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Cosmic Strings Networks & Gravitational Waves

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Outline of the talk

- Introduction to Cosmic Strings and their dynamics.
- Cosmological Networks.
- Stochastic Background of Gravitational Waves.
- Current bounds on Cosmic String tension.
- Open issues and Conclusions.

What is a cosmic string?

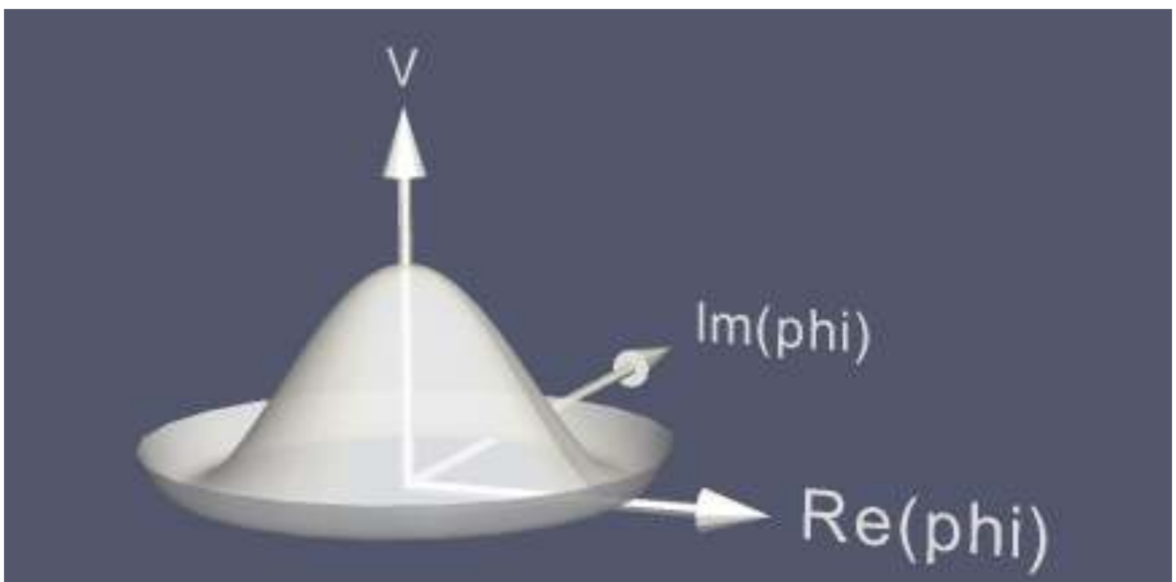
- Topological defects in field theory.
    - Relativistic version of Abrikosov vortices.
    - They exist in many extensions of the standard model.
- Cosmic Superstrings.
    - Fundamental Strings can be stretched to cosmological sizes and behave basically as classical objects.

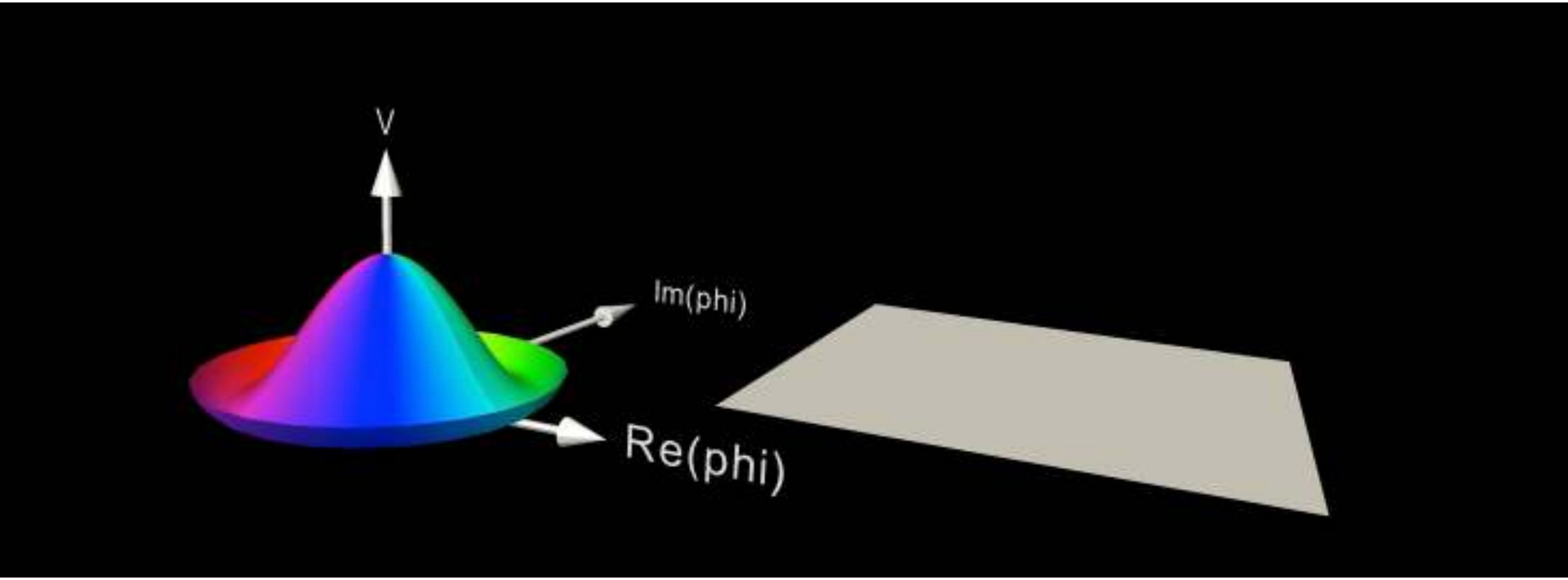
What is a cosmic string?

- Simplest model:

$$S_{U(1)} = \int d^4x \left[\partial_\mu \phi \partial^\mu \phi^* - V(|\phi|^2)\right]$$

$$V(\phi) = \frac{\lambda}{4}(|\phi|^2 - \eta^2)^2$$

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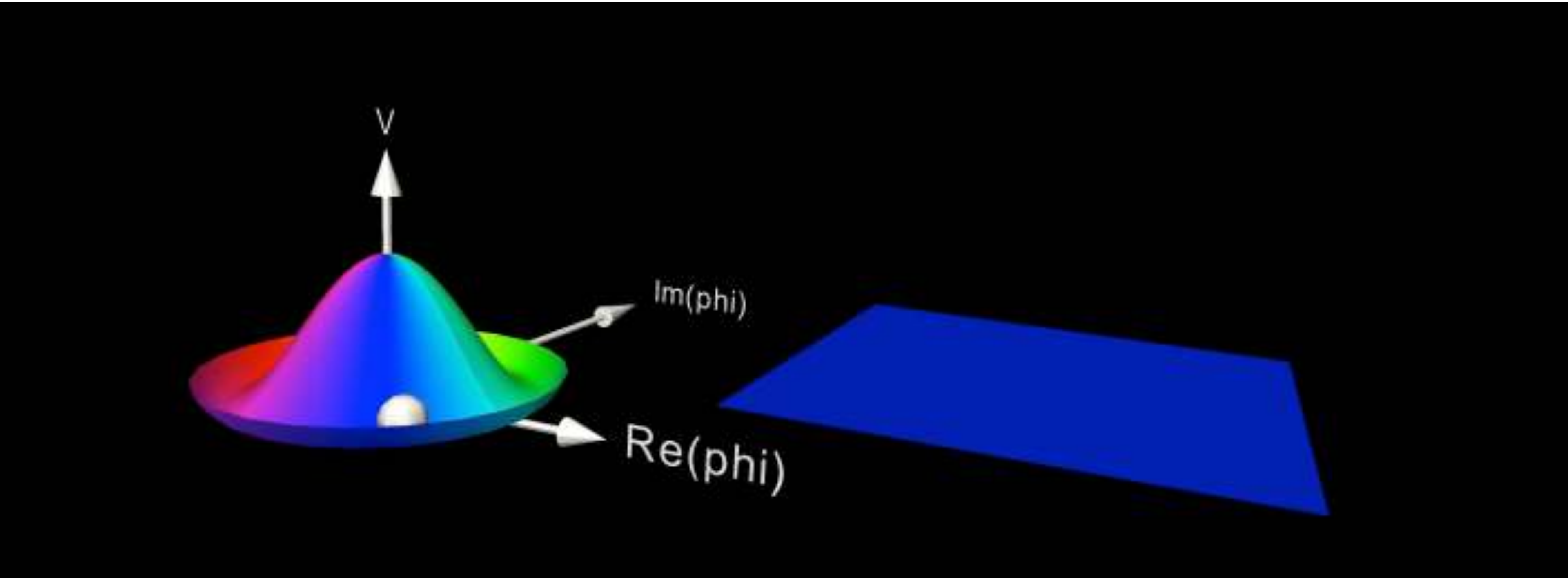
Re(phi)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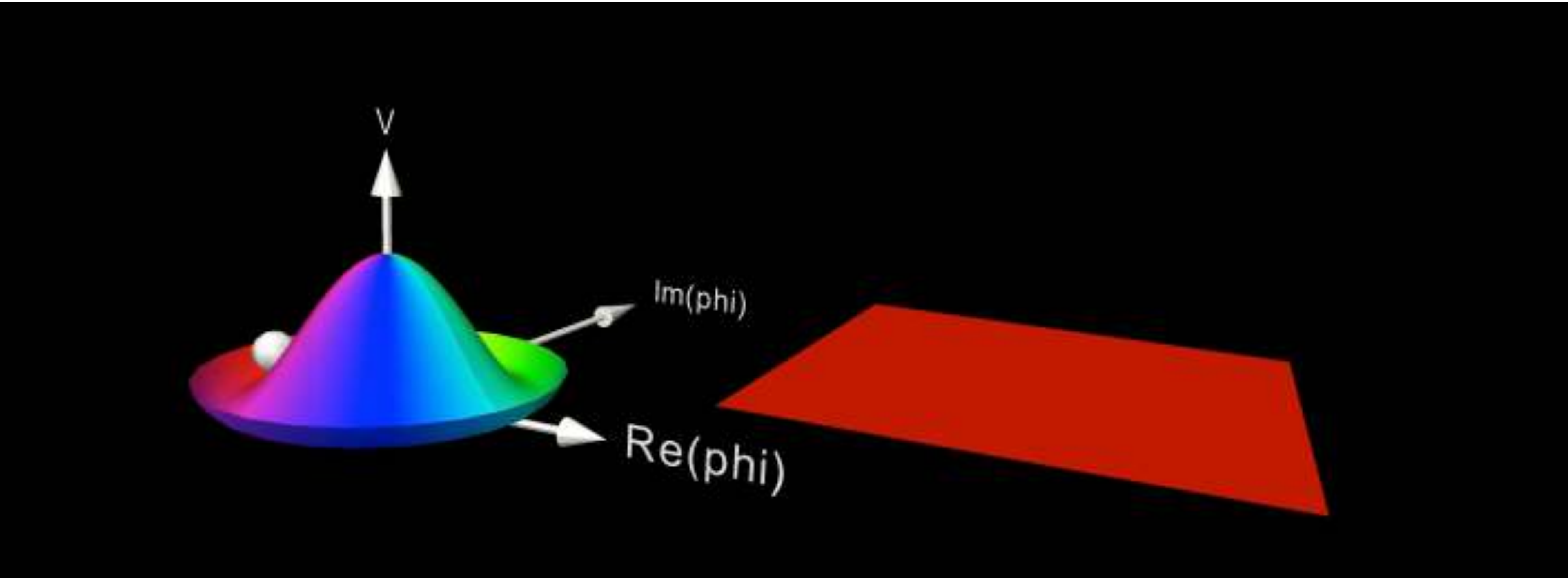
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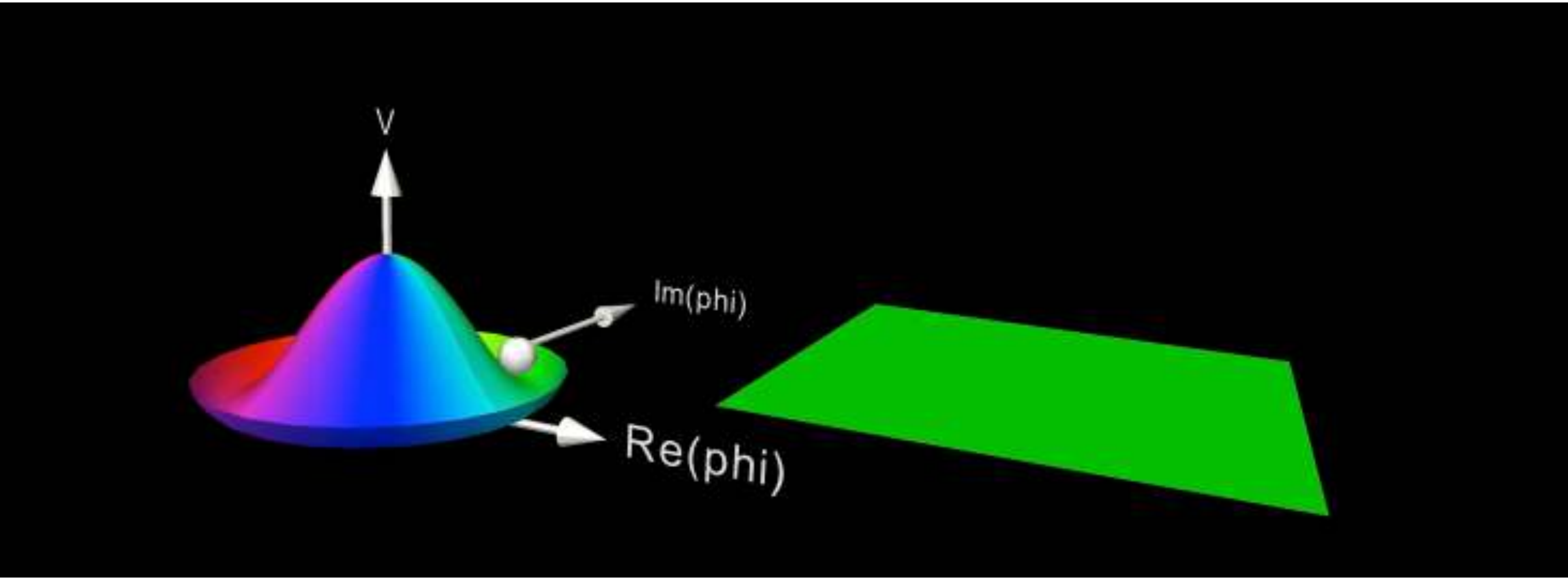
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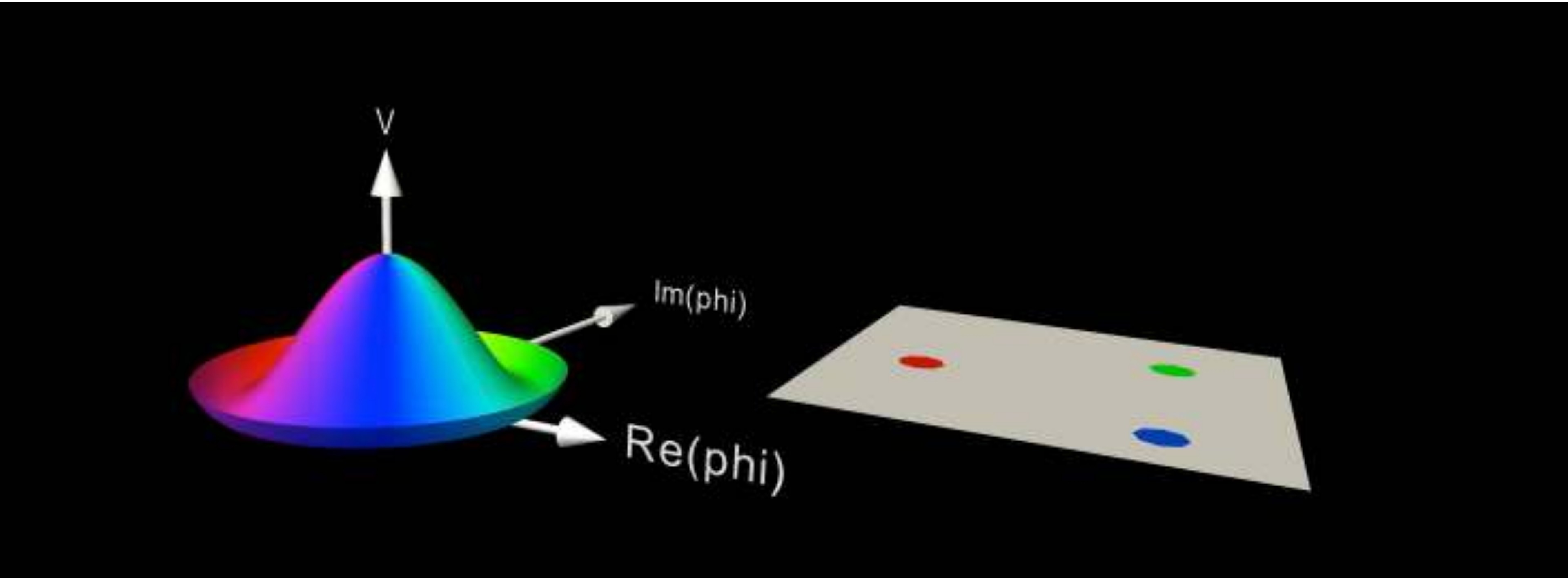
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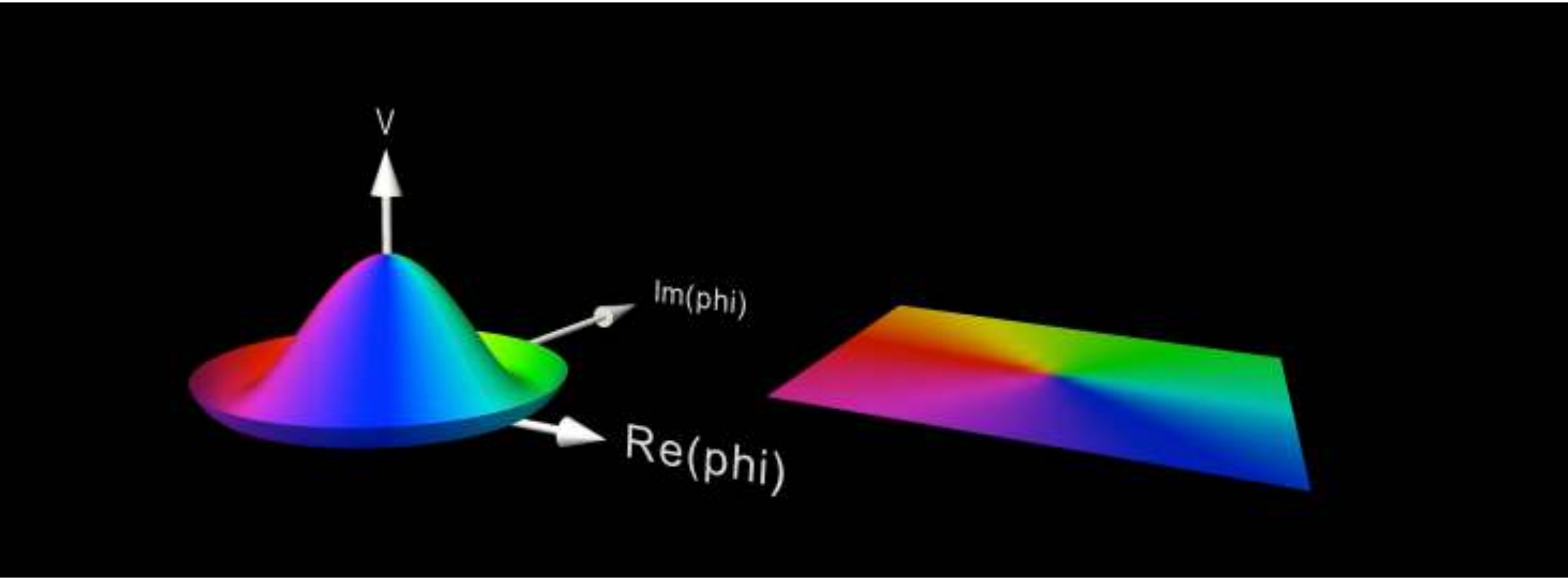

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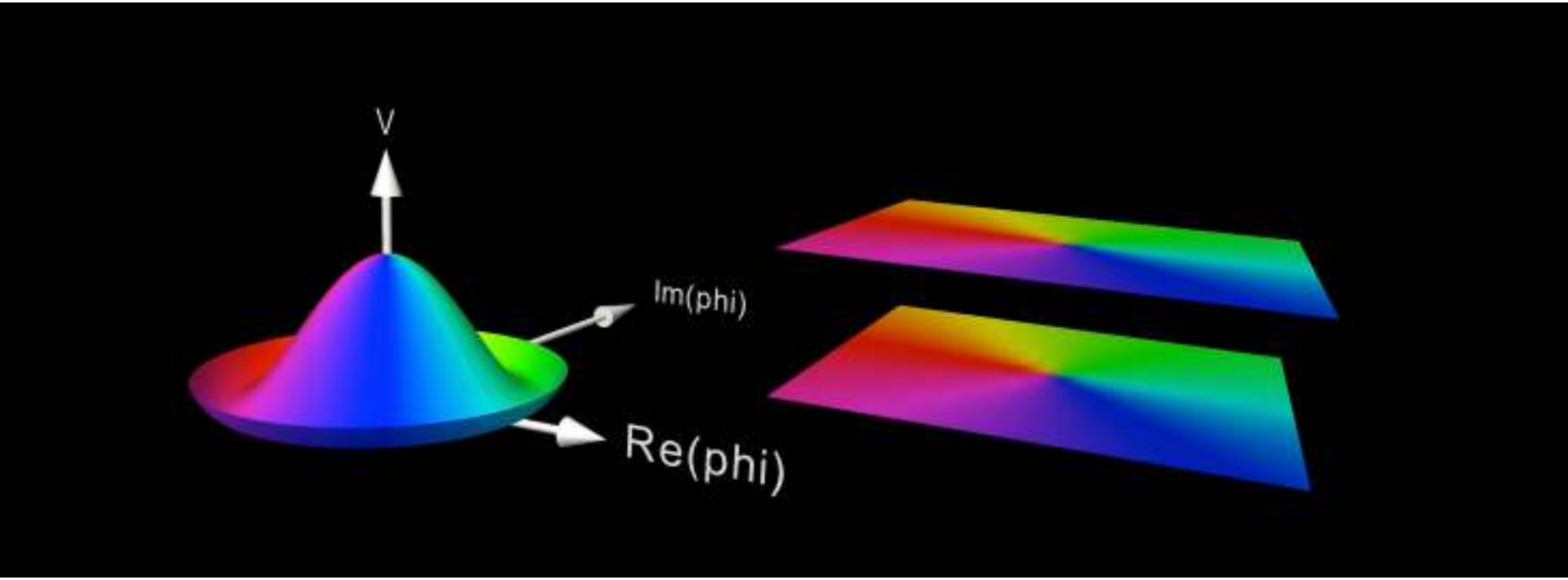
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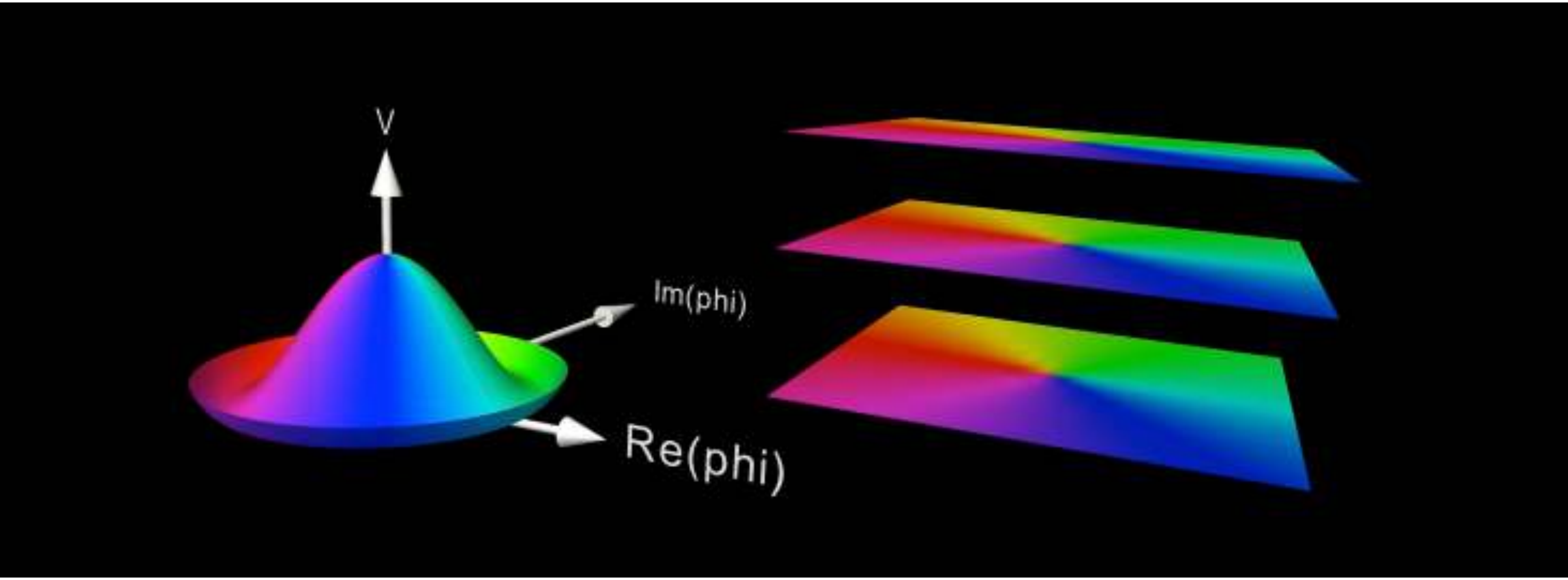
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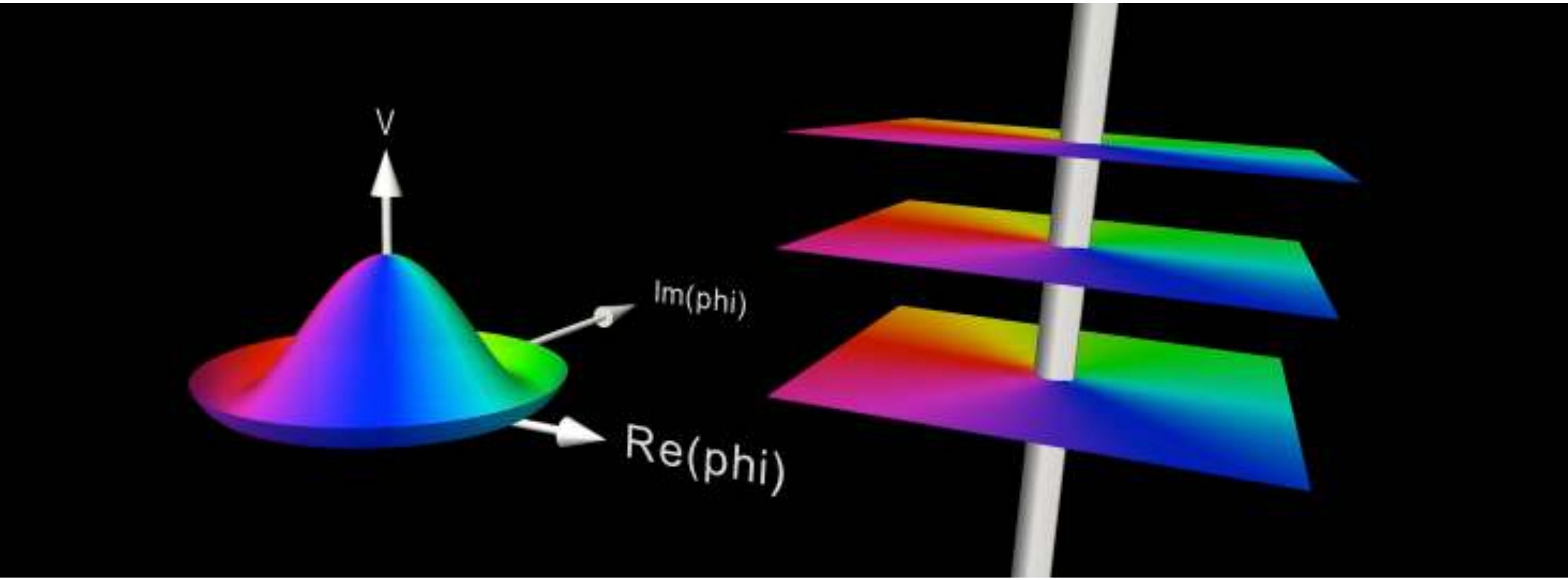

V
Im(phi)
Re(phi)

What is a cosmic string?

- Physical properties of the strings:
  - They are topological stable objects, they have no ends.
  - They are Lorentz invariant.

    Tension = Energy density per unit length

  - They are not coupled to any massless mode, except gravity.

(This is the simplest version of strings that we will consider here)

The String Scale

- Thickness, energy density and tension of the string are controlled by the symmetry breaking scale. η
- For a Grand Unified Theory scale: $\eta \approx 10^{16}\mathrm{GeV}$
- Thickness: $\delta = 10^{-30}\mathrm{cm}$
- Linear mass density: $\mu = 10^{22}\mathrm{gr/cm}$
- Tension : $T = 10^{37}N$
- Gravitational effects depend on:

$$\boxed{G\mu = \left(\frac{\eta}{M_{Pl}}\right)^2 \sim 10^{-6}}$$

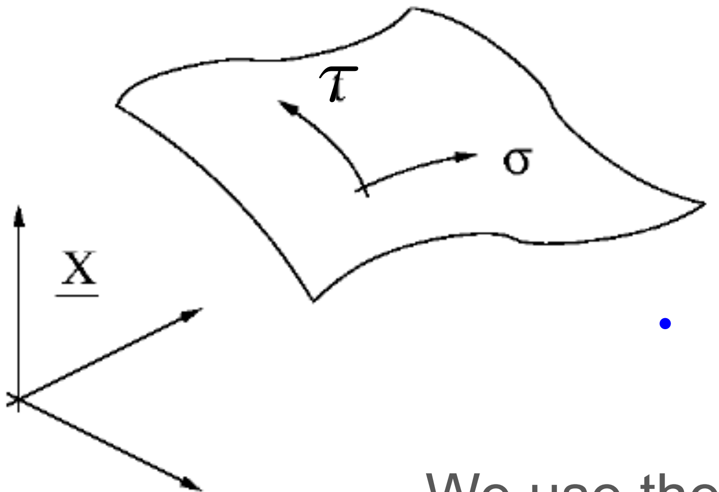
Cosmological Formation

(Kibble '76).

- Strings get formed at a cosmological phase transition.
- In order for strings to be cosmologically relevant this should happen at the end of inflation.
- They could be formed in models of hybrid inflation or during reheating.
- In String Theory they are produced at the end of brane inflation.

Cosmic String Dynamics

(Nambu, '71; Goto '70).



- A relativistic string dynamics has an action of the form,

$$S_{NG} = -\mu \int \sqrt{-\gamma}\, d^2\xi$$

- The string is described by: $\mathbf{X}(\sigma, \tau)$

- We use the gauge conditions: $\mathbf{X}'^{\mathbf{2}} + \dot{\mathbf{X}}^{\mathbf{2}} = \mathbf{1}$

$$\mathbf{X}' \cdot \dot{\mathbf{X}} = \mathbf{0}$$

- The e.o.m. become:

$$\mathbf{X}'' = \ddot{\mathbf{X}}$$

$$\boxed{\begin{gathered}\mathbf{X}(\sigma, \tau) = \frac{1}{2}\left[\mathbf{a}(\sigma - \tau) + \mathbf{b}(\sigma + \tau)\right] \\ |\mathbf{a}'| = |\mathbf{b}'| = \mathbf{1}\end{gathered}}$$

What about interactions?

- Strings interact by exchanging partners, creating loops.

- This mechanism produces kinks on strings.

- This builds up small scale structure on strings.

Cosmic String Dynamics (Loops)

$$\mathbf{X}(\sigma,\tau) = \frac{1}{2}\left[\mathbf{a}(\sigma-\tau) + \mathbf{b}(\sigma+\tau)\right]$$

- The solutions for closed loops are periodic.
- The loops oscillate under their tension.
- The strings move typically relativistically.
- During its evolution a loop may have points where the string reaches the speed of light: A cusp

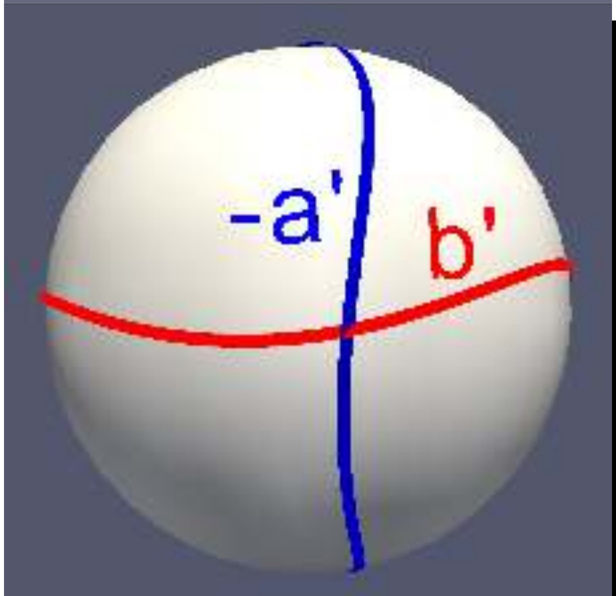
$$|\dot{\mathbf{X}}| = \frac{\mathbf{1}}{\mathbf{2}}|\mathbf{b}' - \mathbf{a}'| \qquad\qquad \mathbf{b}' = -\mathbf{a}'$$

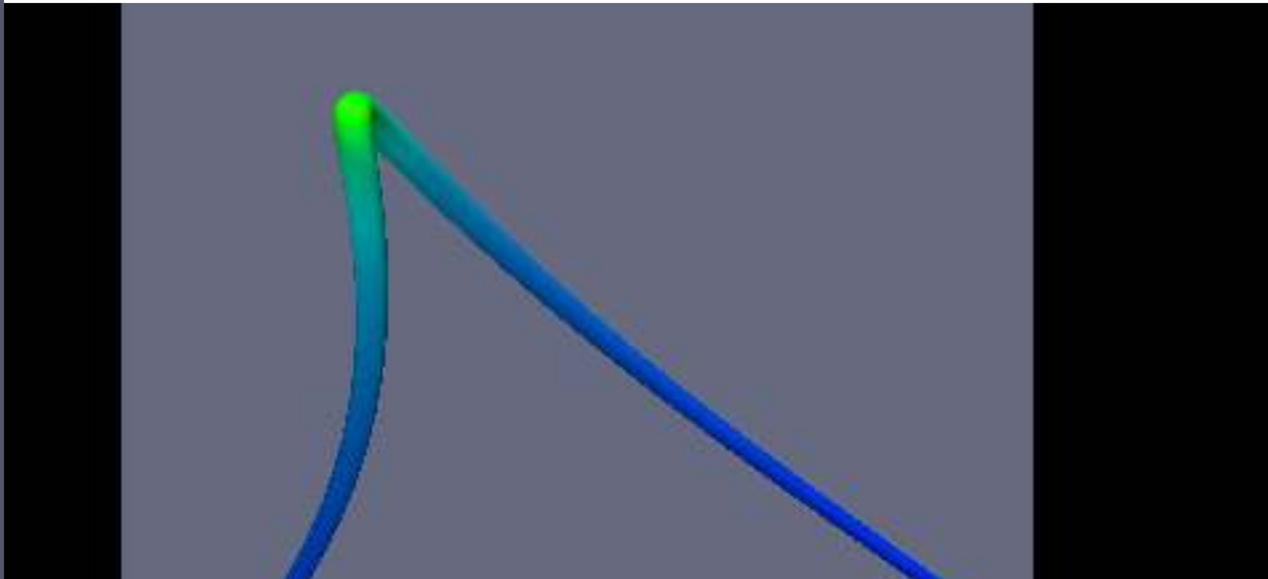
Cosmic String Cusps

(Turok '84).

- Loops will typically have a cusp in each oscillation.
- The string doubles back on itself.

$$\mathbf{X}'^{2} + \dot{\mathbf{X}}^{2} = 1$$





The importance of Loops

- Without any mechanism for energy loss strings would dominate the energy density of the universe.
- Loops oscillate under their tension and lose energy by gravitational radiation.
- This mechanism allows strings to be subdominant part of the energy budget.
- No “monopole problem” for strings.

Gravitational Radiation by Loops

- The power of gravitational waves will affect the size of the loops:

$$\dot{M} \sim G(\dddot{Q})^2 \sim GM^2L^4w^6 \sim \Gamma G\mu^2$$

- The total power has been calculated with several sets of loops:

$$P \sim \Gamma G\mu^2 \qquad \Gamma \sim 50 - 100$$

- Loops will therefore shrink in size so the rest mass of the loop will be:

$$m(t) \sim m(t') - \Gamma G\mu^2(t - t')$$

$$L(t) \sim L(t') - \Gamma G\mu(t - t')$$

Gravitational Waves from the Network

- There are 2 different contributions to gravitational waves from a network of strings:
  - Stochastic background generated by all the modes in the loop.

    (Vilenkin '81, Hogan and Rees '84, Caldwell et al. '92, Siemens et al.; Battye et al., Sanidas et al., Binetruy et al.; Ringeval and Suyama; Kuroyanagi et al…).

  - Burst signals from individual cusps.

    (Damour & Vilenkin '01).

Stochastic background of Gravitational Waves

- The whole network of strings contributes to the stochastic background of GW.

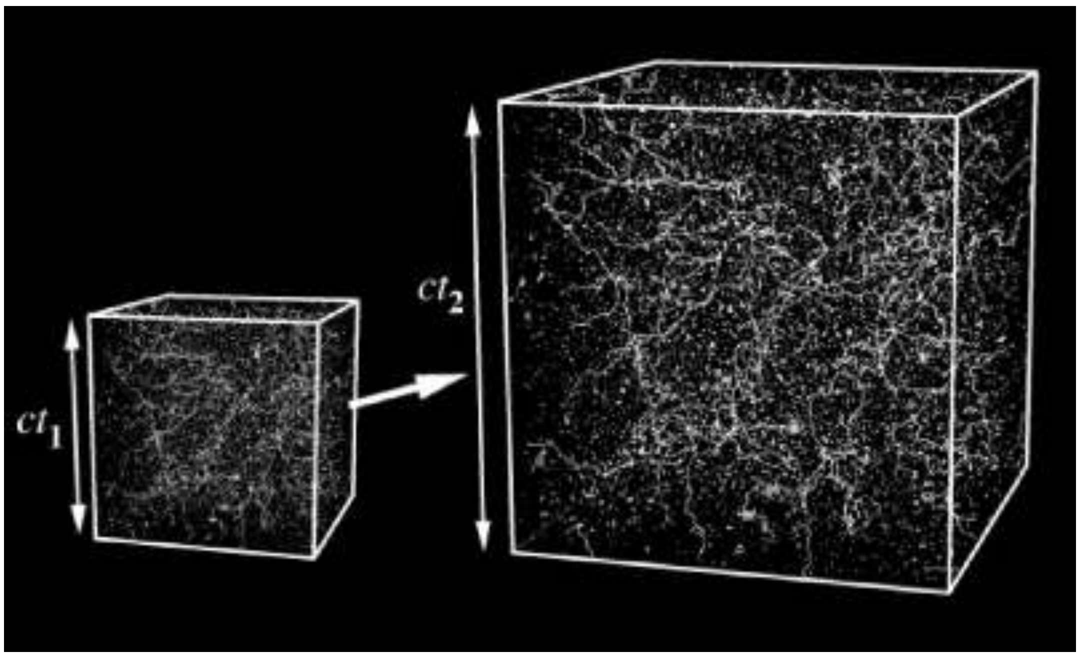
$$\Omega_{gw}(\ln f) = \frac{8\pi G}{3H_o^2} f \int_0^{t_0} dt \left(\frac{a(t)}{a(t_0)}\right)^3 \int_0^{m_{max}} dm \, \boxed{n(t,m)} \boxed{\left(\frac{dP}{df}\right)}$$

$n(t,m)$ ⟸ It depends directly on the number of loops.

$\left(\frac{dP}{df}\right)$ ⟸ It also depends on the spectrum of gw emission by the surviving loops.

Cosmic String Networks

- As the string network evolves it reaches a scaling solution where the energy density of strings is a constant fraction of the energy density in the universe.

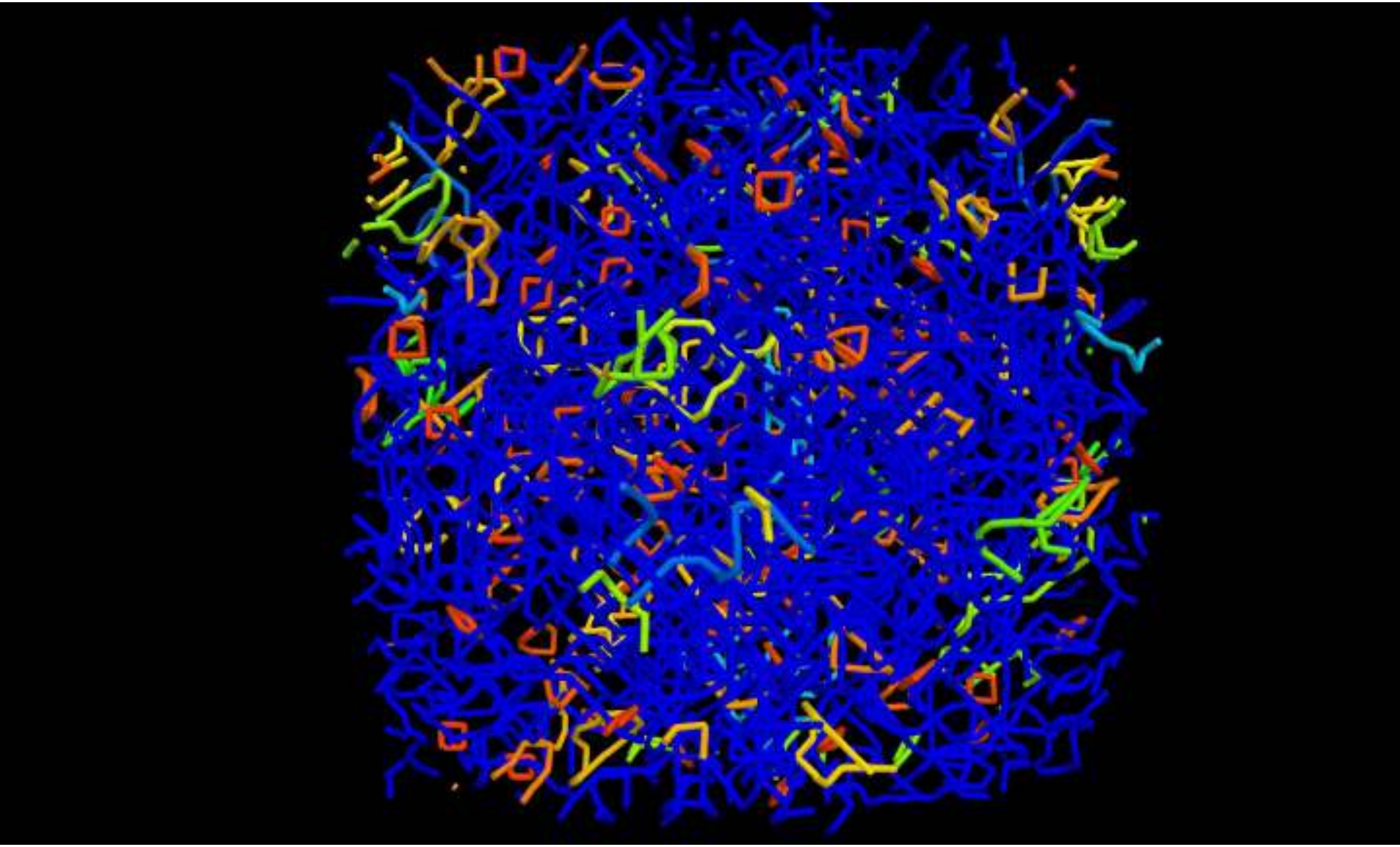


$$\frac{\rho_\infty}{\rho} = \text{constant}$$

- All statistical properties scale with the horizon distance.

Nambu-Goto Cosmic String Networks

(B-P., Olum and Shlaer '12).



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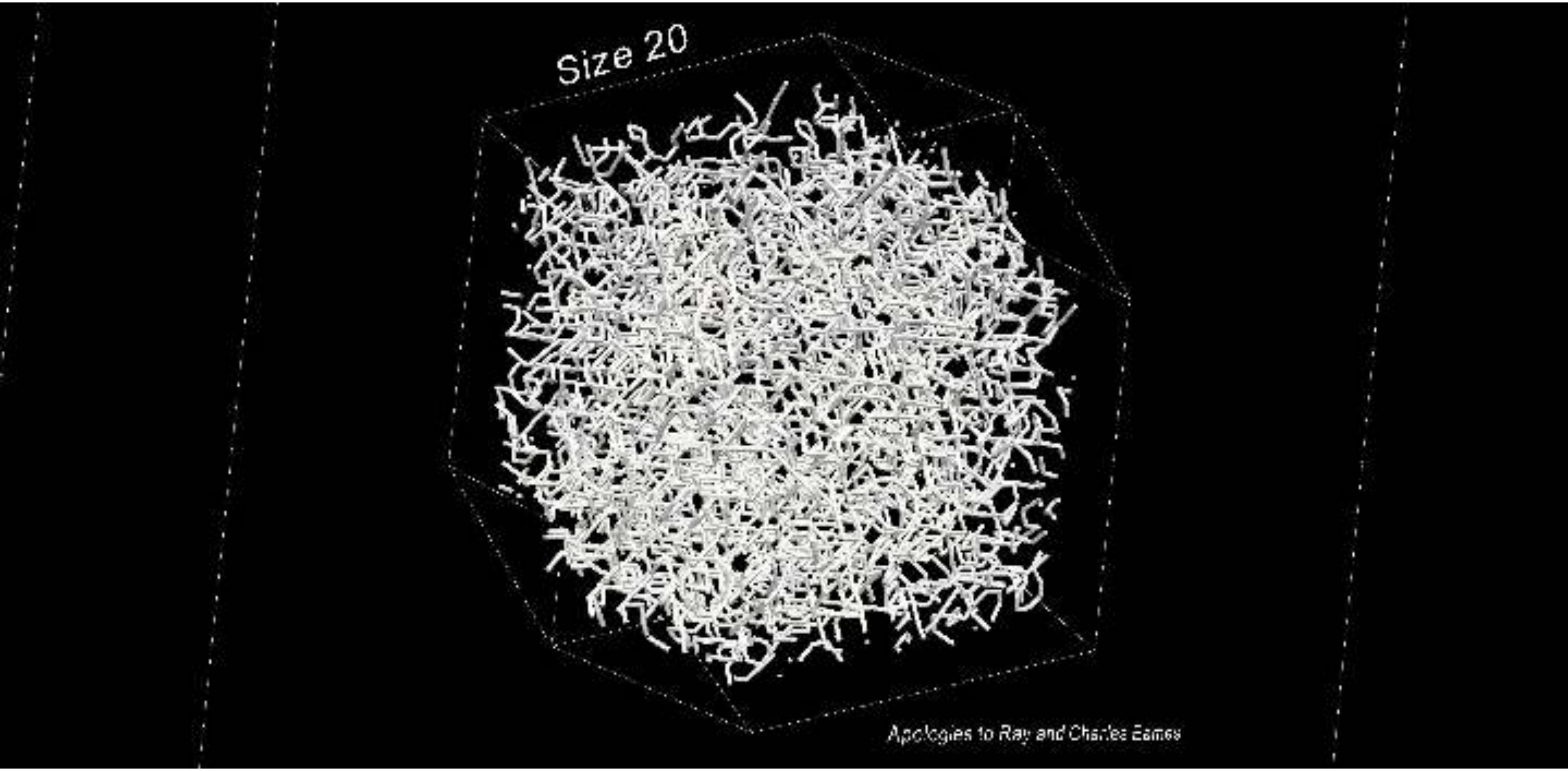

We need to simulate a very large box to find the scaling loops !

The number of cosmic string loops

(B-P., Olum and Shlaer '13).

- We have been able to obtain from the simulations the scaling distribution of loops.

- This allows us to calculate the loop distribution of sizes at any moment in the history of the universe:

$$\boxed{\frac{n_r(t,l)}{a^3(t)} \approx \frac{0.18}{t^{3/2}(l+\Gamma\mu t)^{5/2}}}$$

Stochastic background of Gravitational Waves

- The whole network of strings contributes to the stochastic background of GW.

$$\Omega_{gw}(\ln f) = \frac{8\pi G}{3H_o^2} f \int_0^{t_0} dt \left(\frac{a(t)}{a(t_0)}\right)^3 \int_0^{m_{max}} dm \, \boxed{n(t,m)} \left(\frac{dP}{df}\right)$$

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Gravitational Radiation by Loops

- Loops are periodic sources so they emit at specific frequencies

$$f_n = \frac{2n}{L}$$

- We have to determine the amount of radiation at each frequency and it total power

$$\Gamma = \sum_{n=1}^{\infty} P_n$$

$$L(t) \sim L(t') - \Gamma G\mu(t - t')$$

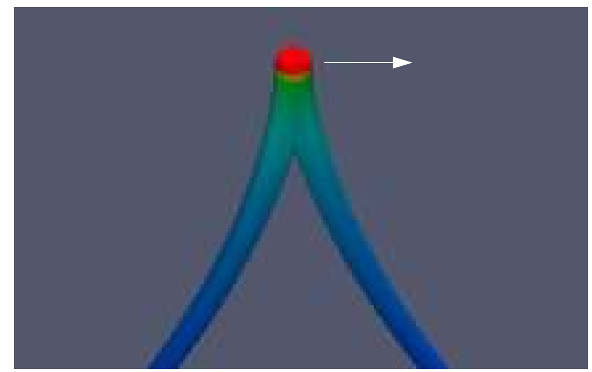
Gravitational Radiation by Loops

- The spectrum of radiation depends on the shape of the string.

- Different structures on the strings have different spectrum:

(Vachaspati and Vilenkin '85).

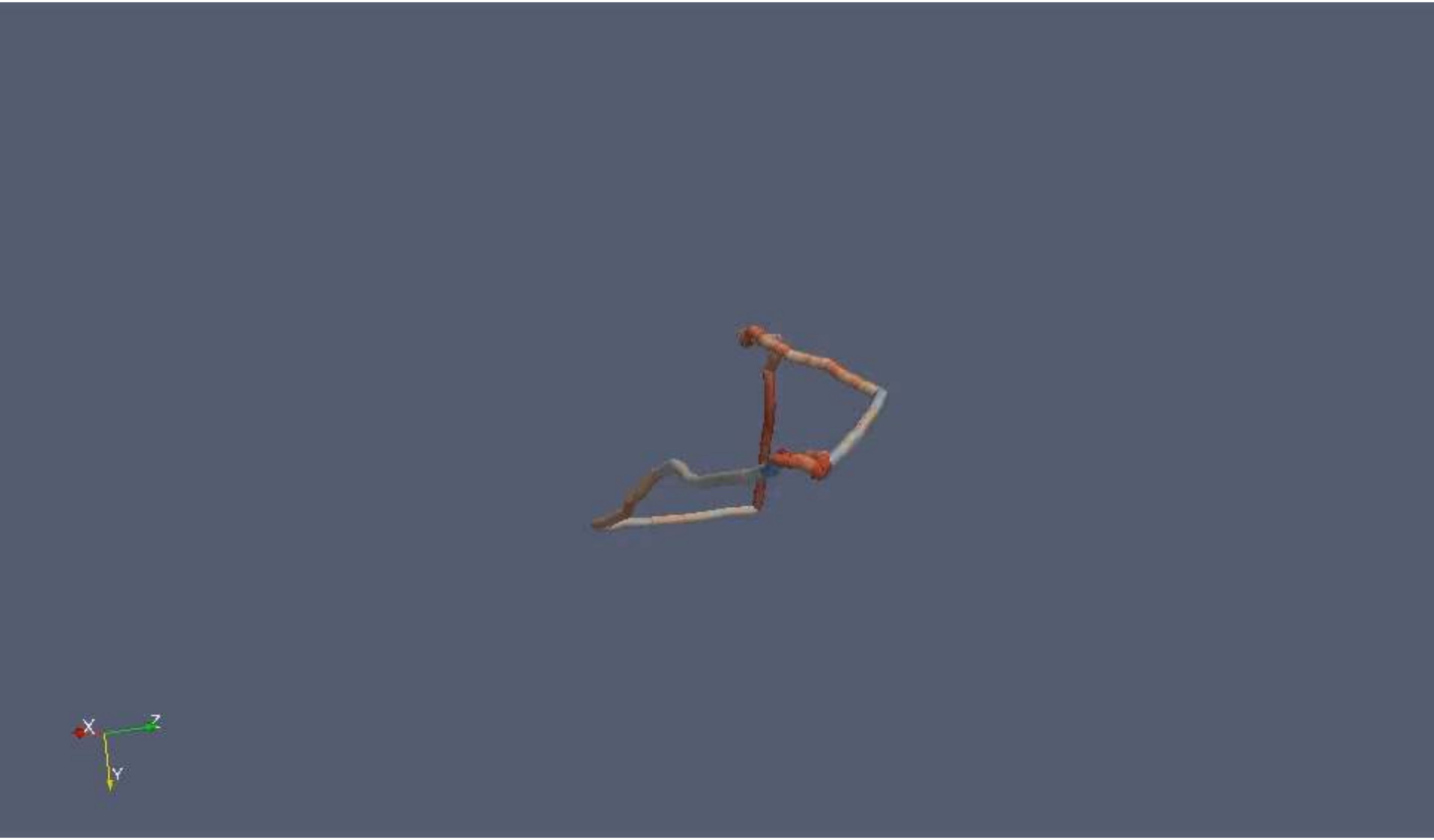
$$P_n^{cusps} \sim G\mu^2 n^{-4/3}$$



$$P_n^{kinks} \sim G\mu^2 n^{-5/3}$$

Loops from the Simulation

(B-P., Olum and Shlaer '12).



Smoothing the loops
(Toy model for backreaction)

(B-P., Olum '15).

- How does the string shape changes with backreaction?

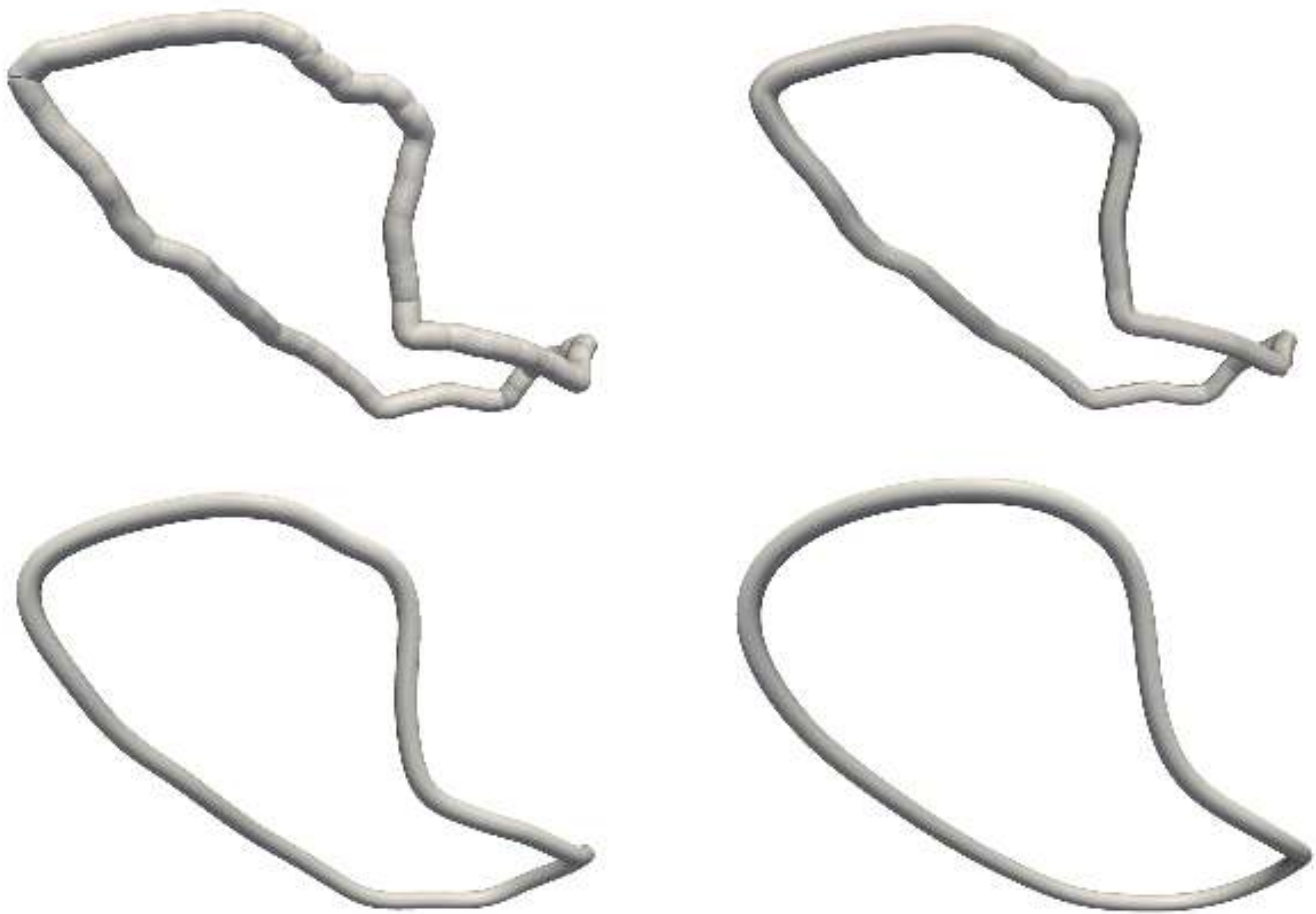
  - We simulate backreaction by smoothing the string at different scales over their lifetime.
  - We evolve the new loop until it falls into a non-self-intersecting trajectory.
  - We repeat these for many loops to get statistical results.
  - We will use these results to compute the typical gw-spectrum.

Smoothing the loops

(B-P., Olum '15).

Smoothing the loops

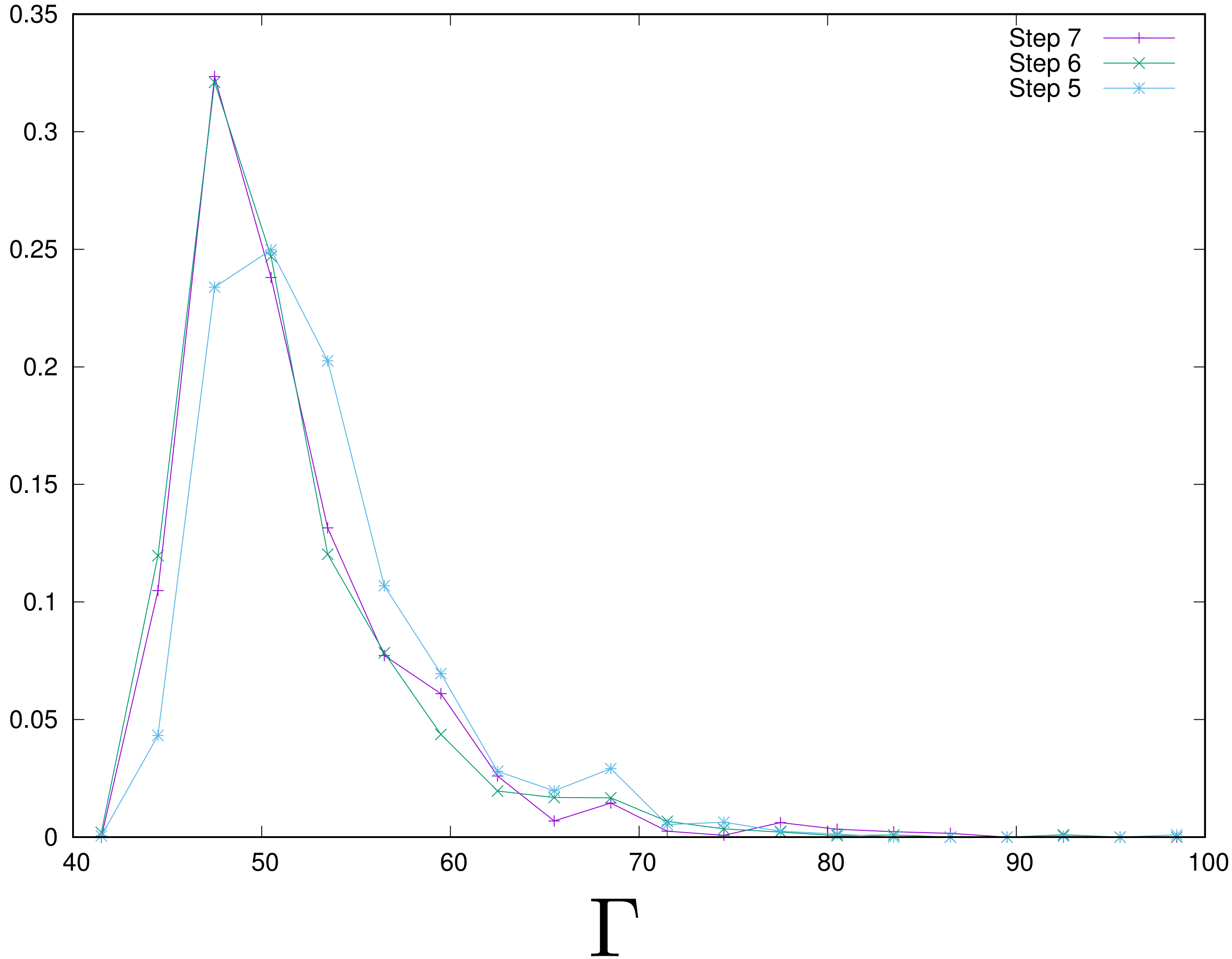
(B-P., Olum '15).



Gravitational Radiation by Loops

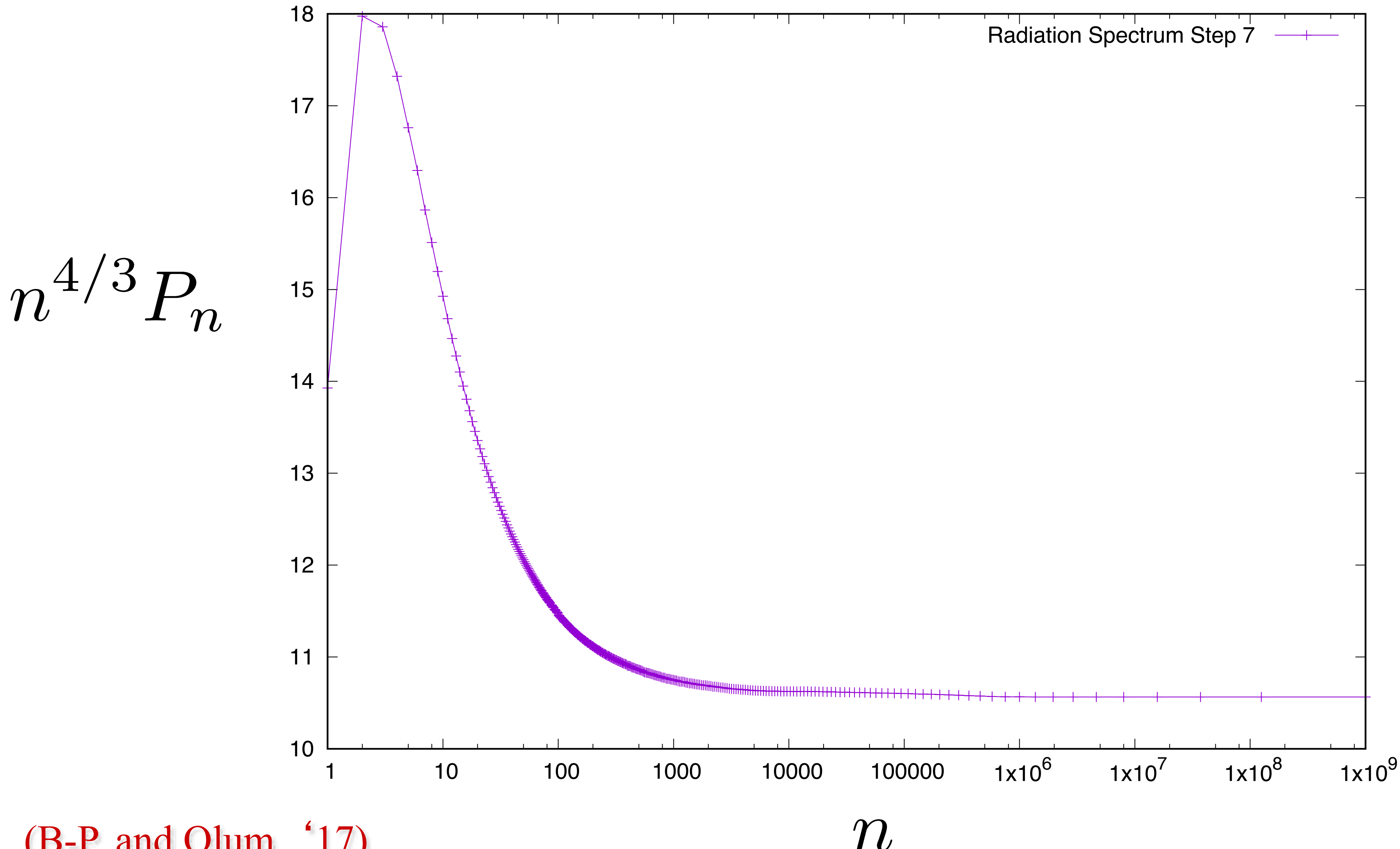
(B-P. and Olum ‘17).

$$L(t) \sim L(t') - \Gamma G\mu(t - t')$$



Gravitational Radiation by Loops

- Averaging over more than 1000 loops we get a spectrum of the form.



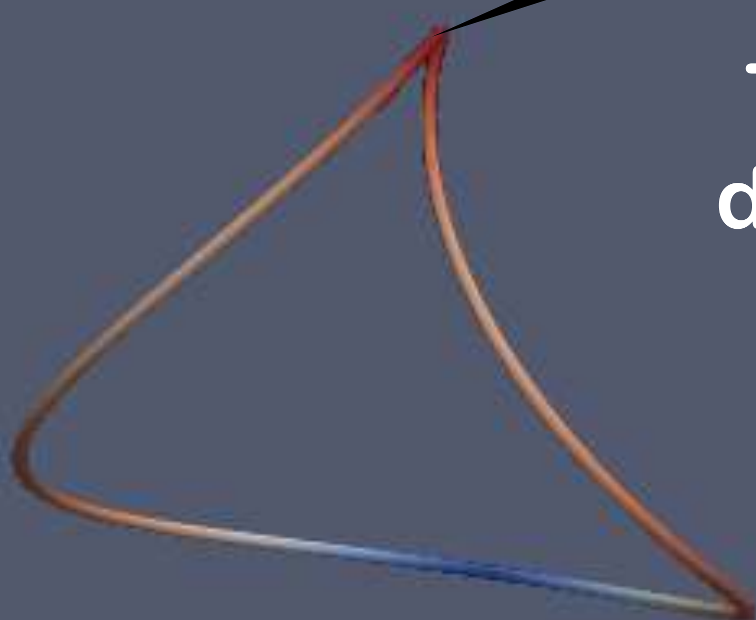
(B-P. and Olum '17).

A typical smoothed Loop

Gravity Waves from cusps

Vachaspati and Vilenkin '85.

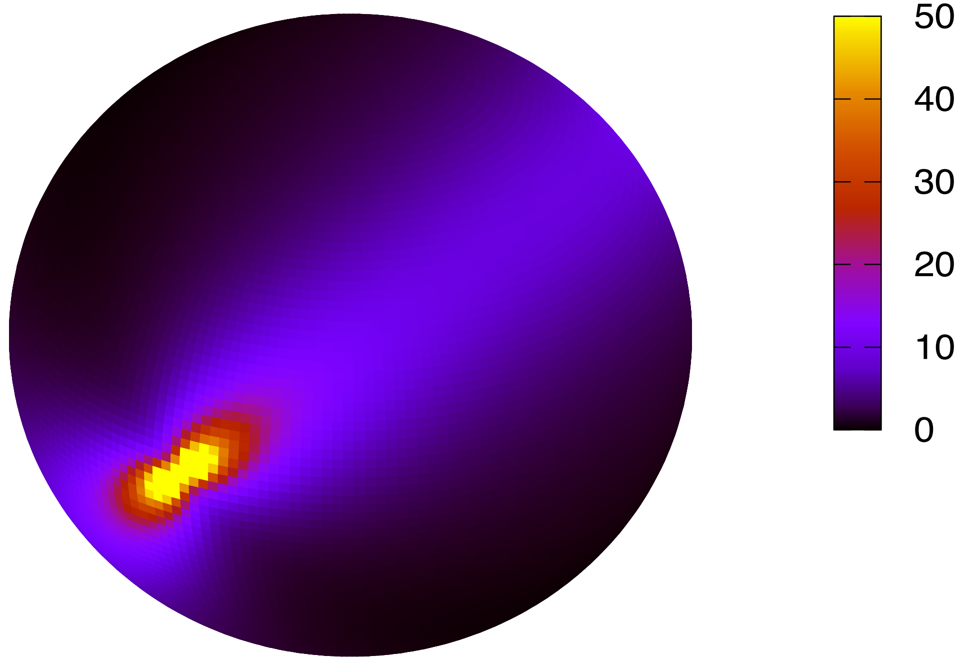
$$P_n^{cusps} \sim G\mu^2 n^{-4/3}$$



This leads to a slow decline of the power.
High frequencies

Gravitational Radiation by Loops

(B-P. and Olum '17).



- Angular distribution is typically highly anisotropic
- This makes the computation of the spectrum a hard problem.

Stochastic background of Gravitational Waves

- The whole network of strings contributes to the stochastic background of GW.

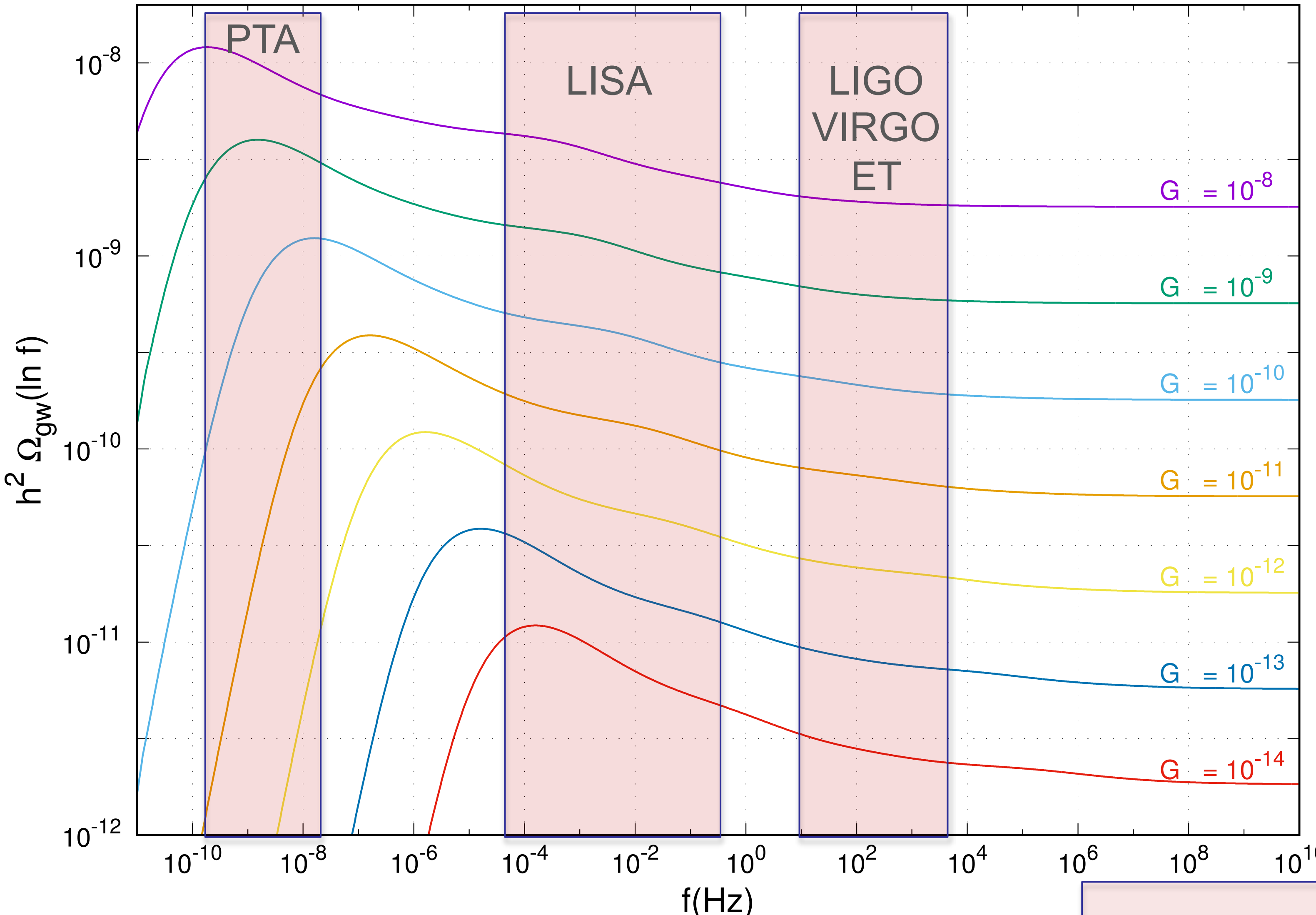
$$\boxed{\Omega_{gw}(\ln f) = \frac{8\pi G}{3H_o^2} f \int_0^{t_0} dt \left(\frac{a(t)}{a(t_0)}\right)^3 \int_0^{m_{max}} dm \; n(t,m) \left(\frac{dP}{df}\right)}$$

✔ $n(t,m)$ ⟸ It depends directly on the number of loops.

✔ $\left(\frac{dP}{df}\right)$ ⟸ It also depends on the spectrum of gw emission by the surviving loops.

Stochastic Background of GWs

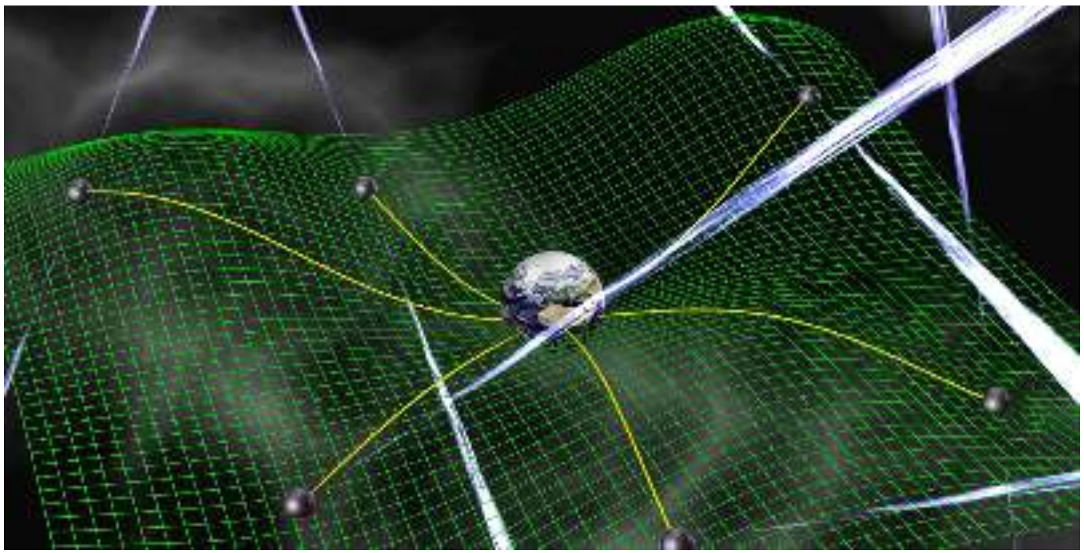
(B-P. and Olum '17).



$$\Omega_{GW}(\infty) \sim \sqrt{\frac{G\mu}{\Gamma}}$$

GW observations from Pulsar Timing Array

There are several PTA observatories that monitor the time of arrival of the pulses that come from many pulsars.



Gravitational waves can create a residual on the time of arrival of these pulses.

These observatories are sensitive to gravitational waves with a frequency of the order:

$$\boxed{f_{\mathrm{PTA}} \approx 10^{-9}\,\mathrm{Hz}}$$

Previous Constraints from PTA limits

(B-P. , Olum and Siemens '17).

- Current limit from Parkes PTA (PPTA) (Australia)

$$G\mu < 1.5 \times 10^{-11}$$

- Similar results from European Pulsar Timing Array (EPTA).

$$G\mu < 1.1 \times 10^{-11}$$

- Previous constraints from NANOGrav collaboration are of the same order

$$G\mu < 4 \times 10^{-11}$$

- All these contraints indicate that these strings would not be seen in the CMB.

NANOGrav 12.5 year results

(Arzoumanian et al. ‘20).

- NANOGrav collaboration has monitored 45 milisecond pulsars for a number of years to search for timing residual signals.

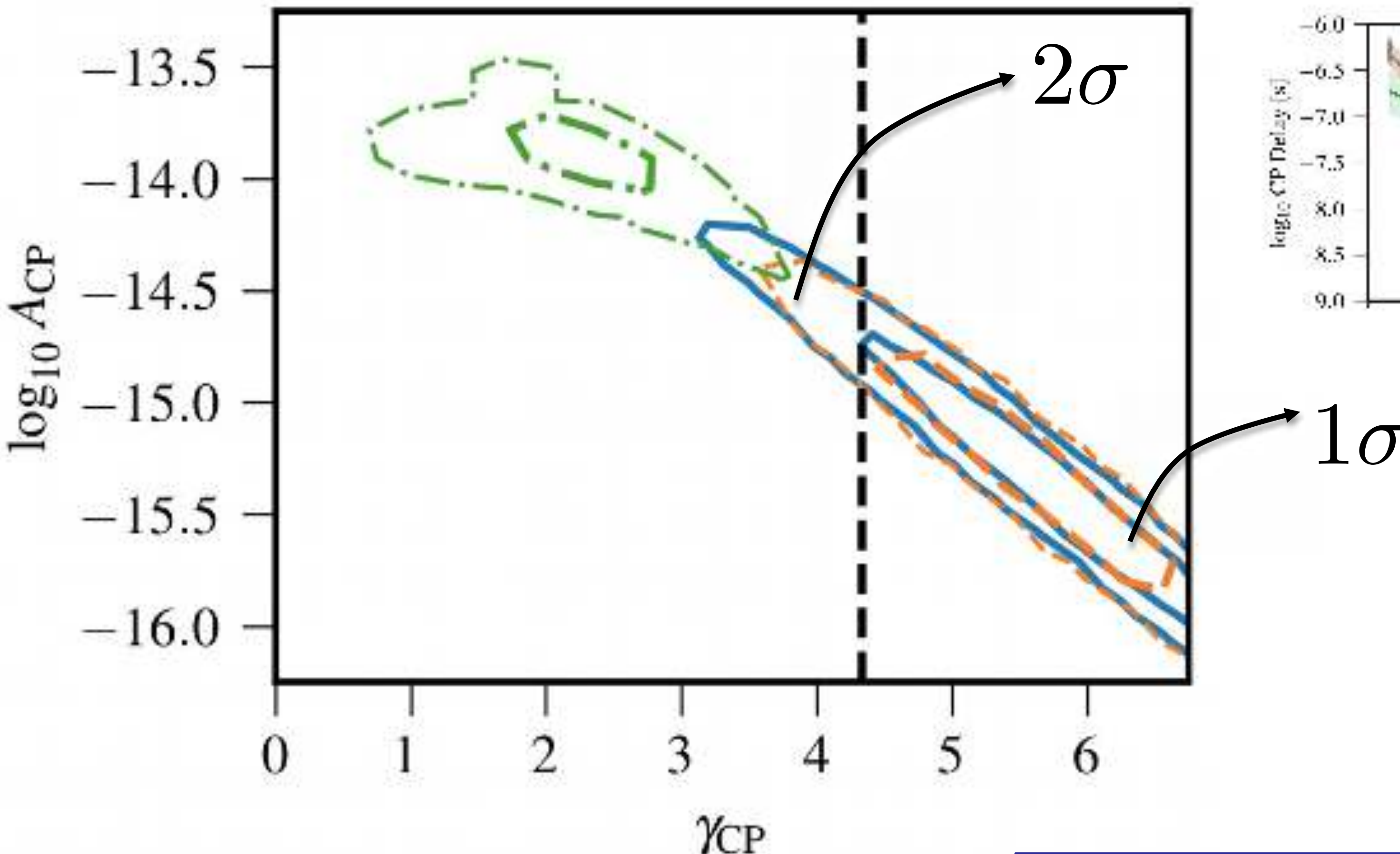
- They report the observation of a common spectrum red noise from these observations.

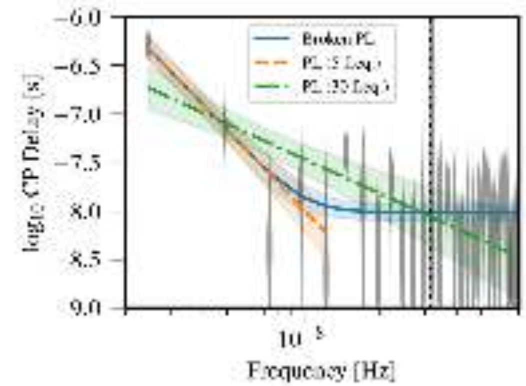
- There is however only a weak evidence of a quadrupole (Hellings-Downs) spatial correlation between the different pulsars.

We will take this results as possibly giving us the first hint of an stochastic gravitational wave background at this frequency.

NANOGrav 12.5 year data

(Arzoumanian et al. ‘20).





- NANOGrav presents its results in terms of the characteristic strain of the form:

$$h_c(f) = A\left(\frac{f}{f_{\rm yr}}\right)^{(3-\gamma)/2}$$

NANOGrav 12.5 year data

(Arzoumanian et al. ‘20).

$$h_c(f) = A\left(\frac{f}{f_{\rm yr}}\right)^{(3-\gamma)/2} \Rightarrow \Omega_{\rm GW} = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)$$

$$\Downarrow$$

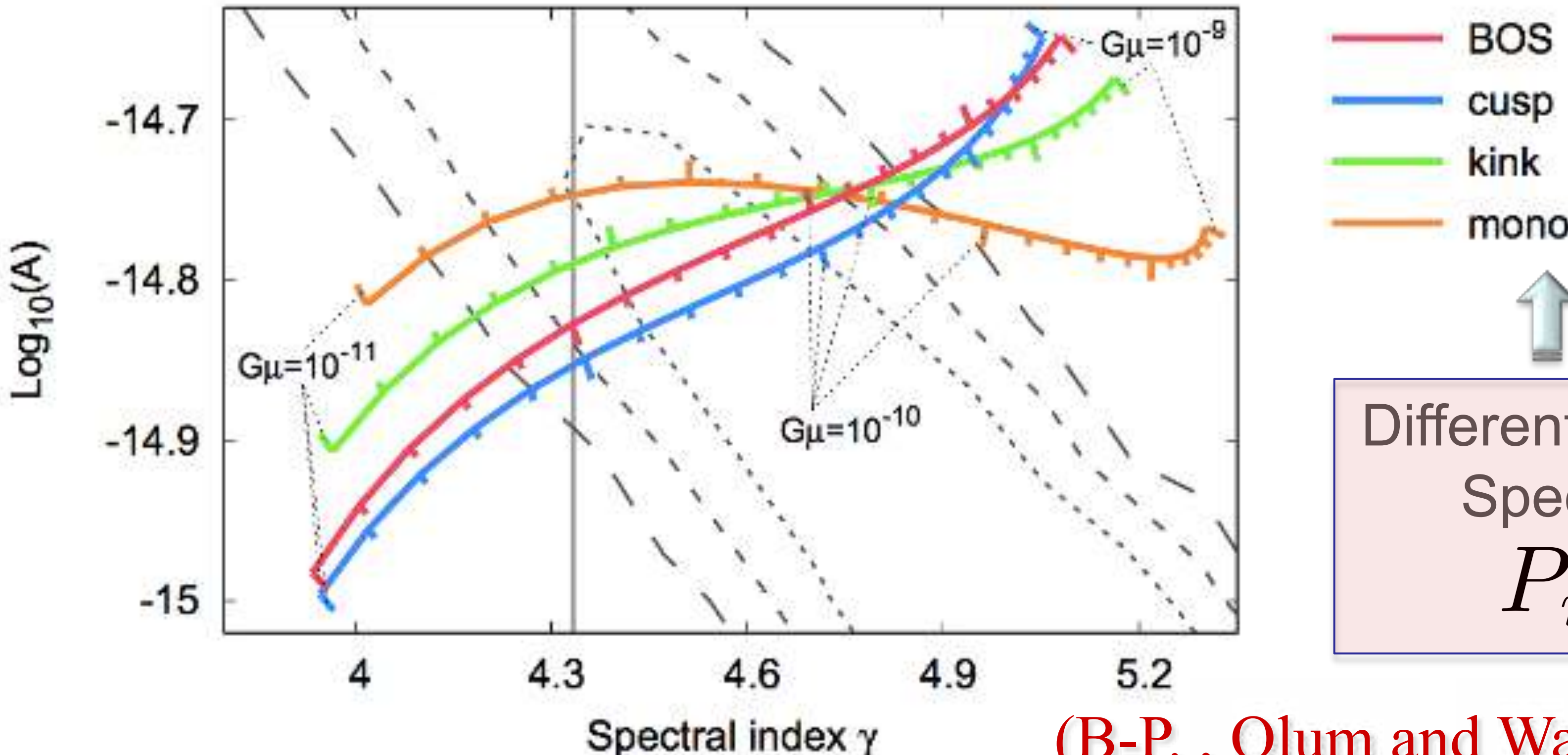
$$\boxed{\Omega_{\rm GW} = \frac{2\pi^2}{3H_0^2} A^2 f_{yr}^2 \left(\frac{f}{f_{yr}}\right)^{5-\gamma}}$$

- Given the prediction for the energy density in gravitational waves at these frequencies one can obtain the corresponding results for the amplitud and the tilt.

- What about cosmic strings?

(Ellis and Lewicki ‘20).
(Blasi, Brdar and Schmitz ‘20).
(B-P. , Olum and Wachter ’21).

Implications from NANOGrav 12.5 year data



Different Power Spectra

P_n

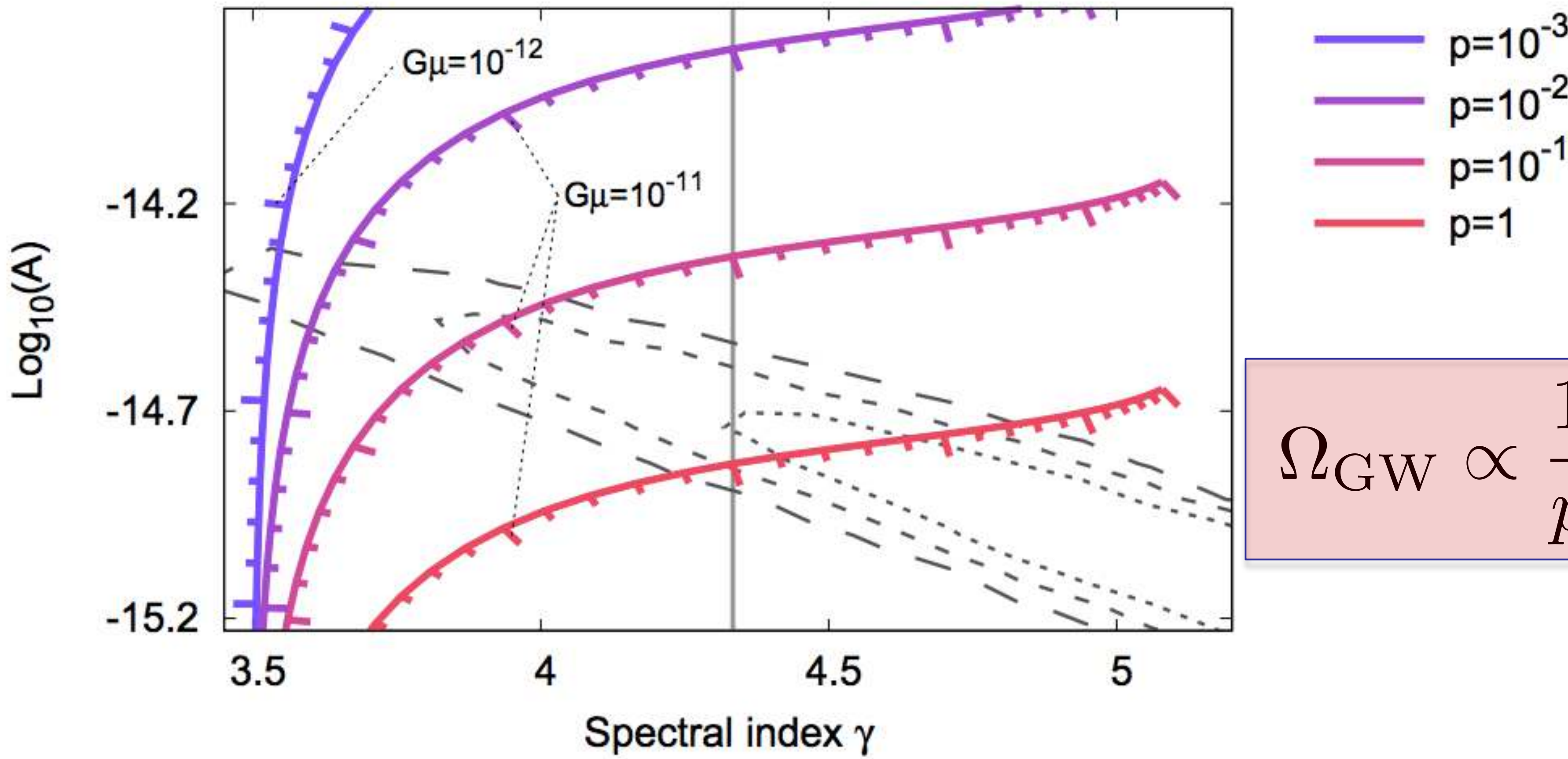
(B-P. , Olum and Wachter '21).

P_n model	1σ range	1.5σ range	2σ range
BOS	$(-10.08, -10.40)$	$(-9.92, -10.52)$	$(-9.77, -10.62)$
cusp	$(-10.02, -10.39)$	$(-9.85, -10.50)$	$(-9.72, -10.60)$
kink	$(-10.24, -10.52)$	$(-10.08, -10.64)$	$(-9.93, -10.74)$
mono	$(-10.45, -10.67)$	$(-10.27, -10.80)$	$(-10.08, -10.90)$

$\log(G\mu)$

Implications from NANOGrav 12.5 year data

- For Superstring networks, the intercommutation probability is lower than one and this boost the signal.

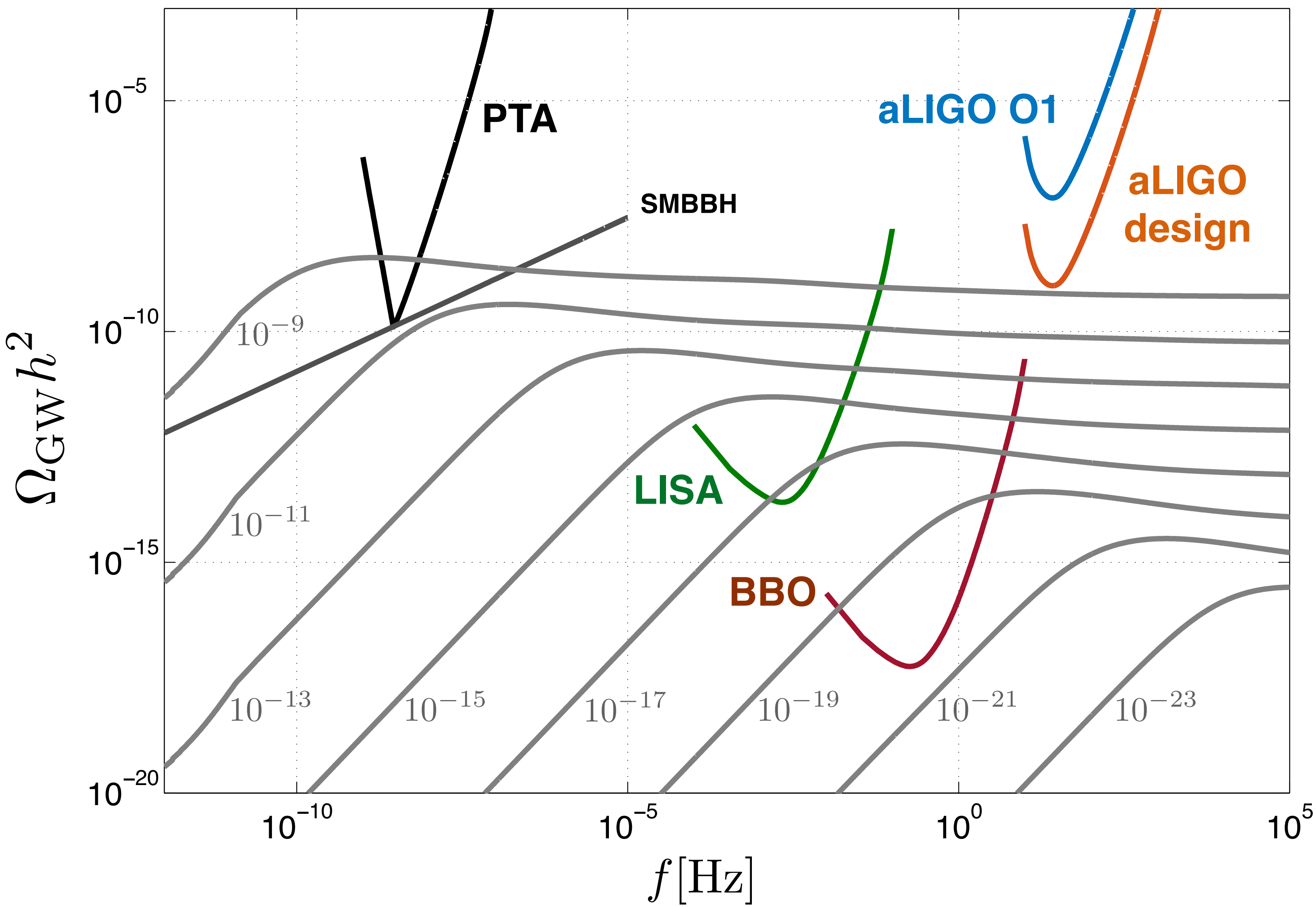


$$\Omega_{\rm GW} \propto \frac{1}{p}$$

(B-P. , Olum and Wachter '21).

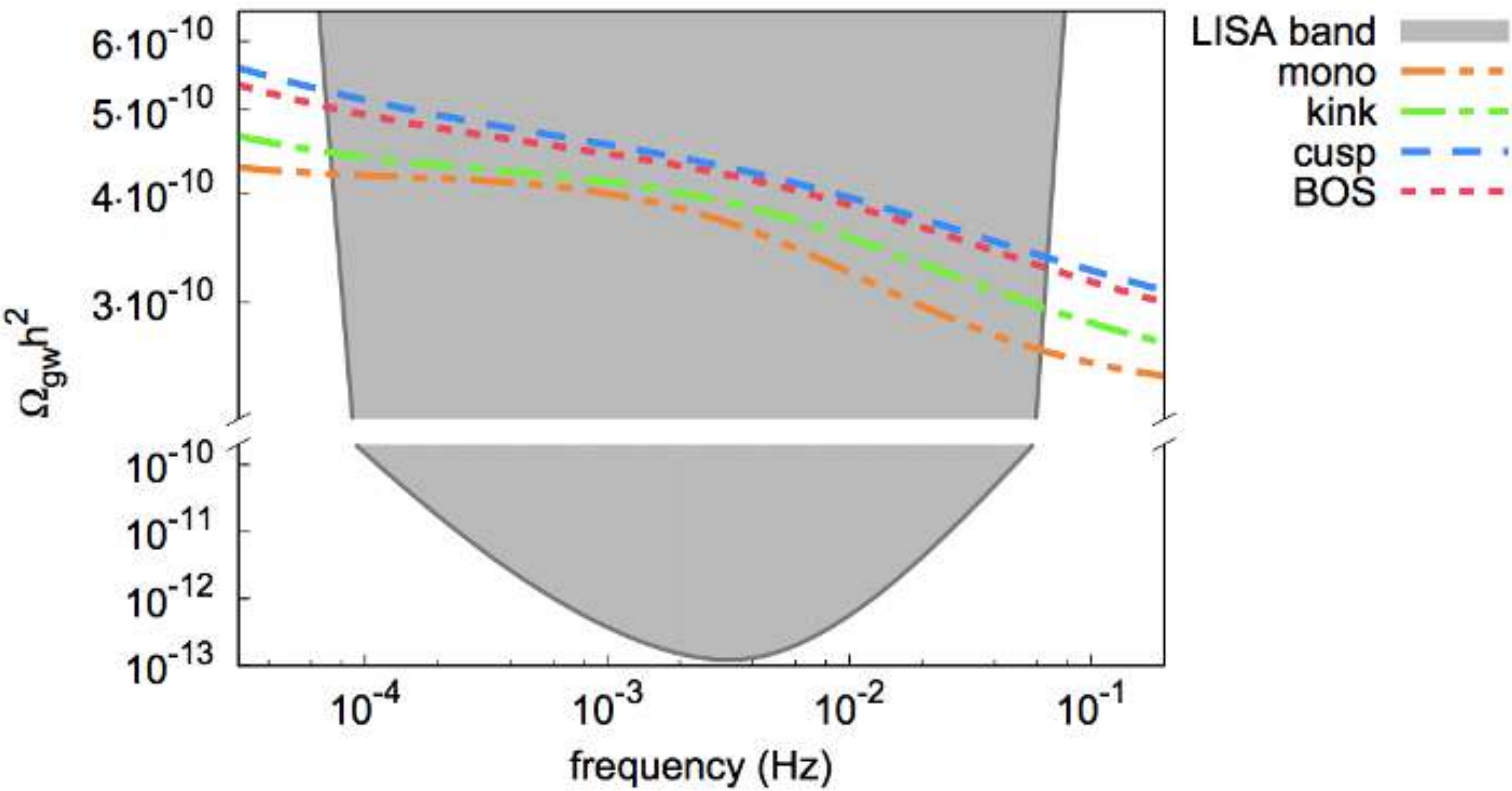
Implications for Future Observations

(B-P. , Olum and Siemens '17).



Implications for Future Observations

(B-P. , Olum and Wachter '21).



Future Developments

- NANOGrav will have more data in the near future.
- Furthermore, other PTA Observations seem to find some evidence of a red noise common spectrum (EPTA and PPTA).
- We will also have a combined analysis by IPTA.
- New observatories like SKA will also increase the sensitivity.
- In the mean time, we should also reduce the theoretical uncertainty in the computation of the SGWB from strings.
- This requires a detailed study of gravitational backreaction.

Real Gravitational Backreaction

(Quashnock and Spergel '90).
(B-P. , Olum and Wachter '18).

- Recall the NG equations:

$$x^\gamma{}_{,\mathrm{uv}} = 0 \qquad\qquad x^\gamma(\mathrm{u},\mathrm{v}) = \frac{1}{2}\left[A^\gamma(\mathrm{v}) + B^\gamma(\mathrm{v})\right]$$

- We want to introduce the gravitational self-interaction at linear order:

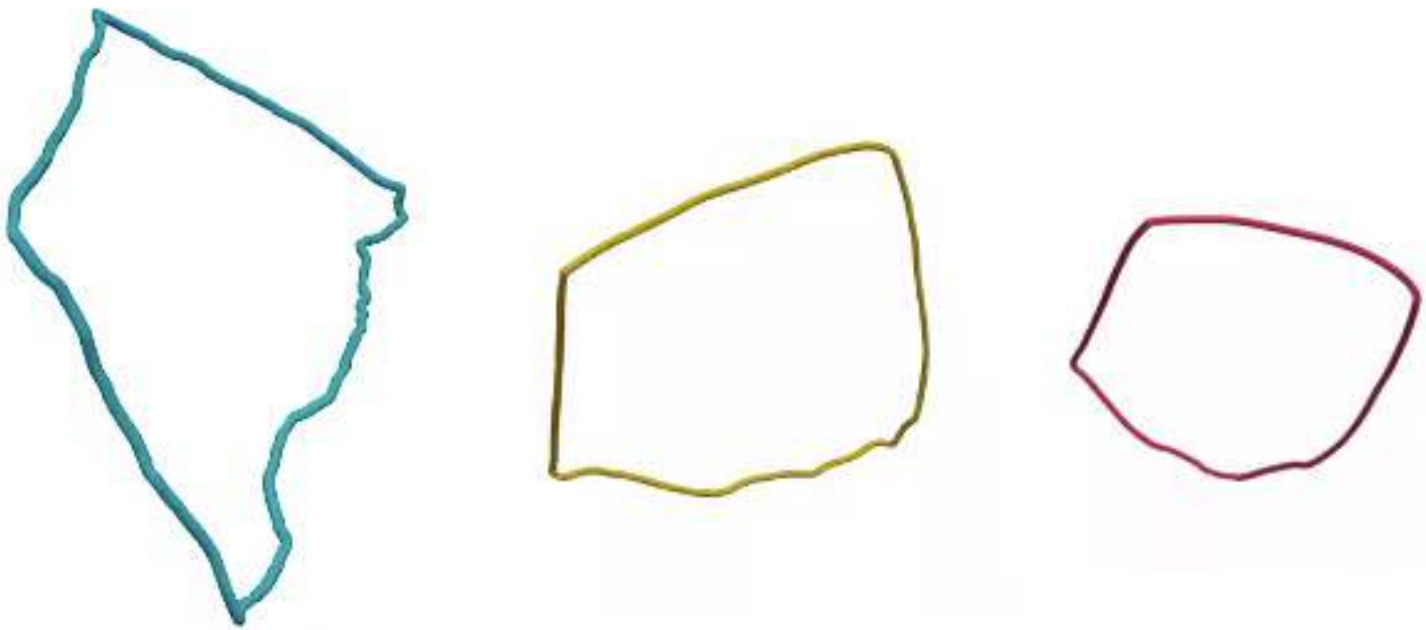
$$x^\gamma{}_{,\mathrm{uv}} = -\frac{1}{4}\Gamma^\gamma_{\alpha\beta}A'^\alpha B'^\beta$$

  - This captures the gravitational effect of the intersection of the worldsheet with the past lightcone of the observation points.

Backreaction on real loops from the simulation

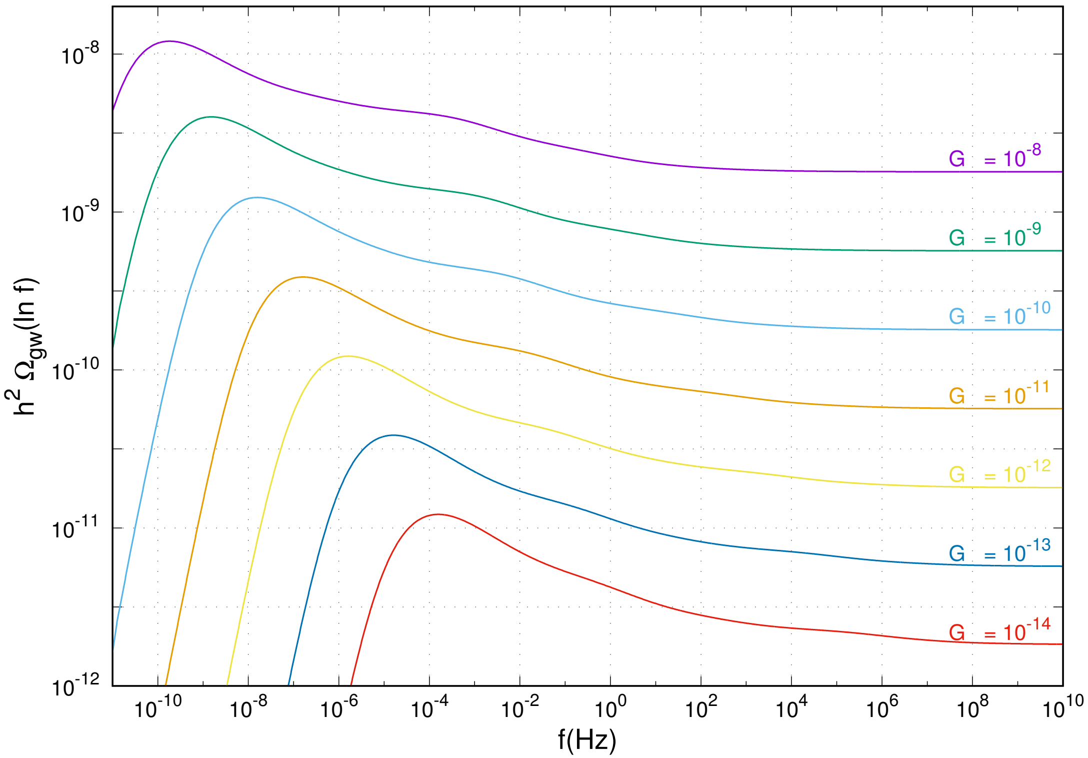
WORK in PROGRESS

Real Loops with Real Backreaction



100% 50% 33%

Effects on Background of GWs



How much would the stochastic background change?

We need to know the effect on:

- Number density of loops. Fragmentation.
- Power spectrum of loops.
- Number of Cusps.

Conclusions

- Cosmic Strings are predicted in many extensions of the SM.
- We are entering an era of precision cosmology in cosmic string simulations.
- All known effects taken into account except real backreaction.
  (Coming soon)
- Cosmic Strings are a good candidate for the NANOGrav 12.5y data.
- Future observatories could confirm this scenario or to the very least constrained these models.
- Observation of Cosmic Strings will have a huge impact on High Energy Physics.
- The lack of observation will constrain models of the Early Universe.

Thank you