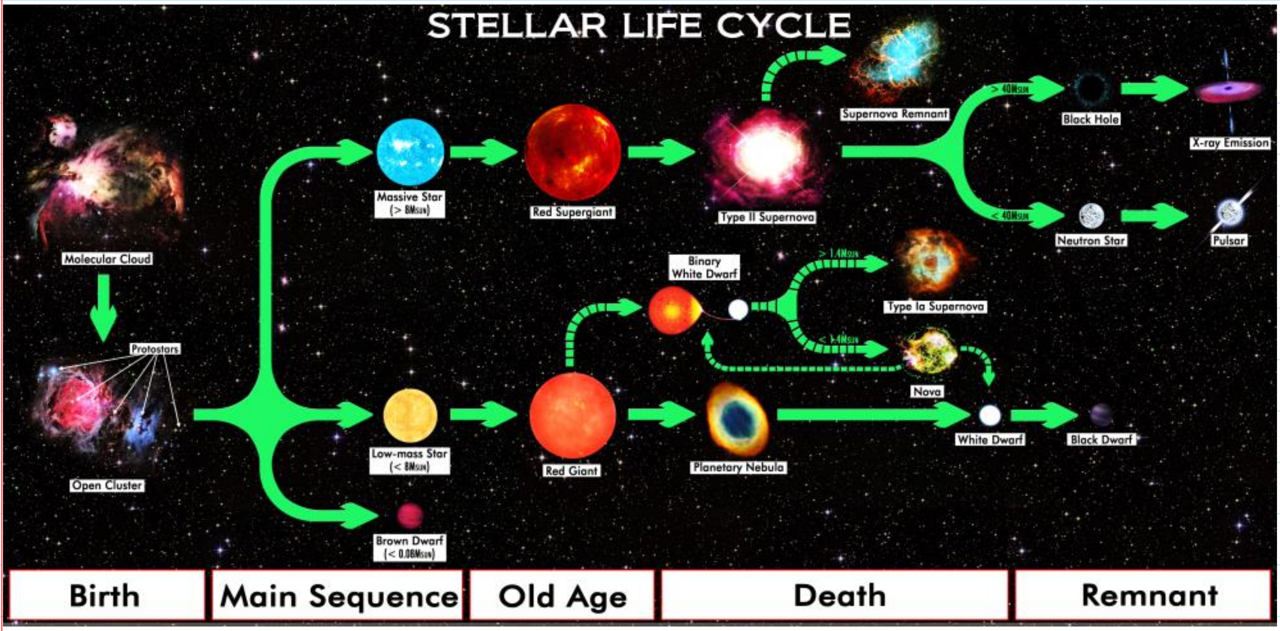
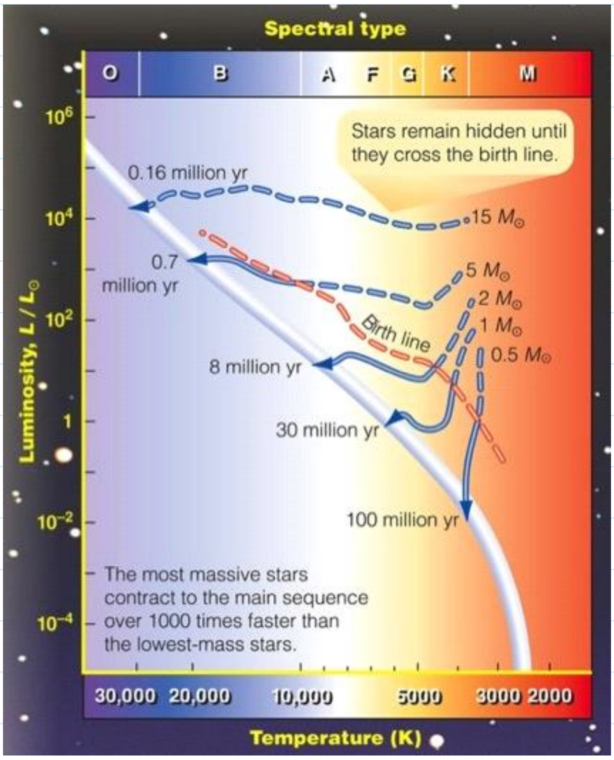
2022 12 22 Star Evolution

2021년 8월 22일 일요일 오후 1:44



Depends on the **mass** of the star, among others.



Birth:

protostar → pre-main-seq. Star→Main Seq star

Protostars & Pre-main-sequence stars :

All stars are formed from collapsing clouds of gas & dust, called nebulae or molecular clouds.

Main Sequence

$0.1 M_\odot \lesssim M_{\text{Main Sequence}} \lesssim 100\ M_\odot$

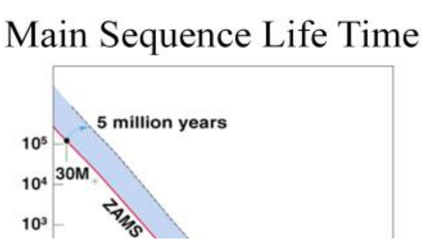
Zero Age Main Sequence (ZAMS)

is the time of joining the main sequence on the HR diagram by burning H in its core (fusion).

Stars spend most of time in MS ⟶

Most stars (about 90%) are Main Sequence Stars.

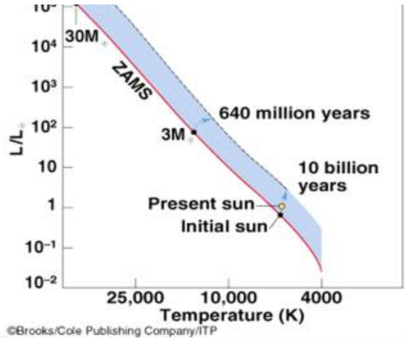
출처: <https://en.wikipedia.org/wiki/Stellar_evolution>



thermal pressure comes from H-fusion to He

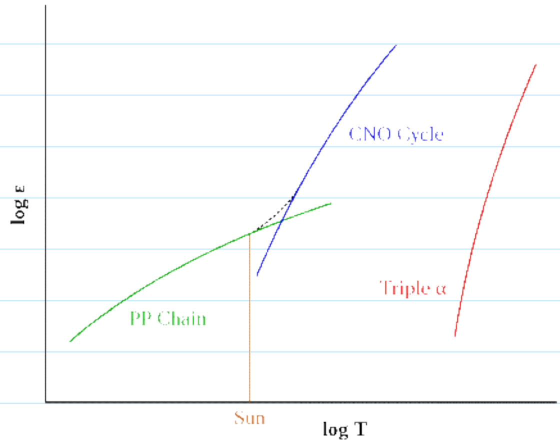
출처: <https://en.wikipedia.org/wiki/Stellar_evolution>

thermal pressure comes from H-fusion to He
thermal pressure ⟷ gravity



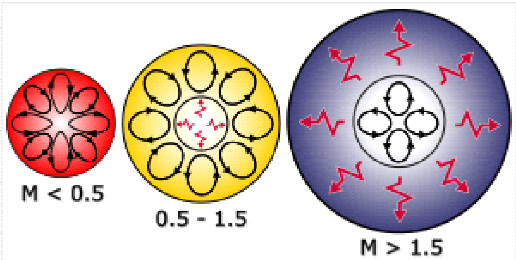
Nuclear reaction
rates are very sensitive to core temp:
p-p Chain rate $\sim T^4$
CNO Cycle rate $\sim T^{18}$

Energy generation **mechanism** in the core
At about 1.1 $M_\odot$ by the P-P chain = by CNO cycle



[Logarithm](#) of the relative energy output (ε) of [proton–proton](#) (PP), [CNO](#) and [Triple-α](#) fusion processes at different temperatures (T). The dashed line shows the combined energy generation of the PP and CNO processes within a star. At the Sun's core temperature, the PP process is more efficient.

출처: <https://en.wikipedia.org/wiki/Stellar_nucleosynthesis>



Internal structures of main-sequence stars, convection zones and radiative zones.
To the left a **low-mass** red dwarf,
in the center a **mid-sized** yellow dwarf and
at the right a **massive** blue-white MS star

$0.08M_\odot$ **$(0.25\sim0.4)M_\odot$** **$1.1\sim1.5M_\odot$**

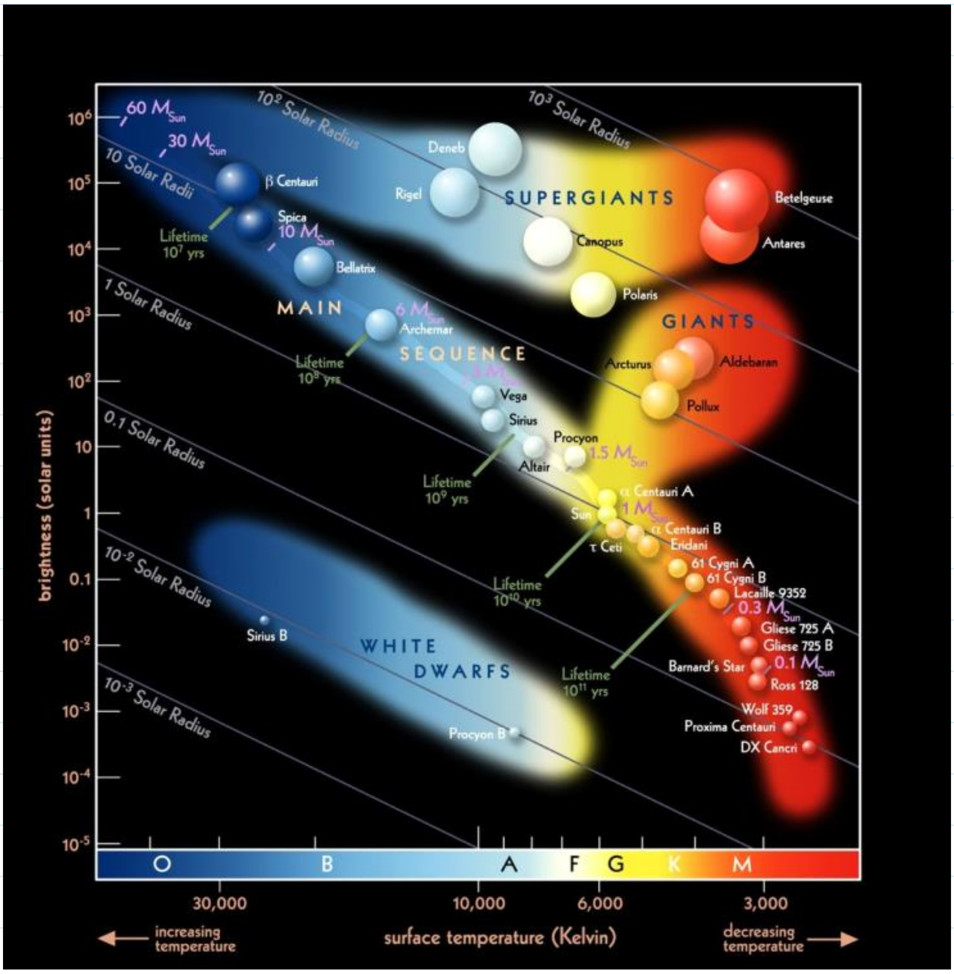
H-fusion no(not a star)⟻ ⟼ yes (Star) $T_{\text{core}} = 18\text{MK}$
brown dwarf ⟻ ⟼ Red Dwarf ⟻
H-fusion via ⟼ p-p chain ⟻ ⟼ CNO cycle

Core ⟼ convective ⟻⟼ radiative ⟻ ⟼ Convection
Envelope ⟼ convective ⟻⟼ Convective ⟻ ⟼ Radiative
(⟼ No RGB ⟻)

Stellar Lifetime

$\boldsymbol{\tau} \sim (\mathbf{1}/M^3 \text{or }) 1/M^2$

Eventually the star exhausts H in its core and begins to evolve off the main sequence.



Note

($\tau \sim$ M/L &

$L \sim (M^4 \text{or }) M^3$,

gives

$\tau \sim (\frac{1}{M^3} \text{ or }) \frac{1}{M^2}$

More massive stars

higher T,

burning more quickly,

to stay in

hydrostatic equilibrium.

and shorter lifetime

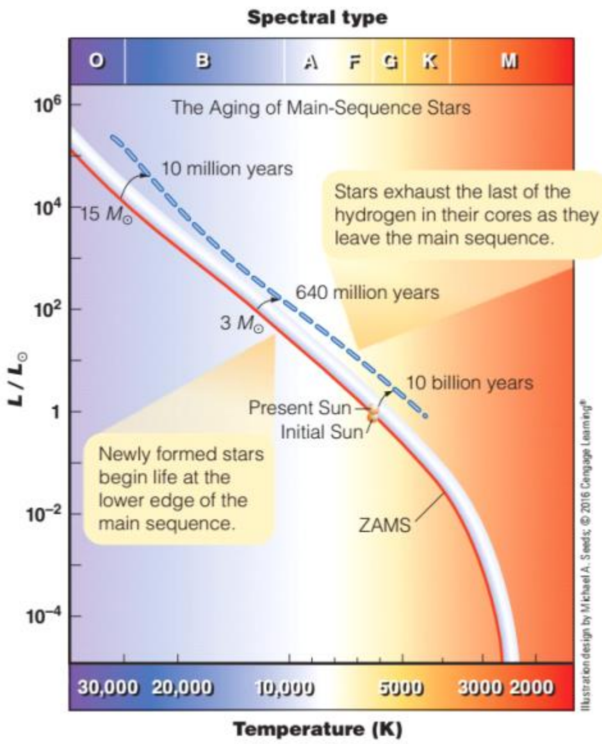
Ex)

1) the Sun

$\tau_\odot \approx 10^{10}$ yr

(5 Gyr old,

will last another 5 Gyr)



2) $M \approx 100 M_\odot$ the most massive stars

$\tau \approx 10^4 \sim 10^6$y

3) $M \approx 10 M_\odot$

$\tau \approx 10^7 \sim 10^8$yr ($\approx$ 32 Myr)

(Hence, all the O & B type stars are young.)

4) $M \approx\ 0.1 M_\odot$, the least massive stars

$\tau \approx 10^{12}$ yr

$\gg$the age of the universe $\approx 1.38 \times 10^{10}$ yr

None of these smaller stars have died yet.

(Hence, can't tell the age of an M-dwarf.)

Mass (solar masses)	Time (years)	Spectral type
60	3 million	O3
30	11 million	O7
10	32 million	B4
3	370 million	A5
1.5	3 billion	F5
1	10 billion	G2 (Sun)
0.1	1000s billions	M7

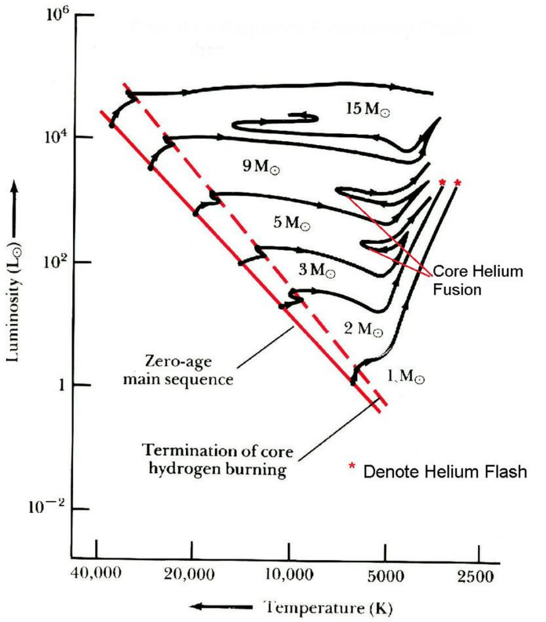
Lifetimes of stars as a function of their masses

출처: <https://en.wikipedia.org/wiki/Stellar_evolution>

Leaving the Main Sequence

The main sequence band

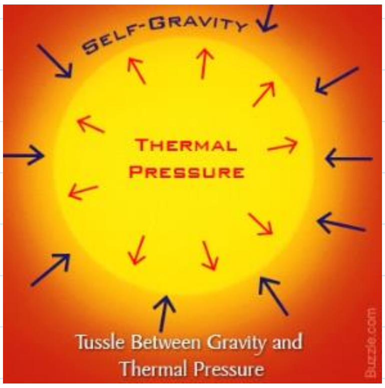
- T_{core} ↗
  As the fraction of hydrogen in the core of a MS star decreases, the core temperature increases and so the rate of fusion increases.
- L ↗
  This causes stars to evolve slowly to higher luminosities as they age and broadens the main sequence band in the Hertzsprung–Russell diagram.
- **Hook** : the end of MS and the start of the SGB



Key concept of Evolution : Energy Conservation

thermal pressure (energy)	↔	gravity force (energy)

Thermal P < Grav ⟶ shrinking (collapsing)
shrinking increases temp leading to (another) fusion
(grav. Energy into thermal energy)
or make degenerate pressure
(White Dwarf, Neutron Star)
or collapse to BH



Thermal P > Grav ⟶ expanding, explosion, blowing off
(thermal energy into grav. Energy, kinetic E of matter)

Evolutionary stages of a 25 M_{SUN} star

Stage	Temperature (K)	Density (g/cm^3)	Duration of stage
Hydrogen burning	4x 10^7	5	7 x 10^6 years
Helium burning	2 x 10^8	700	5 x 10^5years
Carbon burning	6 x 10^8	2 x 10^5	600 years
Neon burning	1.2 x 10^9	4 x 10^6	1 year
Oxygen burning	1.5 x 10^9	10^7	6 months
Silicon burning	2.7 x 10^9	3 x 10^7	1 day
Core collapse	5.4 x 10^9	3 x 10^9	1/4 second
Core bounce	2.3 x 10^{10}	4 x 10^{14}	milliseconds
Explosive	about 10^9	varies	10 seconds

Table 1. Nuclear burning stages in massive stars. We give typical temperatures and time scales for a 20 $M_\odot$ star (Pop I; similar in Pop III) and a 200 $M_\odot$ star (Pop III)

Burning stages		20 $M_\odot$ star		200 $M_\odot$ star	
Fuel	Main product	T (10^9 K)	duration (yr)	T (10^9 K)	duration (yr)
H	He	0.037	8.1×10^6	0.14	2.2×10^6
He	O, C	0.19	1.2×10^6	0.24	2.5×10^5
C	Ne, Mg	0.87	9.8×10^2	$1.1^\dagger$	4.5
Ne	O, Mg	1.6	0.60	$2.4^\dagger$	1.1×10^{-6}
O	Si, S	2.0	1.3	$3.5^\dagger$	3.5×10^{-8}
Si	Fe	3.3	0.031	$4.3^\ddagger$	2.7×10^{-7}

†central radiative implosive burning ‡incomplete silicon burning at bounce

출처: <https://people.highline.edu/iglozman/classes/astronotes/hr_diagram.htm>

Nuclear Burning

H → He → C → Ne → O → Si

$M \lesssim 0.1 M_\odot$: Brown dwarf, too light for H-burning

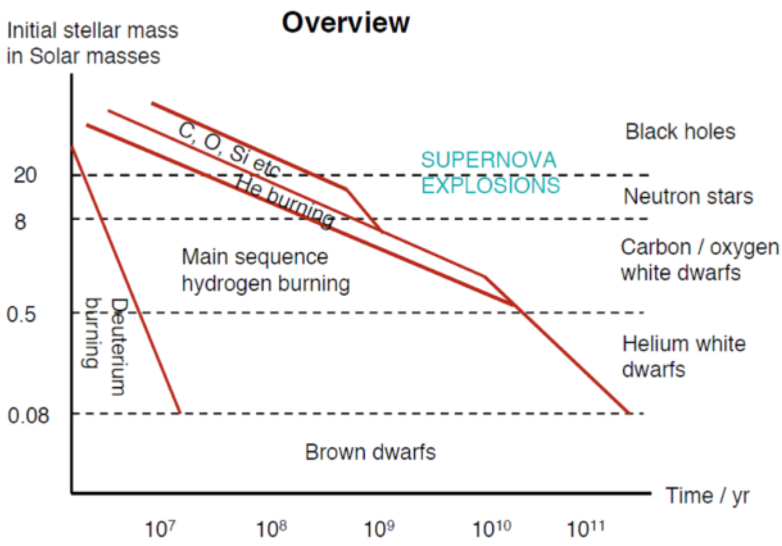
$M \gtrsim 0.1 M_\odot$ H-burning to He

$M \gtrsim 0.6 M_\odot$ He-burning to C & O

$(4M_\odot \lesssim M \lesssim 8M_\odot)$ partial C-burning to Ne & Mg

$M \gtrsim 8M_\odot$ C-burning to Ne & Mg

$M \gtrsim 12M_\odot$ Ne,O,Si-burning



(From P. Armitage)

silicon burning

In astrophysics, is a very brief, about 1 day.

occur with a minimum of about 8–11 solar masses.

No further fusion after Si-burning.

The star catastrophically collapses and may explode as a Type II supernova.

begins when gravitational contraction raises the star's core T to 2.7–3.5 GK(230–300 keV).

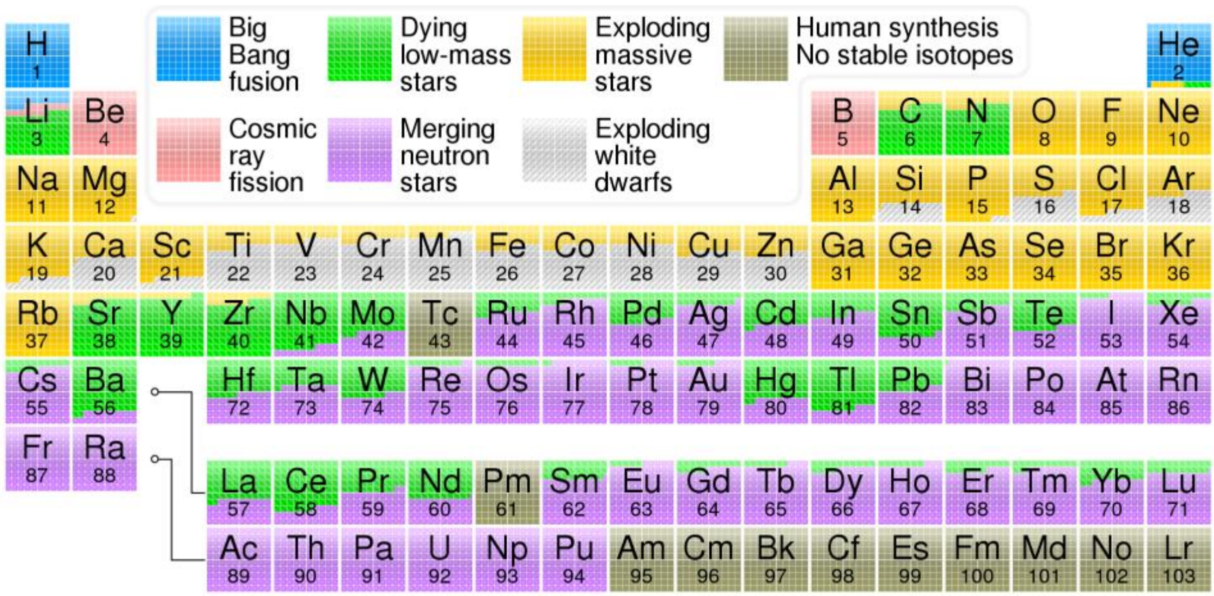
- **the core collapse**

- The silicon-burning sequence lasts about one day
  then struck by the shock wave that was launched by the core collapse.
- Burning then becomes much more rapid at the elevated T
  and stops only when the rearrangement chain has been converted to nickel-56
  or is stopped by supernova ejection and cooling.
- The decay, ${}^{56}_{28}\mathrm{Ni} \rightarrow {}^{56}_{27}\mathrm{NCo} \rightarrow {}^{56}_{26}\mathrm{Fe}$ in a few days or weeks, happens later because only minutes are available within the core.
- The star has run out of nuclear fuel and within minutes its core begins to contract.

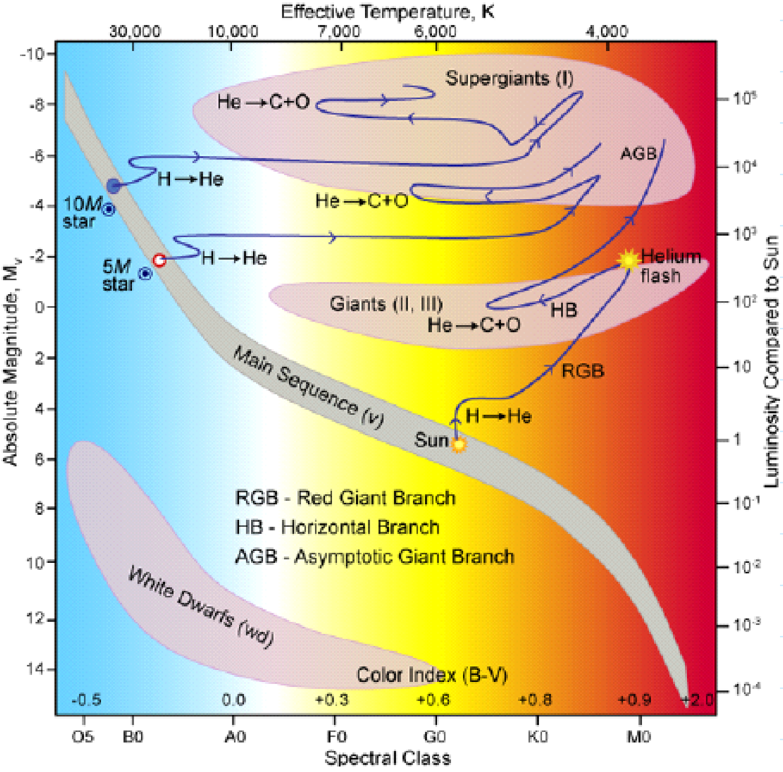
- The interior heats up to 5 GK (430 keV) and this opposes and delays the contraction.

- **After collapse**
- The final unopposed contraction rapidly accelerates into a collapse lasting only a few seconds.
- The central portion of the star
  - is now crushed into either a neutron star or, if the star is massive enough, a black hole.
- The outer layers of the star
  - are blown off in an explosion known as a Type II supernova that lasts days to months.
  - The supernova explosion releases a large burst of neutrons,
  - which may synthesize in about one second roughly half of the supply of elements in the universe that are heavier than iron,
  - via a rapid neutron-capture sequence known as the *r*-process (where the "r" stands for "rapid" neutron capture).

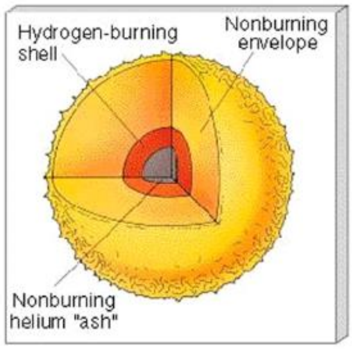
출처: <https://en.wikipedia.org/wiki/Silicon-burning_process>



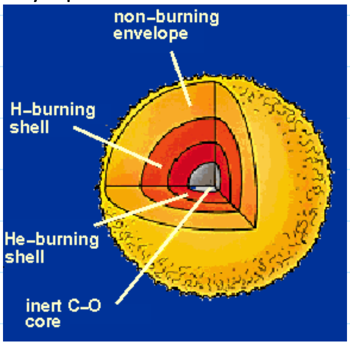
Periodic table showing the currently believed origins of each element.
Elements from carbon up to sulfur may be made in stars of all masses by charged-particle fusion reactions.
Iron group elements originate mostly from the nuclear-statistical equilibrium process in thermonuclear supernova explosions.
Elements beyond iron are made in high-mass stars with slow neutron capture (s-process), and by rapid neutron capture in the r-process, with origins being debated among rare supernova variants and compact-star collisions. Note that this graphic is a first-order simplification of an active research field with many open questions.
출처: <https://en.wikipedia.org/wiki/Nucleosynthesis#/media/File:Nucleosynthesis_periodic_table.svg>

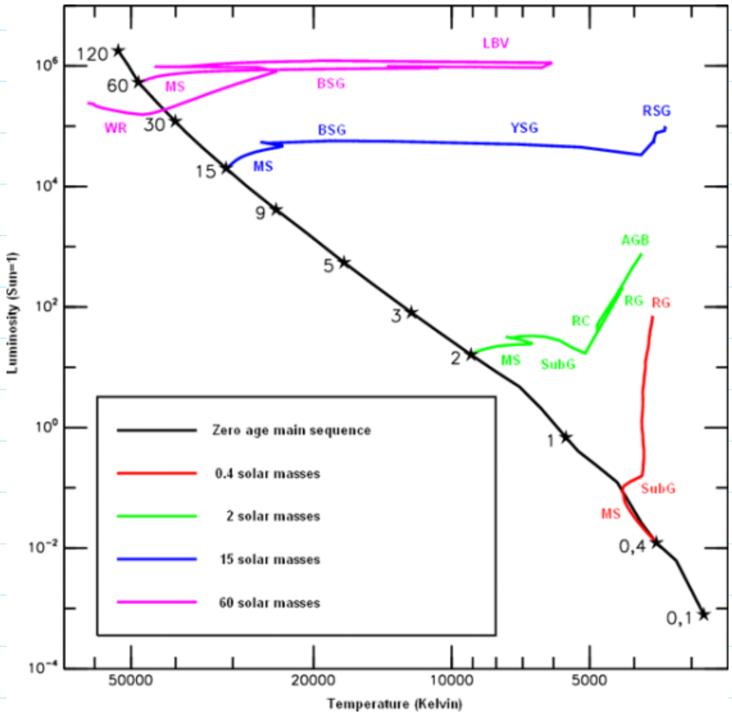


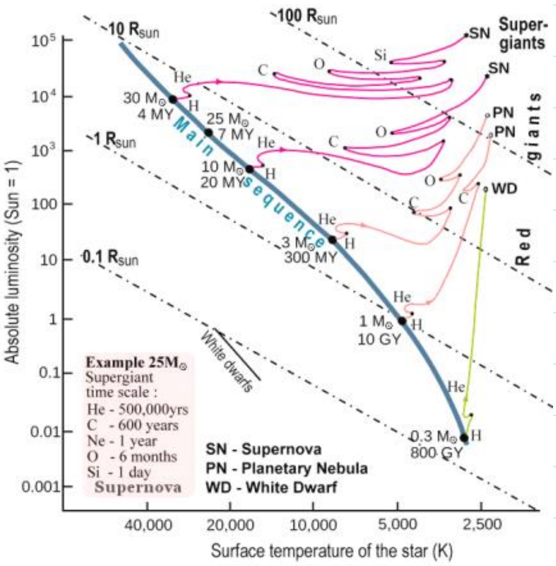
Red Giant Branch

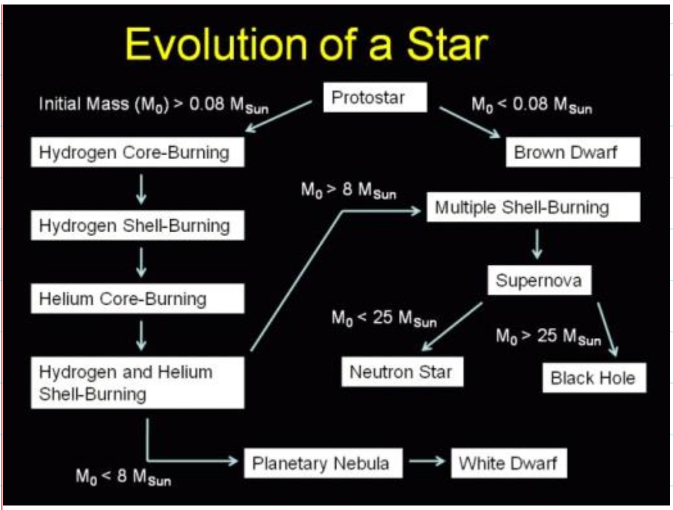


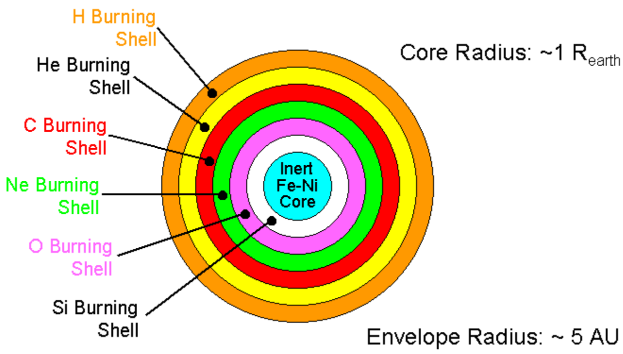
Asymptotic Giant Branch

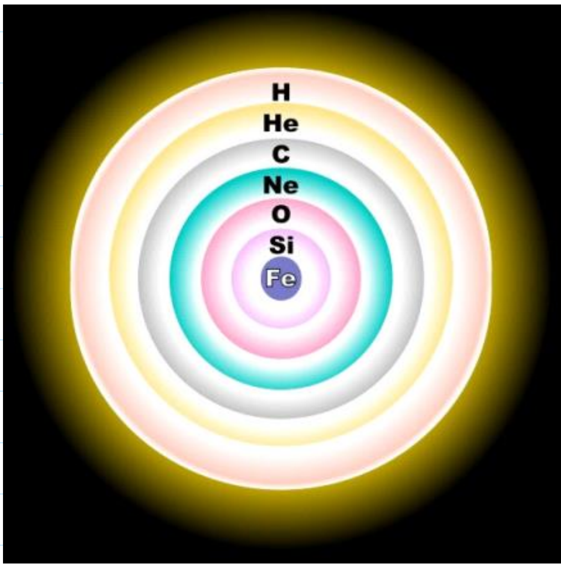












Death/Remnant

$M \lesssim 8{\sim}12M_{\odot}$ $M \gtrsim 12M_{\odot}$ $M \gtrsim 25M_{\odot}$

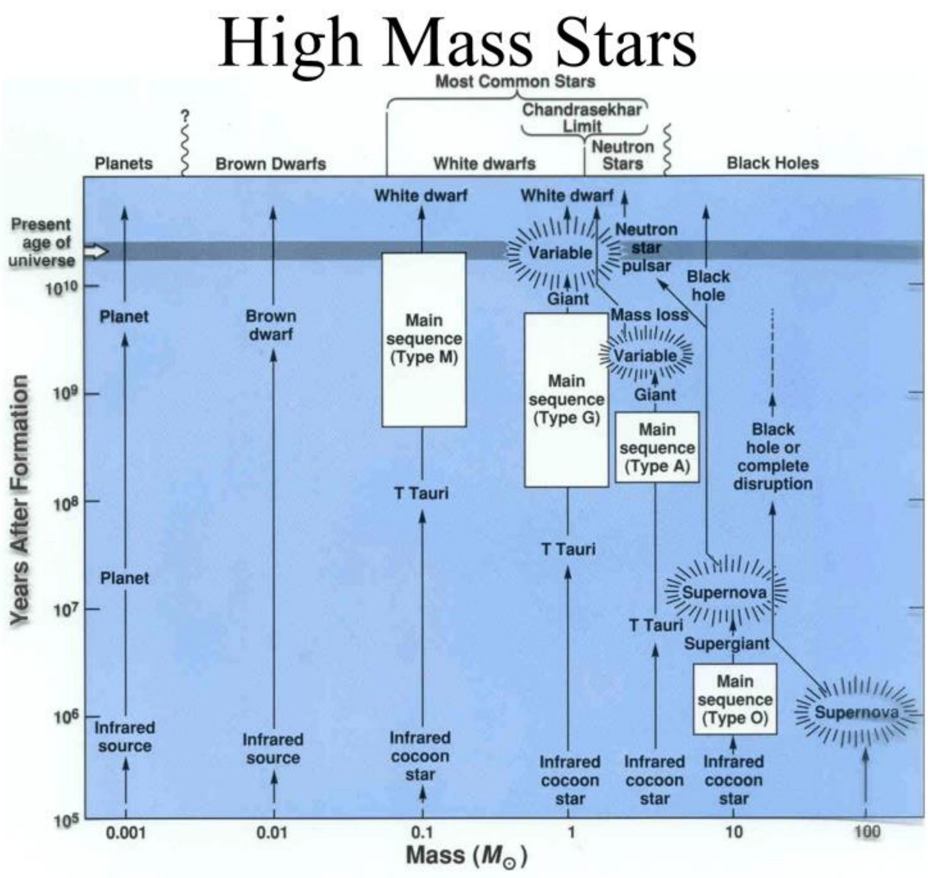
White Dwarf Neutron Star Black Hole

Pressure against gravity

Electron neutron collapsing

-Deg. Deg.

Note : Much of the original mass lost.



Thermal versus Degenerate Pressure

Degenerate pressure important in higer density ("low temp")

Degenerate Matter
- As the helium core contracts it becomes denser.
- The helium core is a mixture of helium nuclei and electrons. (Charge neutral)
- Pauli Exclusion Principle: No two electrons are allowed to have exactly the same properties. If squeeze together too many electrons into a small space they will repel each other.
- This repulsion is called Electron Degeneracy Pressure.
- This repulsion is much stronger (and different) than the usual repulsion which exists between charged particles.
- When degeneracy pressure is stronger than thermal pressure, we call the gas degenerate.
- In the case of the Sun's core, it becomes degenerate when
  - density is larger than 10^6 kg/m^3 (thousand times the density of water)
  - at temperature is near 20 million K.
- The Sun's helium core becomes degenerate early during the red giant phase of its life.
- In all the low mass stars ($M \lesssim 2M_{\odot}$),
  the helium core becomes degenerate during the red giant phase.
- Medium and High mass stars ($M \gtrsim 2M_{\odot}$) are not degenerate while red giants.

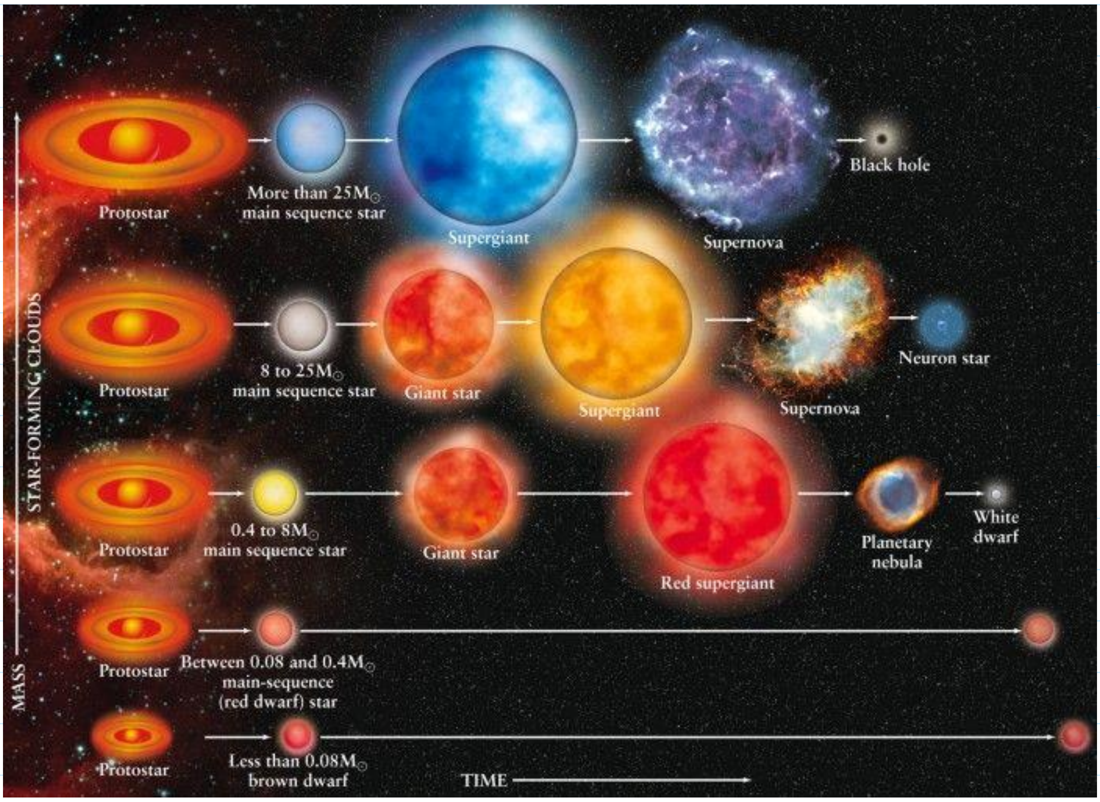
Properties of Degenerate Gases
- A degenerate gas is very different from an ideal gas.
- An ideal gas's pressure depends on the density and the temperature.
- An degenerate gas's pressure only depends on density.
- As long as the temperature is cool enough that the gas is degenerate it doesn't matter how T changes, the pressure only depends on density.

Nuclear Reactions in a Degenerate Gas
- Nuclear reactions in a degenerate gas tend to be explosive.
- Imagine you turn on a nuclear reaction.
- The energy output heats up the gas.
- If the gas is ideal, its pressure increases and it expands and cools down.
- If the gas is degenerate, an increase in temperature doesn't increase the pressure, so it does not expand or cool down.
- Increasing the temperature makes it easier for nuclear reactions to take place, so the cycle is repeated and the reactions occur rapidly.
- This can continue until the temperature gets so high that the core is no longer degenerate.

Evolution Stages : Strongly depends on the Mass (among others such as metalicity, etc)

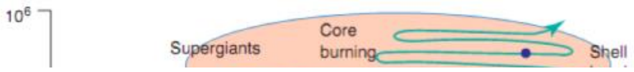
	Low M Star		Mid-sized star					Massive Star	
	0.1$M_\odot$	0.4$M_\odot$	0.6$M_\odot$	1.1$M_\odot$	2$M_\odot$	8$M_\odot$	10 $M_\odot$	12 $M_\odot$	40 $M_\odot$
Main Sequence	MS	MS	MS	MS	MS		MS		
(Hook)	X	X	X	O	O			O	O
SubGiant Branch	X	SGB	SGB	SGB	SGB				
RedGiant Branch	X	RGB	RGB	RGB	RGB			X	X
	(convective)								
(He flash)		(X)	O	O	X				
Horizontal Br			HB	HB					

(Red Clump)			(RC)	(RC)				
(Blue Loop)					(BL)			
Asymptotic GB			AGB	AGB	AGB	X	X	X
(Ther Pulse)			(TP)	(TP)	(TP)			
Super GB							BSG YSG/RSG	BSG LBV
WD,NS, BH	WD	WD	WD	WD	WD	WD	NS	BH
Core	He	He	C&O	C&O	Ne,O,Mg		Fe	
Singularity								

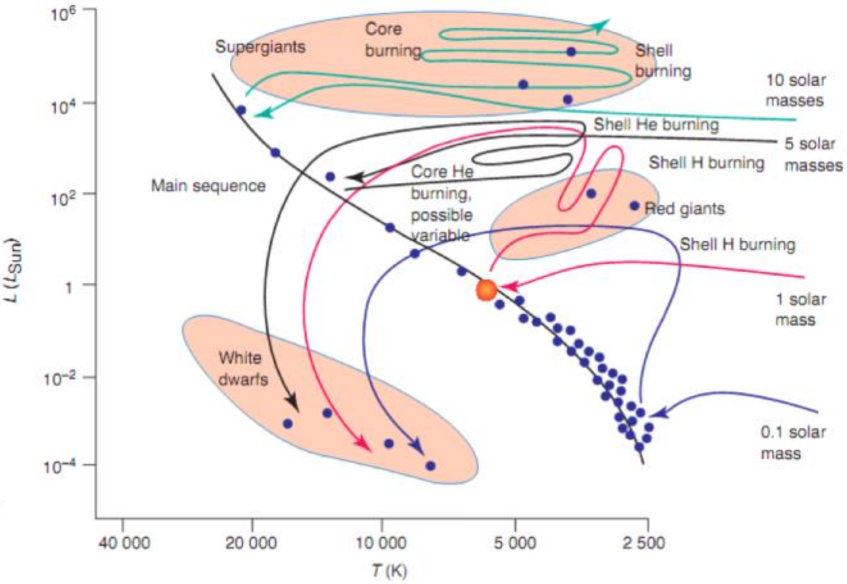


Final Stage

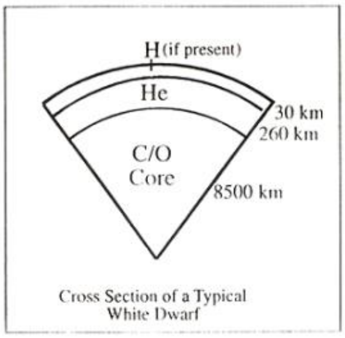
- Stars with $M_{MS} \lesssim 12 M_{\odot}$ ends up as a white dwarf



- Stars with $M_{MS} \lesssim 12 M_\odot$ ends up as a white dwarf
- Low mass stars end up as WDs composed of mainly C & O.
- Medium/High mass stars end up as WDs of the higher mass elements, O, Ne, Na,Mg,Si



- Stars $0.6 M_\odot \lesssim M_{MS} \lesssim 8 M_\odot$
  - evolve into AGB
  - AGBs lose 50~70% mass
  - Ends as WD &Plan Neb
- White Dwarf
  - The left-over (after)
  - Ejection of the matter
    - Ejection of the outer He & H-layers (stellar winds)
    - Planetary Nebula
  - Electron degeneracy p againt grav collapse
  - $M_{WD} \lesssim 1.4 M_\odot$ (Chandrasekhar limit)
  - $r_{WD} \approx r_\oplus$
  - Initial T≈200,000K WD

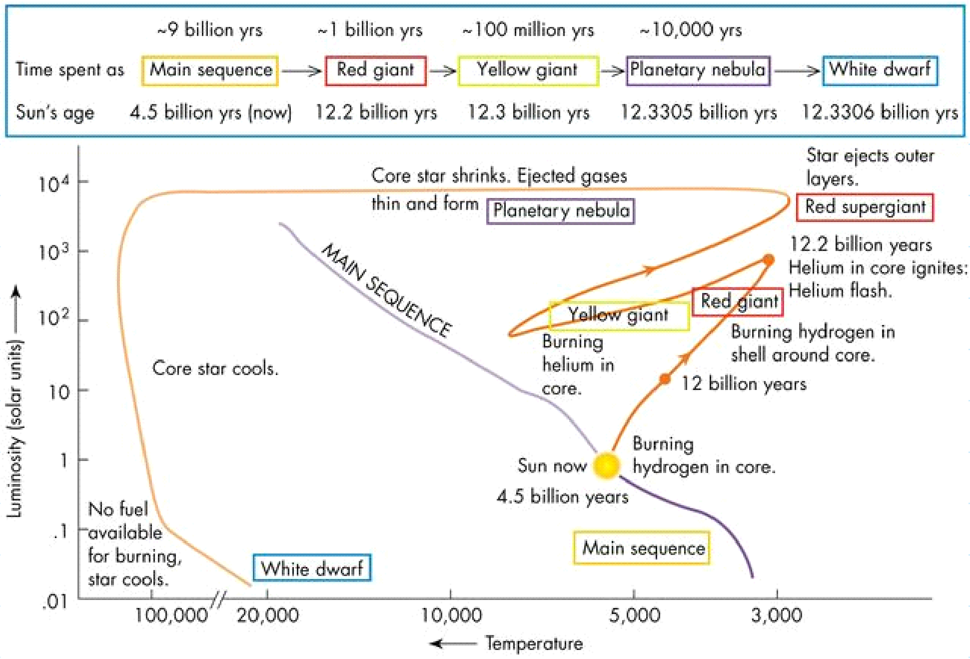


Cross Section of a Typical White Dwarf

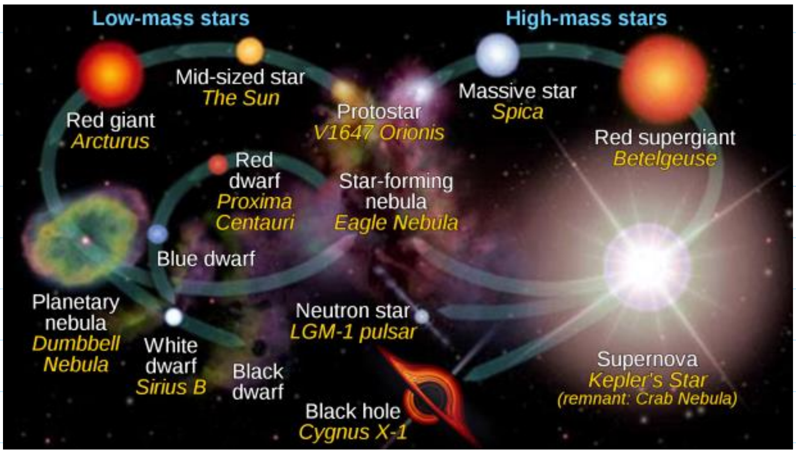
- Low mass : $M \lesssim 0.6 M_\odot$
  White Dwarf
  composed of He

- Mass $0.6 M_\odot \lesssim M \lesssim 8 M_\odot$
  Composed of $^{12}_{6}\mathbf{C}, ^{16}_{8}\mathbf{O}$
  SGB→RGB→(RC/BL)→AGB→
  postAGB→Plan Neb→WD

- Mass $8 M_\odot \lesssim M \lesssim 11 M_\odot$
  Ne & Mg
  O
  Si



Remnant

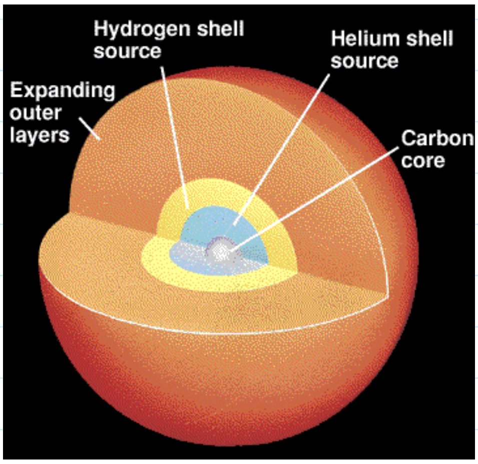


Stellar evolution of low-mass (left cycle) and high-mass (right cycle) stars, with examples in italics

출처: <https://en.wikipedia.org/wiki/Star#Temperature>

Final Stages of Life of a Low-Mass Star

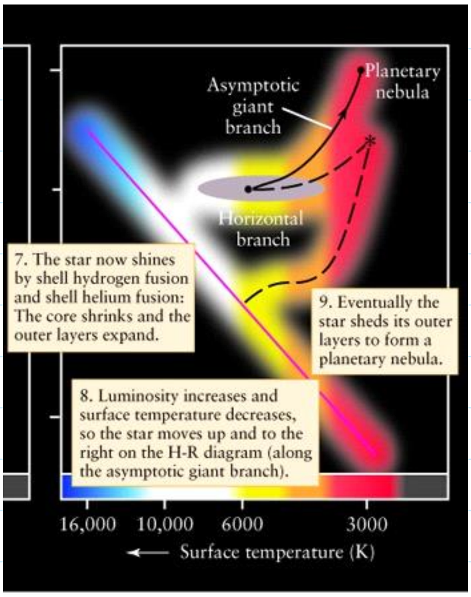
- Burning Helium leaves behind Carbon "ashes".
- the core is depleted of Helium over time and the Carbon collects at the centre of the star.
- The Helium forms a shell around the Carbon and the Hydrogen forms a shell around the Helium.
- Carbon ignition requires hotter temperatures than are available in the core. But Oxygen is usually created in some quantities by **^{12}C + ^{4}He &rarr ^{16}O**



a **second red giant stage**. (Also called an **asymptotic giant branch, AGB**.)

- As in the formation of a red giant, the star begins to contract and heat up.
- Helium in the shell begins to fuse to Carbon.
- Hydrogen fuses in a shell around the Helium.
- The burning in the shell causes the star to expand again, becoming redder.
- This phase doesn't last very long, only one million years for the Sun.

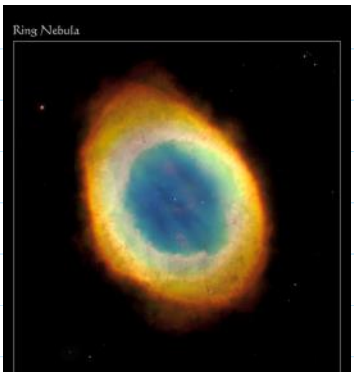
- The core during the last phase never get hot enough for the Carbon to ignite.
- However, the when the Helium shell ignites, it often does so in a series of flashes which tend to push the outer layers outwards.
- The star at this stage is gigantic, so the acceleration due to gravity at the surface of the star is very low. $g = GM/R^2$ = acceleration due to gravity
- It is easy to launch the outer gas of the star so that it escapes from the star!
- The final stage of the Sun's life will be the ejection of the outer Helium and Hydrogen layers.
- These outer layers are hot and glow. The glowing hot ejected gas is called a **planetary nebula**. (Note: nothing to do with planets!)



(c) After core helium fusion ends: An AGB star

Left over central core

- The Carbon (+Oxygen) core left behind is degenerate and supported by degeneracy pressure.
- No nuclear fusion needed to keep the left-over core from collapsing.
- Quantum mechanics won't allow the core to get too dense.
- The left-over Carbon core is called a White Dwarf.
- The White Dwarf's initial surface temperature could be as high as 200,000 K so it will glow blue-white.
- Over time, the White Dwarf cools, becoming redder and



less luminous.

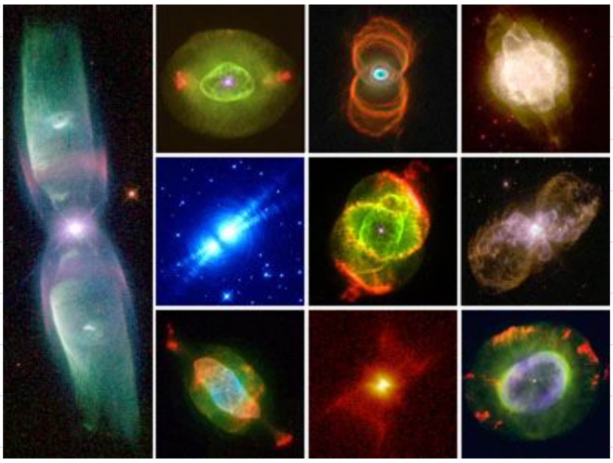
- Typical mass of a white dwarf is similar to the Sun, but its radius is similar in size to the Earth! Very dense!

The Ring Nebula

- This photo of the Ring Nebula shows the glowing ejected outer layers of the Sun-like star.
- The dot in the centre is the White Dwarf.
- Ring Nebula is about 1 light-year in diameter.
- Ultraviolet photons from the hot central White Dwarf ionize the gas in the nebula, which glows when the electrons recombine with the nuclei.
- Blue = emission from Helium (hottest region)
- Green = emission from Oxygen
- Red = emission from Nitrogen (coolest region)
- The Sun may look like this one day

Different Planetary Nebulae

- Planetary Nebulae aren't always spherical in shape.
- These photos were all taken by the Hubble Space Telescope.



Differences between Low and Medium Mass stars

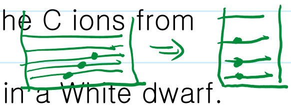
- Low mass stars end up as White Dwarfs composed of mainly Carbon and Oxygen.
- Medium mass stars have higher temperatures in their cores.
- The higher T allows fusion reactions creating Oxygen, Neon, Sodium and Magnesium.
- Medium mass stars end up as White Dwarfs composed of the higher mass elements.

White Dwarf Stars

- White dwarfs are the exposed cores of Red Giant stars.
- The cores are typically composed of mainly Carbon, although Helium, Oxygen, Neon can also exist.
- The Carbon is ionized, so the composition of the star is a plasma of Carbon ions and electrons.
- The Carbon behaves as an ideal gas, so the gas pressure due to the Carbon decreases as the Carbon cools down.
- The electrons obey the Heisenberg Uncertainty principle which keeps the electrons from collapsing to the centre of the star.
- The electrons are negatively charged while the C ions are positively charged, so they are

attracted to each other.

- The attractive electric forces between electrons and positive ions keep the C ions from collapsing to the centre of the the star.



- The force of **electron degeneracy pressure** balances the force of gravity in a White dwarf.
- Electron degeneracy pressure is independent of temperature, so as the star cools, the internal pressure stays constant, and the structure of the star stays constant.
- White Dwarfs cannot exist if the mass exceeds the
  Chandrasekhar limiting mass of M = 1.4 Msun

Life History of a White Dwarf Star

- When the outer layers of the Red Giant are expelled by the dying star, the inner White Dwarf core has a surface temperature over 100,000 K.
- Wein's law for a hot body with this temperature gives a peak wavelength of 2.9×10^{-8}m, corresponding to ultraviolet light.
- The photons emitted from the surface of a hot white dwarf will be very energetic and will easily have enough energy to ionize the gas surrounding the white dwarf. 밝기 ∝ 면적. T^4
- When the electrons recombine with the surrounding ions, they often enter an excited state and then jump down to the ground state emitting visible photons. This process is known as **fluorescence**.
- We call the glowing gas surrounding a hot white dwarf a planetary nebula.
- Planetary nebulae always have a hot white dwarf at their centre.

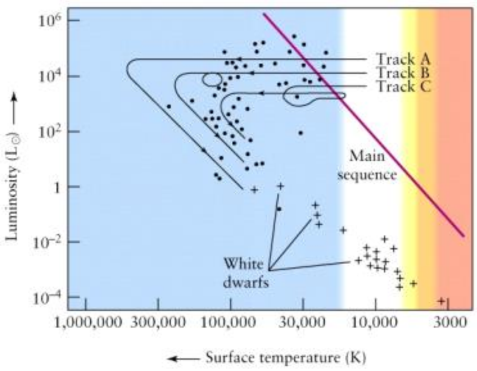
Cooling of the White Dwarf

- Nuclear reactions do not occur inside a white dwarf, so there is no source of energy.
- Over time the white dwarf cools.
- Since electron degeneracy pressure is independent of temperature, the star does not collapse and its radius stays constant.
- From the Stefan-Boltzman law, a star whose radius is contant, but has a surface temperature which changes with time has a luminosity which is proportional to T^4.
- As the star cools, its luminosity will decrease.

- White dwarf stars slowly fade away. If they are born with a luminosity of 1/10 L_{Sun}, after about 5 billion years, their luminosity will be about 10^{-4} L_{Sun}

- As the star cools, the photons it emits have less energy and it is harder for the photons to ionize or excite the planetary nebula, so the nebula will fade with time. As well, the gas in the nebula will be moving outwards, so it will slowly mix with the surrounding interstellar medium.

- When the star becomes cool enough that the thermal kinetic energy of the Carbon ions is less than the electrostatic potential energy of the ions, the Carbon ions can "freeze" (i.e. form bonds) into a crystal lattice structure. After this freezing, the white dwarf is a solid.



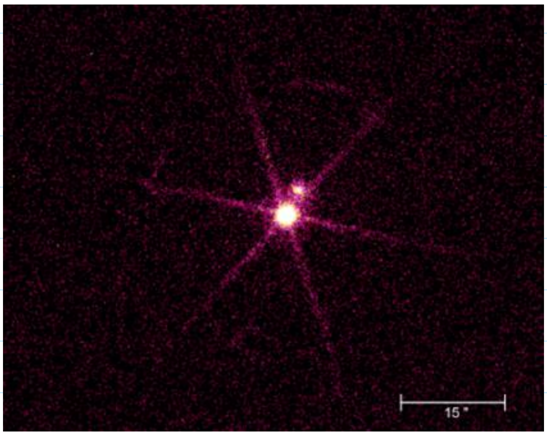
On the H-R diagram, white dwarfs are born hot and luminous, and over time cool and become less luminous. Over time, the white dwarfs move downwards and to the right along an imaginary curve joining all of the white dwarfs.

Sirius B: a sample white dwarf

- The Sirius star system is a binary system consisting of Sirius A and B. Sirius A is a main sequence star and Sirus B is a white dwarf.

- When we look at the star system without a telescope, we only see the very bright Sirius A.
- With a good telescope, you can see the much fainter Sirius B

- With an X-ray telescope, Sirius B is very bright, but Sirius A is very dim.



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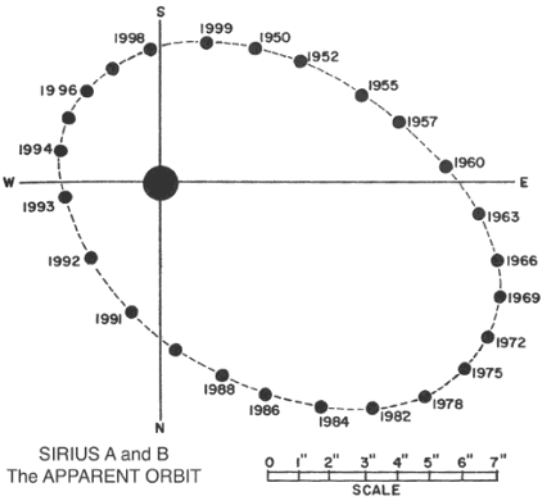
- By comparing the brightness at various wavelengths, we can find that the surface temperature of Sirius B is T = 27,000 K.
- Since we can measure a parallax angle for Sirius, we can find the distance to the star system.
- We can measure the intensity of light from Sirius B, so the luminosity can be calculated.
- By assuming that the luminosity is given by the Stefan-Boltzman equation, we can solve for the radius of Sirius B.
- The radius of Sirius B is calculated to be very close to the radius of the Earth!



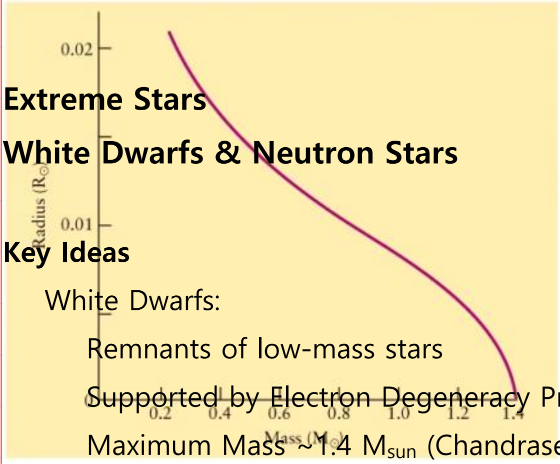
SIRIUS A and B
The APPARENT ORBIT

- The orbital period of the Sirius binary system is close to 50 years.
- We can use Kepler's orbital law and the centre of mass equation to find the masses of the stars.
- Sirius B has a mass almost the same as the Sun's mass.
- The average density of a star is its mass divided by its volume.
- The volume of a sphere is $V = 4/3\ \pi R^3$.
- The volume of Sirius B is
  $V = 4/3\ \pi\ (7 \times 10^6\ m)^3 = 1.4 \times 10^{21}\ m^3$.
- The density of Sirius B is
  density = 2×10^{30} kg/V. = $1 \times 10^9\ kg/m^3$.
- Remember: density of water is 1000 kg/m^3.
- The escape velocity from the surface of a star is $v_{esc} = (2GM/R)^{1/2}$.
- For Sirius B, this corresponds to a velocity of 6×10^6 m/s, about 2/100 c.

Mass-Volume relation for White Dwarf Stars

- For normal matter, if you double the mass of an object, its volume will also double.
- For example in main sequence stars, the higher mass stars also have larger sizes.

- For degenerate matter, if you increase the mass of a star, its radius and volume decreases. Why?
    - For degenerate matter, the source of pressure is density. So it maintain higher mass, one has to increase the density inside.
    - But density = mass/volume, however, mass increase alone does not provide sufficient density (and pressure) increase so the volume has to decrease as well in order to balance gravity.
- For white dwarfs, the masses and volumes are related by **Mass x Volume = constant**.
- This is only approximate, since at the **Chandrasekhar limiting mass** of M = 1.4 M_{Sun}, the electrons would have to move at the speed of light, so the mass-volume relationship does not hold down to zero volume.
- The Chandrasekhar mass is the largest mass that a white dwarf can possibly have.



출처: <https://sites.ualberta.ca/~pogosyan/teaching/ASTRO_122/lect17/lecture17.html>

Extreme Stars

White Dwarfs & Neutron Stars

Key Ideas

White Dwarfs:

Remnants of low-mass stars

Supported by Electron Degeneracy Pressure

Maximum Mass ~1.4 M_{sun} (Chandrasekhar Mass)

Neutron Stars:

Remnants of some post-supernova massive stars

Supported by Neutron Degeneracy Pressure

Pulsar = rapidly spinning magnetized neutron star

The Stellar Graveyard

Question:

What happens to the cores of dead stars?

Answer:

They continue to collapse until either:

- A new pressure law takes hold to halt further collapse & they settle into a new hydrostatic equilibrium.
- If too massive they collapse to zero radius and become a **Black Hole**.

All of these are seen as the remnants of stellar evolution.

Degenerate Gas Law

At high density, a new gas law takes over:

- Pack many electrons into a tiny volume
- These electrons fill all low-energy states
- Only high-energy = high-pressure states left

Result is a "**Degenerate Gas**":

- Pressure is **independent** of Temperature.
- Compression does **not** lead to heating.

This means that the objects could, in principle, be very cold but still have enough pressure to maintain a state of Hydrostatic Equilibrium.

Can such objects exist?

White Dwarfs

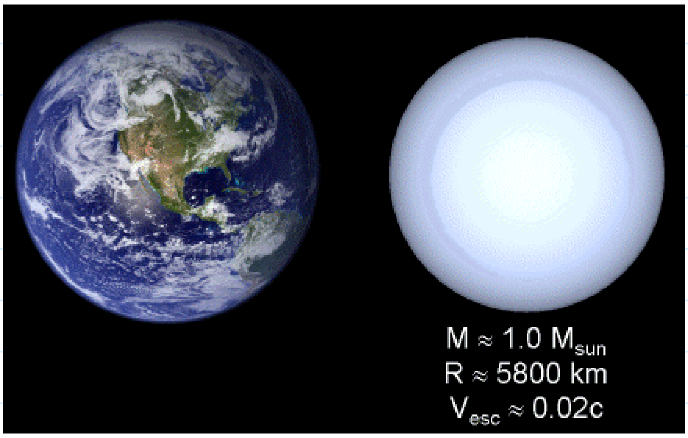
These are the remnant **cores** of stars with M < 8 M_{sun}.

- Supported against gravity by **Electron Degeneracy Pressure**
- M<4 M_{sun}: C-O White Dwarfs
- M = 4-8 M_{sun}: O-Ne-Mg White Dwarfs

Properties:

- Mass < 1.4 M_{sun}
- Radius ~ R_{earth} (<0.02 R_{sun})
- Density ~ 10^{5-6} g/cc
- Escape Speed: 0.02 c (2% speed of light)

No nuclear fusion or gravitational contraction. It shines by residual heat.



Comparison of a White Dwarf Star and the Earth.

Chandrasekhar Mass

Mass-Radius Relation for White Dwarfs:

Larger Mass = **Smaller** Radius

Chadrasekhar Mass:

- **Maximum Mass for** White Dwarf: M_{ch} = 1.4 M_{sun}
- First calculated by Subrahmanyan Chandrasekhar in the 1930s.
- Above this mass, electron degeneracy pressure **fails** & the star collapses.

This prediction of a maximum white dwarf mass is upheld by observations. So far, all white dwarfs we have seen in binary stars have masses below the Chandrasekhar Mass.

Evolution of White Dwarfs

White dwarfs shine by **leftover heat**.

- No energy source (no fusion, nothing)
- Cools off and fades away slowly.
- Forms a **White Dwarf Cooling Sequence** on the H-R diagram

Ultimate State: A "**Black Dwarf**":

- Old, cold white dwarf
- Takes ~10 Tyr to cool off

- Galaxy is not old enough for there to be any Black Dwarfs yet.

Note: Be careful not to confuse Black Dwarfs (old, cold remnant cores of low-mass stars) with "Black Holes" (the extremely collapsed cores of very massive stars).

White Dwarfs: The Other Supernovae

Question:

What would happen if you added enough matter to a White Dwarf to exceed its Chandrasekhar Mass?

Above the Chandrasekhar Mass, the internal electron degeneracy pressure is no longer able to balance Gravity, and it will begin to collapse...

- As the density rises, the temperature does **not** increase
- Eventually ignites carbon-oxygen burning at high enough density
- This begins to generate heat, but no additional pressure to slow the collapse
- The extra heat leads to greater fusion, which leads to greater heat...

The effect is a runaway nuclear explosion:

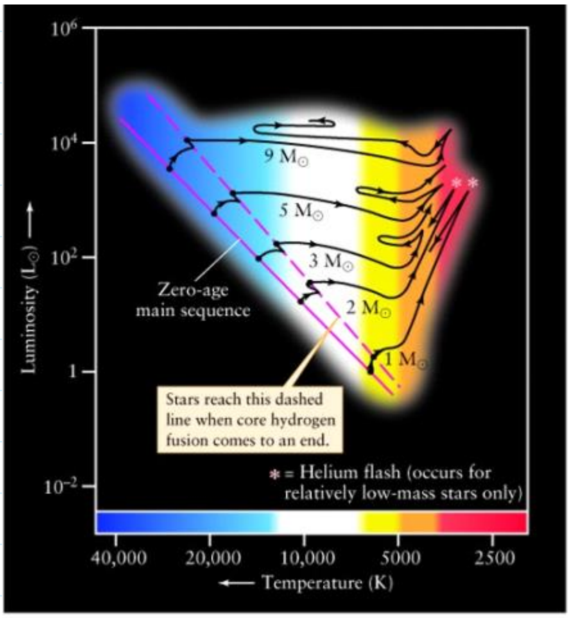
- Fusion of the light elements into Iron and Nickel
- The white dwarf detonates and disrupts completely as a **Type Ia Supernova**.

Type Ia Supernovae leave no remnant behind, and may be responsible for the production of much of the Iron in the Universe.

Evolution of High Mass Stars

High Mass Stars: Mass > 8 M_{Sun}

- The cores of the high mass stars are so large than when the fuel is burnt out, they cannot be stabilized even by denegerate pressure, and collpase further. High mass stars can have many successive stages of
  - fusion of an element in a core and lighter elements in shells around the core
  - exhaustion of the element
  - core collapse and heating
  - fusion of higher mass elements
  - etc...



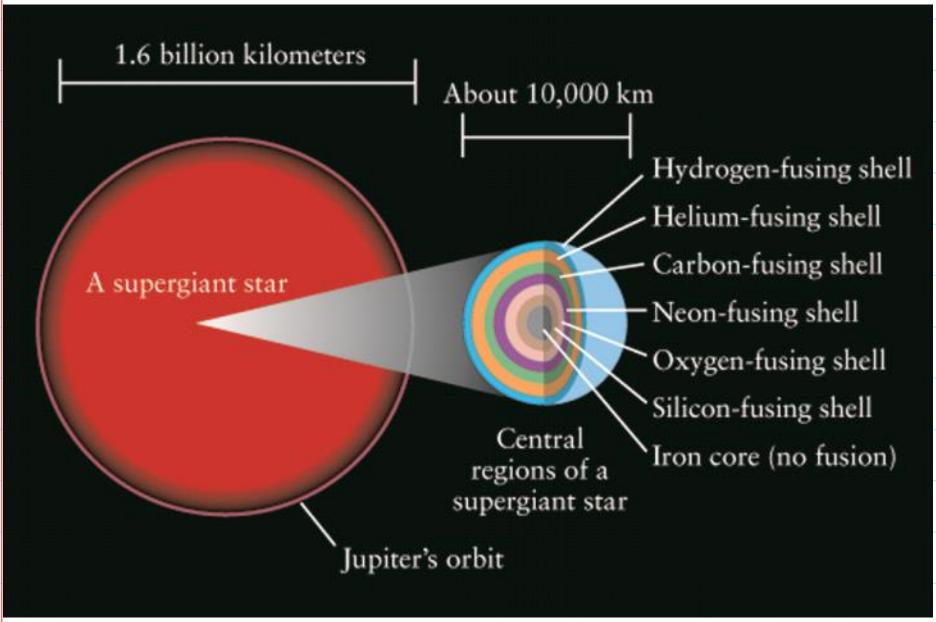
(a) Post-main-sequence evolutionary tracks of five stars with different mass

- The general trend is for the star's surface to become cooler and to become a blue giant and later a red supergiant.

- But there are several oscillations from red (super)giant to blue giant and back phase, correspondent to ignition of the next, heavier, fuel in the core.
- Star loses signifcant mass during super giant stage

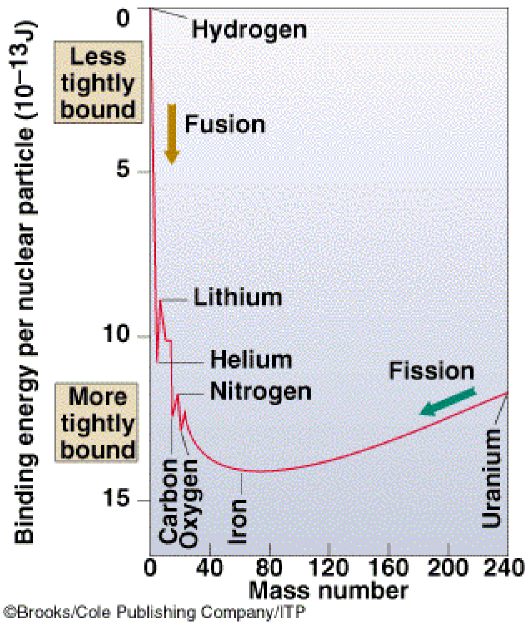
The "Onion-skin" layers of a Supergiant Star

- Over time the internal structure of a high mass star has an "onion-skin" character with layers of elements layered over each other, with highest mass elements at the centre.
- Final structure has inert iron core, outer shells of heavier elements undergoing nuclear fusion.



Iron is the End-Point of Nuclear Reactions

- Under normal circumstances iron doesn't fuse in a star.
- The strong nuclear force which binds nuclei together is a short range force, so there is a limit to how large a nucleus can be.
- Iron is the most stable element. Two types of nuclear reactions in nature:
    - Fusion reactions: light elements are fused (or glued) together to form heavier elements. These reactions are exothermic (release energy) as long as the reactants are lighter than iron. In order to fuse iron into a heavier element, energy would have to be supplied.
    - Fission reactions: heavy elements are are split into lighter elements. (Eg. reactions in



nuclear reactors which use Uranium) These reactions are exothermic if the reactants are heavier than iron.

The Iron Catastrophe

- When a star develops an iron core, an energy crises occurs, since no more exothermic nuclear reactions are possible.
- However, the core is also degenerate, so no heat source is needed to keep it stable.
- The quantum mechanical degeneracy pressure keeps the core from collapsing.
- The core is essentially an "iron white dwarf".
- But... white dwarf stars aren't always stable.
- If the mass of the white dwarf in the core is larger than 1.4 M_{Sun} (called the Chandrasekhar limit) the electrons would have to move faster than the speed of light in order to create enough degeneracy pressure to halt the gravitational collapse.
- Electrons can't move faster than light, so a white dwarf with $M > 1.4\ M_{Sun}$ collapses!
- Main sequence stars with mass larger than about 8 M_{Sun} eventually form white dwarf stars with masses larger than the Chandrasekhar limit and collapse.
- This is the beginning of a **Core Collapse Supernova** also known as a **Type II Supernova**

A Supernova in a star with $8\ M_{Sun} < M < 20\ M_{Sun}$

- When the supernova begins the iron core collapses rapidly under free-fall and becomes denser.
- When the density is very high, protons and electrons can combine together to form neutrons and neutrinos:
  $p + e^{-} \rightarrow n + nu$
- This reaction is called inverse beta decay.
- The neutrinos escape easily since they don't interact well with matter and carry off energy.
- The resulting neutron gas collapses until the density is extremely high.
- Neutrons obey the quantum mechanical Pauli exclusion principle. When the density becomes higher than about 10^{14} g per cubic cm, neutron degeneracy pressure provides an outward pressure which suddenly halts the gravitational collapse.
- The core of neutrons held stable by neutron degeneracy pressure is called a neutron star.
- The outer layers are still collapsing inwards at this point and these collapsing layers collide with the hard surface of the newly formed neutron star.
- This collision causes a violent rebound and a shock wave bouces outwards, colliding with the outer layers of the star.
- This expanding wave carries an extraordinary amount of energy.
- This energy can provide the fuel which allows the endothermic fusion reactions to create very high mass elements such as Uranium. The supernovae are responsible for all the elements with masses larger than iron found on Earth.
- These supernova explosions are extremely bright. Also large amount of energy is carried away by neutrinos

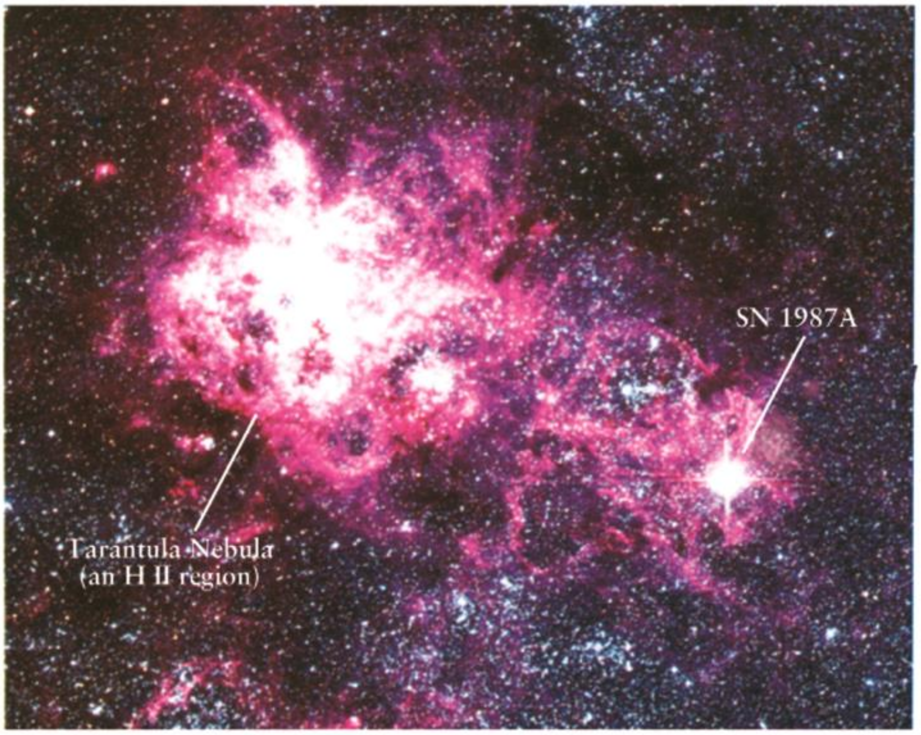
Supernova 1987a

- Core Collapse Supernovae probably occur about once every 50 years in our galaxy, but most of them are hidden by the dust of the galaxy.
- In 1987 a supernova occurred in the nearby galaxy called the Large Magellenic Cloud.

a

b

The B3 I star before the supernova The supernova explosion



A Supernova in a star with M > 20 M_{Sun}

- The evolution of very massive stars is similar with the formation of a neutron star at the core in a supernova.
- However, neutron stars (like white dwarfs) have a maximum mass near 3 M_{Sun}, over which neutron degeneracy pressure can't balance gravity.
- In the very high mass stars, the neutron star goes over the critical mass, and the neutron star collapses.
- No other sources of pressure are available, and the collapsing material forms a black hole.
- Black holes have a "surface" called an event horizon where the force of gravity is so strong that the escape velocity is larger than the speed of light. No light can escape from a black hole, hence its name.

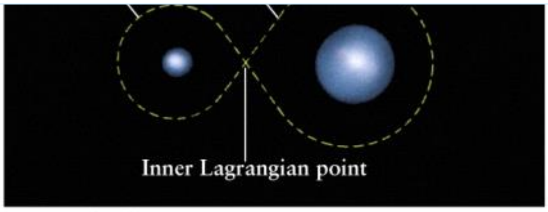
White Dwarf Stars in Binary Systems can also go Supernova

A close binary system

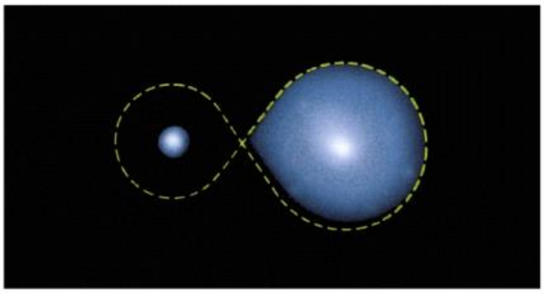
- In any binary system, there is a point

between the stars called the **Inner Lagrangian Point**.

- At the Lagrangian point, the forces acting on a particle exactly cancel out.
- A **Roche Lobe** is an imaginary surface surrounding the stars that goes through the Lagrangian point.
- If a star is large enough to fill up its Roche lobe, matter can flow from the star to the other star.



a Detached binary



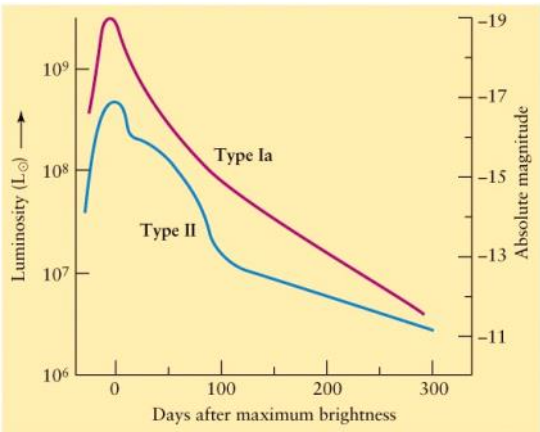
b Semi-detached binary

A White Dwarf in a Semi-Detached Binary

- Suppose that a white dwarf is receiving mass from a companion star.
- The White dwarf's mass will slowly increase.
- If it receives enough mass, the White Dwarf's mass will approach the Chandrasekhar limit and collapse.
- The collapse causes the degenerate Carbon gas in the White dwarf to begin fusing together explosively.
- This type of supernova is essentially a giant Carbon bomb!

The Two Types of Supernovae

- **Type Ia**: Carbon-bomb caused by a white dwarf accreting material.
    - Source of energy is nuclear fusion.
- **Type II**: Core Collapse of a Supergiant Star
    - Source of energy is gravity.
- There are also **Type Ib** and **Ic** , but they are core collapse as Type II are



- Note that our Sun can never go supernova!
- The Sun's mass isn't large enough to become a supergiant star, so it can't undergo a Type II supernova explosion.
- Our Sun will only become a White Dwarf star in the future.
- Since our Sun is not in a binary system, once it becomes a white dwarf, it won't accrete matter and will **not** undergo a Type Ia supernova explosion.

출처: <https://sites.ualberta.ca/~pogosyan/teaching/ASTRO_122/lect18/lecture18.html>

Neutron Stars

End points of stellar evolution

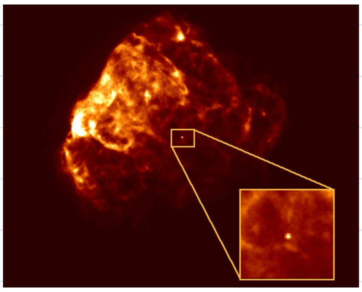
Starting Mass	Outcome	Final Mass	Final Size	Density
≥ 20 M_{Sun}	**Black Hole**	any	2.95 (M/M_{Sun}) km	N/A
8 < M < 20 M_{Sun}	**Neutron star**	< 3-4 M_{Sun}	10~20 km (&darr M)	10^{18} kg/m^3
0.4 < M < 8 M_{Sun}	**White Dwarfs (Carbon)**	< 1.4 M_{Sun}	7000 km (&darr M)	10^9 kg/m^3

$0.08 < M < 0.4\ M_{Sun}$	**White Dwarfs (Helium)**	$0.08 < M < 0.4\ M_{Sun}$	14000 km (&darr M)	
$M < 0.08\ M_{Sun}$	**Brown Dwarfs**	$M < 0.08\ M_{Sun}$	10^5 km	

Birth of a Neutron Star

- The death of a high-mass star (such as Betelgeuse) will leave behind a neutron star.
- Initially, the neutron star will be very hot, about 10^{11} K.
- It will glow mainly in the X-ray part of the spectrum.
- Over its first few hundred years of life, the neutron star's surface cools down to 10^6 K and continues to glow in the x-ray.
- Young neutron stars are found in supernova remnants.

X-ray Image of the Puppis A Supernova Remnant



- The small point-source is a neutron star.
- The neutron star is not at the centre since it was violently **kicked by the supernova** explosion.
- The neutron star moves with a velocity of 1000 km/s.

The Crab Nebula

- In the year 1054 A.D. the Chinese Court astronomer/astrologer Yang Wei-Te noticed a bright new star which suddenly appeared in the constellation Taurus.
- At its brightest (Supernovae explosion), it was almost as bright as Venus
- It was visible during the daytime for 23 days and then continued to be visible to the naked eye at night for another 653 days.
- In the year 1731 John Bevis observed a "fuzzy" white nebula at the same location as the new star.
- Charles Messier observed the nebula in 1758. Messier was interested in finding comets and wanted to make a catalogue of "boring" non-comet fuzzy objects. This nebula became the first object in his catalogue, M1.
- The fuzzy nebula is called the Crab Nebula or M1 today.

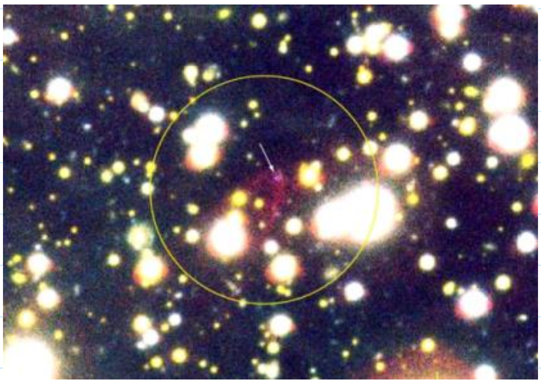
True Colour Photo of the Crab Nebula

- Red = Hydrogen Balmer transition corresponding to ionized hydrogen recombining with electrons.
- Blue = Synchrotron emission as electrons spiral around magnetic field lines.
- photograph made by astrophotographer David Malin
- The total power output by the Crab Nebula is $10^5\ L_{Sun}$
  - This is incredible, since it is almost 1000 years after the supernova explosion.
  - The neutron star inside this nebula rotates once every 33 ms (or about 30 times a second).
  - We see a bright spot on the neutron star, so the star appears to flash once every rotation period.
  - We now say this neutron star is a **pulsar** .

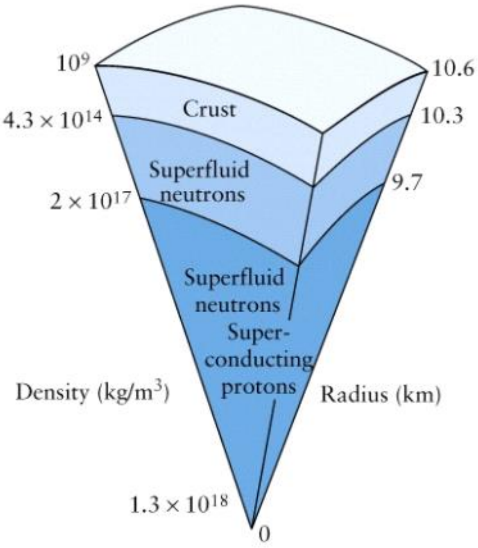
The Nearest Neutron Star

- Nearest to Earth neutron star is in Corona Australis - 200 light-years away.
- This is a more detailed photo (in visible light) of RX J1856.5-3754 made with the ground-based telescope "Kueyen" in Chile.
- Kueyen is an 8 m telescope which is part of 4 telescope array whose light will be combined to make an equivalent 16 m telescope.

- This picture shows a faint red cloud around the neutron star.
- The red light is Hydrogen Balmer Alpha emission.
- Photons emitted by the hot neutron star (T = 700,000 K) are exciting the Hydrogen surrounding the neutron star.



Structure of a Neutron Star



- A neutron star balances gravity with neutron degeneracy pressure.
- The neutrons separated by a distance = d have a velocity given by the Heisenberg Uncertainty principle:
  $v = h/(2 \pi m d)$
  where m is the mass of the neutron and h is Planck's constant.
- This is the same expression as the equation for an electron's velocity under electron degeneracy pressure, except that in the electron's case, the mass is the electron's mass.
- The neutron is about 2000 times more massive than an electron, m_n = 1800 m_e.
- In order for the degenerate neutrons to have the same velocity as the degenerate electrons the neutrons must be 1800 times closer to each other than the electrons in a white dwarf star.
- A neutron star with the same mass as a white dwarf has a radius about 1000 times smaller than a white dwarf.
- Typical radius for a neutron star is 10 km.
- Average density &rho of a 10 km star with a

mass of 2 M_{Sun} is
&rho = 4 x 10^{30} kg x 3/(4 &pi x 10^{12} m^3) =
10^{18} kg/m ^{3}

- This is one billion times more dense than a white dwarf.

Uncertainty about a neutron star's structure

- It is not known what really lies at the core of a neutron star.
- Exotic particles such as pions or unbound quarks might lie in the core.
- Each theory about the dense core provides a correction to neutron degeneracy pressure.
- Since the detailed nature of the core is unknown, the forces opposing gravity are not known exactly and the sizes of neutron stars are not known exactly.
- For example, two different, but reasonable theories of neutron stars predict two different sizes for a neutron star with 1.4 M_{Sun}. One prediction is for a radius of 10 km, the other predicts a radius of 20 km.
- If you could accurately measure the radius of a neutron star and measure its mass, you could rule out certain theories describing dense nuclear matter.
- However, very difficult to measure the radius of a star this tiny.

Maximum Mass of a Neutron Star

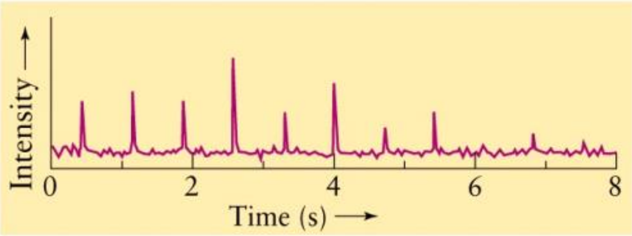
- White dwarfs can't have a mass larger than 1.4 M_{Sun} (the Chandrasekhar limit) since their electrons can't move faster than light.
- Neutron stars have a similar type of limit.
- Each theory of nuclear matter predicts a different maximum mass for neutron stars.
- Maximum masses range from 1.5 to 4 M_{Sun}.
- If you measure a neutron star's mass, you can rule out theories with predicted maxima below your measured mass.
- The maximum mass is important for identifying black holes.
- A black hole in a binary star system has properties very similar to a neutron star, so they are hard to identify.
- Suppose that you observe a mysterious object which is probably either a neutron star or a black hole. If you measure the mass and find out that it is above the maximum mass limit for neutron stars, then it must be a black hole.

The Discovery of the First Neutron Star

- NSs were first theoretically predicted by Walter Baade and Fritz Zwicky in the 1930's.
- The properties seemed so bizarre that nobody took the prediction very seriously.
- In 1967 Jocelyn Bell was observing using a new radio telescope for her Ph.D. thesis.
- She discovered a radio pulse signal with a period of 1.337 s at one particular location.
- She and her supervisor, Antony Hewish, first came to the conclusion that this was a signal from an alien civilisation and called the signal LGM = Little Green Men
- After finding a 2nd similar object at another location they realised these must be real

astronomical bodies, and came to the conclusion that they must be pulsars.

- In 1974 Hewish was awarded the Nobel prize in physics for the discovery of pulsars.
- Over 1,000 NSs have been discovered. Among them 200 very fast **millisecond pulsars.**
- Pulses for some pulsars have been seen in gamma-rays, x-rays, visible light, infrared, and radio



Maximum Spin Rate of a Neutron Star

- A rotating object can't spin too fast, or it will be torn apart by the "centrifugal force".
- The spin period = P is the time for a star to make one rotation.
- Equate gravitational force at the surface and centrifugal force

  $GM/R^2 = R\omega^2$

  to find the max angular velocity ω and, thus, the minimum possible period $P=2\pi/\omega$
- The minimum spin period for an object with mass M and radius R approximately:

  $P_{min} = 2\pi (R^3/(GM))^{1/2}$ = (3 π / (G &rho)) $^{1/2}$
- The minimum spin period for some astronomical objects is:

Object	P_{min}	Actual spin period	Actual period / Minimum period
Earth	5100 s = 1.4 hours	1 day	17 x slower
Jupiter	10,000 s = 2.8 hours	10 hours	4 x slower
Sun	10,000 s = 2.8 hours	25 days	216 x slower
White Dwarf	about 9 s		
Neutron Star	0.5 ms	Fastest is 1.4 ms	3 x slower

- Neutron stars can spin very rapidly because they are tiny and very dense! One can immediately deduce that the density must be high. Even if P=1 s, &rho > 3 &pi/(G P^2) = 10^{11} kg/m^3. Could it be a white dwarf ? Perhaps, but Crab pulsar with 33 millisecond period can't be for sure !
- In 1982 the most rapidly rotating neutron star had P = 1.6 ms (Spin frequency = 600 Hz).
- In 2005 Jason Hessels (BSc. from U of A) discovered a neutron star with P = 1.4 ms (Spin frequency = 715 Hz).

Formation of a Rapidly Rotating, Magnetized Neutron Star

Same principles that we considered during collapse of protostars

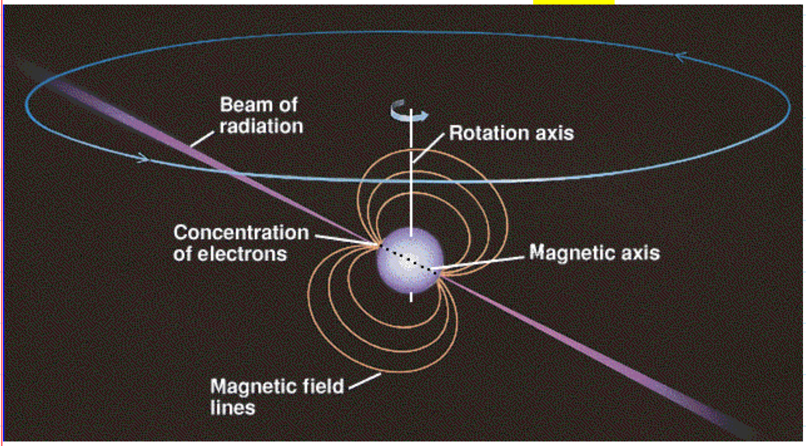
- A neutron star is formed from the collapse of a much larger star.
- **Angular momentum is conserved** during a collapse, so the spin rate increases.
- Spin period is proportional to $(\text{radius})^2$
- For example: The Sun is about 5 orders of magnitude larger than a typical neutron star.
  - If we collapse the Sun down to the size of a neutron star, it will have a spin period 10^{-10} times smaller than the Sun.
  - ie. it would spin with a period of 0.2 ms
  - It is very easy to create a neutron star which spins with a period near a millisecond.
- Similarly, **magnetic flux is conserved** during a collapse so that the magnetic field strength is proportional to $1/(\text{radius})^2$

- If the Sun collapses down to the size of a neutron star, its magnetic field will be 10 billion times stronger.
- Typical magnetic fields on neutron stars are 10^{12} times stronger than the Sun's magnetic field.
- A small number of neutron stars have magnetic fields 10^{14} times the Sun's magnetic field.
  These ultra-strong magnetic field neutron stars are called **magnetars**.
- See Feb. 2003 Scientific American for a great article on magnetars

Even small initial rotation and magnetic field of neutron stars is highly amplified during the collapse

The Pulsar Mechanism

- The strong magnetic field of a neutron star creates a magnetosphere around the neutron star.
- The magnetic poles are not usually aligned with the spin axis.
- Inside the neutron star, the electromagnetic forces rip off the electrons on the surface and the electrons get trapped by the magnetic field.
- The electrons get funnelled along lines of force pointing out of the north and south magnetic poles.
- The electrons are highly accelerated and they radiate synchrotron radiation which is beamed outwards in the directions of the poles.
- The spin of the star causes the beam of radiation to intersect our line of sight once a spin period.
- We see a pulse of light which turns on and off with a regular period. (Light-house mechanism)
- We call this type of neutron star a **pulsar**.

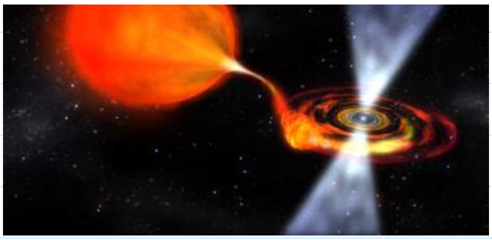


- A magnet which spins about an axis different from its symmetry axis emits radiation which causes it to lose energy.
- This loss of energy causes the magnet's spin to slow down.
- The neutron star must slow down, which means that its spin period must increase slowly with time.

Pulsar Recycling

- Neutron stars are born rapidly rotating but slow down due to the magnetic drain of their energy.

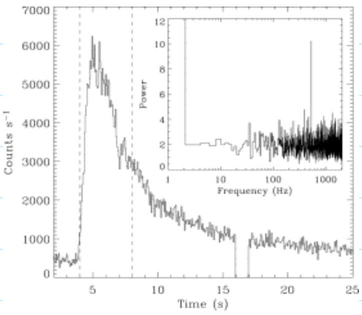
- This would suggest that over time all old pulsars should spin slowly.
- However, if a neutron star is in a binary system things change.



- Matter can flow from the companion to the neutron star through an accretion disk.
- As matter from the disk falls onto the neutron star, it adds mass and angular momentum (or spin) to the neutron star.
- This slowly causes the neutron star to spin faster.
- This process is called **recycling**.
- The accretion disk is very hot and typically radiates x-rays.
- When Hydrogen and Helium are dumped onto the surface, small nuclear explosions occur causing bursts of x-rays.
- When the explosion takes place on only a small part of the star, we see the explosion only once every spin period, so the burst seems to flicker.
- Flickering X-ray Bursting neutron stars have been observed which suggest that they spin with periods in the range of 3 ms to 1.6 ms.

Light Curve for an X-ray Burst

- The large graph shows how brightness varies with time during an X-ray Burst.
- If the time axis was expanded, you would be able to see a periodic wave with a frequency of 530Hz.
- The inset shows a "Fourier Spectrum" which shows the dominant repetition frequency in the data.



The Crab Nebula in Radio, Optical and X-rays

- This shows a recent composite picture of the innermost region of the Crab Nebula (made by combining images from the Chandra X-ray Telescope, Hubble Space telescope and NRAO radio telescopes).
- Red = Radio emission
- Green = Visible emission
- Blue = X-ray emission
- The Crab Pulsar is hidden in the centre of

the rotating disk. The disk is caused by a wind originating from the pulsar.

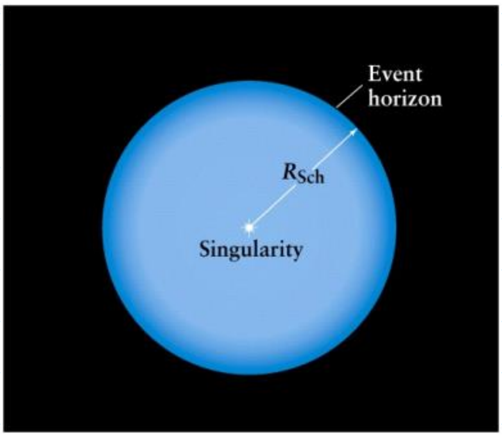
- The pulsar moves in the same direction as its spin axis! Suggests that the supernova gave a peculiar type of "kick" to the neutron star during its birth.

Black Holes

출처: <https://sites.ualberta.ca/~pogosyan/teaching/ASTRO_122/lect19/lecture19.html>

The Black Holes

- Black holes are the astronomical "objects" left over when the outward pressure in a star is insufficient to balance the force of gravity.
- All the matter in the star is attracted to the centre of the star.
- The matter collects in a region with zero volume called a **singularity**.
- Although all the matter from the original star has condensed to the singularity, the mass of the black hole is the same as the mass of the star.
- Far from a black hole, the gravitational attraction between the black hole and other objects is the same as the attraction between a regular star with the same mass as the black hole.



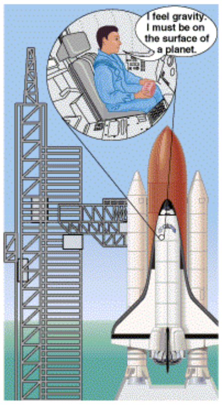
- The most important feature of a black hole is its **event horizon**.
- The event horizon is an imaginary surface which acts as a boundary for the black hole.
- The black hole's "interior" is the region inside of the event horizon.
- The black hole's "exterior" is the region outside the event horizon.
- The event horizon is a sphere with a radius $\mathbf{R_{EH}}$ defined so that the escape velocity from the event horizon is exactly equal to the speed of light.
- At even horison **escape velocity**
  $\mathbf{v = (2GM/R_{EH})^{1/2}} = c$
- So for black hole of mass M, then the event horizon has a radius of
  $\mathbf{R_{Sch} = R_{EH} = 2GM/c^2 = 3km\ M/M_{Sun}}$
- Since nothing can travel faster than light, it is impossible for any particles to escape from the black hole.
- Light also can't escape from the black hole, hence the name.

The Einstein Equivalence Principle

- One of the basic concepts in general relativity is the equivalence between gravity and acceleration.
- Astronaut in a windowless rocket can't tell the difference between:

Rocket sitting at rest on the	Rocket far from any massive bodies moving with a

surface of the Earth. constant acceleration = g.



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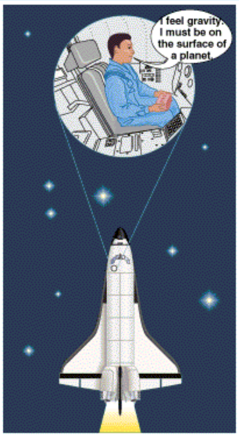

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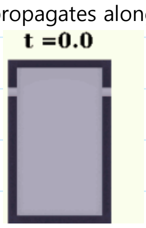
- All experiments done in the two rockets yield the same results.

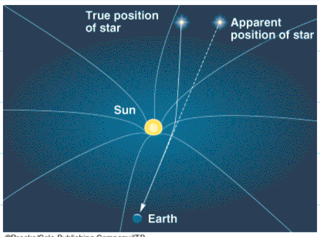
From equivalence principle – gravity is not felt if the object is in free fall.

- Why is this important ? Establishes relation between gravity and acceleration. But acceleration appears when the motion in space time is along curved paths. So link between gravity and curved geometry of space-time is incorporated in this principle.
- Here is for example how one can see from equivalence principle that light must bend in the presence of gravity.

Elevator free-falls. There is no gravity felt inside, special relativity holds, light propagates along the straight line. 	For the observer on Earth, that feels gravity, the light must follow the curve path !, So gravity bends the light rays. 

Bending of Light

- In order for light to travel at the same speed, photons have to move on curved paths when a massive object is present.
- We call this effect the gravitational bending of light.
- This effect was first seen during a solar eclipse in 1919.
- A star which should have been directly behind the Sun was visible, because the star's photons moved on a curved path around the Sun.



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Gravitational Redshift

- Einstein's theory of gravity also predicts that when light is emitted from the surface of a planet or a star, its wavelength gets redshifted as it moves away from the star.
- Equivalent to say that clocks tick slower in a strong gravitational field compared to clocks far from the gravitational field. This is called **gravitational time dilation** . This is not Doppler effect due to

motion of the source !

- This effect is tiny for the Earth (~10^{-15} but was measured in 1960 !) , but it must be taken into account in the GPS satellite system used for finding positions on the Earth!
- This redshift has also been observed from light emitted from the surface of white dwarfs and neutron stars. It is of order ~10^{-4} for white dwarfs and ~0.1 for neutron stars.

Properties of Black Holes

Dark Star

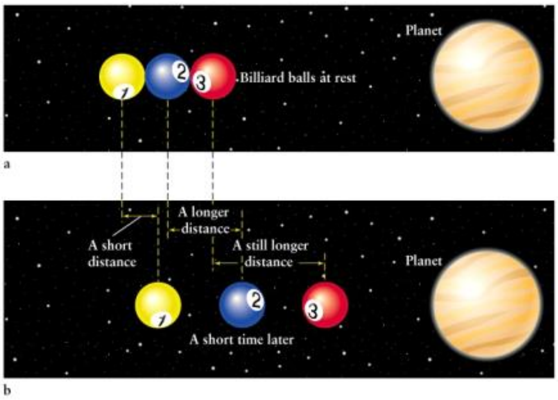
- The concept of a black hole (or dark star) was introduced in the late 1700's by Michell and Laplace.
- They considered a very dense star with an escape velocity greater than light.
- This star would appear black since no light could escape from its surface.

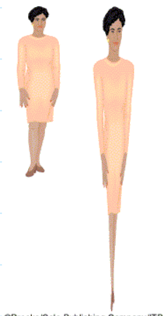
Concept of a Black Hole in Relativity

- The material inside of the dark star can't be at rest. It all falls into the singularity at r=0.
- The event horizon is at the same location as the surface of the dark star conjectured by Michell and Laplace.
- Inside of the event horizon, a massive particle or a photon would have to move faster than light to get outside of the black hole.
- Since nothing travels faster than light, you can't see the inside of a black hole.
- If you are outside the black hole, the gravity is just like the gravity of a star with the same mass as the black hole.
- If our Sun were suddenly replaced by a black hole with Mass = M_{Sun}, the orbits of the planets wouldn't change.
- If you travelled close to the black hole, you could escape, as long as you don't enter the event horizon.
- If you entered the black hole, you could still see everything outside, since light can enter a black hole.
- You wouldn't notice anything special about the event horizon, since it isn't a solid surface.
- Once you enter the event horizon, you will pulled into the singularity.

A trip into a black hole -- Tides

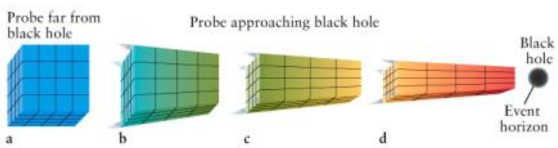
- A trip into a black hole would probably be fatal.
- Tides are created when the force of gravity is stronger on one side of an object than on the other.
- If you were falling feet first into a black hole, the force of gravity acting on your feet would be stronger than the force of gravity acting on your head.
- The effective tidal force would stretch you lengthwise and compress your width.





A trip into a black hole -- Gravitational Redshift and Time Dilation

- Suppose that a rocket decided to enter a black hole.
- The gravitational time dilation effect means that the interval between our detection of the light bursts gets longer as the rocket gets closer to the black hole.



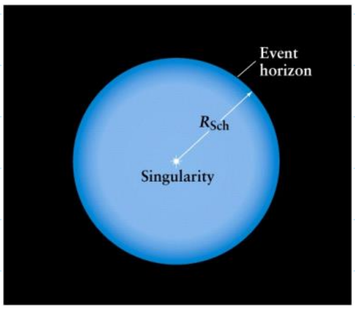
- On our clock, it takes an infinite amount of time for the rocket
- The rocket sends a burst of light every second back towards Earth where we watch the signals.
- The photon's wavelength gets redshifted, so that if the rocket emits blue light, we will detect some redder colour.
- But the astronauts in the rocket think that the

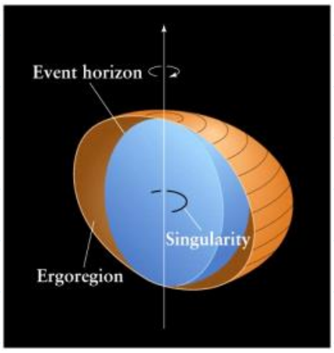
to reach the event horizon.

trip only takes a short, finite amount of time.

Black holes are simple!

- Although a star is approximately a sphere, it also has many bumps (ie. prominences, flares, etc).
- As the density of a star increases, the size of bumps has to decrease, ie. gravity smooths out bumps.
- For instance, mountains on a neutron star can't be much higher than a centimetre.
- In 1967, Werner Israel, (professor at the U of A, now retired in Victoria) proved the "No Hair" Theorem which states that the event horizon has to be perfectly smooth.
- From Israel's theorem, a black hole's properties are given by just **three numbers mass M, electric charge Q and angular momentum L.**
- If the black hole doesn't rotate, it must be exactly spherical.
- If the black hole rotates, it must have a special ellipsoidal shape.



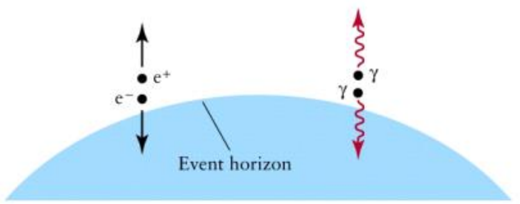


Black Holes Evaporate

- In quantum mechanics, a vacuum really isn't a vacuum.
- Pairs of particles and anti-particles are constantly being created and destroyed.
- The higher the mass of the particles, the shorter the time they exist, through Heisenberg's uncertainty principle:
  mass x c^2 x time = h/(2 &pi)

Hawking Radiation

- 1974 - Stephen Hawking showed that if pairs are created near the event horizon, it is possible for them to be separated so that one falls into the black hole and one escapes.



- Far from the black hole, an observer would see a flux of massive particles coming from the black hole.
- The energy allowing this flux is coming from the mass of the black hole itself, so the black hole must lose mass.
- The flux obeys the Stefan-Boltzmann law, so a temperature can be assigned to the black hole.
- This temperature is inversely proportional to the black hole's mass.

- Consider two masses:
- A black hole with M = 5 M_{Sun}
  - T = 10^{-7} K, which isn't measureable
  - This black hole would take 10^{62} years to evaporate!
- A black hole with M = 10^{10} kg = 10^{-20} M_{Sun}
  - T = 10^{14} K
  - This black hole would take 15 x 10^{9} years to evaporate!
  - But such small mass black holes can't be formed from the cores of stars.
  - Tiny black holes could have been created in the early universe, but no such evidence yet.

How do you detect an invisible object?

Black holes are detected by observing their gravitational effect on other objects.

Stellar mass black holes

Binary systems including a black hole

- Consider a binary system consisting of a regular star and an invisible star.
- By observing the orbit of the regular star, you can find the mass of the invisible star, if you know the angle of inclination of the binary.
- i = angle of inclination = angle between the plane of the binary's orbit and the plane of the sky.
- If angle is unknown you can only find a lower limit on the mass of the invisible star.
- Neutron stars are very dim and could appear to be invisible.
- If the minimum mass measured is larger than the maximum mass for a neutron star (about 3 or 4 times the mass of the Sun), the invisible star is a black hole.
- Since this is an indirect method for finding a black hole, the objects found this way are called Black Hole Candidates.

Six Black-Hole Candidates

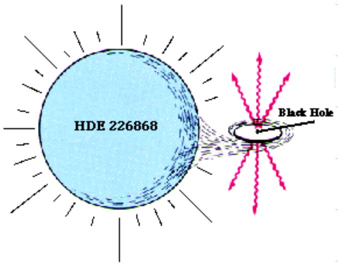
OBJECT	LOCATION	COMPANION STAR	ORBITAL PERIOD	MASS OF COMPACT OBJECT
Cygnus X-1	Cygnus	O Supergiant	5.6 days	10-15 M
LMC X-3	Dorado	B3 main-seq	1.7 days	4-11 M
V616 Mon	Monocerotis	K main-seq	7.75 hrs	3.3-+4.2 M
V404 Cygni	Cygnus	K main-seq	6.47 days	8-15 M
J1655-40	Scorpius	F-G main-seq	2.61 days	4-5.2 M
QZ Vul	Vulpecula	K main-seq	8 hours	5-14 M

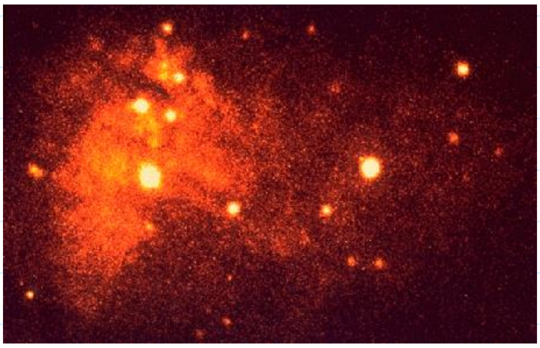
The companion star to Cyg X1

Accretion onto a Black Hole or a Neutron Star

- When gas falls towards a star, gravitational potential energy is converted to kinetic energy.
- The total energy of the gas is E = K − GM/r
- K = kinetic energy
- Energy is conserved as the gas falls.
- Suppose that the gas has zero energy, then K = GM/r
- When the gas is far from the star, the Kinetic energy is small.
- When the gas falls closer to the star, the Kinetic energy increases.
- The kinetic energy of a gas is proportional to its temperature.

- The gas will radiate as it falls, and the higher the Kinetic energy, the higher energy will the photons radiated be.
- Neutron stars and Black holes have such small sizes, that 1/r is very large.
- The gas falling into either a Neutron star or a Black Hole radiates X-rays.
- A binary system which emits X-rays is called an **X-ray Binary**.

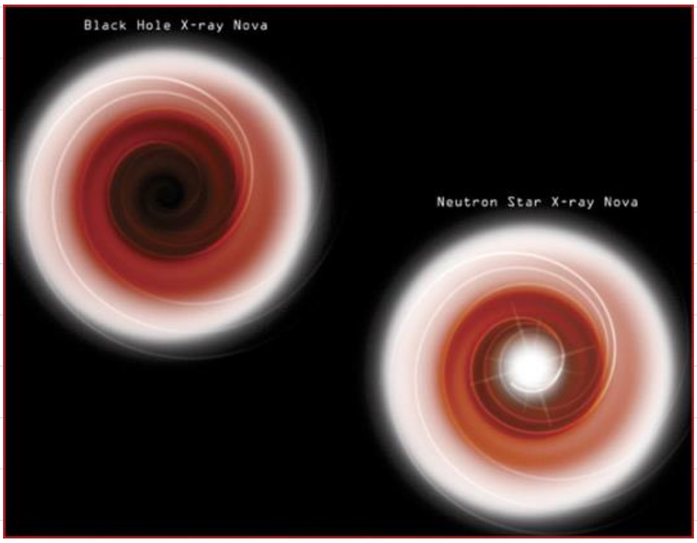




X-ray Picture of the LMC X1 Black Hole Candidate

Differences Between Accreting Neutron Stars and Black Holes

- All of the Black hole candidates listed above are members of X-ray binaries.
- One important difference between neutron stars and black holes:
  - Black holes do not have a hard surface.
  - When Hydrogen falls into the black hole, there is no surface on which nuclear explosions can take place.

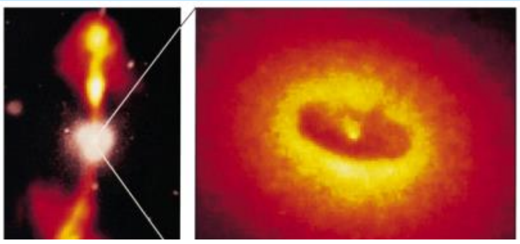


- If X-ray bursts from nuclear explosions take place, the star must be a neutron star, not a black hole.
- (Unfortunately, the converse is not true.)
- It is also possible for gas in the disk to flare. The X-rays from X-ray binaries tend to vary rapidly with time.
- Since gas is lost when it falls into the event horizon, the disks surrounding black holes should be a little dimmer than neutron stars. Some evidence that this signature of less light has been observed.

Supermassive (~ 10^6 M_{Sun}) Black Holes

Black Holes at the Centres of Galaxies

- At the centres of galaxies **supermassive** black holes can form.
- These black holes also have accretion disks



which emit x-rays.

- More about the black holes in the centres of galaxies when we study galaxies.
  - One of the best black hole candidates- **supermassive (10^6 M_{Sun}) black hole** at the center of our Galaxy

a b

[Stars Orbiting Black Hole at Center of Milky Way](#)

[Loose Ã…rrow](#) | [MySpace Video](#)

Other Places Where Black Holes Could Be Found:

Supernova Remnants

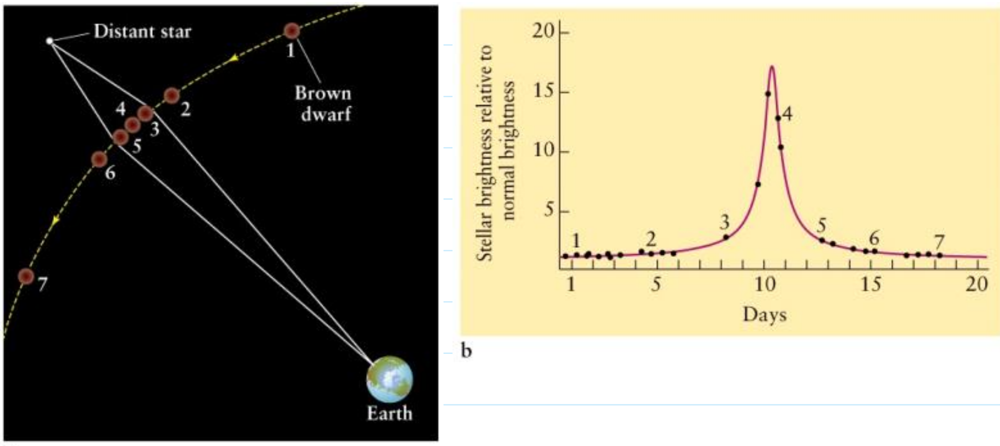
- Black holes are supposed to be formed in supernovae.
- However, no conclusive evidence has been found yet for a black hole found inside of a supernova remnant.
- However, some of the supernova remnants do not have evidence for a neutron star, so perhaps a black hole is in them.

Gamma-Ray Bursts

- The latest theories of gamma-ray bursts (which we will study later on in a couple of weeks) suggests that they occur when a rapidly rotating black hole is formed in a severe type of supernova.

Gravitational Lensing

- The gravitational field of any mass curves space so that light travels on a curved path.
- If a massive object (a black hole, for instance) is in front of a star, we can still see the light from the star since the light moves around the massive object.
- We call this effect **gravitational lensing** since a mass acts like a lens.
- Another effect due to gravitational lensing is that the star being lensed by the foreground mass gets brighter when the mass passes in front of the star.
- It is possible to measure the mass of the "lens".
- A few black holes have been detected as follows:
  - A black hole passes in front of a background star.
  - (The black hole is closer to us, so it moves faster than the star.)
  - The star appears to be brighter when the black hole passes in front.
  - An important fact about the lensing: Both blue and red light get brighter by the same amount.
  - If the star just happened to brighten on its own, the blue and red light would not increase at the same rate.



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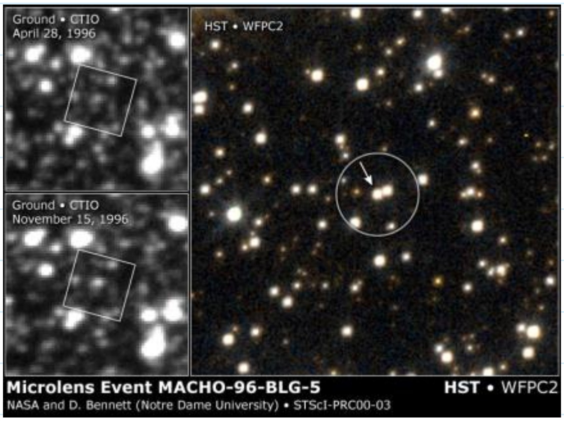

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