Grand Unified Theories: Topological Defects, Inflation, and Gravitational Waves

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@ CQUeST

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Talk based on Publications

- **"Monopoles, Strings and Gravitational Waves in Non-minimal Inflation"**
  R. Maji and Q. Shafi. arXiv:2208.08137 [hep-ph]
- **"Gravitational Waves from Quasi-stable Strings"**
  G. Lazarides, R. Maji and Q. Shafi. JCAP 08 (2022) 042
- **"Cosmic Strings, Inflation, and Gravity Waves"**
  G. Lazarides, R. Maji and Q. Shafi. Phys. Rev. D **104**, no. 9, 095004 (2021)
- **"Primordial Monopoles and Strings, Inflation, and Gravity Waves"**
  J. Chakrabortty, G. Lazarides, R. Maji and Q. Shafi. JHEP 02 (2021) 114
- **"Unification, Proton Decay and Topological Defects in non-SUSY GUTs with Thresholds"**
  J. Chakrabortty, R. Maji and S. F. King. Phys. Rev. D **99**, no. 9, 095008 (2019)

Outline

1. Introduction
2. Predictions of GUTs: Proton Decay & Topological Defects
3. Phase Transitions, Inflation, and Evolution of Monopoles
4. Evolution of Cosmic Strings and Gravitational Waves
5. Quasi-stable Cosmic Strings
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1 *Introduction*

Standard Model ($SU(2)_L \otimes U(1)_Y \otimes SU(3)_C$)

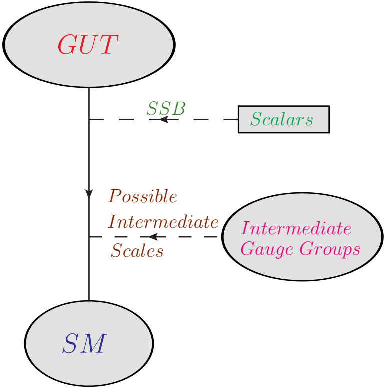
		Fields	Quantum numbers
Spin-$\frac{1}{2}$	Quarks	$Q_g^{i\alpha} = \left\{ \begin{pmatrix} u^\alpha \\ d^\alpha \end{pmatrix}_L, \begin{pmatrix} c^\alpha \\ s^\alpha \end{pmatrix}_L, \begin{pmatrix} t^\alpha \\ b^\alpha \end{pmatrix}_L \right\}$	$(2, \frac{1}{6}, 3)$
		$u^C_{\alpha\,L},\ c^C_{\alpha\,L},\ t^C_{\alpha\,L}$	$(1,-\frac{2}{3}, \overline{3})$
		$d^C_{\alpha L},\ s^C_{\alpha L},\ b^C_{\alpha L}$	$(1, \frac{1}{3}, \overline{3})$
	Leptons	$\ell_g^i = \left\{ \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \right\}$	$(2,-\frac{1}{2}, 1)$
		$e^C_L,\ \mu^C_L,\ \tau^C_L$	$(1, 1, 1)$
Spin-1	$SU(2)_L$	W^a_μ	$(3, 0, 1)$
	$U(1)_Y$	B_μ	$(1, 0, 1)$
	$SU(3)_C$	G^a_μ	$(1, 0, 8)$
Spin-0	Higgs	$\Phi^i = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	$(2, \frac{1}{2}, 1)$

Table: *Fields in the Standard Model.*

Grand Unification beyond the SM

- The basic idea in a Grand Unified Theory (GUT) is that the SM, $SU(2)_L \otimes U(1)_Y \otimes SU(3)_C$, is embedded in a larger simple group, $\mathcal{G}$.

Schematic view



GUT Examples

- $SU(5)$ (rank = 4): $\bar{5} + 10 \Rightarrow$ SM fermions.

  Georgi, Glashow, PRL **32**, 438 (1974)

- $SO(10)$ (rank = 5): $16 \Rightarrow$ SM fermions $\oplus\ \nu_L^C$.

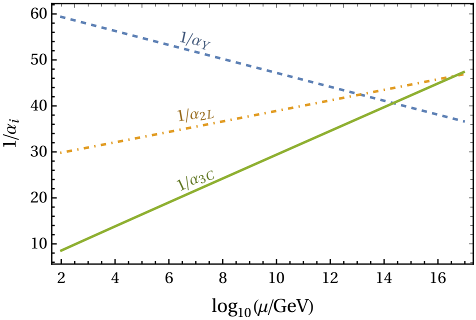
  Fritzsch, Minkowski, Ann. Phys. **93**, 93-266 (1975)

- $E(6)$ (rank = 6): $27 \Rightarrow$ SM fermions $\oplus\ \nu_L^C \oplus$

$$\underbrace{(2, \pm\frac{1}{2}, 1) + (3, -\frac{1}{3}, 1) + (\bar{3}, \frac{1}{3}, 1) + (1, 0, 1)}_{\text{Exotic fermions}}.$$

  Shafi, PLB **79** (1978) 301

Motivation towards the Grand Unification



- Renormalization Group Evolution of Standard Model gauge couplings gives the hint of the possibility of the Grand Unified Theories.

- Imposition of higher symmetry may constraint some free parameters.
- All fermions including $(\nu^c)_L$ can be put in one representation in GUT.
- One way to understand the charge quantization.

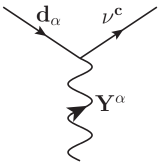
2 *Predictions of GUTs: Proton Decay & Topological Defects*

Prediction of GUTs: Proton Decay

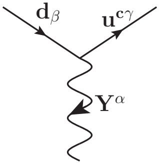
- Mediation of lepto-quark gauge bosons $\Rightarrow$ proton decay into meson plus antilepton :

  $\boxed{p \to M + \bar{l}}$ $\qquad M \in \{\pi^+, \pi^0, K^+, K^0, \eta\}, l \in \{e, \mu, \nu_{e,\mu,\tau}\}.$

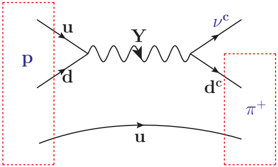
- Example: With $Y^\alpha = (3, 1/3) \in (2, 5/6, 3)$



Lepto-quark vertex



Di-quark vertex



$\mathbf{p} \to \pi^+ \overline{\nu}$

- Selection rules : $\boxed{\Delta B = \Delta L = -1}$ and $\boxed{\Delta S = 0, 1}$.

Machacek, NPB **159** (1979) 37. Langacker, Phys. Rept. **72** (1981) 185.

GUTs and Proton lifetime

- Super-Kamiokande experiment puts a stringent constraint on the partial lifetime for the Golden Channel ($p \to \pi^0 e^+$): $\boxed{\tau_p > 1.6 \times 10^{34} \text{ years.}}$

  Super-K Collaboration, K. Abe et al., PRL 113 (2014), PRD 95 (2017)

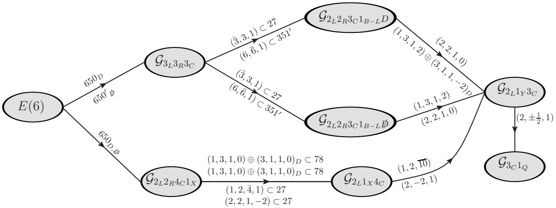
- Many Non-SUSY GUTs in "simplified" forms are ruled out.

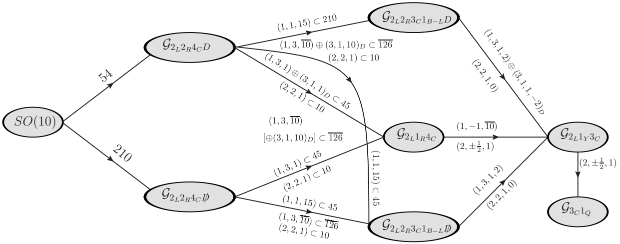
- Way out:
  1. Threshold Corrections,
  2. Dimension-5 operators ($\frac{\eta}{m_{\text{Pl}}} F^{\mu\nu} \langle\phi\rangle F_{\mu\nu}$).

     J. Chakrabortty, **RM**, S. K. Patra, T. Srivastava, S. Mohanty, PRD **97** (2018) 095010

     J. Chakrabortty, **RM**, S. F. King, PRD **99** (2019) 095008

Breaking paths of $E(6)$ and $SO(10)$ to SM





Symmetry breaking and Threshold Correction

- The matching condition for the symmetry breaking, $\mathcal{G}_p \to \mathcal{G}_d$, at the breaking scale μ

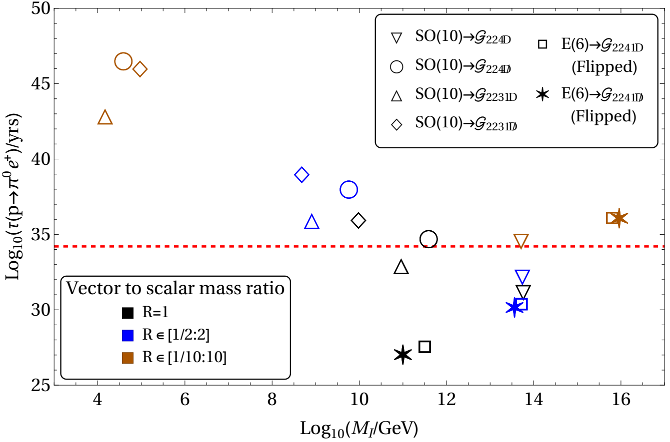
$$\frac{1}{\alpha_d(\mu)} - \frac{C_2(\mathcal{G}_d)}{12\pi} = \left(\frac{1}{\alpha_p(\mu)} - \frac{C_2(\mathcal{G}_p)}{12\pi}\right) - \frac{\Lambda_d(\mu, M_V, M_F, M_S)}{12\pi}$$

Threshold correction after integrating out heavy degrees of freedom.

M_V, M_S and M_F are masses of heavy vector, scalar and fermion fields.

- Parameterize threshold effect by: $R = \frac{M_{S/F}}{M_V}$, with M_V is degenerate with μ.
- We maximize $\tau(p \to \pi^0 e^+)$ for R within $[1/2 : 2]$ and $[1/10 : 10]$.

One Intermediate Scale and Proton Lifetime



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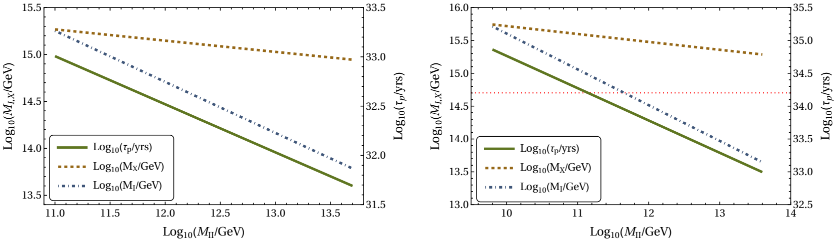
Example: Two step breaking and τ_p compatible with $R \in [1/2 : 2]$



$R = 1$ $\quad$ $R \in [1/2 : 2]$

$$SO(10) \to \mathcal{G}_{2_L 2_R 4_C D} \to \mathcal{G}_{2_L 2_R 3_C 1_{B-L} D} \to \text{SM.}$$

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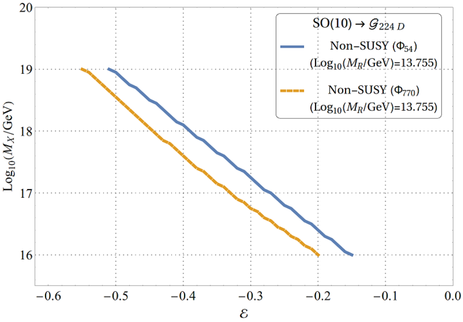
Effect of dimension-5 operator on the unification scenarios

- Dimension-5 opeartor: $-\frac{\eta}{M_{\rm Pl}}\left[\frac{1}{4}{\rm Tr}({\rm F}^{\mu\nu}\phi_{\rm D}{\rm F}_{\mu\nu})\right]$ ϕ_D comes from the symmetric product of two adjoints.

- Gauge couplings will get modified as: $g_U^2 = g_i^2(M_X)(1+\epsilon\delta_i)$
$\epsilon = \eta v_D/(2M_{\rm Pl}) \sim \mathcal{O}(M_X/M_{\rm Pl})$ and δ depends on intermediate gauge group (group theoretic factor)

- Example: $(45 \otimes 45)_S \supset 54 \oplus 210 \oplus 770$

Chakrabortty, **RM**, Patra, Mohanty, Srivastava, PRD **97** (2018) 095010



Predictions of GUTs: Topological Defects

- Topological defects may appear during the SSB of a group $\mathcal{G}$ down to its subgroup $\mathcal{H}$.
- Non-trivial homotopy group $\Pi_k(\mathcal{M})$ of the vacuum manifold ($\mathcal{M} = \mathcal{G}/\mathcal{H}$) implies formation of topological defects.
- Various types of topological defects which can be formed are : domain walls (k = 0), cosmic strings (k = 1), monopoles (k = 2) etc

Topological defects in GUTs

GUT $\to \mathcal{G}_{\rm I} \to \mathcal{G}_{\rm II} \to$ SM	Topological defects		
	GUT $\to \mathcal{G}_I$	$\mathcal{G}_I \to \mathcal{G}_{II}$	$\mathcal{G}_{II} \to$ SM
$E(6) \to \mathcal{G}_{3_L 3_R 3_C D} \to \mathcal{G}_{2_L 2_R 3_C 1_{LR} D} \to$ SM	Unstable $\mathbb{Z}_2$-strings $+\mathbb{Z}_3$-monopoles	Stable monopoles	Domain walls + embedded strings
$E(6) \to \mathcal{G}_{3_L 3_R 3_C} \to \mathcal{G}_{2_L 2_R 3_C 1_{LR} \not{D}} \to$ SM	$\mathbb{Z}_3$-monopoles	Stable monopoles	Embedded strings
$E(6) \to \mathcal{G}_{2_L 2_R 4_C 1_X D} \to \mathcal{G}_{2_L 1_X 4_C} \to$ SM	Unstable $\mathbb{Z}_2$-strings + stable monopoles + unstable $\mathbb{Z}_2$-monopoles	Domain walls	Embedded strings
$E(6) \to \mathcal{G}_{2_L 2_R 4_C 1_X \not{D}} \to \mathcal{G}_{2_L 1_X 4_C} \to$ SM	Stable monopoles + unstable $\mathbb{Z}_2$-monopoles	No defects	Embedded strings
$SO(10) \to \mathcal{G}_{2_L 2_R 4_C D} \to \mathcal{G}_{2_L 2_R 3_C 1_{B-L} D} \to$ SM	$\mathbb{Z}_2$-strings (stable upto M_{II}) $+\mathbb{Z}_2$-monopoles	Stable monopoles	Domain walls + embedded strings
$SO(10) \to \mathcal{G}_{2_L 2_R 4_C D} \to \mathcal{G}_{2_L 2_R 3_C 1_{B-L} \not{D}} \to$ SM	Unstable $\mathbb{Z}_2$-strings + $\mathbb{Z}_2$-monopoles	Domain walls + stable monopoles	Embedded strings
$SO(10) \to \mathcal{G}_{2_L 2_R 4_C \not{D}} \to \mathcal{G}_{2_L 2_R 3_C 1_{B-L} \not{D}} \to$ SM	$\mathbb{Z}_2$-monopoles	Stable monopoles	Embedded strings
$SO(10) \to \mathcal{G}_{2_L 2_R 4_C D} \to \mathcal{G}_{2_L 1_R 4_C} \to$ SM	Unstable $\mathbb{Z}_2$-strings + $\mathbb{Z}_2$-monopoles	Domain walls + stable monopoles	Embedded strings
$SO(10) \to \mathcal{G}_{2_L 2_R 4_C \not{D}} \to \mathcal{G}_{2_L 1_R 4_C} \to$ SM	$\mathbb{Z}_2$-monopoles	Stable monopoles	Embedded strings

J. Chakrabortty, **RM**, S. F. King, PRD **99** (2019) 095008

Observational Constraints on GUTs and Inflation

- Stable domain walls contradict standard cosmology.

  Y. B. Zeldovich, I. Y. Kobzarev, L. B. Okun, Zh. Eksp. Teor. Fiz. **67**, 3-11 (1974)

- Upper bound on comoving monopole number density from MACRO: $\boxed{Y_M = n_M/s \gtrsim 10^{-27}}$.

  M. Ambrosio et al. [MACRO Collaboration], EPJC 25, 511 (2002)

- The Parkes Pulsar Timing Arrays (PPTA) put a contraint on the tension of the “undiluted” cosmic strings : $\boxed{G\mu \lesssim 10^{-11}}$.

  J.J. Blanco-Pillado, K.D. Olum, X. Siemens, PLB 778, 392 (2018)

- Way out $\Rightarrow$ Inflation.

3 *Phase Transitions, Inflation, and Evolution of Monopoles*

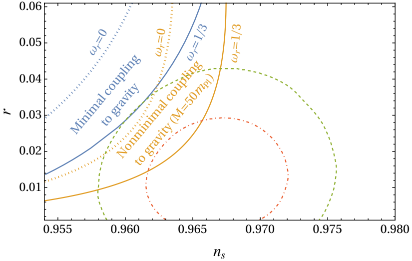
Inflation with GUT-singlet ϕ

- Inflation driven by Coleman-Weinberg potential of GUT-singlet ϕ

$$V(\phi) = A\phi^4 \left[\log\left(\frac{\phi}{M}\right) - \frac{1}{4}\right] + V_0.$$

Shafi, Vilenkin, PRL 52, 691 (1984)

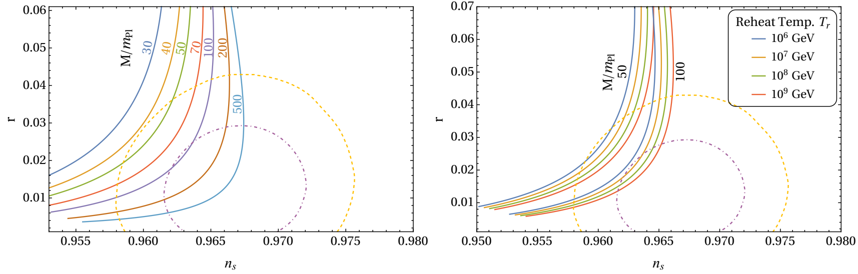
- The BK18 + BAO + Planck 18 data rules out the CW inflation by 2σ. BICEP, Keck, PRL 127 (2021) 151301
- A non-minimal coupling to gravity: $(1+\xi(\phi^2 - M^2))R$ helps to satisfy the current data.



Lazarides, Maji, Roshan, Shafi, 2210.03710

Inflaton with Non-minimal Coupling to Gravity

- $|\xi| \sim 10^{-3}$ brings about inflation below the VEV.
- $-\frac{1}{2}\beta_D \phi^2 \chi_D^2$ couplings in the potential give rise to the spontaneous symmetry breakings via the VEV $\langle \chi_D \rangle$.



$\omega_r = 0, T_r = 10^7$ GeV. $\omega_r = 0.$

Maji, Shafi arXiv:2208.08137

Evolution of Intermediate-mass Monopoles

Number density at production, $\xi = \min\left(H^{-1}, m_{\text{eff}}^{-1}\right)$

Dilution during Inflation

Dilution from Inflaton oscillation

Monopole yield after reheating :

$$Y_M \simeq \frac{\frac{\xi^{-3}}{10} \exp(-3N_M) \left(\frac{\tau}{t_r}\right)^2}{\frac{2\pi^2}{45} g_* T_r^3}$$

Entropy density after reheating

- MACRO bound: $Y_M \lesssim 10^{-27}$.

  Ambrosio et al. [MACRO Collaboration], EPJC 25, 511 (2002)

- Adopted threshold for observability: $Y_M \gtrsim 10^{-35}$.

Intermediate Mass Monopoles and MACRO

$\frac{M}{m_{\rm Pl}}$	$\log_{10}\left(\frac{V_0^{1/4}}{\rm GeV}\right)$	$\phi_+/m_{\rm Pl}$	$\phi_-/m_{\rm Pl}$	H_+	H_-	M_{I+}	M_{I-}	N_+	N_-
				(10^{13} GeV)					
50	16.45	43.62	42.16	3.74	4.15	5.58	6.40	10.9	17.1
70	16.54	63.57	62.07	3.67	4.04	5.27	5.93	10.9	17.1
100	16.64	93.57	92.04	3.51	3.81	4.89	5.39	10.8	17.1
200	16.86	193.71	192.16	3.03	3.21	4.08	4.35	10.8	17.0
500	17.16	494.33	492.79	2.33	2.40	3.07	3.18	10.7	16.9

Table: Values of the various parameters (indicated by a subscript $+$) corresponding to the MACRO bound ($Y_M < 10^{-27}$) on the flux of monopoles formed at the scale M_I and their values (indicated by a subscript $-$) corresponding to the adopted observability threshold ($Y_M > 10^{-35}$) for the monopole flux for a typical choice of nonminimal coupling, $\xi = -0.001$.

Maji, Shafi arXiv:2208.08137

4 *Evolution of Cosmic Strings and Gravitational Waves*

Evolution of Strings in Inflationary Cosmology

- The mean inter-string distance at cosmic time t (temp $= T$):

$$d_{\text{str}} = p\,\xi(\phi_I)\ \exp(N_{\text{str}})\ \left(\frac{t_r}{\tau}\right)^{\frac{2}{3}}\ \frac{T_r}{T}$$

Inter-string seperation at production $\xi = \min\left(H^{-1}, m_{\text{eff}}^{-1}\right)$

Expansion during Inflation

Expansion during Inflaton oscillation

Expansion after reheating

- The string network re-enters the post-inflationary horizon at cosmic time t_F if

$$d_{\text{str}}(t_F) = d_{\text{hor}}(t_F)$$

$$\text{with}\ \ d_{\text{hor}}(t_F) = \begin{cases} 2t_F & \text{(radiation dominance)} \\ 3t_F & \text{(matter domination).} \end{cases}$$

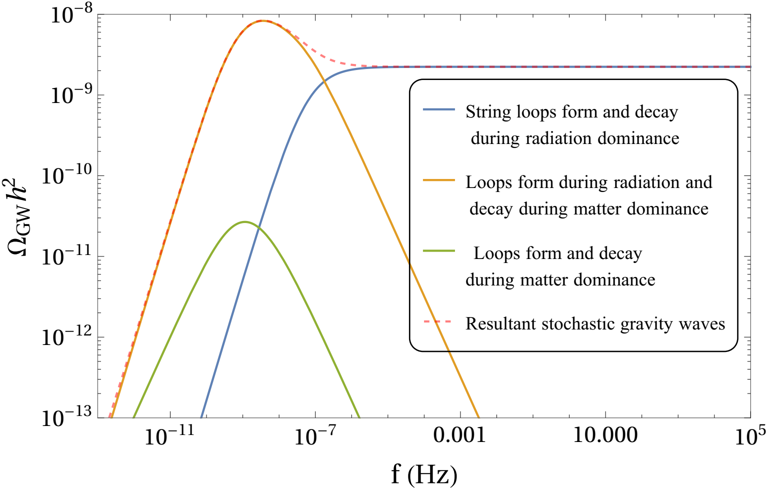
Chakrabortty, Lazarides, Maji, Shafi JHEP 02 (2021) 114

String Loops and Gravitational Waves

- After horizon re-entry, the strings inter-commute and form loops at any subsequent time t_i.
- Loops of initial length $l_i = \alpha t_i$ decay via emission of gravity waves.
- The redshifted frequency of a normal mode k, emitted at time $\tilde{t}$, as observed today, is given by

$$f = \frac{a(\tilde{t})}{a(t_0)} \frac{2k}{\alpha t_i - \Gamma G\mu(\tilde{t} - t_i)}, \quad \text{with} \quad k = 1, 2, 3, \ldots$$

Stochastic Gravitational Wave Background

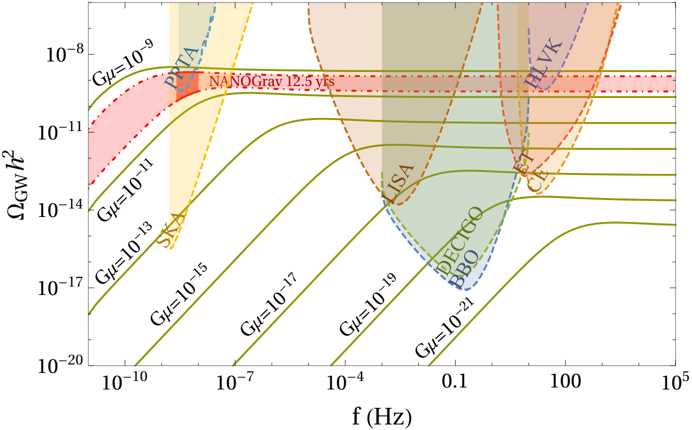


Sousa, Avelino, Guedes, PRD 101 (2020) 10, 103508

Stochastic Gravitational Waves from Strings

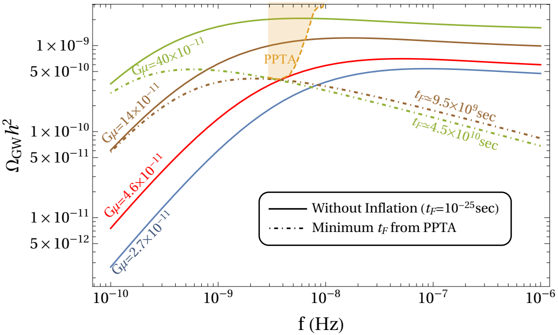
- Unresolved GWs bursts from string loops at different cosmic era produces the stochastic background.
- Loops that are formed and decay during radiation produce a plateau in the spectrum in the high frequency regime.
- Loops that are produced during radiation dominance but decay during matter dominance generate a sharply peaked spectrum at lower frequencies.
- Loops that are produced and decay during matter domination also generate a sharply peaked spectrum which, however, is overshadowed by the previous case.

GWs without Inflation and Observational Prospects



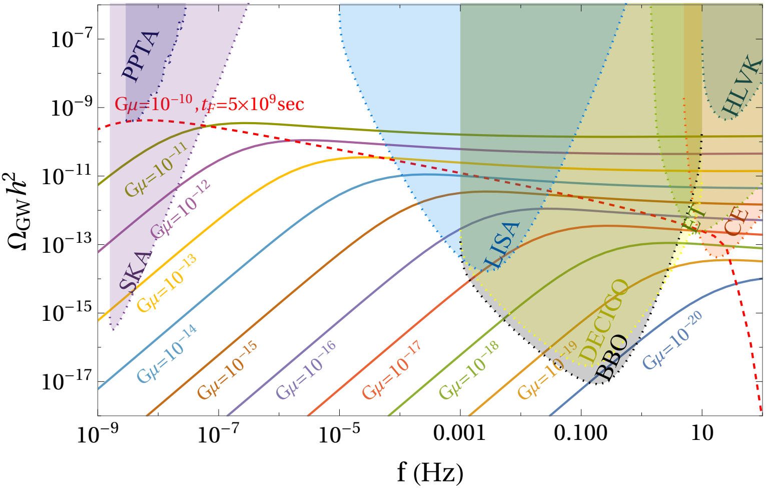
- Strongest constraint has come from PPTA: $G\mu \lesssim 10^{-11}$.
- Provisional GWs signal in NANOGrav: $G\mu \sim 10^{-10}$.

Inflation, GWs and PPTA bound

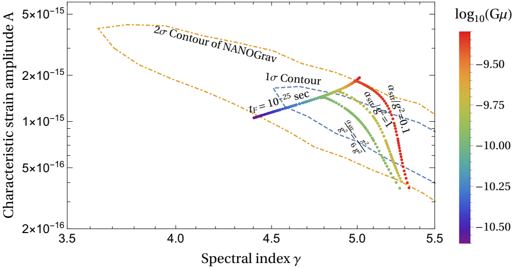


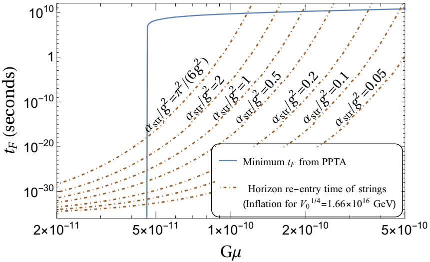
- Partially inflated strings re-enter horizon at a time t_F in post-inflationary universe and can decay via GWs emission.
- Modified GWs spectra from ‘diluted’ strings can satisfy the PPTA bound.

Lazarides, Maji, Shafi, PRD 104 (2021) 9, 095004


PPTA
Gμ=10^−10, t_F=5×10^9 sec
HLVK
Gμ=10^−11
Gμ=10^−12
SKA
Gμ=10^−13
Gμ=10^−14
Gμ=10^−15
Gμ=10^−16
Gμ=10^−17
Gμ=10^−18
Gμ=10^−19
Gμ=10^−20
LISA
DECIGO
BBO
ET
CE
Ω_GW h^2
10^−7
10^−9
10^−11
10^−13
10^−15
10^−17
10^−9
10^−7
10^−5
0.001
0.100
10
f (Hz)

GUT Inflation and NANOGrav Data





5. *Quasi-stable Cosmic Strings*

Quasi-stable Cosmic Strings

- Example:

$$SO(10) \xrightarrow{M_{\text{GUT}}} SU(4)_c \times SU(2)_L \times SU(2)_R$$
$$\xrightarrow{M_I} SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R$$
$$\xrightarrow{M_{II}} SU(3)_c \times SU(2)_L \times U(1)_Y .$$

- Strings formed at M_{II} connect monopole-antimonopole ($M\bar{M}$) pairs formed at M_I.
- Strings are topologically unstable: $\Gamma_d = \frac{\mu}{2\pi} \exp\left(-\pi m_M^2/\mu\right)$ with $\mu \sim \pi M_{II}^2$ and $m_M \sim 10 M_I$.
- Strings are practically stable if $(m_M^2/\mu)^{1/2} \gtrsim 8.7$.

Lazarides, Maji, Shafi JCAP 08 (2022) 042

Gravitational Waves from Quasi-stable Strings

- Intermediate scale magnetic monopoles, created prior to the cosmic strings, experience partial inflation.
- The strings reenter the horizon (t_F) earlier than the $M\overline{M}$ pairs (t_M), form random walks with step of the order of the horizon, and inter-commute generating loops which decay into gravitational waves.
- As monopoles reenter the horizon we obtain monopole-antimonopole pairs connected by string segments which also decay into gravitational waves.

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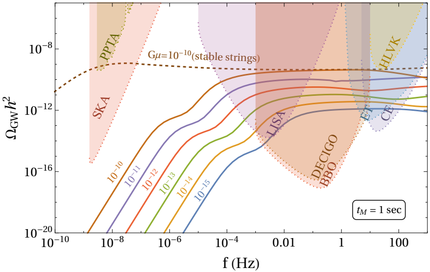
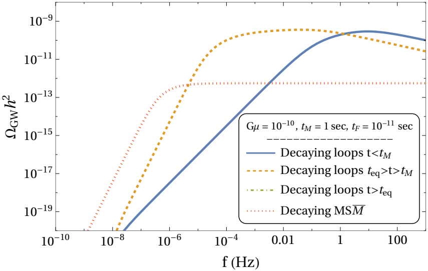
Gravitational Waves from Quasi-stable Strings

- Long string loops and segments are absent.
- Gravitational wave spectrum in the low frequency region is reduced.



GWs spectra.



Different components.

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6 *Summary*

Summary

- Many Non-SUSY GUTs are incompatible with the Super-K proton lifetime bound unless we take the effect of threshold correction into account.
- Effect of dimension-5 operators can improve the proton lifetime for specific breaking paths.
- Topological defects e.g., domain walls, monopoles conflict cosmological observations unless they are inflated away.
- MACRO puts the constraint $\log_{10}(M_I/\mathrm{GeV}) > 13.3$, for the breaking scale generating monopoles.

Summary

- GUT inflation with the CW potential of a singlet with non-minimal coupling to gravity is viable with recent BK18 data.
- Inflation can alleviate PPTA bound on the gravitational wave radiation from cosmic string loops.
- We note that $G\mu$ values lying in a wide range $\sim 10^{-10} - 10^{-20}$ will be probed by a variety of proposed experiments including LISA, SKA, BBO, and ET.
- Quasi-stable strings can produce observable GWs and satisfy the PPTA bound.

Thank You