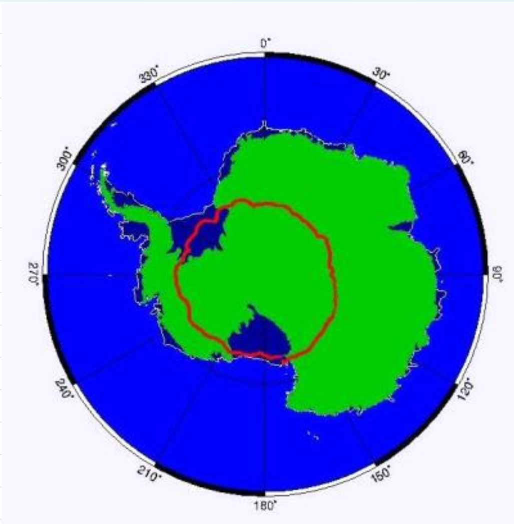


- Photo-montage of the Boomerang balloon in front of Mnt Erebus in Antarctica with the Boomerang data.
- If your eyes were sensitive to microwave radiation, you would "see" these patterns in the sky.



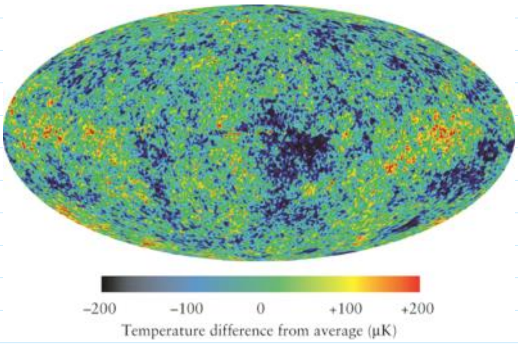
Boomerang's Flight Path

- Boomerang is a microwave detector launched on a balloon in Antarctica.
- A prevailing in Antarctica in summer stable air current moved the balloon kept at 40 km in the air for two weeks (while it gathered data) and then brought the apparatus back within 30 miles of the launch location for relatively easy recovery



WMAP Results (2008)

- WMAP = Wilkinson Microwave Anisotropy Probe
- WMAP is a satellite which has mapped the full sky with a resolution similar to Boomerang.

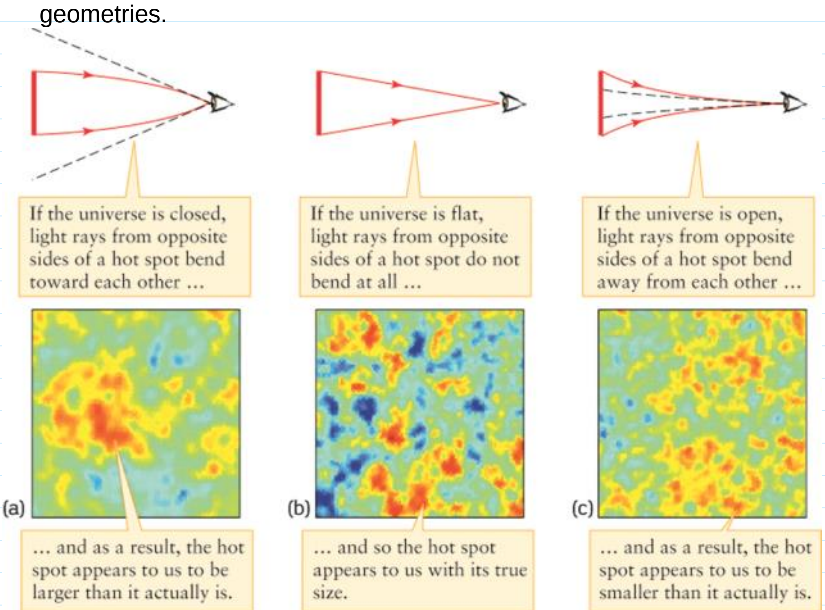


Why are the Boomerang and WMAP Data Important?

- The size of a constant-temperature region is fixed by the size of the horizon at the time of last-scattering. (The horizon is the distance over which a photon can travel during the age of the universe).

The physical size of the spots can be computed - it provides the standard yardstick placed very far at redshift z=1100 when Universe was 370,000 years old

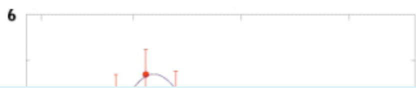
- The apparent angle over which the region is spread depends on the geometry of the universe.
- If the geometry is flat, the average size of spots is about one degree.
- If the geometry of the universe is closed an effective defocusing occurs and the average angular size of the spots is greater than one degree.
- If the geometry is open, corresponding to a hyperbolic geometry like a saddle, the average size of the spots is less than a degree.
- The diagrams show computer simulations of what the sky is predicted to look like for different geometries.



- The actual data looks most like the flat geometry.

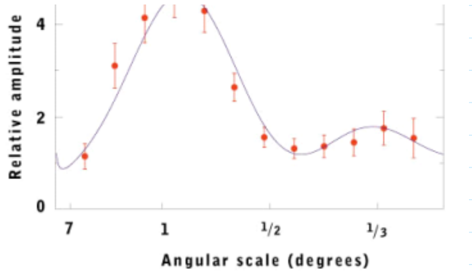
The Angular Power Spectrum

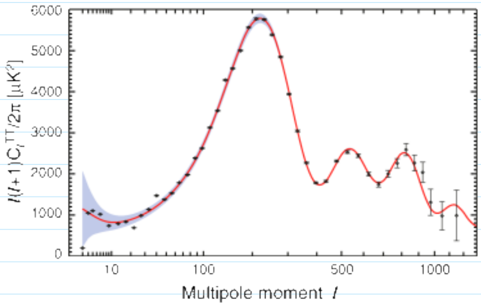
- The analysis is done by taking all pairs of points in the



data set and comparing the difference in temperature for different angular separations.

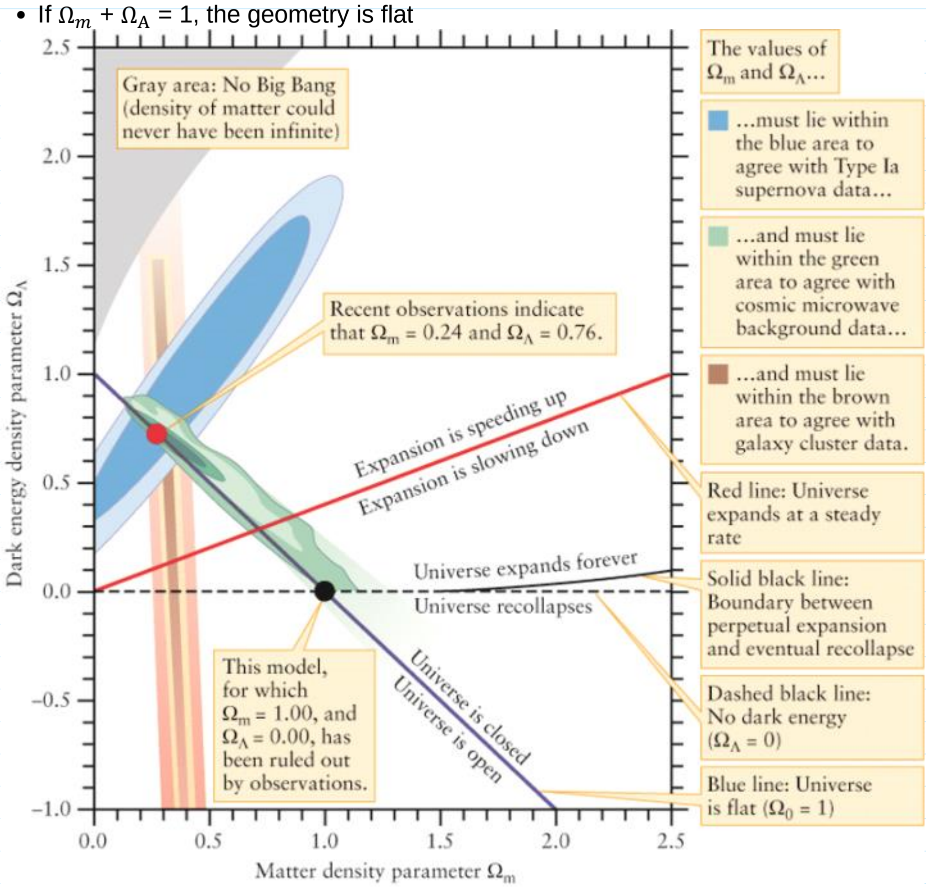
- A plot (called a power spectrum) shows the relative number of spots as a function of the angular size of the spots.
- The curve shows the theoretical prediction for a flat geometry and the red points are the measurement.
- There is a pretty good fit to a universe with a flat geometry, although the geometry could be only slightly curved and still fit.





Current Favourite Cosmological Model

- The Big Bang models are described by two parameters, Ω_m and the Cosmological Constant Λ.
- Ω_Λ is proportional to the Cosmological Constant.
- The two parameters can be plotted against each other as on the diagram below.
- If $\Omega_m + \Omega_\Lambda = 1$, the geometry is flat



Different Big Bang Models

- The vertical axis corresponds to the Cosmological Constant.
- The horizontal axis corresponds to the fraction of the matter density to the critical density.
- Each point on this diagram corresponds to a different type of Big Bang model. Our goal is to figure out which model describes our Universe.

- A flat geometry corresponds to any point on the blue diagonal line.
- 10 years ago, most astronomers thought that the universe corresponded to Ω_m = 1 and zero cosmological constant.
- The Boomerang experiment show that the geometry of the universe is very close to flat, so only

models corresponding to the green region are allowed.

- The Supernovae observations (discussed in the previous lecture) only allow models in the blue region of the diagram.
- The only consistent models are those in the overlapping green region.
- The models favoured by today's observations have Ω_m = 0.27 $\pm$ 0.03 , Ω_{A} = 0.73 $\pm$ 0.03 and H_0=71 $\pm$ 2 km/s/Mpc
- All of the allowed models will expand forever and will not recollapse in a Big Crunch.

Next lecture: The Early Universe

출처: <https://sites.ualberta.ca/~pogosyan/teaching/ASTRO_122/lect31/lecture31.html>